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OJCA Rayleigh distribution, Statistical Properties with Application

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Abstract: In this paper we employment the OJCA family of distributions to introduce a new generalization for Rayleigh distribution named OJCA Rayleigh distribution the new distribution with two parameters denoted by (OJCARay). The purpose of study is to derive some of its statistical properties such as, expansion of probability density function, quntaile function, moments, incomplete moments, entropy. Finally, we use the maximum likelihood method to estimate the parameters for the new distribution.

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توزيع OJCA Rayleigh خصائص إحصائية مع التطبيق

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المستخلص

في هذا البحث قمنا بتوظيف عائلة OJCA لتقديم تعميماً جديد لتوزيع رايلي وسمي هذا التوزيع OJCA Rayleigh حيث قمنا باشتقاق بعض الخواص الإحصائية للتوزيع الجديد مثل توسيع دالة الكثافة والدالة الكمية والعزوم والعزوم الناقصة والانتروبيا كما وتم استخدام طريقة الإمكان الأعظم لتقدير معلمات التوزيع الجديد وفي النهاية تم تطبيق بيانات حقيقة لمقارنة التوزيع الجديد مع بعض التوزيعات الأخرى.

الكلمات المفتاحية: تعميم، معلمات، عزوم ناقصة، طرقة الإمكان الأعظم.

1. Introduction

The primary objective of statistics is to find effective statistical models for real-world events in the form of established probability distributions. Natural life occurrences that are characterized by uncertainty and riskiness are modeled using probability distributions. However, modeling natural events using conventional distributions can be complex and challenging, which has led to the development of many probability distributions. Despite this, some natural events' data continue to be inaccurately represented by recognized probability distributions, which has led to the expansion and alteration of generalized probability distributions to better fit this data.

Researchers have focused on expanding the commonly used probability distributions to create more realistic and flexible models for modeling data. For example, Yousof et al. (2017) introduced the transmuted Topp-Leone G family of distributions, Ristic and Balakrishnan (2012) introduced the gamma exponentiated exponential distribution, and Al-Habib et al., (2023). A New Family of Truncated Nadarajah-Haghighi-G. Alzaatreh and Famoye (2013) introduced a new method for generating families of continuous distributions. Ahmad and Hamedani (2018) proposed a new Weibull-X family of distributions, and Kundu and Raqab (2005) introduced the generalized Rayleigh distribution.

Due to their critical role in explaining a variety of natural phenomena, the Rayleigh probability distribution has been extended in many ways. For instance, Merovci (2013) introduced the transmuted Rayleigh distribution, Dey (2014) proposed the two-parameter Rayleigh distribution, and Elbatal (2015) presented a Weibull-Rayleigh probability distribution. Ateeq et al. (2019) derived the Rayleigh-Rayleigh distribution (RRD), and Al-Babtain (2020) proposed a new extension of the Rayleigh distribution with two parameters called the type I half-logistic Rayleigh distribution. Rana et al. (2022) introduced the Pareto-Rayleigh Distribution, while Lishamol et al. (2019) proposed a generalized Rayleigh distribution. Khaleel et al. (2022) introduced a new [0, 1] truncated inverse Weibull Rayleigh distribution, and Al-Noor et al. (2022) introduced the Rayleigh Gompertz distribution.

The paper is structured as follows: In Section 2, we propose a novel distribution called the OJCA-Rayleigh distribution, which is based on the OJCA family of distributions. We provide the cumulative distribution function (CDF) and probability density function (PDF) for this new distribution. Additionally, we introduce the survival and hazard functions in this section. Section 3 explores the statistical properties of the OJCA-Rayleigh distribution, including its mixture representation, quantile function, moments, incomplete moments, and order statistics. The Rényi entropy of the distribution is also discussed. The maximum likelihood estimation (MLE) method for estimating the parameters of the OJCA-Rayleigh distribution is presented in Section 4. To demonstrate the practicality of the new model, Section 5 applies it to a real dataset, displaying its effectiveness in real-world scenarios. Finally, Section 6 concludes the paper, providing some closing remarks on the findings and contributions of the study.

2. OJCA-Rayleigh distribution

As developed by Iqbal et al., (2023). The odd JCA family of distributions have the following CDF and PDF respectively:

$$F(x.k,\boldsymbol{\varphi}) = 1 - e^{-G(x,\boldsymbol{\varphi})[1-G(x,\boldsymbol{\varphi})]^{k-1}} \tag{1}$$

$$f(x,k,\varphi) = g(x,\boldsymbol{\varphi})\big(1 - kG(x,\boldsymbol{\varphi})\big)[1 - G(x,\boldsymbol{\varphi})]^{k-2}\ e^{-G(x,\boldsymbol{\varphi})[1-G(x,\boldsymbol{\varphi})]^{k-1}} \tag{2}$$

Where $G(x,\boldsymbol{\varphi}), g(x,\boldsymbol{\varphi})$ are respectively the CDF and PDF of the baseline distribution with vector of parameters.

Now if we have the Rayleigh distribution as a baseline distribution then we have:

$$G(x,\theta) = 1 - e^{\frac{-x^2}{\theta^2}} \tag{3}$$

$$g(x,\theta) = \frac{x}{\sigma^2}\, e^{\frac{-x^2}{\theta^2}} \tag{4}$$

So that by inserting Eq. (3) in Eq. (1) we can obtained the CDF of OJCARay distribution as follows:

$$F(x.k,\theta) = 1 - e^{-\left(1-e^{\frac{-x^2}{\theta^2}}\right)\left(e^{\frac{-(k-1)}{2\theta^2}x^2}\right)} \qquad (5)$$

In addition, if we insert Eq. (4) and Eq. (3) in Eq. (2) we obtained the PDF of OJCARay distribution as follows:

$$f(x,k,\theta) = \left\{ \begin{array}{c} \frac{x}{\sigma^2}\; e^{\frac{-x^2}{\theta^2}} \times \left(1 - k\left(1 - e^{\frac{-x^2}{\theta^2}}\right)\right) \times \\ \times\; e^{-\left(1-e^{\frac{-x^2}{\theta^2}}\right)\left(e^{\frac{-(k-1)}{2\theta^2}x^2}\right)} \left(1 - e^{\frac{-(k-2)}{2\theta^2}x^2}\right) \end{array} \right\} \qquad (6)$$

Figures 1 and 2 display various potential configurations of the cumulative distribution function (CDF) and probability density function (PDF) for the OJCARay distribution, corresponding to specific values assigned to two parameters. In **Figure 1,** the CDF exhibits distinctive characteristics, including the range of $0 \leq$ ***F(x)*OJCARay** ≤ 1, strict monotonicity (increasing), and continuity. On the other hand, **Figure 2** illustrates several possible shapes of the OJCARay PDF, such as right-skewed, symmetric, and semi-symmetric. Consequently, the OJCARay distribution proves to be remarkably adaptable for modeling positive data due to its flexibility.

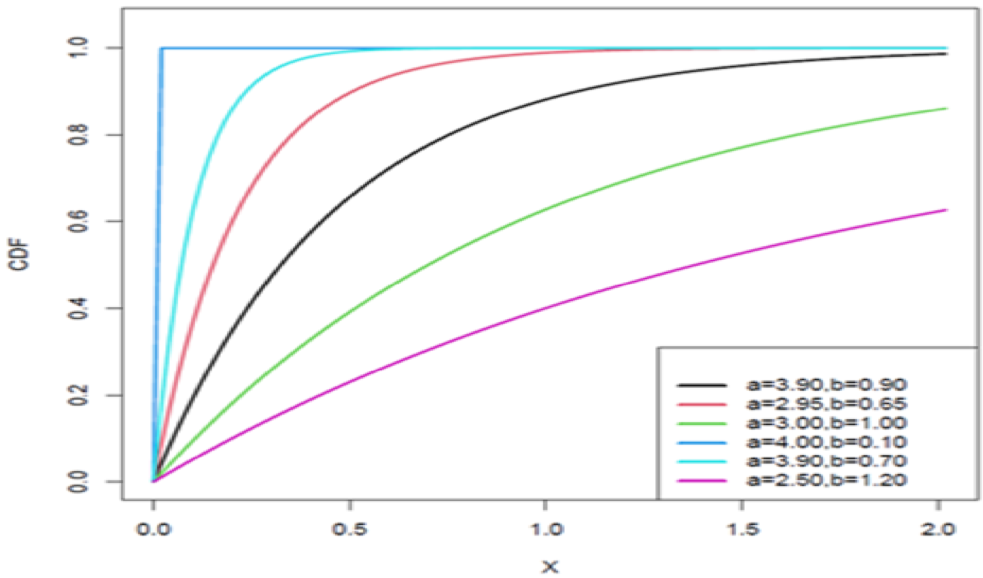

Figure (1) The plot of OJCARay cdf for specific parameters values.

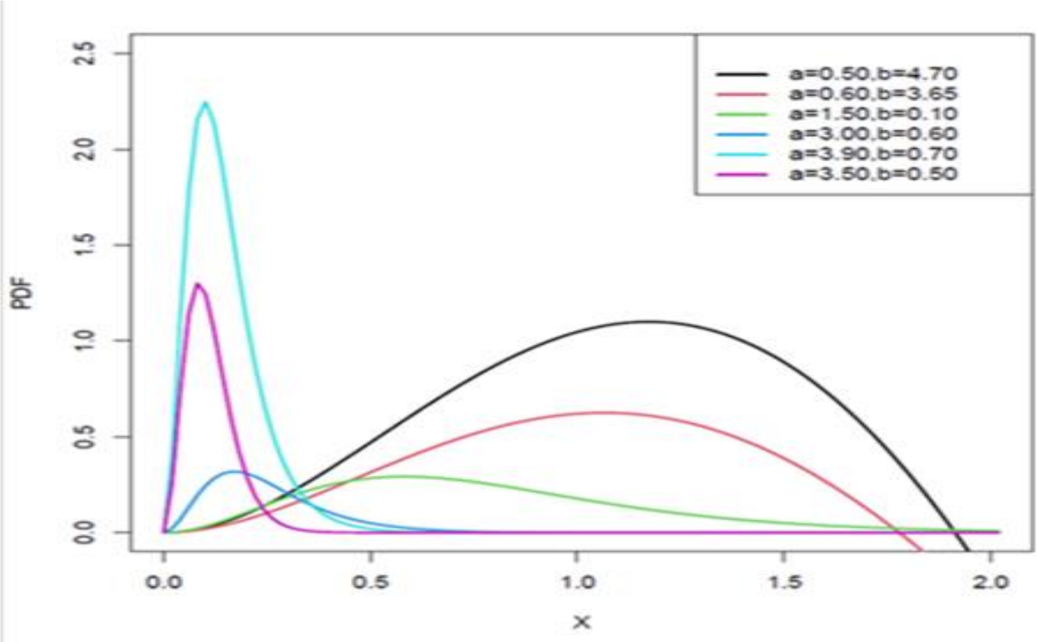

Figure (2): The plot of OJCARay pdf for specific parameters values.

2-1 Survival function

The survival function of OJCARay distribution. This can be done by utilizing the formula presented by Abdal (2020), which states that *S(x) = 1 - F(x),* where *F(x)* represents the distribution function.

$$S(x.k,\theta) = e^{-\left(1-e^{\frac{-1}{2\theta^2}x}\right)\left(e^{\frac{-(k-1)}{2\theta^2}x}\right)} \tag{7}$$

2-2 Hazard function

The hazard function of OJCA distribution as presented by Abdal-hameed et al. (2018), resulting in:

$$h(x,k,\theta) = \frac{x}{\sigma^2}\, e^{-\frac{x}{2\sigma^2}}\left(1-k\left(1-e^{\frac{-1}{2\theta^2}x}\right)\right)\left(e^{\frac{-(k-2)}{2\theta^2}x}\right) \tag{8}$$

3. Statistical properties

3-1 Mixture representation

In this sub-section, we provided the mixture of CDF and PDF for the OJCAChe distribution. By use the expansion of exponential, we get Abdullah et al., (2019), and Oguntunde et al., (2019)

$$e^{-z} = \sum_{j=0}^{\infty}\frac{(-1)^j}{j!}z^j \tag{9}$$

We have get that:

$$e^{-\left(1-e^{\frac{-1}{2\theta^2}x}\right)\left(e^{\frac{-(k-1)}{2\theta^2}x}\right)} = \sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\left(1-e^{\frac{-1}{2\theta^2}x}\right)^j\left(e^{\frac{-j(k-1)}{2\theta^2}x}\right) \tag{10}$$

Eq. (5) can be rewritten as follows:

$$F(x.k,\boldsymbol{\sigma},\theta) = 1+\sum_{j=0}^{\infty}\frac{(-1)^{j+1}}{j!}\left(1-e^{\frac{-1}{2\theta^2}x}\right)^j\left(e^{\frac{-j(k-1)}{2\theta^2}x}\right) \tag{11}$$

Now by the generalized binomial theorem we get that:

$$\left[1-e^{\frac{-1}{\theta^2}x}\right]^j = \sum_{n=0}^{\infty}\binom{j}{n}(-1)^n e^{\frac{-n}{2\theta^2}x} \tag{12}$$

So that Eq. (11) can be written as:

$$F(x.k,\boldsymbol{\sigma},\theta) = 1+\sum_{j=0}^{\infty}\sum_{n=0}^{\infty}\binom{j}{n}\frac{(-1)^{j+n+1}}{j!}\; e^{\frac{-n-j(k-1)}{2\theta^2}x} \tag{13}$$

Eq. (13) represent mixture representation of CDF for OJCARay distribution now by derived Eq. (13) we get the mixture representation for OJCARay distribution probability density function PDF as follows:

$$f(x.k,\theta) = \sum_{j=0}^{\infty}\sum_{n=0}^{\infty}\binom{j}{n}\frac{(-1)^{j+n+1}}{j!}\;\frac{-n-j(k-1)}{2\theta^2}\; e^{\frac{-n-j(k-1)}{2\theta^2}x} \tag{14}$$

Now let

$$T_{j,n} = \sum_{j=0}^{\infty}\sum_{n=0}^{\infty}\binom{j}{n}\frac{(-1)^{j+n+1}}{j!}\;\frac{-n-j(k-1)}{2\theta^2} \tag{15}$$

Then

$$f(x.k,\theta) = T_{j,n}\, e^{\frac{-n-j(k-1)}{2\theta^2}x} \tag{16}$$

3-2 Quntaile function

The quantile function is a statistical technique utilized to identify the threshold that separates a probability distribution into equal or specified segments. In particular, the quantile function calculates the reverse of the cumulative distribution function (cdf), as described by Abdullah et al. (2019). The quantile function of the OJCARay distribution, denoted as $Q(u)$ is derived by employing the equation $Q(u) = F^{-1}(u)$, where F^{-1} represents the inverse of the cdf. Therefore, the expression for the quantile function of the GoIW distribution can be presented as follows**:** Suppose a random variable $X\sim OJCARay(k,\theta)$ then the quantile function provided in Eq. (17) as:

$$\frac{2\theta^2}{(k-1)}\, ln\left\{\frac{1}{(1-\ e^{-1})}\, ln\{1-u\}\right\} = Q(u) \tag{17}$$

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where $u \sim uniform\ (0,1)$. By insert $u = 0.5$ in Eq. (17), we obtained the median (M) of the OJCARay model as follow:

$$\frac{2\theta^2}{(k-1)} ln\left\{\frac{1}{(1-\ e^{-1})} ln\{1-0.5\}\right\} = M$$

3-3 moment

The moments of a probability distribution serve as statistical indicators that condense information about the distribution's characteristics. These moments offer understanding regarding the distribution's central tendency, variability, and shape. They are valuable for comparing various distributions and drawing inferences about the underlying processes, as mentioned by AbuJarad et al. (2020). To calculate the t^{th} moment of the OJCARay distribution can be we can employ the following approach:

For any random variable X we have the t^{th} moment is given by:

$$\mu_t = \int_{-\infty}^{\infty} x^t f(x) dx$$

If $f(x)$ as in (16) we get that:

$$\mu_t = T_{j,n} \int_0^{\infty} x^t\ e^{\frac{-(n+j(k-1))}{2\theta^2}x}\ dx$$

Now let $y = \frac{(n+j(k-1))}{2\theta^2}\ x \ \Rightarrow x = \frac{2\theta^2}{(n+j(k-1))}\ y \ \Rightarrow dx = \frac{2\theta^2}{(n+j(k-1))}\ dy$

Then

$$\mu_t = T_{j,n} \int_0^{\infty} \left(\frac{2\theta^2}{(n+j(k-1))}\ y\right)^t\ e^{-y}\ \frac{2\theta^2}{(n+j(k-1))}\ dy$$

$$\mu_t = T_{j,n} \left(\frac{2\theta^2}{(n+j(k-1))}\right)^{t+1} \int_0^{\infty} (y)^t\ e^{-y}\ dy$$

$$\mu_t = T_{j,n} \left(\frac{2\theta^2}{(n+j(k-1))}\right)^{t+1} \Gamma(t). \tag{18}$$

From Eq. (18) for t = 1, we have get the first moment $E(x)$

$$E(x) = T_{j,n} \left(\frac{2\theta^2}{(n+j(k-1))}\right)^2 \tag{19}$$

And

$$E(x^2) = T_{j,n} \left(\frac{2\theta^2}{(n+j(k-1))}\right)^3 \tag{20}$$

The variance of the OJCARay distribution can be expressed as:

$$Var(x) = E(x^2) - (E(x))^2$$

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$$Var(x) = \left[T_{j,n}\left(\frac{2\theta^2}{(n+j(k-1))}\right)^3\right] - \left[T_{j,n}\left(\frac{2\theta^2}{(n+j(k-1))}\right)^2\right]^2 \quad (21)$$

3-4 incomplete moments

The r^{th} incomplete moments of the OJCARay distribution can be obtained as follows

$$M_r(y) = \int_0^y x^r f(x)dx$$

If $f(x)$ as in (16) we get that:

$$M_r(y) = T_{j,n}\int_0^y x^r\, e^{\frac{n+j(k-1)}{2\theta^2}x}\, dx$$

Then

$$M_r(y) = T_{j,n}\left(\frac{2\theta^2}{(n+j(k-1))}\right)^{r+1} \gamma(r, \frac{n+j(k-1)}{2\theta^2} y) \quad (22)$$

3.5 order statistics

Let $X_1, X_2, \dots, X_n$ be a sequence if *iid* random variables follow the OJCARay distribution then the PDF of j^{th} order statistic Ahmedd et al., (2020), Ahmed et al., (2021). is given by:

$$f_{j:n}(x) = \frac{n!}{(j-1)!\,(n-j)!}\,(F(x))^{j-1}(1-F(x))^{n-j}\, f(x)$$

$$f_{j:n}(x) = \sum_{s=0}^{n-j}\frac{n!}{(j-1)!\,(n-p)!}(-1)^s\binom{n-j}{s}(F(x))^{j+s-1}\, f(x) \quad (23)$$

By inserting equations. (5), (6) in Eq. (23), we get that:

$$f_{j:n}(x) = \sum_{s=0}^{n-j}\frac{n!}{(j-1)!(n-p)!}(-1)^s\binom{n-j}{s}\left(1-e^{-\left(1-e^{\frac{-1}{2\theta^2}x}\right)\left(e^{\frac{-(k-1)}{2\theta^2}x}\right)}\right)^{j+s-1} \times \left(\frac{x}{\sigma^2}\, e^{-\frac{x}{2\sigma^2}}\left(1 - k\left(1-e^{\frac{-1}{2\theta^2}x}\right)\right)\left(e^{\frac{-(k-2)}{2\theta^2}x}\right)\, e^{-\left(1-e^{\frac{-1}{2\theta^2}x}\right)\left(e^{\frac{-(k-1)}{2\theta^2}x}\right)}\right) \quad (24)$$

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Now for $j = 1$ in Eq. (24) we get the smallest order statistic (least value function) and for $j = n$ we get the largest order statistic (big value function).

3.6 Rényi entropy

The Rényi entropy of OJCARay distribution can be obtained according to the form:

$$I_R(\delta)_{OJCARay} = \frac{1}{1-\delta}\log\left\{\int_{-\infty}^{\infty} f^{\delta}(x)dx\right\}, \delta \neq 1, \delta > 0 \qquad (25)$$

Where $f(x)$ as in Eq. (16) We get that:

$$I_R(\delta)_{OJCARay} = \frac{1}{1-\delta}log\left\{(T_{j,n})^{\delta}\int_{0}^{\infty} e^{\frac{-n-j(k-1)}{2\theta^2}\delta x}dx\right\}, \delta \neq 1, \delta > 0 \qquad (26)$$

Then

$$I_R(\delta)_{OJCARay} = \frac{1}{1-\delta}log\left\{(T_{j,n})^{\delta}\left(\frac{2\theta^2}{(n+j(k-1))\delta}\right)^{r+1}\Gamma(r)\right\}, \delta \neq 1, \delta > 0 \qquad (27)$$

4. Estimation Methods for the OJCARay Distribution

In this section, we estimate the unknown parameters of the OJCARay distribution by use the maximum likelihood method. Consider the random sample, $x_1, x_2, \ldots, x_n$ from the OJCARay distribution. Then the likelihood function is for (k, θ) takes the form:

$$L(k,\theta) = \prod_{i=1}^{n}\left\{\frac{x_i}{\sigma^2}e^{-\frac{x_i^2}{2\sigma^2}} \times \left(1-k\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\right) \times \left(e^{\frac{-(k-2)x_i^2}{2\theta^2}}\right) \times e^{-\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\left(e^{\frac{-(k-1)x_i^2}{2\theta^2}}\right)}\right\} \qquad (28)$$

Now let $l = \log L(k,\theta)$, Then

$$l = log\sum_{i=1}^{n} x_i - 2nlog(\sigma) - \sum_{i=1}^{n}\frac{x_i^2}{2\theta^2} + log\sum_{i=1}^{n}\left(1-k\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\right) - \sum_{i=1}^{n}\frac{-(k-2)x_i^2}{2\theta^2} - \sum_{i=1}^{n}\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\left(e^{\frac{-(k-1)x_i^2}{2\theta^2}}\right)$$

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$$\frac{\partial l}{\partial k} = \frac{n\left(-1+e^{\frac{-x_i^2}{2\theta^2}}\right)}{\left(1-k\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\right)} - \frac{n x_i^2}{2\theta^2} + \frac{n\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right) x_i^2 \; e^{\frac{-(k-1)x_i^2}{2\theta^2}}}{2\theta^2} \tag{29}$$

$$\frac{\partial}{\partial\theta} = \frac{-2n}{\theta} + \frac{n x_i^2}{\theta^3} + \frac{n(k-2)x_i^2}{\theta^3} + \frac{n k x_i^2 e^{\frac{-x_i^2}{2\theta^2}}}{\theta^3\left(1-k\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\right)} + \frac{n x_i^2 e^{\frac{-x_i^2}{2\theta^2}} \; e^{\frac{-(k-1)x_i^2}{2\theta^2}}}{\theta^3} + \frac{n(k-1)x_i^2\left(1-e^{\frac{-x_i^2}{2\theta^2}}\right)\left(e^{\frac{-(k-1)x_i^2}{2\theta^2}}\right)}{\theta^3} \tag{30}$$

By setting, the score function to zero for the equations $\frac{\partial}{\partial\theta}$ and $\frac{\partial}{\partial k}$w e can find the parameter values that maximize the likelihood. However, solving these equations algebraically can be extremely challenging. Therefore, we resort to numerically iterative methods, such as Newton-Raphson type algorithms, implemented using the R software package "Newdistns." This approach allows us to obtain estimators for the parameters, as described in the subsequent section (Mohammad, 2020).

5. Application

In this section, we utilized the OJCA Rayleigh distribution to fit real data sets and evaluate its goodness of fit in comparison to other competing distributions. The analysis was conducted using the R Statistical Software. To compare the proposed model with alternative models like TEER, BeR, KuR, EGR, WeR, GoR, and R, we employed various statistical criteria, namely *-l* (log-likelihood), AIC (Akaike Information Criterion), AIC (corrected Akaike Information Criterion), BIC (Bayesian Information Criterion), and HQIC (Hannan-Quinn Information Criterion) implemented in R. The study was conducted by Mohammad et al. (2020) and Ibrahim and Khaleel (2018) and Khaleel et al. (2020, 2022). The data is about the total milk production in the first birth of 107 cows from SINDI race Hammed and Khaleel, (2023). And the data given as

0.4365, 0.4260, 0.5140, 0.6907, 0.7471, 0.2605, 0.6196, 0.8781, 0.4990, 0.6058, 0.6891, 0.5770, 0.5394, 0.1479, 0.2356, 0.6012, 0.1525, 0.5483,

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0.6927, 0.7261, 0.3323, 0.0671, 0.2361, 0.4800, 0.5707, 0.7131, 0.5853, 0.6768, 0.5350, 0.4151, 0.6789, 0.4576, 0.3259, 0.2303, 0.7687, 0.4371, 0.3383, 0.6114, 0.3480, 0.4564, 0.7804, 0.3406, 0.4823, 0.5912, 0.5744, 0.5481, 0.1131, 0.7290, 0.0168, 0.5529, 0.4530, 0.3891, 0.4752, 0.3134, 0.3175, 0.1167, 0.6750, 0.5113, 0.5447, 0.4143, 0.5627, 0.5150, 0.0776, 0.3945, 0.4553, 0.4470, 0.5285, 0.5232, 0.6465, 0.0650, 0.8492, 0.8147, 0.3627, 0.3906, 0.4438, 0.4612, 0.3188, 0.2160, 0.6707, 0.6220, 0.5629, 0.4675, 0.6844, 0.3413, 0.4332, 0.0854, 0.3821, 0.4694, 0.3635, 0.4111, 0.5349, 0.3751, 0.1546, 0.4517, 0.2681, 0.4049, 0.5553, 0.5878, 0.4741, 0.3598, 0.7629, 0.5941, 0.6174, 0.6860, 0.0609, 0.6488, 0.2747

Table 1: The K-S value with its corresponding p-value and W value of the data set.

Model	W	A	K-S	p-value
OJCARay	0.0938	0.5961	0.0513	0.9442
TEER	0.3519	2.2158	0.1185	0.1045
BER	0.3292	2.0785	0.1141	0.1295
KuR	0.2505	1.6046	0.0875	0.3964
EGR	0.3403	2.1451	0.1186	0.1041
WeR	0.2366	1.5185	0.0854	0.4278
GoR	0.0960	0.6087	0.0520	0.9382
R	0.3325	2.0975	0.1517	0.0158

- Based on the values presented in Table 1. It can be observed that the proposed OJCARay distribution has the smallest values for K-S, A, and W, and the largest p-value. This suggests that the proposed distribution is a better fit compared to the other competing distributions.

Table (2): The values of -2 l, AIC, CAIC, BIC, HQIC.

Model	MLEs	-2 l	AIC	CAIC	BIC	HQIC
OJCARay	$\hat{a}$ = 0.125 $\hat{b}$ = 2.219	-27.0239	-50.0478	-49.9302	-44.7399	-47.8969
TEER	$\hat{a}$ = 66.022 $\hat{b}$ = 1.269 $\hat{\lambda}$ = 0.070	-18.1841	-30.3628	-30.1306	-22.4064	-27.1419
BER	$\hat{a}$ = 1.278 $\hat{b}$ = 13.361 $\hat{\lambda}$ = 0.376	-18.9877	-31.9755	-31.7379	-24.0136	-28.7492

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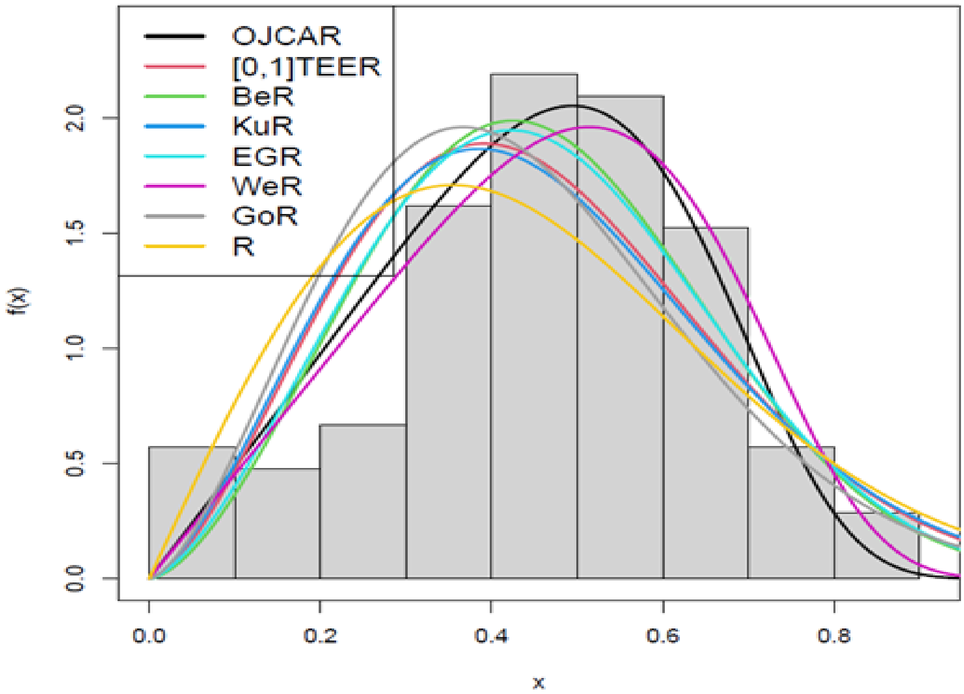
Model	MLEs	-2 l	AIC	CAIC	BIC	HQIC
KuR	$\hat{a}$ = 1.297 $\hat{b}$ = 43.284 $\hat{\lambda}$ = 0.208	-21.2130	-36.4260	-36.1884	-28.4641	-33.1997
EGR	$\hat{a}$ = 2.132 $\hat{b}$ = 1.250 $\hat{\lambda}$ = 2.132	-18.5903	-31.1806	-30.9429	-23.2187	-27.9543
WeR	$\hat{a}$ = 1.293 $\hat{b}$ = 1.070 $\hat{\lambda}$ =3.979	-21.6121	-37.2242	-36.9865	-29.2623	-33.9978
GoR	$\hat{a}$ = 1.262 $\hat{b}$ = 1.463 $\hat{\lambda}$ =1.788	-27.006	-48.0132	-47.7755	-40.0513	-44.7868
R	$\hat{\lambda}$ = 3.974	-17.151	-32.3039	-32.2651	-29.6499	-31.2285

❖ From Table 2, it can be observed that the log-likelihood value of the proposed OJCARay distribution is smaller compared to the log-likelihood values of the other distributions. Additionally, the AIC value for the proposed distribution is smaller compared to the AIC values of the other distributions. This indicates that the proposed distribution is more suitable for fitting this dataset.



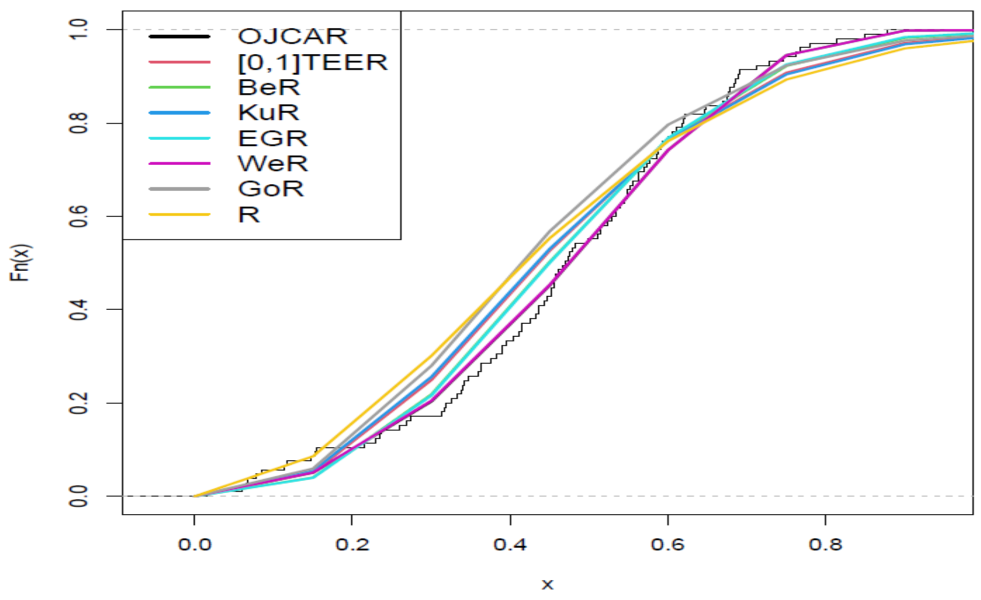


Figure 3. The CDFs and PDFs for the selected models for the data set.

From Figure 3, it is evident that the fitted density of the new distribution closely matches the empirical histogram, outperforming the TEER, BeR, KuR, EGR, WeR, GoR and R distributions.

6. Conclusion

This paper proposes a novel extension of the Rayleigh distribution, known as the OJCA Rayleigh distribution, which is based on the OJCA family of distributions. The OJCA Rayleigh distribution, with two parameters, offers greater flexibility compared to other distributions like the Rayleigh distribution where from table 2 the new distribution have the smallest values for the information's criterion (AIC, CAIC, BIC, HQIC) in addition from table 1 the OJCARay distribution have the largest values for K-S and P-value. Moreover, from Figure 3, it is evident that the fitted density of the new distribution closely matches the empirical histogram, outperforming the TEER, BeR, KuR, EGR, WeR, GoR and R distributions.

The study derives several statistical properties for the new distribution, including the quantile function, moments, incomplete moments, and entropy. Additionally, the maximum likelihood method is utilized to estimate the parameters for the new distribution. In summary, the proposed OJCA Rayleigh distribution presents a more versatile alternative to the conventional Rayleigh distribution, and the derived statistical properties enhance its potential applicability in various fields.

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