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Optimizing Wavelet Selection for VARX Models: Insights from Iraq's Government Expenditures and Foreign Reserves

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Abstract: This study seeks to improve the accuracy of the VARX model used for analyzing monthly economic data through noise processing techniques using wavelet transform. A set of wavelet types, including Coiflets, Daubechies, Symlets, biorthogonal, and inverse biorthogonal, are evaluated by testing all different wavelet orders and levels to improve the reliability of the model and its ability to simulate real economic patterns and predict future trends. This research aims to identify the optimal wavelets and appropriate levels and orders based on performance criteria (AIC and BIC). To achieve this goal, a Universal threshold-based algorithm with soft rules is proposed to determine the optimal value, as well as its performance is compared with traditional and non-wavelet-based methods. In the traditional method, the level of analysis (L = 3) and order (N = 3) were applied to all types of wavelets, except for Biorthogonal and Reverse Biorthogonal, where the level (L = 3) and order (N = 8) were applied, which are the default values in MATLAB23. This method was tested using real data. The methodology included applying the study to the monthly data of the Iraqi economy during the period from January 31, 2010, to July 31, 2024, to analyze the impact of external variables such as money supply and oil revenues on internal variables such as government expenditures and official reserves (OR). The results demonstrated the efficiency of the proposed method in reducing noise and estimating the parameters of VAR models, which contributed to improving the overall performance of the model. This comprehensive framework provides significant improvements in the analysis of economic data, by providing accurate forecasts and deep insights to decision-makers, which enhances the reliability of estimates and helps in understanding economic trends more effectively.

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تحسين اختيار المويجات لنماذج VARX : رؤى من نفقات الحكومة العراقية واحتياطيات النقد الأجنبي

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المستخلص

تسعى هذه الدراسة إلى تحسين دقة نموذج VARX المستخدم لتحليل البيانات الاقتصادية الشهرية من خلال تقنيات معالجة الضوضاء باستخدام تحويل المويجات. يتم تقييم مجموعة من أنواع المويجات، بما في ذلك Coiflets، Daubechies، Symlets، biorthogonal و inverse biorthogonal، من خلال اختبار جميع الرتب ومستويات المويجات المختلفة لتحسين موثوقية النموذج وقدرته على محاكاة الأنماط الاقتصادية الواقعية والتنبؤ بالاتجاهات المستقبلية. يهدف هذا البحث إلى تحديد المويجات المثلى والمستويات واالرتب المثلى بناءً على معايير الأداء (BIC,AIC) ولتحقيق هذا الهدف، تم اقتراح خوارزمية تعتمد على العتبة الشاملة ذات القواعد الناعمة لتحديد القيمة المثلى، وكذلك مقارنة أدائها بالطرق التقليدية وغير المعتمدة على المويجات. في الطريقة التقليدية، تم تطبيق مستوى التحليل (L = 3) والترتيب (N = 3) على جميع أنواع المويجات، باستثناء المويجات ثنائية التعامد (biorthogonal) والمويجات ثنائية التعامد العكسية (inverse biorthogonal)، حيث تم تطبيق المستوى (L = 3) والترتيب(N = 8) ، وهي القيم الافتراضية في برنامج MATLAB23. تم اختبار هذه الطريقة باستخدام بيانات الواقعية. تضمنت المنهجية تطبيق الدراسة على البيانات الشهرية للاقتصاد العراقي خلال الفترة من 31 يناير 2010، إلى 31 يوليو 2024، لتحليل تأثير المتغيرات الخارجية مثل المعروض النقدي وإيرادات النفط على المتغيرات الداخلية مثل النفقات الحكومية والاحتياطيات الرسمية .(OR) وقد أظهرت النتائج كفاءة الطريقة المقترحة في تقليل الضوضاء وتقدير معلمات نماذجVAR ، مما ساهم في تحسين الأداء العام للنموذج. ويوفر هذا الإطار الشامل تحسينات كبيرة في تحليل البيانات الاقتصادية، من خلال توفير توقعات دقيقة ورؤى عميقة لصناع القرار، مما يعزز موثوقية التقديرات ويساعد في فهم الاتجاهات الاقتصادية بشكل أكثر فعالية.

الكلمات المفتاحية: السلاسل الزمنية، نموذجVARX ، المويجة المثلى، مستوى وترتيب المويجات، العتبة.

1. Introduction

The Iraqi economy is considered one of the developing economies that depends mainly on the oil sector for export. Oil revenue is Iraq's most significant source of income, influencing government spending levels and the amount held in foreign reserves. The Broad Money Supply (M2) reflects the money circulating in the market, providing insight into the overall health of the economy. Within this context, government expenditures and official reserves demonstrate tangible economic impacts primarily influenced by fluctuations in oil revenues and monetary liquidity.

This study presents monthly financial and monetary data of the Iraqi economy, measured in billion Iraqi dinars, spanning from January 31, 2010, to July 31, 2024, with a total of 175 observations (n=175), covering essential indicators used to evaluate economic performance and analyze its dynamics. These primary variables include Oil Revenues (ORv), Broad Money Supply (M2), Government Expenditures (Exp), and Official Foreign Reserves (OR) (Central Bank of Iraq :2024).

The VARX model is a special case of the VAR model that measures exogenous and endogenous variables to improve forecast accuracy. It is more flexible than basic Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) and Stochastic Volatility (SV) models, which represent expected value and asset variability. The VARX model loosens limitations on the economic process, enabling better forecasts. However, these models typically feature steady mean equations, and strict assumptions, and are complicated to apply and understand (Pesaran, 2015: 563; Parra et al.2024: 2).

One crucial issue is lowering noise in time series data. A novel and effective technique for financial time series analysis is wavelet analysis, which lowers data noise. On various levels, wavelets can break down a signal or a time series. Consequently, the structure of the underlying signal as well as trends, periodicities, leaps, or singularities that are not originally visible are revealed by this decomposition (Ali et al., 2022: 433; Ali et al., 2024: 345; Liu& Cheng, 2024: 2-3).

The research problem is to improve the accuracy of VARX models used in analyzing monthly economic data by reducing noise using wavelet transformation. The study aims to identify the optimal wavelet with the most efficient level and order, to achieve the lowest possible values of performance criteria (AIC and BIC), which contributes to improving the accuracy of forecasts and providing clearer economic insights.

The study also seeks to compare the performance of the proposed methodology using wavelets with traditional methods, and without wavelet transforms, in order to develop a more effective and reliable model. This methodology was applied to Iraqi economic data during the period 2010-2024 to analyze the impact of external variables, such as money supply and

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oil revenues, on internal variables. These results aim to support decision-makers and researchers in better understanding economic trends and providing more accurate analysis tools.

The research focused on two main aspects including the theoretical aspect, which dealt with the VARX model and its selection criteria, wavelets, wavelet families, universal threshold, soft threshold rule, and the proposed method. While the applied aspect, also included a case study on real data from the Iraqi economy to verify the algorithm's validity. Statistical tools such as Minitab V21, EViews V12, and MATLAB R2023a, alongside custom MATLAB programs, were utilized for analysis.

MATLAB and EViews were specifically chosen for their advanced capabilities in time series analysis and constructing models like VARX. Their selection was based on their precision in handling complex economic models, computational efficiency, flexibility in data processing (e.g., seasonal differentiation and noise reduction), and comprehensive support through extensive documentation and libraries for implementing diagnostics like stability and residual tests. This ensured the robustness and reliability of the analytical framework used in the study.

2. **Theoretical Aspect:** The theoretical aspect presented some basic concepts about research from the statistical side, as shown in the following paragraphs.

2-1. VARX Model: The Vector Autoregressive model with exogenous variables (VARX) is an extension of the traditional Vector Autoregressive (VAR) framework by incorporating both endogenous and exogenous variables to capture their dynamic relationships. This extension lets the model examine the interdependencies between multiple time series while considering the influence of external variables. The VARX model employs a multivariate system where past values of endogenous variables, alongside relevant exogenous variables, serve as inputs to predict multiple future outputs. This dynamic interaction allows the model to capture interdependencies within the system while accounting for the influence of external variables.

Therefore, the mathematical formula representation of the VARX (p, q) model, where p indicates the number of lags for the endogenous variables and q for the exogenous variables, can be described by the following formula (Lütkepohl, 2004: 92; Lütkepohl, 2005: 386).

$$Y_t = C + \sum_{i=1}^{p} \Phi_i Y_{t-i} + \sum_{j=1}^{q} \beta_j X_{t-j} + \epsilon_t \qquad (1)$$

Alternatively, it can be expressed using the lag operator B, as follows:
Where

$$\left(I - \sum_{i=1}^{p} \Phi_i B^i \right) Y_t = C + \left(\sum_{j=1}^{q} \beta_j B^j \right) X_{t-j} + \epsilon_t \qquad (2)$$

Where

Y_t: is a $k \times 1$ vector of dependent (endogenous) variables at a time t.

Y_{t-i}: is the lagged endogenous variables for all lags $(i = 1,2, \dots, p)$.

X_{t-j}: is an $m \times 1$ vector of external (exogenous) predictors at lag $t - j$.

Φ_i: $k \times k$ matrices representing the coefficients of the endogenous variables' lags.

β_j: $k \times m$ matrices representing the coefficients of the exogenous variables' lags.

C: is a $k \times 1$ vector of intercept terms.

ϵ_t: is a $k \times 1$ vector of error terms, accounting for unexplained randomness or residual influences.

B: is the Back shift Operator، where $B^k Y_t = Y_{t-k}$. The Back shift Operator B simplifies time-series modeling by representing past values of variables compactly, enabling the representation of relationships across time lags in models like VARX.

In the mathematical framework above, any number of dependent variables (k) can be selected, along with p lags for these dependent variables, m external predictors, and q lags for these predictors. This structure enables the modeling of complex interactions among endogenous variables while simultaneously capturing the influence of exogenous predictors. The VARX model is particularly notable for its ability to compactly represent dynamic relationships over time using the Back Shift Operator (B), which simplifies the inclusion of lagged variables. As such, it is widely employed in time series studies to understand evolving interconnections and the impact of external factors.

2-2. Model Selection Criteria: Use these criteria for choose the best model from a collection of models, defined as follows (Burnham& Anderson, 2004:

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268, 275; Acquah & Carlo,2010: 2-3; Issa, 2021, 493; Hussein, 2021, p.518; Mdloul & Hussein, 2021: 81).

Akaike Information Criterion (AIC): The lower AIC value represents the best model, which indicates a good fit with fewer parameters and is calculated according to the following equation:

$$AIC = 2k - 2Ln(L) \qquad (3)$$

Where:

k: is the number of parameters in the model for each k = 1, 2, and 3,4.

L: is the log-likelihood function of the model.

A lower AIC value indicates a better model, as it implies a good fit with fewer parameters.

2-3. Schwarz Information Criterion (SIC or BIC): The Bayesian Information Criterion (BIC) or Schwarz information criterion called by (SIC, SBC, and SBIC) is another criterion used for model selection, that also takes into account the goodness of fit and model complexity but suppose a stricter penalty for the number of parameters compared to AIC. It is calculated according to the following equation:

$$BIC = kLn(n) - 2Ln(L) \qquad (4)$$

Where:

- n: is the number of observations.
- k: is the number of parameters in the model for each k = 1, 2, and 3,4.
- L: is the log-likelihood function of the model.

Similar to AIC, a lower BIC value indicates a better model, but BIC tends to favor simpler models compared to AIC due to the $Ln(n)$ term.

2-4. Wavelets: Wavelets are oscillatory functions with an amplitude that starts at zero, increases or decreases, and then repeats the cycle back to zero. Modern signal processing and time series analysis rely heavily on these short oscillations, called wavelets (Omer et al., 2024: 114).

These wavelets are advanced mathematical tools in signal processing and time series analysis, known for effectively handling non-stationary signals. Unlike Fourier transformations, which decompose signals into sine and cosine functions, wavelets provide dual localization in time and frequency domains. The mother wavelet, $\psi(t)$, serves as the primary function from which all other wavelets are generated through scaling and translation (Mallat, 2009: 57-60; Daubechies, 1992: 30-35).

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$$\psi^{*}_{s,r}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-r}{s}\right) \qquad (5)$$

Thus (s) is the scaling parameter that controls the frequency, and (r) is the translation parameter that controls the time localization.

The discrete wavelet transform (DWT) provides a set of coefficients in both time and frequency domains, summarizing the information of all observations with a reduced number of coefficients. DWT is widely used, especially when data contains contaminants or noise (Ali & Saleh, 2022: 926).

The discrete wavelet transform (DWT) denotes the coefficients utilized to represent the signal in terms of wavelets, as follows (Percival & Walden, 2000: 1-19):

$$Y(t) = \sum_{s,r} W_{s,r}\,\psi_{s,r}(t) \qquad (6)$$

where $W_{s,r}$ are the wavelet coefficients and $\psi_{s,r}(t)$ are the wavelet basis functions at scale a and translation b.

2-5. Wavelet Families: The following are some of the wavelet families used in the application section of this study:

2-5-1. Coiflets Wavelets: Coiflets wavelets are a major development in wavelet theory, especially for signal processing tasks. Ingrid Daubechies introduced them in the spring of 1989 at Ronald Coifman's request. Coifman's proposal went beyond the previous emphasis on the wavelet function (Ψ) and expanded the idea of vanishing moments to scaling and wavelet functions. Coiflets, denoted as Coif$N$ (where N is the rank), have properties tied to their rank: the candidate length is $6N$, the number of vanishing moments for the wavelet function is $L = 2N$, and for the scaling function is $L_1 = 2N - 1$. Coiflets exhibit compact support, orthogonality, and near symmetry (Ali & Mustafa, 2016: 436).

The mathematical form for the Coiflets wavelet ψ (t) that derived from the mother wavelet through scaling and translation, and it is calculated according to the following equation:

$$\psi_{s,r}(t) = 2^{s/2}\,\psi(\,2^{s}\,t - r) \qquad (7)$$

Where (s) represents the scale parameter and (r) is the translation parameter.

The following equation show explains how the denoised signal $Y_{denoised}$ is reconstructed using wavelet coefficients and wavelet basis functions after applying thresholding in the Coiflet wavelet denoising process. The equation is defined as:

$$Y_{denoised} = \sum_{s,r} \widehat{W}_{s,r}\, \psi_{s,r}(t) \qquad (8)$$

Thus, $\widehat{W}_{s,r}$ are the wavelet coefficients after applying thresholding to remove noise, and $\psi_{s,r}(t)$ are the wavelet basis functions used for signal reconstruction.

Equation (8) represents the wavelet noise removal equation or wavelet reconstruction equation after thresholding, which clarifies the notion of wavelet denoising, in which the signal is processed to remove unwanted noise while preserving its major components. Thus, Wavelet denoising, particularly utilizing Coiflets wavelets, is most applied in signal processing and time series analysis due to its efficiency in noise repression and keeping signal safety (Mallat, 2009: 74).

2-5-2. Daubechies wavelets: The Daubechies wavelet, established by Ingrid Daubechies in 1988, is a subclass of wavelets used in signal processing and time series analysis. Denoted as D or dbN, these wavelets are compactly supported and efficient, making discrete wavelet analysis practical in various fields (Daubechies, 1992: 167; Ali et al, 2023: 11; Jalal et al., 2024: 16; Omer et al., 2024: 447).

The mathematical representation of Daubechies wavelets is based on the scaling function $\phi(t)$and the wavelet function $\psi(t)$, defined as follows:

$$\phi(t) = \sum_{b=0}^{N-1} a_b\, \phi(2t-b) \qquad (9)$$

$$\psi(t) = \sum_{b=0}^{N-1} c_b\, \phi(2t-b) \ \text{ where } \ c_b = (-1)^b a_{N-1-b} \qquad (10)$$

Where a_k are the scaling coefficients, c_b are the wavelet coefficients, and N denotes the number of coefficients, directly related to the vanishing moments. The wavelet coefficients c_b are derived from the scaling coefficients a_k using the relation $c_b = (-1)^b a_{N-1-b}$.

As N increases, Daubechies wavelets demonstrate improved regularity and stronger localization in the frequency domain; however, this also results in higher computational complexity.

Daubechies proposed Symlets and Coiflets to improve symmetry and signal distortion in reconstruction, preserving essential properties of dbN wavelets while enhancing their utility in multiresolution analysis and signal decomposition (Daubechies, 1992: 167; GUO et al., 2022: 588732).

2-5-3. Symlets Wavelets: Symlets wavelets, introduced by Ingrid Daubechies in 1992, improve symmetry and phase characteristics, offering linear phase responses. They are useful for applications like edge detection, signal reconstruction, and retaining details in biomedical signals, ensuring accurate diagnostics and data analysis. Thus, the mathematical framework of Symlets wavelets is based on the same formulation as Daubechies wavelets, defined by the scaling function $\phi(t)$ and the wavelet function $\psi(t)$, ensuring orthogonality and compact support. The main difference is the optimization of the scaling coefficients akin to Symlets to achieve better symmetry while maintaining the essential qualities of Daubechies wavelets, like compact support and the number of vanishing moments (Daubechies, 1992: 198; Mallat, 1999: 254).

2-5-4. Biorthogonal Wavelets: Biorthogonal wavelets are a subclass of wavelets that are not strictly orthogonal but have biorthogonality. This property allows separate scaling and wavelet functions for analysis and synthesis, providing greater flexibility in signal compression and denoising applications.

The biorthogonal wavelet functions $\phi(t)$ and $\psi(t)$ are defined alongside their dual functions $\tilde{\phi}(t)$ and $\tilde{\psi}(t)$, as follows:

$$\phi(t) = \sum_r h_k\, \phi(2t-r), \quad \tilde{\phi}(t) = \sum_r \tilde{h}_r\, \tilde{\phi}(2t-r) \qquad (11)$$

$$\psi(t) = \sum_r g_r\, \phi(2t-r), \tilde{\psi}(t) = \sum_r \tilde{g}_r\, \tilde{\phi}(2t-r) \qquad (12)$$

Here, h_r, g_r, $\tilde{h}_r$, and $\tilde{g}_r$ are filter coefficients satisfying the biorthogonal conditions. Structural characteristics of the biorthogonal wavelets allow greater flexibility in the filter design providing better symmetry- vanishing moments-complexity trade-off (Strang & Nguyen, 1996: 208) (Cohen et al., 1992: 488-491).

2-5-5 Reverse Biorthogonal Wavelets: Reverse biorthogonal wavelets are one of the extensions of the biorthogonal wavelet family. In comparison to the standard biorthogonal wavelets, the analysis and synthesis roles are switched for this kind of wavelet, hence making them particularly efficient for certain signal processing tasks, including edge detection and data compression.

Mathematically, the reverse biorthogonal wavelets share the same formulation as biorthogonal wavelets but with reversed filter roles for analysis and synthesis, as follows:

$$\tilde{\phi}(t) = \sum_b \tilde{h}_b\, \phi(2t-b), \psi(t) = \sum_b g_b\, \tilde{\phi}(2t-b) \qquad (13)$$

The reverse setup has special features that help maintain important properties such as vanishing moments and compact support while processing the symmetrical parts of signals (Cohen et al., 1992: 488-491; Tilak & Krishnakumar, 2018: 66).

2-6. Universal Threshold Method: Universal thresholding is commonly applied in wavelet shrinkage for the purpose of denoise data or images. This method focuses on determining the best threshold value based on the statistical properties of the data. The concept behind universal thresholding selection involves evaluating the noise level present in the data and implementing a threshold that adjusts according to this level of noise. Universal thresholding is considered one common method for Stein's unbiased risk estimate (SURE), which represents an unbiased estimate of the mean squared error (MSE) of the denoised signal. The threshold value was chosen to decrease and minimize this estimated MSE. The SURE threshold is commonly called the "universal threshold" since it does well across an extensive domain of data and noise kinds without required prior knowledge of the noise properties. Then adaptively choosing the threshold based on the same data and universal thresholding can effectively remove noise while keeping important data characteristics (Karamikabir et al., 2021: 1729; Jalal et al., 2024: 162).

The following equation calculates the universal threshold method:

$$\lambda_u = \hat{\sigma}_{MAD}\sqrt{2log(n)} \qquad (14)$$

Where (n) is the number of observations while ($\hat{\sigma}_{MAD}$) is the standard deviation estimator of the detail coefficients and this estimate is calculated as follows:

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$$\hat{\sigma}_{MAD} = \frac{MAD}{0.6745} \qquad (15)$$

The wavelet coefficients' median absolute deviation (MAD) at the delicate scale is calculated as follows:

$$MAD = median\left[\left|W_{1,0}\right|, \left|W_{1,1}\right|, \dots, \left|W_{1,\frac{N}{2}-1}\right|\right] \qquad (16)$$

Here, $W_1 = W_{1,0}\ W_{1,1}, \dots, W_{1,\frac{N}{2}-1}$ represent the discrete wavelet transformation coefficients at the first level for observations while the constant (0.6745) is the median of the standard normal distribution (Omer et al., 2024: 117).

In the context of this research, the universal threshold method was applied to denoise the data set related to the Iraqi economy. This approach ensured data accuracy and reliability by removing unwanted noise while preserving critical economic patterns. The results confirmed the efficiency of this method through key statistical metrics, including AIC and BIC, supporting its role in enhancing prediction accuracy and model reliability.

2-7. Soft Threshold Rule: A method used in signal and image processing, especially about wavelet shrinkage, is called soft thresholding. Introduced by Donoho and Johnstone, it extends the hard thresholding method by shrinking wavelet coefficients toward zero to reduce the impact of noise. This method is mathematically defined as follows (Jalal, et al., 2024: 163; Ali et al., 2023: 13):

$$\widehat{W}_n = sign(W_n).max(|W_n| - \lambda, 0)_+ \qquad (17)$$

or

$$\widehat{W}_n = Sign(W_n)(|W_n| - \lambda)_+ \qquad (18)$$

Where:

$$Sign\ (W_n) = \begin{Bmatrix} +1 & if\ W_n > 0 \\ 0 & if\ W_n = 0 \\ -1 & if\ W_n < 0 \end{Bmatrix} \qquad (19)$$

and

$$(|W_n| - \lambda)_+ = \begin{Bmatrix} (|W_n| - \lambda) & if\ (|W_n| - \lambda) \geq 0 \\ 0 & if\ (|W_n| - \lambda) < 0 \end{Bmatrix} \qquad (20)$$

In these equations, W_n represents the wavelet coefficients and λ is the threshold value. Soft thresholding is particularly effective at noise suppression while preserving significant signal features, although it may introduce some signal distortion. Therefore, it is the main focus of the application side of wave noise removal (Li et al., 2023: 1244).

2-8. Proposed Method: The proposed algorithm includes data de-noise of VAR time series using a variety of wavelets with the Universal thresholding estimation method and applying the soft rule through the following steps:

1. Level of wavelet decomposition, specified as a positive integer less than or equal to $(log_2 n)$ where n is the number of observations in the time series. Set L = 1, 2, …, l, where l returns the maximum level L possible for a wavelet decomposition of data using the wavelets specified by Coiflets, Daubechies, Symlets, Biorthogonal, and Reverse Biorthogonal (default value equal to 3).
2. Denoises VARX time series data for all L, as defined in equation (8).
3. Estimating the parameters of the VAR Model for each L, according to equation (1), and computing AIC and BIC for all Models using equations (5-6).
4. Determine the optimal level (OL) that gives the least AIC and BIC.
5. Denoises VARX time series data at OL for a variety of wavelets (Table 1) and Universal thresholding estimation method using equations (14-16) according to the Universal thresholding and applying the soft rule according to equations (17-20).
6. Determine the optimal order (OO) that gives the least AIC and BIC.
7. Use OL and OO for Coiflets wavelet with Universal thresholding estimation methods and apply the soft rule in data de-noise.
8. Use data de-noise from point (7) in estimating the VARX Model according to equation (1).

2-9. Limitations of the Proposed Method: Although the proposed method provides an effective and innovative approach for analyzing time series data using wavelets and thresholding estimations, there are several limitations that should be acknowledged. First, the reliability of the method is highly dependent on the quality and accuracy of the data. Any missing, incomplete, or biased data can significantly impact the precision and validity of the results. Second, implementing this method requires technical expertise and familiarity with advanced software tools, such as MATLAB and EViews, which may pose a challenge for researchers who lack sufficient technical skills or resources. Additionally, the method assumes linear relationships among variables, which may not adequately reflect the complexity of real-world phenomena, particularly in cases involving nonlinear or dynamic

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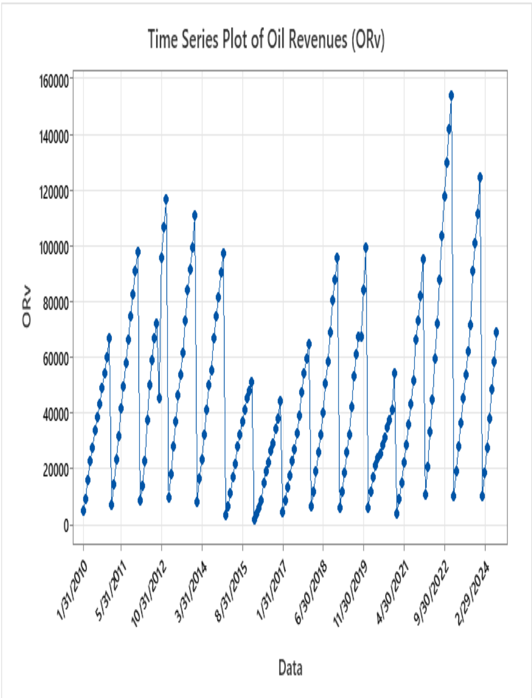
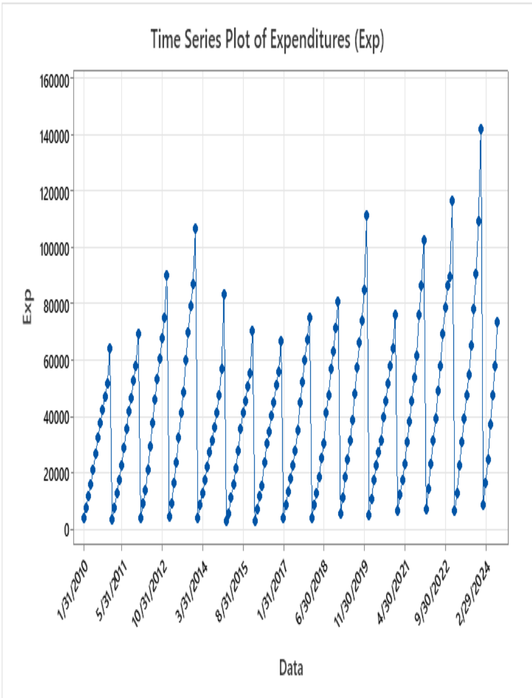
interactions. Lastly, the approach is sensitive to the selection of the number of time lags, requiring extensive testing and evaluation to determine optimal values. Given these limitations, it is recommended to apply the method critically, ensuring it is adapted to the specific requirements and context of the study while considering potential improvements to address its constraints.

3. **Numerical Results:** This study applies multivariate wavelet shrinkage to the VARX model, comparing its performance to both traditional approaches and models without wavelet transformations the target is to determine the optimal wavelet and improve model accuracy that achieves the lowest AIC and BIC values, using real-world data.

3-1. Real Data: The graphs display the monthly time series data of the Iraqi economy, measured in billions of Iraqi dinars, covering the period from January 31, 2010, to July 31, 2024, and include indicators such as Official Reserves (OR), Oil Revenues (ORv), Expenditures (Exp), and Broad Money (M2), known as a time series plot (See Figures 1). The first step in time series analysis is plotting the observations over time, which provides initial insights into the series' characteristics, such as its stability and the presence of trends or seasonal variations. This preliminary evaluation helps prepare for more accurate stability tests later on.





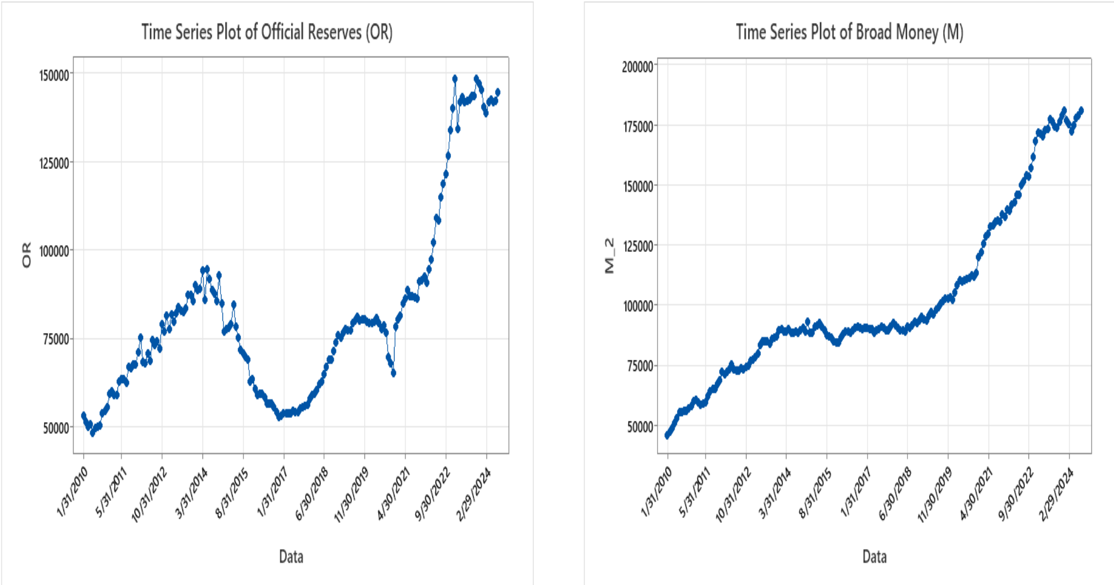


Figure (1): Time series plot for monthly financial and monetary dataset of the Iraqi economy (Billion IQD)

Source: Prepared by the researchers based on the outputs generated by the Minitab V21program.

Discussion: Figure (1) shows non-stationarity in all four-time series, indicating an upward trend. Seasonal patterns in Oil Revenues and Expenditures recur every 12 months, reflecting cyclical factors. Validating these findings requires stationarity tests like the Augmented Dickey-Fuller test. If non-stationarity is identified, differencing methods can remove trends. Stationarity is achieved through ADF test and VARX modeling, ensuring accuracy and reliability in future data analysis.

Unit Root Test for Trend Detection: After performing the logarithmic transformation (Ln) of the four-time series represented by the monthly financial and monetary dataset of the Iraqi economy (Billion IQD) to stabilize the variance of those series. Then specialized tests, such as the Augmented Dickey-Fuller (ADF) tests, are employed to evaluate the stationarity of time series data around a mean. These tests are conducted based on the optimal lag periods (Maxlag =13, based on (SIC) Schwarz information criterion) as shown in Table 1, to determine whether a time series is stable around a mean or linear trend or if it remains unstable due to the presence of a unit root. The hypotheses tested are as follows:

- Null Hypothesis H0: The data contain a unit root (it is non-stationary).
- Alternative Hypothesis H1: The data do not contain a unit root (it is trend stationary).

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Table (1): Augmented Dickey-Fuller Test for monthly financial and monetary dataset of the Iraqi economy (Billion IQD)

Data	Case	Form	lag	Test Statistics	Critical Value	P-Value at 5%	Decision I(d)
Ln (M2)	level	Constant	0	-1.543287	-2.878015	0.5094	Non- Stationary I(0)
D1 (Ln (M2))	First Difference	Constant	0	-10.62810	-2.878113	0.0000*	stationary I(1)
Ln (ORv)	level	Constant	12	-2.693238	-2.879267	0.0774	Non- Stationary I(0)
D2 (Ln (ORv))	Second Difference	None (Constant, Linear Trend)	10	-55.24727	-1.942805	0.0001*	stationary I(2)
Ln(EXP)	level	Constant, Linear Trend	12	-1.790039	-3.437801	0.7053	Non- Stationary I(0)
D1(Ln(EXP))	First Difference	None (Constant, Linear Trend)	11	-3.330144	-1.942805	0.0010*	stationary I(1)
Ln(OR)	level	None (Constant, Linear Trend)	0	1.983990	-1.942666	0.9888	Non- Stationary I(0)
D1 (Ln(OR))	First Difference	Constant	0	-14.42290	-2.878113	0.0000*	stationary I(1)

Stationary I(1) ***:** The null hypothesis is rejected ($|Test\ statistic\ | \geq |critical\ value|$) at significance level = 0.05. Or p-value ≤ 0.05.

Source: Prepared by the researchers based on the outputs generated by the EViews V12 program.

Discussion

Table 1 shows the ADF stationarity test results showing that all variables (M2, ORV, Exp, and OR) are non-stationary at their levels as the null hypothesis of unit root cannot be rejected. However, when these variables differ for the first and second time, the null hypothesis is rejected, indicating that the first and second differences of all variables are stationary and integrated from the first order, I(1) for variables(M2, Exp, and OR) and the second order, I(2) for variable (ORV). The ADF stationarity test results are as follows:

A. **First Difference for Ln (M2) (Broad Money):** The absolute test statistic is 10.62810, greater than the absolute critical value of 2.878015 (the p-value is 0.0000 < 0.05), indicating that the null hypothesis is rejected. Therefore, the first difference of Ln (M2) is stationary I (1)).

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B. **Second Difference** for **Ln (ORV) (Oil Revenues):** The absolute test statistic is 55.24727, greater than the absolute critical value of 1.942805 (the p-value is 0.0001 < 0.05), indicating that the null hypothesis is rejected. Therefore, the second difference of Ln (Orv) is stationary (I (2)).

C. **First Difference** for **Ln (**Exp) (Expenditures): The absolute test statistic is 3.330144, greater than the absolute critical value of 1.942805 (the p-value is 0.0010 < 0.05), indicating that the null hypothesis is rejected. Therefore, the first difference of Ln (Exp) is stationary (I (1)).

D. **First Difference for Ln(OR) (Official Reserves):** The absolute test statistic is 14.42290, greater than the absolute critical value of 2.878113, (the p-value is 0.0000 < 0.05), indicating that the null hypothesis is rejected. Therefore, the first difference of Ln(OR) is stationary (I (1)).

To address this pattern, seasonal differencing of the second and first differences for both ln(ORV) and ln(EXP) should be applied to stabilize the series in terms of the mean and eliminate the seasonal effect. This approach is further supported by applying the ADF test in the case using None (Constant and Linear Trend) Form at the optimal Lag Length is 5 (based on SIC, maxlag =13) on the seasonal differences of the second and first differences for both ln (ORV) and ln(EXP), reinforcing the stability of the time series, as follows:

A. **First Seasonal Difference for Second Difference (Ln (ORv)) (Oil Revenues):** The absolute test statistic is 9.564246, greater than the absolute critical value of 1.942896 (the p-value is 0.0000 < 0.05), indicating that the null hypothesis is rejected. Therefore, the first seasonal difference for the second difference (Ln (ORv)) is stationary (I (1)).

B. **First Seasonal Difference for First Difference Ln(EXP) (Expenditures):** The absolute test statistic is 14.86825, greater than the absolute critical value of 1.942818 (the p-value is 0.0000 < 0.05), indicating that the null hypothesis is rejected. Therefore, the first seasonal difference for the first difference Ln (Exp) is stationary (I (1)).

After the stationarity of the four-time series above, these results are essential in constructing two-dimensional VARX models with two predictors because these models require stationary data to produce accurate and interpretable results. In this case, different noise removal methods applied to this stable model can be applied to evaluate the impact of independent Variables (represents predictors), such as 'M2' (Broad Money) and 'ORv' (Oil Revenues), along with dependent Variables (represents dimensional), including 'Exp' (Expenditures) and 'OR' (Official Reserves), on the internal economic system. To evaluate the performance of multivariate wavelet shrinkage using a variety of wavelets, as shown in Table2.

Table (2): Multivariate Wavelet Shrinkage with Various Wavelet Types

Wavelet Type	Order Range
Coiflets	coif1 to coif5
Daubechies	db1 to db45
Symlets	sym1 to sym30
Biorthogonal	bior1.1 to bior6.8
Reverse Biorthogonal	rbio1.1 to rbio6.8

Source: Prepared by the researchers**.**

In Table 2, all ranks of each of the above wavelets were tested to determine the optimal order, the optimal level, and the best wavelet type for these wavelets that achieve the lowest AIC and BIC values**.** The threshold level was estimated using the Universal method with soft thresholding to enhance data quality and reduce noise.

In Table 3, the performance of the VARX models was evaluated using multiple noise reduction techniques through wavelet transforms, in addition to testing without using wavelets. For each tested model, the lowest AIC and BIC values were recorded, enabling the identification of the optimal level (L), the optimal order (N), and the best wavelet type, as well as, identifying the optimal model. The results also demonstrated the superiority of certain selected wavelets over classical methods and models without wavelet transformations, supporting the effectiveness of wavelet transforms in enhancing model accuracy and suitability for analyzing complex time-series data. Sample sizes for n=175 were used in the actual data experiment. These sample sizes were defined as $n = 2^j$, where j is a positive integer that represents the level and L is the maximum level. For instance, n=128, which is a lower sample size than the original sample size of n=175 is obtained when j=7 (max L=7).

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Table (3): Average AIC and BIC criteria for a Two-dimensional VARX (p) model with Two External Predictors

Model	Method	Optimal Wavelet Selection	Level (L)	Order (N)	AIC	BIC
	Classical	Coif3	3	3	-1693.3069	-1662.6179
	Proposed	Coif4	*7	*4	*-2884.7215	*-2854.0325
	Classical	db3	3	3	-770.5639	-739.8749
	Proposed	**db38	**7	**38	**-4051.961	**-4021.2719
	Classical	sym3	3	3	-770.5639	-739.8749
VARX(1)	Proposed	Sym30	*7	*10	*-3537.3961	*-3506.7071
	Classical	bior3.3	3	8	-681.494	-650.805
	Proposed	bior6.8	*7	*14	*-3034.6321	*-3003.9431
	Classical	rbio3.3	3	8	-717.7111	-687.0221
	Proposed	rbio6.8	*5	*14	*-1960.5546	*-1929.8656
	Without wavelet		-	-	507.1522	537.8412
	Classical	coif3	3	3	-2068.1229	-2012.8826
	Proposed	coif5	*7	*5	*-4242.1024	*-4186.8621
	Classical	db3	3	3	-1466.2446	-6712.4936
	Proposed	**db33	**7	*33	**-6712.4936	**-6657.2533
	Classical	sym3	3	3	-1466.2446	-1411.0043
VARX (2)	Proposed	Sym30	*7	*30	*-5067.6283	*-5012.388
	Classical	bior3.3	3	8	-1427.8368	-1372.5965
	Proposed	bior5.5	*7	*13	*-3683.4798	*-3628.2395
	Classical	rbio3.3	3	8	-254.7074	-199.4671
	Proposed	rbio6.8	*7	*14	*-3364.5472	*-3309.307
	Without wavelet		-	-	284.1116	339.3519
	Classical	coif3	3	3	-2360.1095	-2280.318
	Proposed	coif5	*7	*5	*-5333.3632	*-5253.5717
	Classical	db3	3	3	-1052.2623	-972.4708
	Proposed	**db39	**7	**39	**-14226.058	**-14146.2665
	Classical	Sym3	3	3	-1052.2623	-972.4708
	Proposed	Sym25	*7	*25	*-7096.4107	*-7016.6192
VARX (3)	Classical	bior3.3	3	8	-1239.7264	-1159.9349
	Proposed	bior6.8	*7	*14	*-4418.0755	*-4338.284
	Without bior wavelet		-	-	153.7824	233.5739
	Classical	rbio3.3	3	8	-302.0351	-222.2436
	Proposed	rbio6.8	*7	*14	*-4226.6051	*-4146.8136
	Without wavelet		-	-	153.7824	233.5739
	Classical	coif3	3	3	-2129.5683	-2025.2256
	Proposed	coif5	*7	*5	*-5324.401	*-5220.0582
	Classical	db3	3	3	-1352.4795	-1248.1368
	Proposed	**db33	**7	**33	**-14275.1049	**-14170.7622
	Classical	sym3	3	3	-1352.4795	-1248.1368
VARX (4)	Proposed	Sym30	*7	*30	*-8332.6538	*-8228.311
	Classical	bior3.3	3	8	-1442.5963	-1338.2535
	Proposed	bior5.5	*6	*13	*-3821.1894	*-3716.8466
	Classical	rbio3.3	3	8	-234.5647	-130.222
	Proposed	rbio6.8	*7	*14	*-3788.2875	*-3683.9448
	Without wavelet		-	-	-78.3609	25.9818

An asterisk (*) denotes the optimal level and order for all the wavelet corresponding to the minimum AIC and BIC values.

An asterisk (**) denotes the optimal wavelet value corresponding to the minimum AIC and BIC values.

Source: Prepared by the researchers based on the outputs generated using the MATLAB programming language.

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Discussion:

Table (3) highlights the effectiveness of the proposed method, particularly when using universal thresholding with the soft rule. The method achieved the lowest AIC and BIC values, as indicated by the asterisks (*), signalling optimal performance. This demonstrates that, with carefully selected levels (L) and orders (N), the best wavelet type for these wavelets. The proposed method excels at denoising, significantly enhancing the model's accuracy and efficiency. Table 3 presents an in-depth analysis of VARX models using the proposed method, incorporating wavelet transforms. The findings reveal that this approach, specifically employing the db wavelet types, offers significant performance improvements over traditional methods and models that do not integrate wavelet transformations. Notably, for each VARX model tested, the following configurations were identified as optimal, as indicated by the asterisk (**) and highlighted in yellow. The results in the table (4) are as follows:

Table (4): Optimal Wavelet based on lowest AIC and BIC criteria for a Tow-dimensional VARX (P) model with Two Predictors

Model	Method	Wavelet	Level (L)	Order (N)	AIC	BIC
VARX(1)	Proposed	**db38	**7	**38	**-4051.961	**-4021.2719
VARX(2)	Proposed	**db33	**7	*33	**-6712.4936	**-6657.2533
VARX(3)	Proposed	**db39	**7	**39	**-14226.058	**-14146.2665
VARX(4)	Proposed	**db33	**7	**33	**-14275.1049	**-14170.7622

An asterisk (**) denotes the optimal wavelet value corresponding to the minimum AIC and BIC values.

Table 4 showcases the optimal wavelets, carefully selected from a range of orders between 1 and 45, which consistently attained the lowest AIC and BIC values, as indicated by the asterisks (**). This consistent achievement of minimal AIC and BIC values underscores the superior performance of these wavelets in noise reduction across VARX models with orders 1, 2, 3, and 4.

The optimal db wavelets (db38, db33, db39, and db32) emerged as the most efficient for denoising the data, reinforcing the effectiveness of the proposed method in enhancing model accuracy. By significantly minimizing noise, these wavelets demonstrated improved model reliability, positioning the proposed approach as a robust alternative for analyzing time-series data compared to traditional VARX models and those without wavelet integration.

Utilizing the optimal db33 wavelet at the optimal level (L=7) and order (N=33) for the VARX (4) model demonstrates superior performance over other VARX models with orders 1, 2, and 3, as indicated by the asterisk (**) and highlighted in yellow. This configuration achieves the lowest AIC and BIC values, recorded at (**-14779.9389) and (**-14675.5962), respectively. Such results underscore the two-dimensional VARX model with two predictors as the most accurate and efficient model, making it an optimal choice for forecasting and data analysis. Although classical methods remain practical, they yield higher AIC and BIC values, indicating less effectiveness in enhancing model precision.

Table 3 presents the results, which clearly demonstrate the consistent superiority of the proposed method utilizing the universal threshold with the soft rule over traditional methods and those without wavelet transformations. Across VARX models of orders 1, 2, 3, and 4, the proposed approach achieved significantly lower AIC and BIC values, highlighting its efficiency in noise reduction and its contribution to enhancing model accuracy. Although the classical method showed a degree of effectiveness, it fell short of the performance achieved by the proposed approach when optimal wavelets were employed.

In contrast, methods that did not incorporate wavelet transformations yielded higher AIC and BIC values, reflecting lower efficiency and a poorer model fit compared to wavelet-based techniques. Without the application of wavelet transformations, the models struggled to effectively remove noise from the data, resulting in less accurate parameter estimates and weaker predictive performance. These comparisons underscore the limitations of traditional methods in handling noise effectively and highlight the potential of wavelet-based approaches as a superior alternative in time series modeling, particularly for complex datasets requiring robust noise reduction and enhanced predictive capabilities.

Proposed VARX (4) Model for Stationary Time Series Data: As described in the theoretical framework, a two-dimensional VARX(4) model with two - external Predictors for stationary time series data, using the

Proposed Method, demonstrates superior performance, particularly when applying the optimal db33 wavelet at the optimal level (L=7) and order (N=33), and universal thresholding with the soft rule. Consequently, the VARX (4) model with a constant term was selected based on achieving the lowest AIC and BIC values. Table 9 presents the estimation results for this model With the estimated number of parameters N = 34 parameters, and the mathematical form of the 2-dimensional VARX (p=4,q=4) model with 4 lags and 2 external Predictor, including lags from t−1 to t−4 for both the dependent variables (Expenditures (Exp) and Official Reserves (OR)) and the external predictors (Broad Money (M2) and Oil Revenue(ORv)), according to equations (2), **is as follows:**

$$(\mathbf{1} - \boldsymbol{\Phi}_1\boldsymbol{B} - \boldsymbol{\Phi}_2\boldsymbol{B}^2 - \boldsymbol{\Phi}_3\boldsymbol{B}^3 - \boldsymbol{\Phi}_4\boldsymbol{B}^4)\begin{bmatrix} \boldsymbol{Exp_t} \\ \boldsymbol{OR_t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C_1} \\ \boldsymbol{C_2} \end{bmatrix} + \left(\sum_{j=1}^{q=4} \boldsymbol{\beta_j}\,\boldsymbol{B^j}\right)\begin{bmatrix} \boldsymbol{M2_t} \\ \boldsymbol{ORv_t} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\epsilon_{1,t}} \\ \boldsymbol{\epsilon_{2,t}} \end{bmatrix}$$

$$\left(\mathbf{1} - \begin{bmatrix} 4.0123 & 2.1789e-07 \\ 0.1358 & 4.0023 \end{bmatrix}\boldsymbol{B} - \begin{bmatrix} -6.0377 & -6.6438e-07 \\ -0.20568 & -6.0217 \end{bmatrix}\boldsymbol{B^2} - \begin{bmatrix} 4.0384 & 6.7687e-07 \\ 0 & 4.03654.0365 \end{bmatrix}\boldsymbol{B^3} - \begin{bmatrix} -1.013 & -2.3043e-07 \\ 0.069948 & -1.0172-1.0172 \end{bmatrix}\boldsymbol{B^4}\right)\begin{bmatrix} \boldsymbol{Exp_t} \\ \boldsymbol{OR_t} \end{bmatrix}$$
$$= \begin{bmatrix} -1.7428e-09 \\ -2.5472e-06 \end{bmatrix} + \begin{bmatrix} 2.0271e-14 & 4.6588e-15 \\ -8.3725e-10 & -2.6196e-10 \end{bmatrix}\begin{bmatrix} \boldsymbol{M2_{t-1}} \\ \boldsymbol{ORv_{t-1}} \end{bmatrix} + \begin{bmatrix} 4.7283e-15 & 4.0698e-14 \\ -4.3105e-11 & -2.2426e-09 \end{bmatrix}\begin{bmatrix} \mathbf{M2_{t-2}} \\ \mathbf{ORv_{t-2}} \end{bmatrix} + \begin{bmatrix} 6.7501e-15 & 8.2568e-14 \\ -1.6361e-10 & -4.4078e-09 \end{bmatrix}\begin{bmatrix} \mathbf{M2_{t-3}} \\ \mathbf{ORv_{t-3}} \end{bmatrix} + \begin{bmatrix} 1.3627e-14 & 4.7736e-14 \\ -5.6543e-10 & -2.4644e-09 \end{bmatrix}\begin{bmatrix} \mathbf{M2_{t-4}} \\ \mathbf{ORv_{t-4}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\epsilon_{1,t}} \\ \boldsymbol{\epsilon_{2,t}} \end{bmatrix}$$

Table 5: Estimation Results for the Proposed 2-dimensional VARX (p=4, q=4) Model with 2-External Predictors

Parameter	Value	Standard Error	t Statistic	P-Value
Constant(1)	-1.7428e-09	5.3684e-13	-3246.4	0
Constant(2)	-2.5472e-06	3.4472e-08	-73.891	0
AR{1}(1,1)	4.0123	5.5406e-07	-7.2418e+06	0

Parameter	Value	Standard Error	t Statistic	P-Value
AR{1}(2,1)	0.1358	0.058715	-2.3129	0.020729
AR{1}(1,2)	2.1789e-07	3.17e-09	68.735	0
AR{1}(2,2)	4.0023	0.00020373	19645	0
AR{2}(1,1)	-6.0377	1.6707e-06	3.614e+06	0
AR{2}(2,1)	-0.20568	0.17682	1.1632	0.24474
AR{2}(1,2)	-6.6438e-07	9.7234e-09	-68.328	0
AR{2}(2,2)	-6.0217	0.00062492	-9635.9	0
AR{3}(1,1)	4.0384	1.6762e-06	-2.4093e+06	0
AR{3}(2,1)	0	0.1774	0	1
AR{3}(1,2)	6.7687e-07	9.9678e-09	67.905	0
AR{3}(2,2)	4.0365	0.00064065	6300.7	0
AR{4}(1,1)	-1.013	5.5954e-07	1.8105e+06	0
AR{4}(2,1)	0.069948	0.05929	-1.1797	0.2381
AR{4}(1,2)	-2.3043e-07	3.4157e-09	-67.463	0
AR{4}(2,2)	-1.0172	0.00021954	-4633.2	0
Beta(1,1)	2.0271e-14	1.2927e-14	1.5681	0.11685
Beta(2,1)	-8.3725e-10	8.312e-10	-1.0073	0.3138
Beta(1,2)	4.6588e-15	1.0739e-13	0.04338	0.9654
Beta(2,2)	-2.6196e-10	6.9057e-09	-0.037934	0.96974
Beta(1,3)	4.7283e-15	1.2688e-14	0.37265	0.70941
Beta(2,3)	-4.3105e-11	8.1588e-10	-0.052833	0.95787
Beta(1,4)	4.0698e-14	2.7534e-13	0.14781	0.88249
Beta(2,4)	-2.2426e-09	1.7705e-08	-0.12666	0.89921
Beta(1,5)	6.7501e-15	1.2495e-14	0.54023	0.58904
Beta(2,5)	-1.6361e-10	8.0345e-10	-0.20364	0.83864
Beta(1,6)	8.2568e-14	2.8161e-13	0.2932	0.76937
Beta(2,6)	-4.4078e-09	1.8108e-08	-0.24342	0.80768
Beta(1,7)	1.3627e-14	1.2191e-14	1.1177	0.26369
Beta(2,7)	-5.6543e-10	7.8393e-10	-0.72128	0.47074
Beta(1,8)	4.7736e-14	1.1519e-13	0.41442	0.67857
Beta(2,8)	-2.4644e-09	7.4069e-09	-0.33272	0.73935

Source: Prepared by the researchers based on the outputs generated using the MATLAB programming language.

Discussion: Table 5 presents a comprehensive analysis of the parameter estimation results for the proposed 2-dimensional VARX (p=4, q=4) model with two external predictors. It includes key parameters such as autoregressive coefficients (AR) and external predictor coefficients (Beta), along with their estimated values, standard errors, t-statistics, and p-values. The autoregressive coefficients (AR) show strong statistical significance, as evidenced by very low p-values and small standard errors. For instance, AR{1}(1,1) has an estimated value of 4.0123 with a standard error of 5.5406×10^{-7} and a p-value of 0.000, while AR{2}(1,1) has an estimated value of -6.0377, a standard error of 1.6707×10^{-6}, and a p-value of 0.000. Similarly, AR{3}(1,1) has an estimated value of 4.0384 with a standard error of 1.6762×10^{-6} and a p-value of 0.000, further confirming its importance in capturing the temporal dynamics of the model. These findings underscore the substantial role of the AR coefficients in capturing the temporal dynamics of the model and defining relationships among endogenous variables.

These results highlight the robustness of the AR coefficients in effectively capturing the core dynamics of the system and defining the relationships among endogenous variables. While the Beta coefficients exhibit limited statistical significance in some cases, their impact does not undermine the overall performance of the model. Revisiting the selection of external predictors and incorporating alternative variables could further enhance the model's explanatory power and predictive accuracy, offering potential avenues for improvement without affecting the model's current strength.

Figure 2 illustrates the performance of the VARX (4) model in capturing the dynamics of the Exp and OR variables. The observed values (blue lines) and estimated values (red lines) for Exp show strong alignment, with minor deviations evident in the peaks and troughs. For Exp, the model achieves a Root Mean Square Error (RMSE) of 1.3453e-13 and a Mean Absolute Percentage Error (MAPE) of 2.6866e-09%**,** indicating its effectiveness in capturing seasonal patterns and fluctuations.

For the OR variable, the alignment between observed and estimated values is reasonable, though slightly less precise compared to Exp. Some deviations are noticeable, particularly around the middle of the series, where the model slightly underestimates or overestimates sudden changes. These

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deviations suggest that while the model captures the general trends well, there is room for improvement in handling abrupt changes. The RMSE for OR is 8.6502e-09, and the MAPE is 2.2213e-05%, highlighting the need for potential refinements in the model to better address these variations.

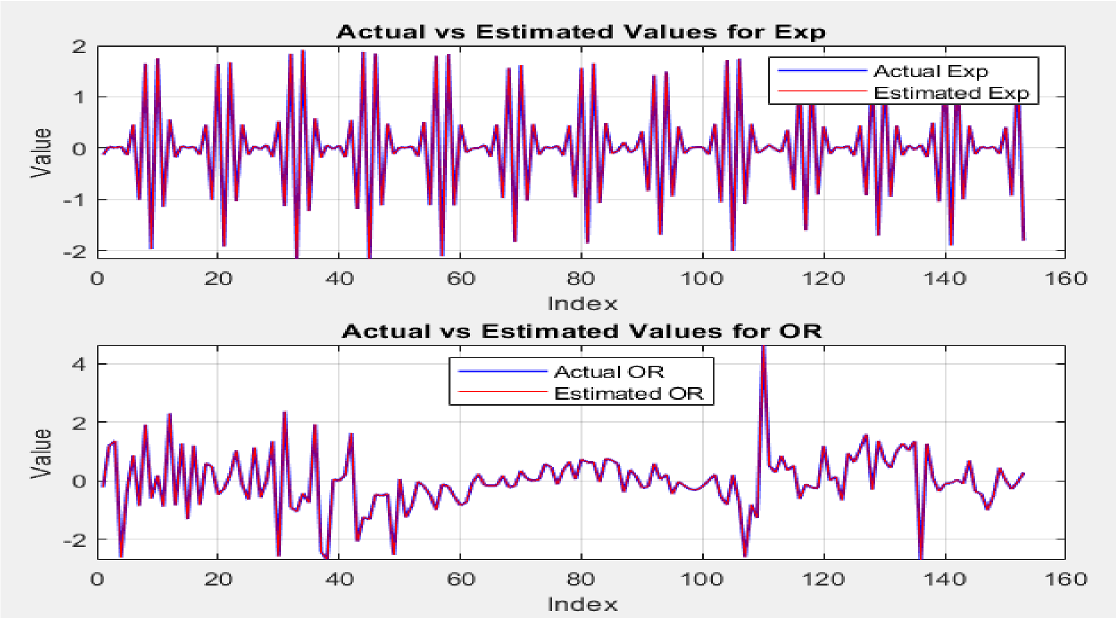

Figure (2): Fit of the Proposed VARX (4) Model for Dependent Variables Exp and OR

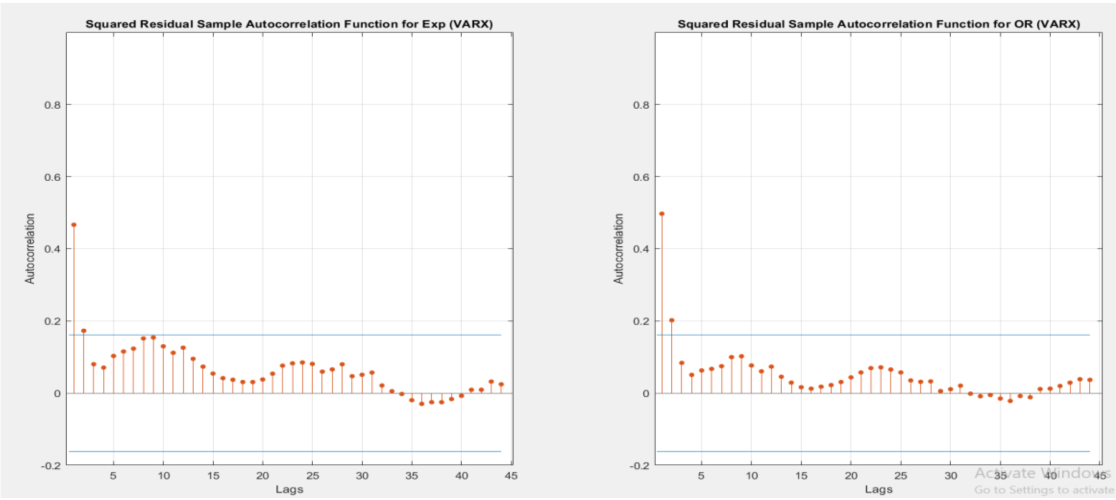

Figure (3): Square Residual Sample Autocorrelation Function the Proposed VARX (4) Model

Source: Prepared by the researchers based on the outputs generated using the MATLAB programming language.

Figure 3 illustrates the squared residual autocorrelation functions for Exp and OR. For Exp, residuals oscillate around zero, mostly within the confidence intervals, indicating the model's effectiveness in capturing its core dynamics. Similarly, for OR, most residuals fall within the confidence bounds, with minor deviations suggesting minimal unexplained patterns that do not significantly affect performance.

The Box-Ljung Test results confirm these findings:

- ❖ For Exp, $\mathrm{Q-Stat} = 58.84 < \chi^2(0.05,44) = 60.46$, with $\mathrm{p} = 0.14$ $(\mathrm{p} > 0.05)$, indicating no significant autocorrelation at a 5% significance level.
- ❖ For OR, $\mathrm{Q-Stat} = 44.84 < \chi^2(0.05,44) = 60.46$, with $\mathrm{p} = 0.437$ $(\mathrm{p} > 0.05)$, also indicating no significant autocorrelation at a 5% significance leve.

These results validate the residual independence, highlighting the robustness of the proposed VARX (4) model for forecasting.

4. Conclusions: The research led to the following conclusions from Real Data Analysis:

A. Applying non-stationarity Seasonal and non-seasonal differentiation in VARX models, improving reliability and interpretability by stabilizing variance and mean through Dickey-Fuller tests

B. The higher-order VARX models (p=1, 2, 3, and 4) always achieve lower AIC and BIC values, which indicates their strong effectiveness in capturing the underlying time series dynamics.

C. The proposed method outperforms by using universal thresholding with the soft rule consistently classical and without variety wavelet transformation methods, that achieves the lowest AIC and BIC values across VARX models with orders1,2, 3, and 4.

D. The study reveals that optimal wavelets db33, db38, db39, and db32 are most effective in removing noise from data, thereby improving the accuracy of economic models in forecasting events.

E. The db33 wavelet at the optimal level (L=7) and optimal order (N=33) proved best, significantly improving VARX (4) model performance with the lowest AIC (-14779.94) and BIC (-14675.60) values.

F. Diagnostic tests validate the model's strength, with minimal autocorrelation in residual analysis, enhancing its reliability and accuracy, and indicating its suitability for future predictions.

G. The VARX (4) model effectively captures economic indicators like expenditures and reserves, with minor deviations, offering room for refinement.

H. VARX (4) demonstrates strong predictive power for Iraqi economic variables, validated by diagnostic tests, with potential for enhanced accuracy.
I. Incorporating wavelet techniques into economic models improves forecasting quality, supporting decision-makers in analyzing economic trends.

5. Recommendations: Through the research, we conclude with the following recommendations:

A. Future research should compare VARX and ARIMAX models to evaluate their effectiveness in addressing external variable impact, enabling the selection of the model that best suits the data's nature.
B. Future research should compare VARX models with VAR to assess the impact of external variables on prediction accuracy and reliability, particularly in analyses requiring comprehensive interpretation of internal and external interactions.
C. Higher-order VARX models are recommended for complex economic data to better match data and capture time series patterns.
D. . Evaluating denoising method efficacy using statistical criteria like AIC and BIC is crucial.
E. Addressing the non-stationarity of economic time series through seasonal and non-seasonal differentiation improves forecast reliability.
F. Diagnostic testing like residual autocorrelation analysis confirms model accuracy and creates more reliable economic forecasts.

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