ON C- Monotone Extension of Mapping

Ali I. Mansour

Hula M. Salih

College of Education

University of Al-Mustansirya

Abstract:

In this work, We study C-Monotone map, Monotone map and the relation between them, and also we prove that every function not necessary C- Monotone we can extension for C-Monotone function and every map not necessary C-Monotone we can extension for C- Monotone map.

1. Introduction:

One of the very important concepts in topology is the consept of mapping there are several types of mapping .in this work we study an important class of mapping, namely, monotone extension of mapping, the class of monotone extension mapping was first introduction by Urysohn (1959)and extension theorem for Titze (1965) and it was introduction and studied by Frankline (1970), Whyburn (1978), Dikman, Coban(1980), T.Mackowiak (1978).

In the last time introduction by J.ETERS(1991) L.norici (1996), Daniel .J(1997), Hans(2000).

In this work, we have studied (extension of mapping) we show that by extension we can get anew properties not exist at the main map. We prove that every map can be extended to a C- Monotone map. In section two recall the basic definitions and study the relation between Monotone map and C- Monotone map. In section three we prove that every function not necessary C- Monotone we can extension for C- Monotone function and every map not necessary C- Monotone we can extension for C- Monotone map Finally in this work the word mapping means , continuous function.

2. Basic definition :

In this section, we recall the basic definition and study the relation between C- Monotone map and Monotone map.

2-1 Definition [2]:

Let X and Y be two topological space, a map f: $x \to y$ is said to be (Monotone map) if and only if for each y EY then

$f^1(y)$ is compact and connected set.

2-2 Definition [4] :

Let X and Y be two topological space, a map f: x→y is said to be (C- Monotone map) if and only if for each y ∈Y then $f^{1}(y)$ is connected set.

2-3 Remark :

Every Monotone map is C-Monotone map. But the converse is not true in general.

2-4 Example :

(1) let X= ([2,4], Tu) and Y = (١٦،٤], the map f: x→y be defined by $f(x) = x^2$ ∀ xe X Clearly yeY then f^{1} (y) is compact and connected set.

∴ f is Monotone map

Hence f is C-- Monotone map

(2) let X= ((0,1), T_1) and Y= ({5}, T_1) and the map f: x→y be defined by f(t)=5, ∀ t∈ (0,1) f is C-Monotone map because $f^{-1}(5)$ =(0,1) is connected set. But f is not Monotone map because f (5)=(0,1) is not compact set.

2-5 Definition [6]:

The point y ∈ Y is non C-Monotone point for the function no

f: x→y if $f^{1}(y)$ is disconnected set in X.

2-6 Example:

Let X=(R,T_1) and Y=(R+,Tu) be two topological spaces, R+={y∈R, y>0} set of positive real numbers Let the map f: X → Y is defined by $f(x)=x^2$, ∀ xeX if y ∈ Y then f(y) is disconnected set, This mean f is not C-Monotone map and (16 ∈Y) is non C-Monotone point because $f^{-1}(16)$={-4,+4} is disconnected set in X.

2-7 Definition [6]:

Let X and Y be two topological space a function

f:XY is defined the set Mf by

Mf={y ∈ Y : y is non C-Monotone point for f} ={ y ∈ Y : f^1 (y) is disconnected set in X}

2-8 Remark:

The map f: x→y is C-Monotone if Mf=

(in example 2-4 Mf =) then f is not C-Monotone map if Mf≠(in example 2-6 Mf =y>0)

3- Main Results :

In this section, we study C-Monotone extension through some theorems and corollaries and show that every map can be extended to C-Monotone map.

3-1 Definition [6]:

Let X and Y be a topological spaces, XI Y= A function f:X → Y defined the set X* as following X*= X Y Mf and T x denote the family of all subsets G of X satisfy two condition

(1) GI X open in X

(2) f^1 (CI X) 1 Mf ⊂ G → f^{-1} (CI X) Then fT x is topology defined on the set X

3-2 Definition[5]:

Let X,Y,X' topological spaces, a function f: X→Y defined a function f: X → Y as follow Z ze Mf f(Z)=

F* (z)= z ∀ZЄf

f (z) ∀ZЄX

The following example give you function non C-o Monotone has function C-Monotone extension.otosaib

3-3 Example:

Let f: XY be a function defined by f (a) = f (b) = x

f (c) = z, f (d) = w, such that X = {a,b,c,d}, Tx = {$,X} indiscrete topology on X, Y = {x,y,z, w}, Ty = {, Y, {x},{y}, {x,y}, {y,z,w} }, since f`(x) = { {a}, {b}}

Then x non C- Monotone point for the function f Mf= {x}, since X*=X0Mf then X*= {a,b,c,d,x} Now define the topology Tx on X as follow Tx = {, X, X} define f* : X → Y as follow f(a)=f(b) = f(x) = x, f* (c) = z, f* (d) = w then f*: X → Y is C-Monotone extension function for f and Monotone extension function *

3-4 Corollary[6]

Let x,y two point in topological space X*, x ≠y if y e

Mf and x ∈ f¹ (y) then F*(y)=f¹ (y) Y {y} and any open set in X* contain x contain also y.

3-5 Corollary [6]

Let Ce X, then a set C is closed in X* iff satisfy two condition

1) CI x is closed set in X

2) ∀y ∈ CI Mf then f ¯1(y) c C

3-6 Theorem:

If f:X → Y is closed function then f :X → Y is closed function.

Proof:

Let C be closed subset in X Hence (CI X) closed subset in Xe U 19.1 Since f closed function,.. f(CI X) closed set in Y By (2) from corollary (3-5).

Then f^1 (U) satisfy two condition (1) and (2) from (3-1)

(1)To proof f^{*-1} (U) 1 X open set in X. f^{*1} (U) 1 X\ = [f^1(U) Y (UI Mf)] I X = [f^1(U) 1 X] Y[(UI Mf) I X] = f(U) 1 X = f'(U) Since f ¯¹(U) open set in X, Hence f^{**1} (U) 1 X rosqmoo also open set in X.

(2) To proof f (f^{*1} (U) IX) 1 Mfc f^{*1} (U) *-1 Paroo By condition (1) (S) Hence f (f^{*1} (U) IX) 1 Mf = f (f^{*1} (U) I Mf C

(UI Mf) cf^{*1}

its Y

(U) .. f^{*-1} (U) open set in X* Hence f is mapping.

3-9 Theorem:

If f: X → Y is mapping then f*: X → Y is C-Monotone extension map for f.

Proof:

By theorem (2-8) and (2-9) we get this theorem.

3-10 Theorem :

If f: X → Y is open function then f* : X → Y is open function only if Mf open set in Y. 192

Proof:

Let U open set in X*

f'(U)=f*(U 1 X) Y f*(UI Mf) = f(U1 X) Y

(UI Mf)

Clearly f* (U) open set in Y .. f is open function

3-11 Theorem :

If f: X → Y is compact function then f*: X → Y is compact function only If Mf closed set in Y.

Proof:

Let k compact set in Y then $f^{**1}(k) = f^{1}$ (k) Y (ki Mf)

(1) Since f is compact function then f^{-1} (k) is compact set in X and compact in X*.

(2) Let Mf closed set in Y, Hence (kI Mf) compact set in Y and compact in X From (1) and (2) Hence f^{*1} (k) compact set in X*

3-12 Theorem :

If f: X → Y is point compact map f*: X → Y is point compact map.

Proof:

Let y ∈Y

(1) if ye Mf then $f^{-1}(y) = f^{**1}(y)$ and since f is point compact map .. $f^{-1}(y)$ compact set in X for each y∈ Y also $f^{1}(y)$ compact set in X for each yeY

.. fis point compact map (2) if ye Mf then $f^{*}(y)=f(y)$ Y {y} since f^{1} (y) compact set in X, so that {y} compact set X, and $f^{**1}(y)$ compact set in X .. fis point compact map. *

2-13 Example :

Let X={a,b}, Y= {c} Tx = {0,X, {a}, {b}}is defined on X and Ty = {, {c}}is defined on Y f:XY is function defined by f(x) = c∀ x∈X and f is point compact map since Mf = {y∈ Y : $f^{-1}(y)$ is disconnected set }

then Mf = {c} and the set X = (a,b,c)

and Tx'= {, X', {c}, {a,c}, {b,c}}M

defined f': X → Y by f(x) = c ∀ xeX

Hence the function f is point compact map.

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