

Robust Low Pass Filter-PID Controller for 2-DOF Helicopter System

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Abstract

In this article, a robust control technique for 2-DOF helicopter system is presented. The 2-DOF helicopter system is 2 inputs and 2 outputs system that is suffering from the high nonlinearity and strong coupling. This paper focuses on design a simple, robust, and optimal controller for the helicopter system. Moreover, The proposed control method takes into account effects of the measurement noise in the closed loop system that effect on the performance of controller as well as the external disturbance. The proposed controller combines low pass filter with robust PID controller to ensure good tracking performance with high robustness. A low pass filter and PID controller are designed based H_∞ weighted mixed sensitivity. Nonlinear dynamic model of 2-DOF helicopter system linearized and then decoupled into pitch and yaw models. Finally, proposed controller applied for each model. Matlab program is used to check effectiveness the proposed control method. Simulation results show that the proposed controllers has best tracking performance with no overshoot and the smallest settling time with respect to standard H_∞ and optimized PID controller.

Keywords

Helicopter system, H infinity, Low pass filter, PID, Robust control.

I. INTRODUCTION

Helicopters are now increasingly used in a variety of industries, including agricultural, civil operations, and the military. This increase in helicopter applications has prompted researchers to develop the two-rotor aerodynamically system (TRAS), which is an experimental setup that can be used for several investigations. The major goal is to manage the pitch and yaw angles to pursue a particular trajectory while rejecting disruptions. It resembles a helicopter in some ways, but the angle of attack of the rotor blades is fixed. The mechanism of a typical helicopter is controlled by adjusting the angle of attack of the rotor blades. When the angle of attack is fixed, the aerodynamic force is regulated by adjusting the engine speed. It exhibits a higher-order nonlinear system with strong cross-coupling from a control standpoint. These nonlinearities and cross-coupling between the pitch and yaw axes motivate

researchers to propose different types of controllers [1, 2].

Because of its simplicity, stability, and robustness, LQR is used extensively to create an optimal controller for the TRMS. In [3], adaptive PSO algorithm has been presented to select optimal weighting matrices (Q and R). Rajaa and Vinodh improved tracking performance of 2-DOF helicopter system by designing LQR combined with model reference control based inverse Lyapunov function to overcome the system uncertainties and external disturbance challenges [4]. Karthick, S., et al presented adaptive robust control method by combining linear quadratic integral with model reference adaptive control (MRAC) scheme to deal with problem of parameter variations [5]. Ali, et al proposed robust PID controller for control 2-DOF helicopter by tuning the PID parameters gain base H_∞ specifications [1]. Garcia-Castro used radial basis function with wavelet transform as activation function to tune PID controller for control the Quanser helicopter [6].



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Published by Iraqi Journal for Electrical and Electronic Engineering | College of Engineering, University of Basrah.

Sliding mode control (SMC) and backstepping have been effectively used to regulate a variety of linear and nonlinear systems that suffering from system uncertainties problem [7, 8]. Thus, several control methods based on the SMC have been presented for controlling the helicopter system [9, 10]. However, chattering due to control signal discontinuity was a potential drawback in SMC, as it could harm the controlled system's actuator [11, 12].

Applied successfully of intelligent control technique based fuzzy logic and neural networks in control different types of nonlinear systems, attract designer to use them for control different complex systems [13, 14]. In this context, Naderi used fuzzy logic controller with adaptive control theory for control 3-DOF helicopter system with metaheuristic algorithms for determining the optimum values for the parameters of the proposed controller [15]. Hu, Yanpeng, et al improved genetic algorithm to tune fuzzy rules to implement fuzzy PID controller [16]. A fuzzy logic is selected to adaptively determine the optimum parameters for the sliding mode controller to control nonlinear helicopter system [17]. Haoxiang, et al estimated the system uncertainties of helicopter by fuzzy logic model and design nonlinear observer to handle the external disturbance [18]. Most proposed methods ignore effect of the measurements noise that effect on the accuracy of the control system. In practice, it's not easy to measure the outputs of system perfectly due to the measurement noise. In classical feedback control system, the tracking performance may degraded due to the measurement noise [19, 20]. Moreover, a measurement noise may cause system instability. In this paper hybrid controller that tuning parameters of PID controller and design robust low pass filter based H_∞ are presented. The proposed control method improving the good features of robust PID controller such as simplicity, fast responding, small overshoot by increasing the robustness against the measurement noise during design robust LPF based H_∞ . The main contribution of this paper are: 1) design a robust and optimal controller for 2-DOF helicopter system based LPF-PID 2) improving robustness of PID controller by taking in account the H infinity requirements in tuning PID gains and design the low pass filter 3) achieve good tracking performance with highly robustness against external disturbance, measurement noise, and parameter variations with simple structure controller.

II. DOF HELICOPTER DYNAMIC MODEL

As seen in Fig. 1, there are two degrees of freedom for the 2-DOF Helicopter model and the angles pitch (ψ) and yaw (θ) can be used to represent them. Furthermore, yaw and pitch angles refer to rotation around the Z axis and Y axis respectively. The helicopter system contains two DC motors to drive two blades. By controlling these motors, it can be adjust the yaw and pitch angles to track a desired trajectory.

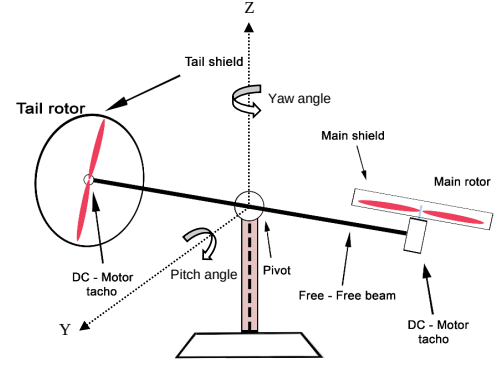


Fig. 1. 2-DOF helicopter system

III. MATHEMATICAL MODELING OF HELICOPTER SYSTEM

The non-linear system of the helicopter dynamic model can be expressed as follows:

$$(J_p + ml^2)\ddot{\theta} = K_{p1}V_p + K_{p2}V_y - B_p\dot{\theta} + \zeta(t) \quad (1)$$

$$(J_y + ml^2 \cos^2(\theta))\ddot{\psi} = K_{y1}V_p + K_{y2}V_y - B_y\dot{\psi} + \varkappa(t) \quad (2)$$

$$\zeta(t) = -ml^2 \sin(\theta) \cos(\theta) \dot{\psi}^2 - mgl \cos(\theta) \quad (3)$$

$$\varkappa(t) = 2ml^2 \dot{\theta} \sin(\theta) \cos(\theta) \dot{\psi} \quad (4)$$

where $\dot{\psi}(t)$ and $\dot{\theta}(t)$ represent the pitch and yaw velocities respectively. K_{p1} , K_{p2} , K_{y1} , and K_{y2} are the thrust torque constants, and V_p and V_y are the input voltages to DC motors. J_p , J_y , B_p , and B_y denote the moment of inertia and viscous damping about pitch and yaw axes respectively. Table I lists the nominal values of these parameters [21].

The linearized system for the dynamic model of the helicopter system can be represented as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (5)$$

$$y(t) = Cx(t) \quad (6)$$

TABLE I.
PARAMETERS VALUES

Parameter	Value
K_{p1}	0.204 Nm/V
K_{p2}	0.006 Nm/V
K_{y1}	0.021 Nm/V
K_{y2}	0.072 Nm/V
B_p	0.800 N/V
B_y	0.318 N/V
J_p	0.038 kg.m ²
J_y	0.043 kg.m ²
m	1.387 kg
l	0.186 m

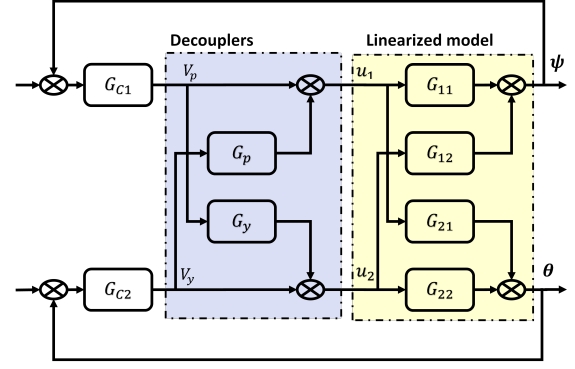


Fig. 2. Decoupling approach

$$x(t) = [\psi(t) \quad \theta(t) \quad \dot{\psi}(t) \quad \dot{\theta}(t)]^T \quad (7)$$

$$u = [V_p \quad V_y]^T, y = [\psi(t) \quad \theta(t)]^T \quad (8)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-B_p}{J_p + ml^2} & 0 \\ 0 & 0 & 0 & \frac{-B_y}{J_y + ml^2} \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{p1}}{J_p + ml^2} & \frac{K_{p2}}{J_p + ml^2} \\ \frac{K_{y1}}{J_y + ml^2} & \frac{K_{y2}}{J_y + ml^2} \end{bmatrix} \quad (10)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (11)$$

IV. DECOUPLING INTO 2 SISO SYSTEMS

The linearized state space for the 2-DOF helicopter system represents multi input multi output (MIMO) system with 2 inputs and 2 outputs. However, this state space can be converted into two single input single output (SISO) systems by applying linear decoupling technique as shown in Fig. 2. The matrix transfer function for the controlled system can be written as follows:

$$\begin{bmatrix} \psi(s) \\ \theta(s) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_p \\ V_y \end{bmatrix} \quad (12)$$

Decoupling controllers G_p and G_y will be added.

$$\psi(s) = (G_{11} + G_{12}G_y)V_p \quad (13)$$

$$\theta(s) = (G_{22} + G_{21}G_p)V_y \quad (14)$$

$$G_p = -\frac{G_{12}}{G_{11}} \quad (15)$$

$$G_y = -\frac{G_{21}}{G_{22}} \quad (16)$$

Finally, the following decoupled transfer function for pitch and yaw models are obtained:

$$\frac{\psi(s)}{V_p(s)} = \frac{0.924s + 13.64}{s^3 + 29.52s^2 + 217.9s} \quad (17)$$

$$\frac{\theta(s)}{V_y(s)} = \frac{3.677s + 0.04055}{s^3 + 0.02206s^2 + 0.0001217s} \quad (18)$$

V. PROPOSED ROBUST LPF-PID CONTROLLER DESIGN

This section presents design procedures of the proposed controller that integrate LPF with PID controller to achieve simple and good robust control method. The parameters of the proposed controller are obtained by solving the H[∞] weighted mixed sensitivity problem.

A. PID Controller

In this paper, PID controller has been used to control the 2-DOF helicopter system and its parameters will be tuned in such way that ensure a good performance with high robustness against external disturbance and measurement noise. The transfer function of PID controllers is:

$$G_c = K_p + K_d s + K_d \frac{s}{Ns+1} \quad (19)$$

where K_p , K_i , K_d and N are the proportional, derivative, integral gains, and time constant parameters respectively.

B. Filtering

Filtering the signals that come from the measurement devices is very important producers to reduce the effect of measurement noise. However, adding filter will effect negatively on the performance of the controlled system and reduce its robustness. Thus, design a filter with minimum effects on the performance and robustness of the controlled system is a big challenge. First order LPF with the following transfer function has been used for filtering process:

$$F = \frac{1}{1+T_f s} \quad (20)$$

T_f represents the time constant for the filter.

C. LPF-PID Design based H^∞ Mixed Sensitivity Approach

Since the helicopter system is expected to meet a high external load disturbance, measurement noise and parameter variations, a proposed LPF-PID control method has been designed to ensure stability of controlled system with high robustness to these challenges. High efficiency of H^∞ mixed sensitivity control method in design robust controller for different complex system motivated us to employ it in designing the robust LPF-PID for the 2-DOF helicopter system.

The objective of this work is to tune the four parameters of the PID controller as well as the time constant of the LPF so that the feedback control system achieves the desired performance in presence of the load disturbance and measurement noise.

This paper aims to design robust controller with the following specifications:

- Stable closed loop system.
- High robustness against external disturbance and parameters variations.
- Good transient specifications (i.e. low overshoot, small rise time and settling time).
- Good tracking performance.

Fig. 3 shows the block diagram for the mixed sensitivity

control model used in this paper, where W_1 , W_2 , and W_3 are weighting functions. The transfer functions between error signal $e(t)$, control signal $u(t)$, output signal $y(t)$, and the input reference $r(t)$ are called sensitivity function S , Control signal sensitivity function U , and Complementary sensitivity function T respectively.

These functions can be define as follows:

$$S = (1 + GKF)^{-1} \quad (21)$$

$$U = K(1 + GKF)^{-1} \quad (22)$$

$$T = GK(1 + GKF)^{-1} \quad (23)$$

where K is the controller that needs to be designed and in this paper it refers to the PID controller, while G is the transfer function of the controlled system, and F denotes the low pass filter.

Thus, the constraints that must be applied to achieve robustness are:

Robust performance constraint: $\|W_1 S\|_\infty \leq 1$

Robust stability constraint: $\|W_2 U\|_\infty \leq 1$

Control signal constraint: $\|W_3 T\|_\infty \leq 1$

By combining these constraints as follows, the robust PID controller is obtained:

$$\left\| \begin{matrix} W_1 S \\ W_2 U \\ W_3 T \end{matrix} \right\|_\infty \leq \gamma < 1 \quad (24)$$

where γ is the performance index.

D. Weighting Function Design

There are no clear and specific steps to design weighting function but there are some rules that discussed in details in [22,23], which applied in this paper to select the weighting functions.

W_1 has the characteristics of low pass filter with high gain at low frequencies to reduce effect of the disturbance and increase the performance of reference tracking. W_3 refers to the robust stability of the closed loop control system and its designed in such way that eliminates the measurement noise in high frequencies. In general, W_2 select as constant after select W_1 and W_3 . It refers to the limitation on additive uncertainties.

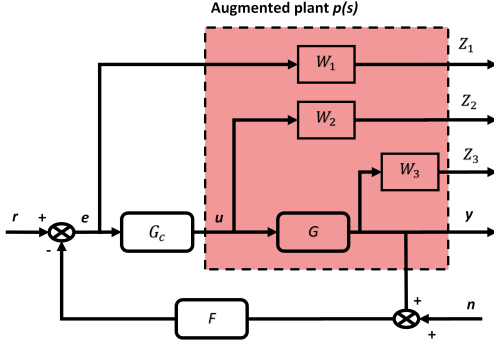


Fig. 3. Weighting functions for controlled system

VI. SIMULATION AND RESULTS ANALYSIS

The proposed optimal robust controller is applied on the dynamic model of the twin rotor helicopter system, and the performance and robustness of the controlled system are discussed by simulating the proposed controller using Matlab 2022. The parameters of PID controller and the low pass filter had been tuned based fixed-structure H^∞ control method and robust control MATLAB toolbox was used for tuning. Simulations are applied for main and tail rotors with different cases to check the performance of the tracking and robustness condition. A comparison is made between the proposed controller and standard H^∞ and optimal PID controller based particle swarm optimization (PSO) method. After selecting suitable weighting functions, the parameters of the robust LPF-PID controller are determined.

$$W_1 = \frac{(s+6)^2}{(s+0.0006)(s+0.6)} \quad (25)$$

$$W_2 = 0.0001 \quad (26)$$

$$W_3 = \frac{2000(s+10)(s+50)}{(s+1000)^2} \quad (27)$$

1) Proposed controller parameters:

• For pitch model:

PID parameters: $K_p = 1.26 \times 10^3$, $K_i = 0.000126$, $K_d = 112$, $N = 209 \times 10^{-6}$

LPF parameter: $T_f = 0.2 \times 10^{-3}$

• For yaw model:

PID parameters: $K_p = 0.00353$, $K_i = 0.000216$, $K_d = 27.3$,

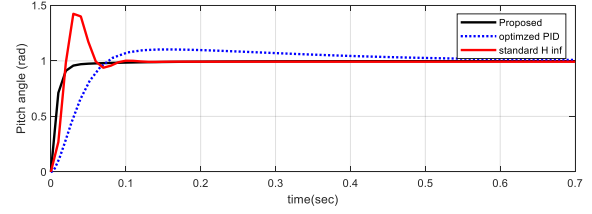


Fig. 4. Step response for pitch model

$$N = 463 \times 10^{-7}$$

LPF parameter: $T_f = 0.1 \times 10^{-3}$

2) The gains of others controller that used for comparison are:

• For pitch model:

Optimal PID: $K_p = 492.32$, $K_i = 1688.88$, $K_d = 25.02$, $N = 95.87$

Standard H^∞ : $\frac{3.59 \times 10^6 s(s+42.31)(s^2+29.52s+21)}{(s+14.76)(s+5)^2(s^2+307.1s+4.304 \times 10^4)}$

• For yaw model:

Optimal PID: $K_p = 6.9 \times 10^{-5}$, $K_i = 1.7727 \times 10^{-7}$, $K_d = 0.004699$, $N = 0.0716$

Standard H^∞ : $\frac{9.3287 \times 10^5 s(s+42.81)(s^2+0.02206s+0.0001217)}{(s+0.01103)(s+5)^2(s^2+308.4s+4.304 \times 10^4)}$

A. Reference Tracking

The main control objective of tracking process is minimizing the difference between the desired input positions and actual positions of the helicopter. A unit step signal is applied on the helicopter system and the tracking performance of the proposed and other controllers are shown in Figs. 4 and 5. The integral absolute time error (IAE) and transient specification have been calculated to compare between the controllers. The obtained results indicate that the proposed optimal robust control method for helicopter system has no overshoot with shortest settling time. The transient specifications for all controllers are listed in Tables II and III for the pitch and yaw models respectively. Integral absolute time error (IAE) for all controllers are shown in Fig. 6 and Fig 7 for the pitch and yaw models respectively. It can be notice from these figures the proposed controller has the minimum IAE. These results illustrated superior of the proposed controller in tracking the reference input.

TABLE II.

TRANSIENT SPECIFICATIONS FOR PITCH MODEL

Method	M_p	t_r	t_s
Proposed	0.0	0.0179	0.0599
PID	10.1429	0.0512	0.5382
H^∞	43.5204	0.0151	0.0852

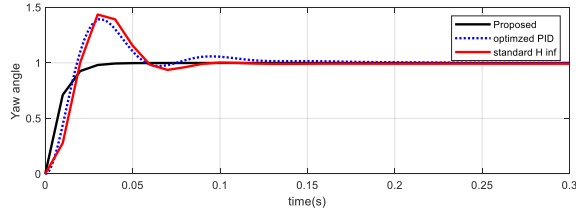


Fig. 5. Step response for yaw model

TABLE III.

TRANSIENT SPECIFICATIONS FOR YAW MODEL

Method	M_p	t_r	t_s
Proposed	0.0	0.0173	0.0297
PID	39.3804	0.0124	0.1230
H∞	44.6372	0.0149	0.0838

B. Robustness Analysis

To illustrate the robustness of the proposed controller against external disturbance, a pulse disturbance signal with amplitude 0.3 and period 0.1 sec is applied on pitch and yaw models where the reference input is unit step. Responses of the proposed and other controllers to reject this disturbance are shown in Fig. 8 and Fig 9 for the pitch and yaw models respectively. These figures reveal high efficiency for the proposed controller in rejecting disturbance. It needs less period with respect other controllers to achieve a steady state.

VII. CONCLUSION

This paper proposed a simple approach to design a robust optimal control of the decoupled 2-DOF helicopter system. At first, the dynamic model of the helicopter is linearized and decoupled into pitch and yaw models. Simplicity in structure, optimal, and robust are the important properties of the proposed controller. Thus, it can handle the system uncertainties, external disturbance, and measurement noise as shown in the simulation results. Effectiveness of the presented control method is approved by comparing its performance with other

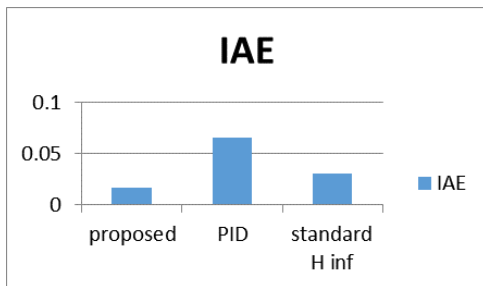


Fig. 6. IAE variations for pitch model

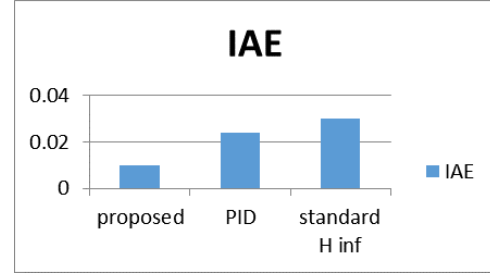


Fig. 7. IAE variations for yaw model

controllers. Comparison is made between controllers in terms of signal tracking and disturbance rejection. This comparison shows clearly superior of the proposed method in terms of smaller settling time, smallest overshoot, and high ability in rejecting external disturbance. High robustness and simplicity of the control method presented in this paper can motivate the researchers to apply this method in the industrial applications.

CONFLICT OF INTEREST

The authors have no conflict of relevant interest to this article.

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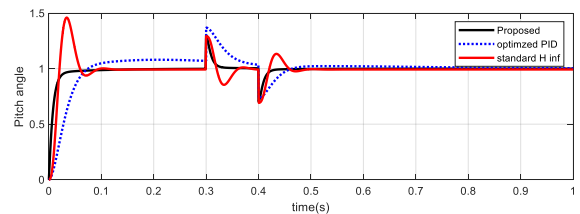


Fig. 8. Response pitch model for disturbance

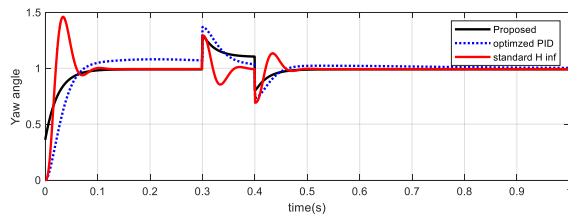


Fig. 9. Response yaw model for disturbance

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