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ORIGINAL STUDY

Bayesian Analysis of Left Censored Regression With Normal-compound Gamma Priors

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Abstract

This paper presents a Bayesian estimation of left censored regression models with scale mixture of normal-compound gamma priors. We presented a new hierarchical modeling for Bayesian inference in left censored regression models. We derived a Gibbs sampling algorithm from this Bayesian hierarchical modeling to estimate the regression parameters with an efficient EM algorithm for updating the hyperparameters. We illustrated the new model using simulation studies and a real data analysis. The results show that the proposed model performs very well in comparison to the other existing models.

Keywords: Tobit regression, Normal-compound gamma prior, Gibbs sampler

1. Introduction

The issue with censored data is that traditional methods can no longer be used in the standard problems of Statistics. Tobit regression introduced in Ref. [14] is a special case of censored regressions which covers a large class of models where the dependent variable is censored beyond some threshold. These models in addition to the Cox proportional hazard model [7], the accelerated failure time [10] and several others where proposed to overcome the bias caused by the censoring of the data. The main attractiveness and reason for wide use of the Cox model is the possibility do variable selection without any assumption on the duration variable. Kalbfleisch et al. [11] and Efron, B [8]. established that the Cox model produces nearly fully efficient estimator. However, this model is very limited in some cases especially where the proportionality of the hazard function is not verified by the data. For this, the accelerated failure time models, unlike the Cox model, suffers from the problem of the need of setting a distribution for the duration. Additionally, these models are not consistent, that is, the bias doesn't vanish as the size of sample increases. There are several examples of censored data such as the demand of capital goods in econometrics [14], the time of recovery after surgery in biomedicine or the number of repeat arrests of prisoners [16]. These examples and others demonstrate the need for developing methods that provide consistent estimates for censored data.

In this paper, we will be concerned with the Tobit model which modifies the likelihood function depending on the value of the latent dependent variable such that the resulting function has different sampling probability for each observation which is a consistent estimator for large samples as was shown in Ref. [2]. While the method of maximum likelihood and other related methods maybe used for uncensored data, these methods cannot be applied for censored data. The main problem stems from the fact that the method of least square is not valid for censored observations. There are several variations of the Tobit model depending on where censoring occurs, we will focus in the standard Tobit model

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$$y_i = \max\{y_i^*, 0\}, i = 1, \cdots, n \tag{1}$$

and

$$y_i^* = x_i^T \beta + \epsilon_i \tag{2}$$

where the observable y_i as equal to the latent random variable y_i^* for positive values and zero otherwise. The above regression model is defined by the matrix of covariates $X^{n\times p} = (x_1, \cdots, x_p)$, an unknown regression coefficient vector $\beta^{p\times 1} = (\beta_1, \cdots, \beta_p)^T$ and the vector $\epsilon^{n\times 1} = (\epsilon_1, \cdots, \epsilon_n)^T$ where each ϵ_i has a normal distribution with mean zero and variance σ^2.

In the following sections, we will introduce the mormal-compound gamma (NCG) prior for Tobit regression. In addition, the Markov chain Monte Carlo (MCMC) and Monte Carlo Expectation-Maximization (MCEM) sampler for the Tobit model will be derived. Finally, we will use simulation studies and real data set to compare our model with different priors.

2. Normal-compound gamma prior

In recent years, different priors of the form of scale mixture of normals for the regression coefficients has been used in sparse regression [5,6,13,17]. Perhaps one the most famous of these priors is the horseshoe prior [6] which is defined as $\beta_i\big|\text{rest} \sim \mathcal{N}(0, \sigma^2 z_1), z_1^{1/2} \sim \mathrm{C}^+(0, z_2)$, and $z_2^{1/2} \sim \mathrm{C}^+(0, 1)$, where C^+ is the half-Cauchy distribution or alternatively:

$$z_1 \sim \mathcal{G}(1/2, z_2),$$

$$z_2 \sim \mathcal{IG}(1/2, z_3),$$

$$z_3 \sim \mathcal{G}(1/2, z_4),$$

$$z_4 \sim \mathcal{IG}(1/2, 1)$$

where $\mathcal{G}$ is the gamma distribution, $\mathcal{IG}$ is the inverse-gamma distribution and z_i indicates a vector of length p while z_i is a scalar. Another example is the Three-Parameter Beta Distribution [3,4] (or the Scaled Beta2 (SBeta2) family of distributions [12]) which is given by:

$$\beta_i\big|\text{rest} \sim \mathcal{N}(0, \sigma^2 z_1),$$

$$z_1 \sim \mathcal{G}(a, z_2),$$

$$z_2 \sim \mathcal{IG}(b, \phi)$$

where ϕ is some constant. This is sometimes called the Normal-Beta Prime (NBP) prior for $\phi = 1$. In this paper, we seek to generalize these compound gamma priors for the scale mixture and propose the prior of the form

$$\pi(x) = \int_0^\infty \ldots \int_0^\infty \left[\prod_{i=1}^{N} \frac{z_{i+1}^{c_i}}{\Gamma(c_i)} z_i^{c_i - 1} \exp\{-z_i z_{i+1}\}\right] dz_2 \ldots dz_N \tag{3}$$

where $z_1 = x$, z_i are the latent variables, c_i are the hyperparameters for our prior and $z_{N+1} = \phi$ is some constant. In Alhamzawi, A. and G. S. Mohammad [1], we have presented a study of this prior in the contest of posterior consistency, where it was shown that this model achieves posterior consistency under some conditions. Furthermore, we derived the Markov chain Monte Carlo (MCMC) and the Variational Inference (VI) methods for this model. In this paper we seek to further study the properties of this model in the context of censored data. Fig. 1 clearly shows the shape of the prior for different values of N and c_1. In order to simplify our prior, we use the following

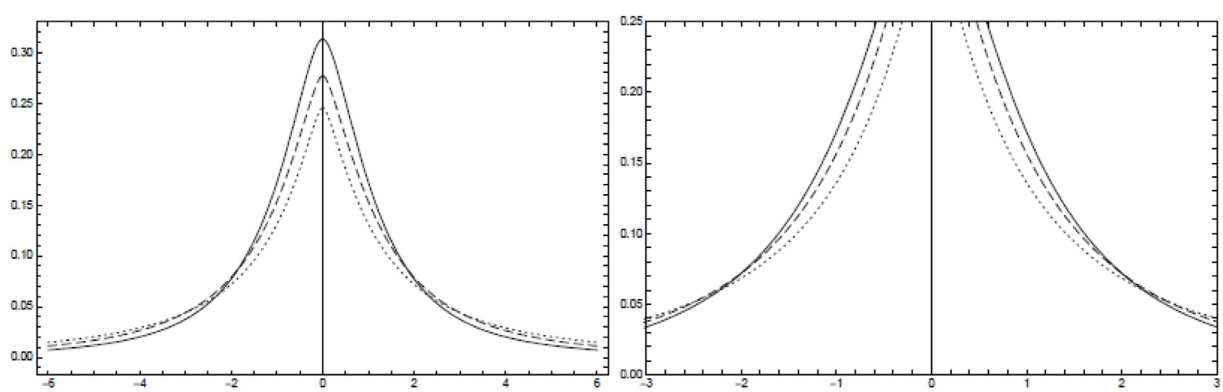

Fig. 1. A plot showing the behavior of the density (3) with changing parameters for N and c_1 (left: $c_1 = 0.3$ and right $c_1 = 1.3$). The solid, dashed and dotted lines represent $N = 3, N = 4$ and $N = 5$, respectively.

Proposition 1. If $z_1 \sim \mathcal{CG}(c_1,\ldots,c_N,\phi)$, where $\mathcal{CG}$ is the compound gamma distribution, then

$$(1)\ z_1 \sim \mathcal{G}(c_1,z_2), z_2 \sim \mathcal{G}(c_2,z_3), \ldots, z_N \sim \mathcal{G}(c_N,\phi) \tag{4}$$

$$(2)\ z_1 \sim \mathcal{G}(c_1,1), z_2 \sim \mathcal{IG}(c_2,1), \ldots, z_N \sim \mathcal{AG}(N,c_N,\phi) \tag{5}$$

where $\mathcal{AG}(N,a,b) = \begin{cases} \mathcal{G}(a,b) & \text{odd} \quad N \\ \mathcal{IG}(a,b) & \text{even} \quad N \end{cases}$
$\mathcal{CG}(c_1,\ldots,c_N,\phi)$ is compound gamma of order N and $\mathcal{G}(a,b)$ is the gamma distribution with shape a and inverse scale (rate) parameter b.

proof of Proposition 1. The proof is provided in [1]. Thus, our hierarchal model is given by

$$y_i = \max\{0, y_i^*\}, i = 1, \cdots, n,$$

$$y_i^* = x_i^T\beta + \epsilon_i, \epsilon_i \sim N(0,\sigma^2),$$

$$\beta_i \Big| z_1,\ldots,z_N,\sigma^2 \sim N\left(0, \sigma^2 \prod_{i=1}^{N} z_i\right),$$

$$z_i \sim \mathcal{AG}(i,c_i,\phi),$$

$$\sigma^2 \sim \mathcal{IG}(c_0,d_0).$$

3. The full conditionals

The full conditional probability is given by

1. Update y_i^*

$$y_i^* \Big| y_i,\beta,\sigma^2 \sim \begin{cases} \delta(y_i) & y_i^* > y_i, \\ N(x_i'\beta,\sigma^2) I(y_i^* \le y_i) & \text{otherwise}, \end{cases}$$

where the distribution of $\delta(y_i)$ is that it concentrates all of its mass on y_i and I is the indicator function.

2. Update β

$$P(\beta|X,y^*,y,z_1,\ldots,z_N,\sigma^2) \propto P(y^*|\beta,\sigma^2)\pi(\beta),$$

$$\propto \exp\left\{-\frac{1}{2\sigma^2}(y^* - X\beta)^T(y^* - X\beta) - \frac{1}{2\sigma^2}\beta^T \mathbf{Z}^{-1}\beta\right\},$$

$$\propto \exp\left\{-\frac{1}{2\sigma^2}\left(-2y^{*T}X\beta + \beta^T X^T X\beta + \beta^T \mathbf{Z}^{-1}\beta\right)\right\}, \tag{6}$$

$$= \exp\left\{-\frac{\Sigma}{2\sigma^2}\left(-2\mu_\beta^T\beta + \beta^T\beta\right)\right\},$$

where $\mu_\beta = \Sigma^{-1}X^T y^*$, $\mathbf{Z} = \text{daig}(\prod_{i=1}^{N} z_{i1}, \ldots, \prod_{i=1}^{N} z_{ip})$ and $\Sigma = X^T X + \mathbf{Z}^{-1}$. Therefore, we have the normal distribution $\mathcal{N}(\mu_\beta, \Sigma^{-1}\sigma^2)$. Forodd k we have

3. Update z_k (for odd k)

$$P(z_k|X,y^*,\sigma^2,z_1,\ldots,z_N) \propto \pi(\beta_i|z_1,\ldots,z_N,\sigma^2)\pi(z_k)$$

$$\propto \frac{1}{\sqrt{z_k}} \exp\left\{-\frac{\beta^T \mathbf{Z}^{-1}\beta}{2\sigma^2}\right\} \times (z_k)^{c_k - 1} \exp\{-z_k\varphi_k\}, \tag{7}$$

$$\propto (z_k)^{\left(c_k - \frac{1}{2}\right)-1} \exp\left\{-\frac{1}{2}\left[\frac{\beta^T \mathbf{Z}_{-k}^{-1}\beta}{\sigma^2}(z_k)^{-1} + 2z_k\varphi_k\right]\right\},$$

where $\varphi_k = 1 + (\phi - 1)I(k - N)$ and $\mathbf{Z}_{-k} = \text{daig}(\prod_{i=1,i\ne k}^{N} z_{i1}, \ldots, \prod_{i=1,i\ne k}^{N} z_{ip})$. Thus, we have the generalized inverse-gaussian distribution $\mathcal{GIG}\left(\frac{\beta^T \mathbf{Z}_{-k}^{-1}\beta}{\sigma^2}, 2\varphi_k, c_k - \frac{1}{2}\right)$. For even k, we have

4. Update z_k (for even k)

$$P(z_k|X,y^*,\sigma^2,z_1,\ldots,z_N) \propto \pi(\beta_i|z_1,\ldots,z_N,\sigma^2)\pi(z_k)$$

$$\propto \frac{1}{\sqrt{z_k}} \exp\left\{-\frac{\beta^T \mathbf{Z}^{-1}\beta}{2\sigma^2}\right\} \times (z_k)^{-c_k - 1} \exp\left\{-\frac{\varphi_k}{z_k}\right\}, \tag{8}$$

$$\propto (z_k)^{-\left(c_k + \frac{1}{2}\right)-1} \exp\left\{-\left[\frac{\beta^T \mathbf{Z}_{-k}^{-1}\beta}{2\sigma^2} + \varphi_k\right](z_k)^{-1}\right\},$$

which is the inverse-gamma distribution $\mathcal{IG}\left(c_k + \frac{1}{2}, \frac{\beta^T \mathbf{Z}_{-k}^{-1}\beta}{2\sigma^2} + \varphi_k\right)$.

5. Update σ^2

$$P(\sigma^2|X,y^*,z_1,\ldots,z_N) \propto P(y|\beta,z_1,\ldots,z_N,\sigma^2)\ \pi(\beta_i|z_1,z_2,\ldots,z_N,\sigma^2)\pi(\sigma^2)$$

$$\propto (\sigma^2)^{-n/2} \exp\left\{-\frac{(y^* - X\beta)^T(y^* - X\beta)}{2\sigma^2}\right\}$$
$$\times (\sigma^2)^{-p/2} \exp\left\{-\frac{\beta^T \mathbf{Z}^{-1}\beta}{2\sigma^2}\right\} \tag{9}$$
$$\times (\sigma^2)^{-c_0 - 1} \exp\left\{-\frac{d_0}{\sigma^2}\right\}$$

Table 1. Results for example 1.

Methods	σ^2	MSE (sd)	FPR (FPRsd)	FNR (FNRsd)
tNCG2	1	0.0454 (0.0344)	0.0100 (0.1000)	0.0000 (0.0000)
tNCG10	1	0.0291 (0.0303)	0.0100 (0.1000)	0.0000 (0.0000)
tHorseShoe	1	0.0418 (0.0330)	0.0100 (0.1000)	0.0000 (0.0000)
Btqr	1	0.2026 (0.1016)	0.5400 (0.8459)	0.0000 (0.0000)
BALtqr	1	0.1386 (0.0746)	0.2600 (0.4845)	0.0000 (0.0000)
Tobit	1	0.1860 (0.1451)	0.0000 (0.0000)	0.0000 (0.0000)
tNCG2	9	1.4470 (0.8713)	0.1600 (0.3949)	0.2400 (0.4292)
tNCG10	9	1.2977 (0.8091)	0.0600 (0.2387)	0.2900 (0.4560)
tHorseShoe	9	1.3789 (0.8180)	0.0700 (0.2564)	0.2300 (0.4230)
Btqr	9	2.1650 (0.9438)	0.4500 (0.5752)	0.1400 (0.3487)
BALtqr	9	1.6910 (0.8286)	0.2100 (0.4333)	0.2100 (0.4094)
Tobit	9	1.8141 (0.9094)	0.0000 (0.0000)	0.0000 (0.0000)
tNCG2	25	1.1154 (0.8142)	0.0300 (0.1714)	0.0000 (0.0000)
tNCG10	25	0.6907 (0.6219)	0.0100 (0.1000)	0.0000 (0.0000)
tHorseShoe	25	1.0170 (0.7498)	0.0200 (0.1407)	0.0000 (0.0000)
Btqr	25	4.3371 (2.0923)	0.6200 (0.8261)	0.0000 (0.0000)
BALtqr	25	2.4953 (1.4605)	0.2600 (0.5049)	0.0000 (0.0000)
Tobit	25	3.6516 (1.5782)	0.0000 (0.0000)	0.0000 (0.0000)

$$\propto \left(\sigma^2\right)^{-\left(\frac{n+p+2c_0}{2}\right)-1} \exp\left\{-\frac{(y^*-X\beta)^T(y^*-X\beta)+\beta^T\mathbf{Z}^{-1}\beta+2d_0}{2\sigma^2}\right\}$$

this is again the inverse-gamma distribution $\mathcal{IG}\left(\frac{n+p+2c_0}{2}, \frac{(y^*-X\beta)^T(y^*-X\beta)+\beta^T\mathbf{Z}^{-1}\beta+2d_0}{2}\right)$.

As for the hyperparameters c_k, we follow the method provided in Ref. [1] by proposing an Expectation Maximization Monte Carlo (MCEM) method [15] with

$$\Gamma'(c_k) = \sum_{i=1}^{p}(-1)^{k+1}\mathbb{E}_{c_k^{\text{old}}}[\log(z_{ki})|y] + c_N\log(\phi)I(k=N)$$

4. Simulation studies

In this section, we compare the performance of our Gibbs sampler (**tNCG10**) with the Beta prime prior for scale parameters for tobit data with the NCG prior (**tNCG2**), the tobit horseshoe (**tHorseShoe**), Bayesian tobit quantile regression (**Btqr**), Bayesian adaptive Lasso tobit quantile regression (**BALtqr**) and the classical tobit method (**Tobit**). For tobit horseshoe we use our Gibbs sampler with $N=4$ and fix $c_1 = c_2 = c_3 = c_4 = 0.5$. All the results in these simulations will be averaged over 100 replications and presented in Tables 1−3 with their associated standard deviations (sd) listed in the parentheses with the mean squared error (MSE), the false positive rate (FPR) and the false negative rate (FNR).

Table 2. Results for example 2.

Methods	σ^2	MSE (sd)	FPR (FPRsd)	FNR (FNRsd)
tNCG2	1	0.1469 (0.0877)	0.0500 (0.2190)	0.0000 (0.0000)
tNCG10	1	0.1404 (0.0850)	0.0500 (0.2190)	0.0000 (0.0000)
tHorseShoe	1	0.1637 (0.0883)	0.0800 (0.2727)	0.0000 (0.0000)
Btqr	1	0.2525 (0.1254)	0.3800 (0.6159)	0.0000 (0.0000)
BALtqr	1	0.2051 (0.1072)	0.1900 (0.4191)	0.0000 (0.0000)
Tobit	1	0.3769 (0.9249)	0.0000 (0.0000)	0.0000 (0.0000)
tNCG2	9	1.4470 (0.8713)	0.1600 (0.3949)	0.2400 (0.4292)
tNCG10	9	1.2977 (0.8091)	0.0600 (0.2387)	0.2900 (0.4560)
tHorseShoe	9	1.3789 (0.8180)	0.0700 (0.2564)	0.2300 (0.4230)
Btqr	9	2.1650 (0.9438)	0.4500 (0.5752)	0.1400 (0.3487)
BALtqr	9	1.6910 (0.8286)	0.2100 (0.4333)	0.2100 (0.4094)
Tobit	9	1.8141 (0.9094)	0.0000 (0.0000)	0.0000 (0.0000)
tNCG2	25	3.2497 (1.6826)	0.1100 (0.3145)	0.6100 (0.4902)
tNCG10	25	2.9618 (1.5235)	0.0500 (0.2190)	0.7600 (0.4292)
tHorseShoe	25	3.0524 (1.5256)	0.0700 (0.2564)	0.7100 (0.4560)
Btqr	25	4.5313 (2.1146)	0.2600 (0.5049)	0.4900 (0.5024)
BALtqr	25	3.5570 (1.7871)	0.1300 (0.3380)	0.6200 (0.4878)
Tobit	25	3.9734 (1.7510)	0.0000 (0.0000)	0.0000 (0.0000)

Table 3. Results for example 3.

Methods	σ^2	MSE (sd)	FPR (FPRsd)	FNR (FNRsd)
tNCG2	1	0.1718 (0.0803)	0.0000 (0.0000)	0.0100 (0.1000)
tNCG10	1	0.1854 (0.0875)	0.0000 (0.0000)	0.0100 (0.1000)
tHorseShoe	1	0.1914 (0.0914)	0.0000 (0.0000)	0.0200 (0.1407)
Btqr	1	0.2144 (0.0977)	0.0000 (0.0000)	0.0300 (0.1714)
BALtqr	1	0.2328 (0.1069)	0.0000 (0.0000)	0.0300 (0.1714)
Tobit	1	0.1774 (0.0851)	0.0000 (0.0000)	0.0000 (0.0000)
tNCG2	9	1.1757 (0.5697)	0.0000 (0.0000)	4.4800 (1.3520)
tNCG10	9	1.4622 (0.6475)	0.0000 (0.0000)	5.5000 (1.3890)
tHorseShoe	9	1.4911 (0.6296)	0.0000 (0.0000)	5.7900 (1.2333)
Btqr	9	1.5406 (0.8226)	0.0000 (0.0000)	4.8200 (1.4521)
BALtqr	9	1.5260 (0.7439)	0.0000 (0.0000)	5.1900 (1.3977)
Tobit	9	1.3243 (0.6334)	0.0000 (0.0000)	0.0000 (0.0000)
tNCG2	25	2.6075 (1.2051)	0.0000 (0.0000)	7.9500 (1.1580)
tNCG10	25	3.1410 (1.4552)	0.0000 (0.0000)	8.5200 (0.9690)
tHorseShoe	25	2.9559 (1.2974)	0.0000 (0.0000)	8.4100 (0.9438)
Btqr	25	3.9399 (1.7057)	0.0000 (0.0000)	7.2600 (1.2522)
BALtqr	25	3.0383 (1.3227)	0.0000 (0.0000)	7.5800 (1.1822)
Tobit	25	3.3154 (1.4806)	0.0000 (0.0000)	0.0000 (0.0000)

4.1. Example 1 (very sparse model)

Here we consider a simple sparse model. We set $\beta = (6.5, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ and $\sigma^2 \in \{1, 9, 25\}$. One can see as demonstrated in Fig. 1 and displayed in Table 1 that our approach tendsto work better than the other approaches in terms of variable selection and estimation.

4.2. Example 2 (sparse model)

Here we consider a sparse model by setting $\beta = (1.5, 4, 0, 5, 0, 0, 7, 0, 9, 0)$ and $\sigma^2 \in \{1, 9, 25\}$. From Table 2, we can see clearly that the proposed approach perform better than the other approaches.

4.3. Example 3 (dense model)

In this example, we consider a dense model with $\beta = \underbrace{(0.95, 0.95, \ldots, 0.95)}_{p=10}$ and $\sigma^2 \in \{1, 9, 25\}$.

We can clearly see from Table 3, that with dense data, our model has better prediction accuracy than other approaches considered in this paper.

5. Real data

In this section, we consider the extramarital affairs which consist of 601 ovservations and 9 variables from Ref. [9]. The results are presented in Table 5. The outcome variable is the number of extramarital sexual intercourse during the past year (affairs). The other eight covariates are described in Table 4.

Table 4. Description of the extramarital affairs covariates.

Covariates	Description
Affairs	Number of extramarital sexual intercourse last year
Gender	Indicates gender
Age	Indicates age (in years)
Yearsmarried	Number of years married
Children	Number of children
Religiousness	Indicates religiousness (1–5)
Occupation	The Hollingshead occupation classification
Rating	Indicates the happiness rate of marriage (1–5)

Table 5. Results for extramarital affairs data.

Methods	MSE	sd
tNCG2	79.0599	75.9117
tNCG10	78.9466	75.1048
tHorseShoe	122.1099	101.2448
Btqr	104.5170	103.0127
BALtqr	99.5817	94.6092
Tobit	118.4258	107.6908

Table 5 shows that the proposed method perform better than the other methods. Hence, the simulation examples and real data analyses show that the proposed approach perform better than the others. Fig. 2 shows that the different methods lie in the confidence interval of our model. Furthermore, The trace plots in Fig. 3 and histograms in Fig. 4 demonstrate that our prior converges very fast compared to the stationary distribution.

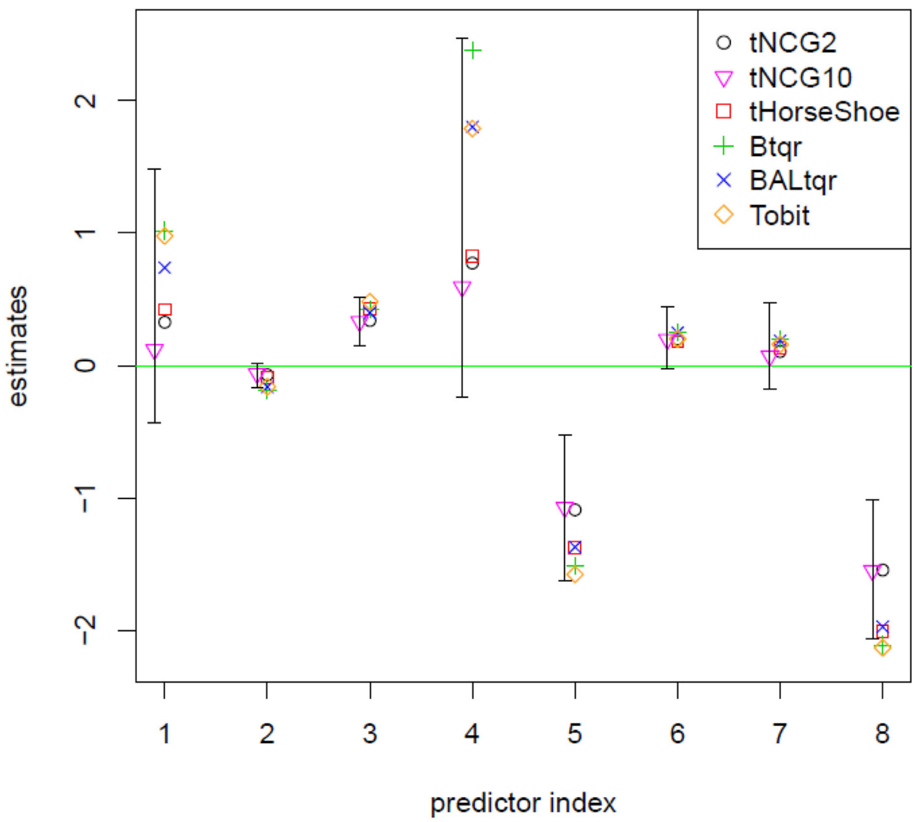

Fig. 2. The confidence interval of NCG10 for the predictors of the marriage affairs data compared with different methods.

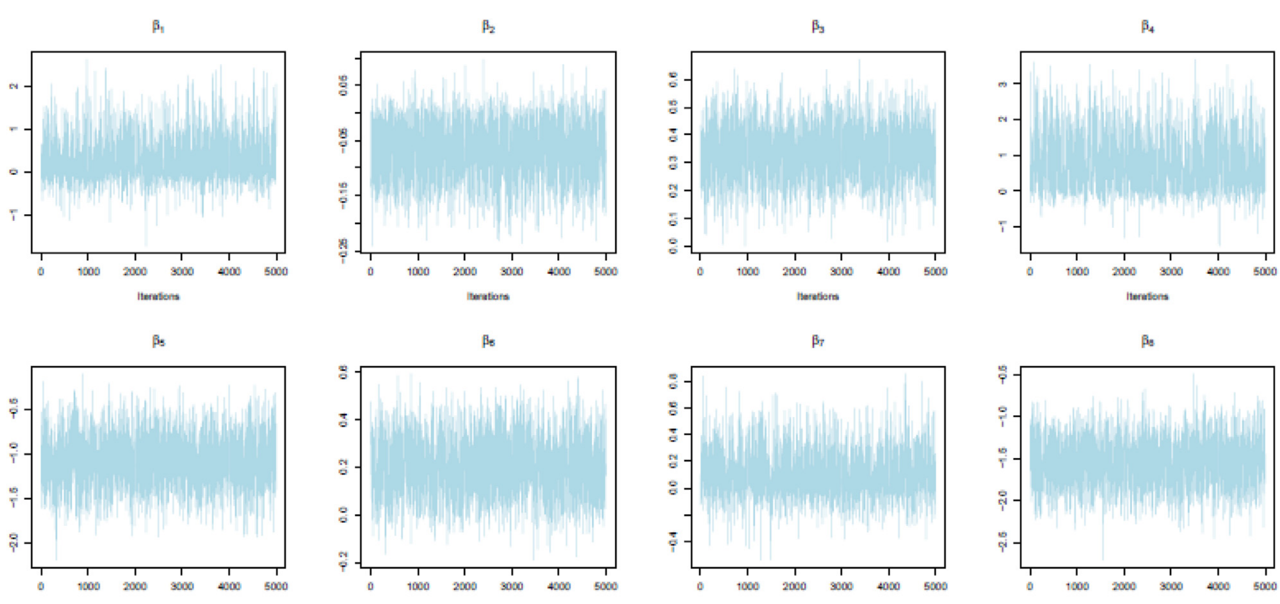

Fig. 3. Trace plots of marriage affairs data covariates.

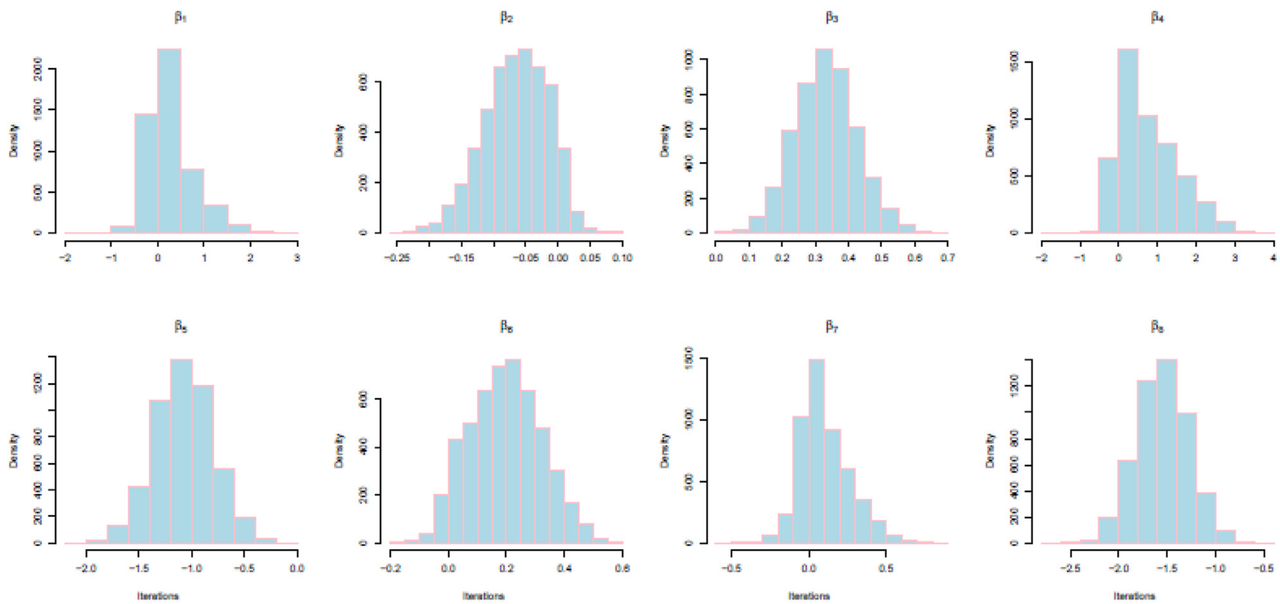

Fig. 4. Histograms of marriage affairs data covariates.

6. Concluding Remarks

In this paper, we have proposed a new method for variable selection and estimation in tobit model. We also extend the Beta prime prior for scale parameters and the horseshoe for tobit data. We have proposed a new Bayesian hierarchical modeling and derived the Gibbs sampling algorithm from this Bayesian hierarchical modeling to estimate the regression parameters with an efficient EM algorithm for updating the hyperparameters. The results of the simulation studies and real data analyses show that the proposed method perform better than the others.

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