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ORIGINAL STUDY

Solving Multidimensional Fractional Telegraph Equation by Using Yang Hussein Jassim Method

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ABSTRACT

This study employs the Young Hussein Jassim (YHJ) technique to examine the exact solutions of the space-time telegraph equation analytically (ST-TE). The YHJ approach is an innovative and appealing hybrid transformation integration method, effectively combining the HJ and Young methods. Through a simplified iterative process with minimal computational requirements, this approach quickly provides convergent, sequential solutions. The reliability of the method is demonstrated by applying it to two case studies of the ST-TE within the framework of the Tania derivative, which includes the definition of non-singular kernel functions. The study also includes extensive comparisons between approximate, exact, and relevant literature-based solutions to assess the technique's accuracy and effectiveness. Graphical representations illustrate the effect of non-integer temporal and spatial parameters on solution behavior. The findings suggest that this method is easy to implement and suitable for exploring complex physical models governed by nonlinear partial differential equations with fractional time components.

Keywords: Hussein Jassim method, Yang transform, The Caputo fractional derivative of function, Multidimensional fractional telegraph equation equations

1. Introduction

Fractional differential equations are widely used across various scientific disciplines. Numerous phenomena in engineering, physics, chemistry, and other fields can be effectively modeled using fractional calculus. Nonlinear vibrations, such as those resulting from earthquakes, acoustics, electromagnetism, electrochemistry, wave propagation, and signal processing, can all be represented by fractional equations [1–3]. Additionally, telegraph equations play a crucial role in both physics and engineering applications. In recent years, this topic has attracted significant attention from researchers, making it more accessible to the scientific and engineering communities and deepening our understanding of the fundamental nature of matter. Fractional calculus offers a more efficient framework for interacting with natural phenomena. Mathematical tools, such as fractional differential operators, have recently gained prominence in reformulating and analyzing many new partial differential equations. These operators, fundamental in fractional calculus, generalize traditional integer-order integral and differential operators to fractional or non-integer orders. The incorporation of these non-local operators into partial differential equations results in a new class known as fractional partial differential equations (FPDEs). Due to its flexibility, fractional calculus has become increasingly important and finds broad applications in science and engineering. However, finding exact solutions to fractional PDEs can be challenging. As a result, many researchers rely on various analytical techniques to obtain sequential or iterative solutions for

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these problems. Several iterative methods have been applied to solve differential equations in general, and the telegraph equation in particular, including the Variational Iteration Method (VIM), the Adomian Decomposition Method (ADM)and other method [4–51].

In this paper, the Hussein Jasim Yang method (YHJ) is extended and applied to both spatial and temporal fractional telegraph equations. Section 2 introduces the preliminary materials, while Section 3 discusses the concept of the Husayn Jasim method. Section 4 presents the development of the Husayn Jasim method using the Yang transform. Section 5 provides illustrative examples, and Section 6 concludes with a summary of the findings.

2. Preliminaries

Definition 1: [1, 44]. If $f(x) \in C([a, b])$, $\ltimes. > 0$, and $a. < x < b$, then The Riemann-Liouville fractional integral of order, $\ltimes$ is defined as follows

$$I_x^{\ltimes} f(x) = \frac{1}{\Gamma(\ltimes)} \int_0^x \frac{f(t)}{(x-t)^{1-,\ltimes}} dt. \tag{1}$$

Here, Γ represents The well-known Gamma function

The Riemann-Liouville fractional integral has the following properties:

$$\begin{aligned} &1.\ I_x^{\ltimes} I_x^{\mathcal{B}} f(x) = I_x^{\ltimes+\mathcal{B}} f(x).\\ &2.\ I_x^{\ltimes} I_x^{\mathcal{B}} f(x) = I_x^{\mathcal{B}} I_x^{\ltimes} f(x).\\ &3.\ I_x^{\ltimes} x^{\mathcal{B}} = \frac{\Gamma(\mathcal{B}+1)}{\Gamma(\ltimes+\mathcal{B}+1)} x^{\ltimes+\mathcal{B}}. \end{aligned} \tag{2}$$

Definition 2: [1, 3]. The Caputo fractional derivative of function $f(x)$, $x > 0$ is defined by

$$D_x^{\ltimes} u(x,t) = \begin{cases} \frac{1}{\Gamma(n-\ltimes)} \int_0^x (x-t)^{n-\ltimes-1} f^{(n)}(t)\,dt & n-1 < \ltimes \leq n \in \mathbb{N}. \\ \frac{d^n}{dx^n} f(x), & \ltimes = n \in \mathbb{N}. \end{cases} \tag{3}$$

Note 1. Based on Definition 2, the following result can be derived:

$$D_t^{\ltimes} t^{\beta} = \begin{cases} \frac{\Gamma(\beta+1)}{\Gamma(\beta-\ltimes+1)} t^{\beta-\ltimes}. & n-1 < \ltimes \leq n, \beta > n-1,\ \beta \in \mathcal{R} \\ 0. & n-1 < \ltimes \leq n, \beta < n-1,\ \beta \in \mathbb{N} \end{cases} \tag{4}$$

Definition 3: [14]. The Yang transform YT is stated as.

$$Y_a\{u(t)\} = \int_0^{\infty} e^{-\frac{t}{v}} u(t)\, dt, t > 0, \tag{5}$$

with v representing the transform variable.

Definition 4: [14]. The Yang transform of fractional order derivative is defined by

$$Y_a\left\{D_x^{\ltimes} u(x,t)\right\} = \frac{Y_a\{u(t)\}}{v^{\ltimes}} - \sum_{k=0}^{n-1} \frac{u^{(k)}(0)}{v^{\ltimes-k-1}}, \qquad n-1 < \ltimes \leq n. \tag{6}$$

Few properties

$Y_a\{1\} = v.$

$Y_a\{t\} = v^2.$

$Y_a\{t^n\} = v^{n+1} n!.$

$Y_a\left\{t^{\ltimes}\right\} = v^{\ltimes+1}\Gamma(\ltimes+1).$

Definition 5: [1, 44]. The Mittag-Leffler function with two parameters is defined by

$$\mathbb{E}_{\ltimes,\wp}(\mathcal{Z}) = \sum_{n=0}^{\infty} \frac{\mathcal{Z}^n}{\Gamma(n\ltimes+\wp)}, \ltimes, \wp, \mathcal{Z} \in \mathbb{C},\ Re(\ltimes) > 0, Re(\wp) > 0 \tag{7}$$

Note 2. Based on Definition 1, the following result can be derived:

$$\begin{aligned} &(1)\quad \mathbb{E}_{2,1}(x^2) = \cos\hbar(x.) \\ &(2)\quad \mathbb{E}_{2,2}(x^2) = \frac{\sin\hbar(x)}{x}. \\ &(3)\quad \mathbb{E}_{2,3}(x^2) = \frac{1}{x^2}[-1+\cos\hbar(x)]. \end{aligned} \tag{8}$$

3. Hussein Jassim method (HJM)

The idea of the fractional analysis method using Hussein Jassim's method: We will use Hussein Jassim's method [8] to obtain accurate data or approximate solutions to linear and nonlinear problems. The Hussein Jassim method (HJM) provides solutions in an infinite series that converge quickly. To clarify the concept of HJM, we will consider the general nonlinear equation with the provision of certain conditions.

$$D_x^{\ltimes}u(x,t) + Lu(x,t) + Nu(x,\ t) = f(x,t), \tag{9}$$

where u the analytical function, $D_x^{\ltimes}$ is Caputo fractional derivative and N denote the nonlinear operators, and f is an function given.

$$u(x,t) = g(t) + \frac{x^{\ltimes}}{\Gamma(\ltimes+1)}\hbar(t) + I_x^{\ltimes}\left[a_1\frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2\frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3u(x,t) + f(x,t)\right]. \tag{10}$$

with

$$u(0,t) = g(t),\ u_x(0,t) = \hbar\ (t).$$

4. Yang Hussein Jassim method (YHJM)

We examine the following general form of the telegraph fractional polynomial equation:

$$\frac{\partial^{2\ltimes}u}{\partial x^{2\ltimes}}(x,t) = a_1\frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2\frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3u(x,t) + f(x,t), \tag{11}$$

where $0 < \ltimes. \le 1$, $0 < \beta \le 1$, $0 < \vartheta \le 1$, $x, t \ge 0$, $u(0,t) = g(t)$, $u_x(0,t) = \hbar(t)$
and a_1, a_2, a_3 are constants

To solve using Hussein Jassim's method with Yang transformation, we follow the following steps:

1. Taking Yang Transform of the Fractional Derivative: For a given function $u(x,t)$, denote the Yang transform of $u(x,t)$ with respect to x *is denoted* as $Y_a\{u(x,t)\}$. The Yang transform of the Caputo fractional derivative $D_x^{\ltimes}u(x,t)$ of order $\ltimes$ with respect to x is expressed as:

$$Y_a\left\{D_x^{2\ltimes}u(x,t)\right\} = Y_a\left\{a_1\frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2\frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3u(x,t) + f(x,t)\right\}. \tag{12}$$

2. Implementing the Yang Transform: Apply the Yang Transform the entire differential equation involving The fractional Derivative with respect to x. This converts The fractional derivative $D_x^{2\ltimes}u(x,t)$ into a polynomial expression in (v) of degree $(\ltimes)$ alphaα, simplifying the equation to an algebraic form.

$$\frac{Y_a\{u(t)\}}{v^{2\ltimes}} - \frac{u(x,0)}{v^{2\ltimes-1}} - \frac{u_x(x,0)}{v^{2\ltimes-2}} = Y_a\left\{a_1\frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2\frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3u(x,t) + f(x,t)\right\}. \tag{13}$$

3. Solving for $Y_a\{u(x.,t)\}$: Solve The transformed Equation to find $Y_a\{u(x,t)\}$ in The (v)-domain. This step transforms the problem into an algebraic equation, making it easier to isolate $Y_a\{u(x,t)\}$ without the need for the fractional derivative.

$$Y_a\{u(x,t)\} = vg(t) + v^2 k(t) + \quad v^{2\ltimes} Y_a\left\{a_1 \frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2 \frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3 u(x,t) + f(x,t)\right\}. \tag{14}$$

4. Inverse Yang Transform: After solving for $Y_a\{u(x,t)\}$, use the inverse Yang transform to return to the original x-domain and obtain the solution $u(x,t)$. The inverse Yang transform, denoted as Y_a^{-1}, returns the function from the (v)-domain back to the x-domain.

$$u(x.,t) = g(t) + \frac{x^{\ltimes}}{\Gamma(\ltimes+1)} k(t) + Y_a^{-1}\left[v^{2\ltimes} Y_a\left\{a_1 \frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2 \frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3 u(x,t) + f(x,t)\right]\right.. \tag{15}$$

Following these steps allows you to handle and eliminate the fractional derivative in the transformed domain, thereby simplifying the process of solving for (x,t).

Now, we apply Hussain Jassim's method by rewriting Express The right-hand side of Eq. (15) as an infinite series of fractional powers

$$\begin{aligned} g(t) + \frac{x^{\ltimes}}{\Gamma(\ltimes+1)} k(t) + Y_a^{-1}\left[v^{2\ltimes} Y_a\left\{a_1 \frac{\partial^\beta u}{\partial t^\beta}(x,t) + a_2 \frac{\partial^\vartheta u}{\partial t^\vartheta}(x,t) + a_3 u(x,t) + f(x,t)\right\}\right] \\ = \eta_0 + \eta_1 x^{\ltimes} + \eta_2 x^{2\ltimes} + \cdots = \sum_{i=0}^{\infty} \eta_i x^{i\ltimes}, \end{aligned} \tag{16}$$

where $\eta_0, \eta_1, \eta_2, \ldots$ are coefficients. BY taking the fractional derivative of Caputo $D_x^{i\ltimes}$ where $i = 0, 1, 2, \ldots$, for both sides of Eq. (16) at $x = 0$; we obtain

$$\begin{cases} \eta_0 = g(t). \\ \eta_1 = \frac{x^{\ltimes}}{\Gamma(\ltimes+1)} k(t) \\ \eta_2 = \frac{D_x^{2\ltimes}\left\{g(t)+\frac{x^{\ltimes}}{\Gamma(\ltimes+1)}k(t)+Y_a^{-1}\left[v^{2\ltimes}Y_a\left[a_1\frac{\partial^\beta(u_0+u_1)}{\partial t^\beta}+a_2\frac{\partial^\vartheta(u_0+u_1)}{\partial t^\vartheta}+a_3(u_0+u_1)+f(x,t)\right]\right]\right\}_{x=0}}{\Gamma(2\ltimes+1)}. \\ \eta_3 = \frac{D_x^{3\ltimes}\left\{g(t)+\frac{x^{\ltimes}}{\Gamma(\ltimes+1)}k(t)+Y_a^{-1}\left[v^{2\ltimes}Y_a\left[a_1\frac{\partial^\beta(u_0+u_1+u_2)}{\partial t^\beta}+a_2\frac{\partial^\vartheta(u_0+u_1+u_2)}{\partial t^\vartheta}+a_3(u_0+u_1+u_2)+f(x,t)\right]\right]\right\}_{x=0}}{\Gamma(3\ltimes+1)}. \\ \vdots \\ \eta_i = \sum_{i=0}^{\infty} \frac{D_x^{i\ltimes}\left\{g(t)+\frac{x^{\ltimes}}{\Gamma(\ltimes+1)}k(t)+Y_a^{-1}\left[v^{2\ltimes}Y_a\left[a_1\frac{\partial^\beta(u_0+u_1+u_2+\cdots)}{\partial t^\beta}+a_2\frac{\partial^\vartheta(u_0+u_1+u_2+\ldots)}{\partial t^\vartheta}+a_3(u_0+u_1+u_2+\cdots)+f(x,t)\right]\right]\right\}_{x=0}}{\Gamma(i\ltimes+1)}. \end{cases} \tag{17}$$

Substituting Eq. (10) into Eq. (11) yields the following result:

$$\begin{aligned} u(x.,t) \\ = \sum_{i=0}^{\infty} \frac{D_x^{i\ltimes}\left\{g(t)+\frac{x^{\ltimes}}{\Gamma(\ltimes+1)}k(t)+Y_a^{-1}\left[v^{2\ltimes}Y_a\left[a_1\frac{\partial^\beta(u_0+u_1+u_2+\cdots)}{\partial t^\beta}+a_2\frac{\partial^\vartheta(u_0+u_1+u_2+\ldots)}{\partial t^\vartheta}+a_3(u_0+u_1+u_2+\cdots)+f(x,t)\right]\right]\right\}_{x=0}}{\Gamma(i\ltimes+1)} x^{i\ltimes}. \end{aligned} \tag{18}$$

Assume now that Eq. (17) is a solution to Eq. (16), which can be written as follows:

$$u(x,t) = \sum_{i=0}^{\infty} u_i(x,t). \tag{19}$$

We substitute Eq. (19) into Eq. (18) and get

$$\begin{aligned}&\sum_{i=0}^{\infty} u_i(\mathfrak{x},t)\\ &= \sum_{i=0}^{\infty} \frac{D_{\mathfrak{x}}^{(i-1)\ltimes ..}\left\{g(t)+\frac{\mathfrak{x}^{\ltimes}}{\Gamma(\ltimes+1)}k(t)+Y_a^{-1}\left[v^{2\ltimes}Y_a\left[a_1\frac{\partial^{\beta}(u_0+u_1+u_2+\cdots)}{\partial t^{\beta}}+a_2\frac{\partial^{\vartheta}(u_0+u_1+u_2+\ldots)}{\partial t^{\vartheta}}+a_3(u_0+u_1+u_2+\cdots)+f(\mathfrak{x},t)\right]\right]\right\}x=0}{\Gamma(i\ltimes+1)}\mathfrak{x}^{i\ltimes}.\end{aligned} \tag{20}$$

$$\begin{aligned}&u_0(\mathfrak{x},t)=\eta_0=g(t).\\ &u_1(\mathfrak{x},t)=\eta_1\frac{\mathfrak{x}^{\ltimes}}{\Gamma(\ltimes+1)}=\frac{\mathfrak{x}^{\ltimes}}{\Gamma(\ltimes+1)}k(t).\\ &u_2(\mathfrak{x},t)=\eta_2\frac{\mathfrak{x}.^{2\ \ltimes}}{\Gamma(2\ \ \ltimes.+1)}.\\ &u_3(\mathfrak{x}.,t)=\eta_3\frac{\mathfrak{x}^{3\ltimes}}{\Gamma(3\ltimes+1)}.\\ &\sum_{i=0}^{\infty}u_i(\mathfrak{x},t)=\sum_{i=0}^{\infty}\eta_i\frac{\mathfrak{x}^{i\ltimes}}{\Gamma(i\ltimes+1)}.\\ &u(\mathfrak{x},t)=u_0+u_1+u_2+u_3+\cdots=\sum_{i=0}^{\infty}u_i.\end{aligned} \tag{21}$$

5. Convergence for Yang transform with Caputo operator

Theorem: Let $u(\mathfrak{x},t)\in H$, where is a Hilbert space, and $\ltimes\in(0,1)$. Assume that $u(\mathfrak{x},t)$ is the precise solution to Eq. (21). The resulting series $\sum_{i=0}^{\infty}u_r(\mathfrak{x},t)$ converges to $u(\mathfrak{x},t)$ $if: u_r(\mathfrak{x},t)\le u_{r-1}(\mathfrak{x},t)\,\forall r>A$, i.e., for every $\beta>0$ there exists $A>0$, in which

$\|u_{r+n}(x,t)\|\le\beta,\ \forall\, r,n\in N.$

Proof. Consider a sequence of $\sum_{r=0}^{\infty}u_r(x,t)$.

$$\begin{aligned}&T_0(x,t)=u_0(x,t),\\ &T_1(x,t)=u_0(x,t)+u_1(x,t),\\ &T_2(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t),\\ &T_3(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t),\\ &\vdots\\ &T_r(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots+u_r(x,t).\end{aligned} \tag{22}$$

We have to show that $\{T_r(x,t)\}_{r=0}^{\infty}$ forms a "Cauchy sequence" under the obtained Observe that

$$\begin{aligned}&\|T_{r+1}(x,t)-T_r(x,t)\|=\|u_{r+1}(x,t)\|\le\varepsilon\|u_r(x,t)\|\le\varepsilon^2\|u_{r-1}(x,t)\|\le\varepsilon^3\|u_{r-2}(x,t)\|\ldots\\ &\qquad\le\ \varepsilon^{r+1}\|u_0(x,t)\|.\end{aligned} \tag{23}$$

For $r,n\ \in N$, it follows that

$$\|T_r(x,t)-T_n(x,t)\|=\|u_0(x,t)\|\sum_{k=n+1}^{r}\varepsilon^k. \tag{24}$$

Using the formula for geometric series, we get

$$\|T_r(x,t)-T_n(x,t)\|=\|u_0(x,t)\|\frac{\varepsilon^{n+1}}{1-\varepsilon}. \tag{25}$$

Since $u_0(x,t)$ is bounded and $0 < \varepsilon < 1$, the sequence $\{T_r(x,t)\}_{r=0}^{\infty}$ is a "Cauchy sequence" in H. Therefore

$$\lim_{r\to\infty} u_r(x,t) = u(x,t) \text{ for } \exists\, u(x,t) \in H. \tag{26}$$

This completes the proof.

6. Example illustrations

Instance 1. *Consider the time – fractional 2D telegraph equation*

$$\begin{aligned} &\frac{\partial^{2\ltimes} u}{\partial t^{2\ltimes}} = \frac{\partial^2 u}{\partial\, x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - 2u., \qquad 0 < \ltimes \le 1 \\ &With,\ \ u(x,y,0) = e^{x+y}, \qquad\qquad u_t(x,y,0) = -3\, e^{x+y} \end{aligned} \tag{27}$$

Solution 1. *Applying the Y_aT transform and differentiating both sides of Eq. (27) with respect to x, we obtain*

$$\begin{aligned} &\frac{Y_a\{u(t)\}}{v^{2\ltimes}} - \frac{u(x,\mathcal{J},0)}{v^{2\ltimes-1}} - \frac{u_t(x,\mathcal{J},0)}{v^{2\ltimes-2}} = Y_a\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - 2u\right]. \\ &Y_a\{u(x,\mathcal{J},t)\} = v\ \ u(x,\mathcal{J},0) + v^2\ u_t(x,\mathcal{J},0)\ + v^{2\ltimes} Y_a\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - 2u\right]. \end{aligned} \tag{28}$$

Taking the invers Y_aT of (28) yields

$$u(x,\mathcal{J},t) = \ e^{x+\mathcal{J}} - 3e^{x+\mathcal{J}}\frac{t^{\ltimes}}{\Gamma(\ltimes+1)} + Y_a^{-1}\left\{v^{2\ltimes} Y_a\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - 2u\right]\right\}. \tag{29}$$

Applying an algorithm (H J) Equation (29) It can be articulated as follows

$$\sum_i^{\infty} u_i = \sum_i^{\infty} \frac{D_t^{i\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - 2u\right]\right\}\right\} t = 0}{\Gamma(i\ltimes+1)} t^{i\ltimes}.$$

$$u_0(x,\mathcal{J},t) = u(x,\mathcal{J},0) = e^{x+\mathcal{J}}.$$

$$u_1(x,\mathcal{J},t) = u_t(x,\mathcal{J},0) = -3e^{x+\mathcal{J}}\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}.$$

$$u_2(x.,\mathcal{J},t) = \frac{D_t^{2\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2(u_0+u_1)}{\partial x^2} + \frac{\partial^2(u_0+u_1)}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes}(u_0+u_1)}{\partial t^{\ltimes}} - 2(u_0+u_1)\right]\right\}\right\}t=0}{\Gamma(2\ltimes+1)} t^{2\ltimes}$$

$$u_2(x,\mathcal{J},t) = \frac{D_t^{2\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\{9v^{2\ltimes+1}e^{x+\mathcal{J}.}\}\right\}t=0}{\Gamma(2\ltimes+1)} t^{2\ltimes}$$

$$u_2(x,\mathcal{J},t) = \frac{D_t^{2\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + \frac{9t^{2\ltimes}}{\Gamma(2\ltimes+1)}e^{x+\mathcal{J}}\right\}t=0}{\Gamma(2\ltimes+1)} t^{2\ltimes}$$

$$u_2(x,\mathcal{J},t) = 9e^{x+\mathcal{J}}\frac{t^{2\ltimes}}{\Gamma(2\ltimes+1)} = e^{x+\mathcal{J}}\frac{(3t^{\ltimes})^2}{\Gamma(2\ltimes+1)}.$$

$$u_3(x,\mathcal{J},t) = \frac{D_t^{3\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2(u_0+u_1+u_2)}{\partial x^2} + \frac{\partial^2(u_0+u_1+u_2)}{\partial \mathcal{J}^2} - 3\frac{\partial^{\ltimes}(u_0+u_1+u_2)}{\partial t^{\ltimes}} - 2(u_0+u_1+u_2)\right]\right\}\right\}t=0}{\Gamma(3\ltimes+1)} t^{3\ltimes}$$

$$u_3(x,\mathcal{J},t) = \frac{D_t^{3\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\{9v^{2\ltimes+1} - 27e^{x+\mathcal{J}}\}\right\}t=0}{\Gamma(3\ltimes+1)} t^{3\ltimes}$$

$$u_3(x,\mathcal{J},t) = \frac{D_t^{3\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + 9\frac{t^{2\ltimes}}{\Gamma(2\ltimes+1)}e^{x+\mathcal{J}} - 27\frac{t^{3\ltimes}}{\Gamma(3\ltimes+1)}e^{x+\mathcal{J}}\right\}t=0}{\Gamma(3\ltimes+1)} t^{3\ltimes}$$

$$u_3(x,\mathcal{J},t) = -27e^{x+\mathcal{J}}\frac{t^{3\ltimes}}{\Gamma(3\ltimes+1)} = -e^{x+\mathcal{J}}\frac{(3t^{\ltimes})^3}{\Gamma(3\ltimes+1)}.$$

.

$$u(x,\mathcal{J},t) = \sum_i^{\infty} u_i = u_O + u_1 + u_2 + u_3 + u_4 + \cdots\cdots.$$

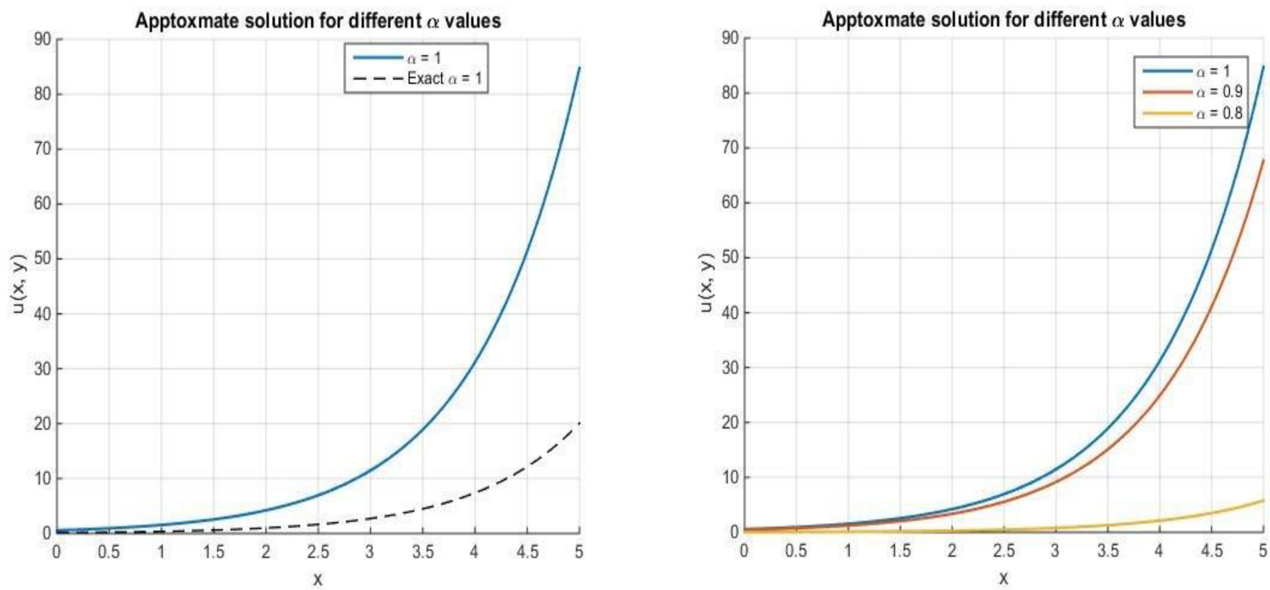

Fig. 1. Graphs of the approximate solution $u\,(x, \mathcal{J}, t)$ for various values of $\propto alpha\alpha$ while keeping x constant.

Table 1. Quantitative results of the approximate and exact solutions for various values of x and t when $\ltimes = 0.8, 0.9, 1$.

x	$\ltimes = 1$	$\ltimes = 0.9$	$\ltimes = 0.8$	$\ltimes = 0.7$
−2	2.0522e-05	0.038388	0.081755	0.15046
−1.7778	2.5629e-05	0.047941	0.1021	0.1879
−1.5556	3.2006e-05	0.059871	0.12751	0.23466
−1.3333	3.9971e-05	0.07477	0.15924	0.29306
−1.1111	4.9918e-05	0.093377	0.19886	0.36598
−0.88889	6.234e-05	0.11661	0.24835	0.45706
−0.66667	7.7853e-05	0.14563	0.31015	0.5708
−0.44444	9.7227e-05	0.18187	0.38733	0.71284
−0.22222	0.00012142	0.22713	0.48372	0.89023
0	0.00015164	0.283865	0.60409	1.1118

$$u\,(x, \mathcal{J}, t) = \quad e^{x+\mathcal{J}} - e^{x+\mathcal{J}}\,\frac{3t^{\ltimes}}{\Gamma(\ltimes + 1)} + \quad e^{x+\mathcal{J}}\,\frac{(3t^{\ltimes})^2}{\Gamma(2 \ltimes +1)} - \quad e^{x+\mathcal{J}}\,\frac{(3t^{\ltimes})^3}{\Gamma(3 \ltimes +1)} + \cdots\cdots$$

$$u\,(x, \mathcal{J}, t) = \quad e^{x+\mathcal{J}}\left(1 - \frac{3t^{\ltimes}}{\Gamma(\ltimes + 1)} + \quad \frac{(3t^{\ltimes})^2}{\Gamma(2 \ltimes +1)} - \quad \frac{(3t^{\ltimes})^3}{\Gamma(3 \ltimes +1)} + \cdots\cdots\right).$$

When $\ltimes = 1$

$$u\,(x, \mathcal{J}, t) = e^{x+\mathcal{J}}\left(1 - 3t + \frac{(3t)^2}{2!} - \frac{(3t)^3}{3!} + \cdots\cdots\right). \tag{30}$$

$$\begin{aligned} u\,(x, \mathcal{J}, t) &= e^{x+\mathcal{J}}.e^{-3t} \\ u\,(x, \mathcal{J}, t) &= e^{x+\mathcal{J}-3t}. \end{aligned} \tag{31}$$

In Fig. 1, we plot the approximate solution curve for the two Eq. (27) when ($\ltimes = 0.8, 0.9, 1$) In Fig. 2, the surface solution for the two Eq. (27) and is represented when ($\ltimes = 0.8, 0.9, 1$). In Table 1, we have evaluated the numerical values of the approximate solution $u\,(x, \mathcal{J}, t)$ for the problem (27) obtained using the (YHJM) method and compared them with the exact solution given in Eq. (31) through different values of $(x, \mathcal{J}, t)$ when ($\ltimes = 0.8, 0.9, 1$).

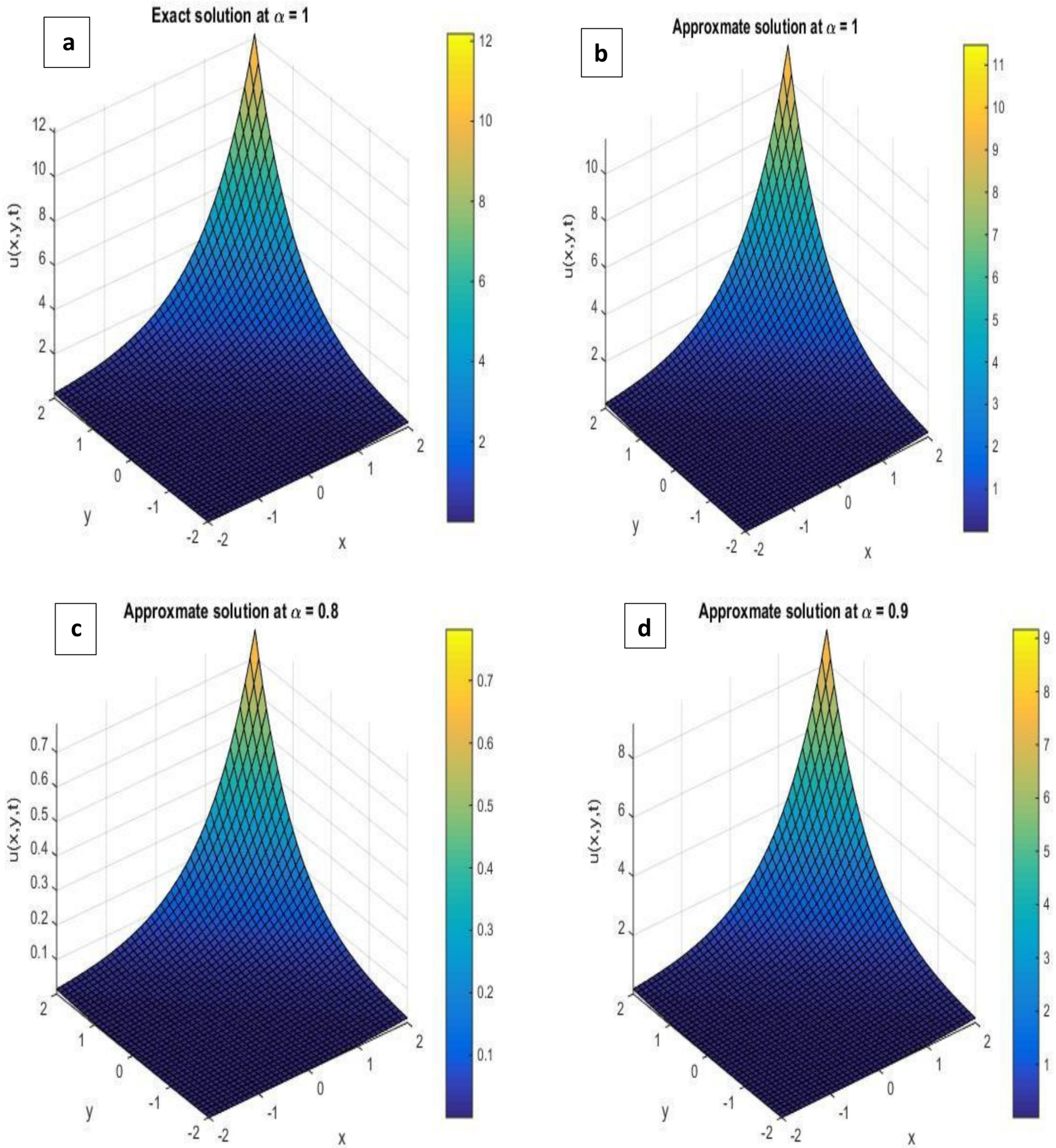

Fig. 2. The surface graph of the approximate solution $u\,(x, \mathcal{J}, t)$ of Eq. (31); (a) $u\,(x, \mathcal{J}, t)$ when exact solution, (b) $u\,(x, \mathcal{J}, t)$ when $\alpha = 1$, (c) $u\,(x, \mathcal{J}, t)$ when $\alpha = 0.8$ (d) $u\,(x, \mathcal{J}, t)$ when $\alpha = 0.9$.

Instance 2. *Consider the time – fractional 3D telegraph equation*

$$\frac{\partial^{2\ltimes} u}{\partial t^{2\ltimes}} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} + \frac{\partial^2 u}{\partial \mathbb{Z}^2} - 2\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - 3u, \qquad 0 < \ltimes \leq 1 \tag{32}$$

With, $u\,(x, \mathcal{J}, \mathbb{Z}, 0) = \sinh\,(x) \sinh(\mathcal{J}) \sinh\,(\mathbb{Z})$, $u_t\,(x, \mathcal{J}, \mathbb{Z}, 0) = -2\, \sin k(x) \sin k\,(\mathcal{J}) \sin k\,(\mathbb{Z})$.

Solution 2. *Applying the Y_aT transform and differentiating both sides of Eq. (32) with respect to $\mathfrak{x}$, we obtain*

$$\frac{Y_a\{u(x,\mathcal{J},\mathbb{Z},t)\}}{v^{2\ltimes}}-\frac{u(\mathfrak{x},\mathcal{J},\mathbb{Z},0)}{v^{2\ltimes-1}}-\frac{u_t(\mathfrak{x},\mathcal{J},\mathbb{Z},0)}{v^{2\ltimes-2}}=Y_a\left[\frac{\partial^2 u}{\partial\mathfrak{x}^2}+\frac{\partial^2 u}{\partial\mathcal{J}^2}+\frac{\partial^2 u}{\partial\mathbb{Z}^2}-2\frac{\partial^{\ltimes}u}{\partial t^{\ltimes}}-3u\right].$$
$$Y_a\{u(x,\mathcal{J},\mathbb{Z},t)\}=vu(x,\mathcal{J},\mathbb{Z},,0)+v^2u_t(\mathfrak{x},\mathcal{J},\mathbb{Z},0)+v^{2\ltimes}Y_a\left[\frac{\partial^2 u}{\partial\mathfrak{x}^2}+\frac{\partial^2 u}{\partial\mathcal{J}^2}+\frac{\partial^2 u}{\partial\mathbb{Z}^2}-2\frac{\partial^{\ltimes}u}{\partial t^{\ltimes}}-3u\right]. \tag{33}$$

Taking the invers Y_aT of (33) yields

$$u(x,\mathcal{J},\mathbb{Z},t)=\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})-\frac{2t^{\ltimes}}{\Gamma(\ltimes+1)}\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})$$
$$+Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial\mathcal{J}^2}+\frac{\partial^2 u}{\partial\mathbb{Z}^2}-2\frac{\partial^{\ltimes}u}{\partial t^{\ltimes}}-3u\right]\right\}. \tag{34}$$

Applying an algorithm (HJ) Equation (34) It can be articulated as follows

$$\sum_i^{\infty}u_i$$
$$=\sum_i^{\infty}\frac{D_t^{i\ltimes}\left\{\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})-2\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})+Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2 u}{\partial\mathfrak{x}^2}+\frac{\partial^2 u}{\partial\mathcal{J}^2}+\frac{\partial^2 u}{\partial\mathbb{Z}^2}-2\frac{\partial^{\ltimes}u}{\partial t^{\ltimes}}-3u\right]\right\}\right\}t=0}{\Gamma(i\ltimes+1)}t^{i\ltimes}.$$
$$u_0(\mathfrak{x},\mathcal{J},\mathbb{Z},t)=u(x,\mathcal{J},\mathbb{Z},0)=\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z}).$$
$$u_1(\mathfrak{x},\mathcal{J},\mathbb{Z},t)=\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}u_t(\mathfrak{x},\mathcal{J},\mathbb{Z},0)=-\frac{2t^{\ltimes}}{\Gamma(\ltimes+1)}\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z}).$$
$$u_2(\mathfrak{x},\mathcal{J},\mathbb{Z},t)=\frac{4t^{2\ltimes}}{\Gamma(2\ltimes+1)}\times\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})=\frac{(2t^{\ltimes})^2}{\Gamma(2\ltimes+1)}\times\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z}). \tag{35}$$
$$u_3(\mathfrak{x},\mathcal{J},\mathbb{Z},t)=\frac{-8t^{3\ltimes}}{\Gamma(3\ltimes+1)}\times sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})=-\frac{(2t^{\ltimes})^3}{\Gamma(3\ltimes+1)}\times sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z}).$$
.

$$u(x,\mathcal{J},\mathbb{Z},t)=\sum_i^{\infty}u_i=u_0+u_1+u_2+u_3+u_4+\cdots\cdots.$$
$$u(x,\mathcal{J},\mathbb{Z},t)=\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})\left[1-\frac{2t^{\ltimes}}{\Gamma(\ltimes+1)}+\frac{(2t^{\ltimes})^2}{\Gamma(2\ltimes+1)}-\frac{(2t^{\ltimes})^3}{\Gamma(3\ltimes+1)}+\cdots\cdots\cdots\right].$$

When $\ltimes=1$

$$u(x,\mathcal{J},\mathbb{Z},t)=\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z})\left[1-2t+\frac{(2t)^2}{2!}-\frac{(2t)^3}{3!}+\cdots\cdots\right)$$
$$u(x,\mathcal{J},\mathbb{Z},t)=\sin\hbar(x)\sin\hbar(\mathcal{J})\sin\hbar(\mathbb{Z}).e^{-2t}. \tag{36}$$

In Fig. 3, we plot the approximate solution curve for the two Eq. (32) when ($\ltimes=0.8,0.9,1$) In Fig. 4, the surface solution for the two Eq. (32) and is represented when ($\ltimes=0.8,0.9,1$). In Table 2, we have evaluated the numerical values of the approximate solution $u(x,\mathcal{J},Z,t)$ for the problem (32) obtained using the (YHJM) method and compared them with the exact solution given in Eq. (36) through different values of $(x,\mathcal{J},Z,t)$ when ($\ltimes=0.8,0.9,1$).

Instance 3. Consider the time – fractional 2D nonlinear telegraph equation

$$\frac{\partial^{2\ltimes}u}{\partial t^{2\ltimes}}=\frac{\partial^2 u}{\partial\mathfrak{x}^2}+\frac{\partial^2 u}{\partial\mathcal{J}^2}-2\frac{\partial^{\ltimes}u}{\partial t^{\ltimes}}-u^2+e^{2(x+\mathcal{J})-4t}-2e^{(x+\mathcal{J})-2t},\qquad 0<\ltimes\leq 1 \tag{37}$$

With, $u(x,\mathcal{J},0)=e^{x+\mathcal{J}}$, $u_t(\mathfrak{x},\mathcal{J},0)=-2\,e^{x+\mathcal{J}}$.

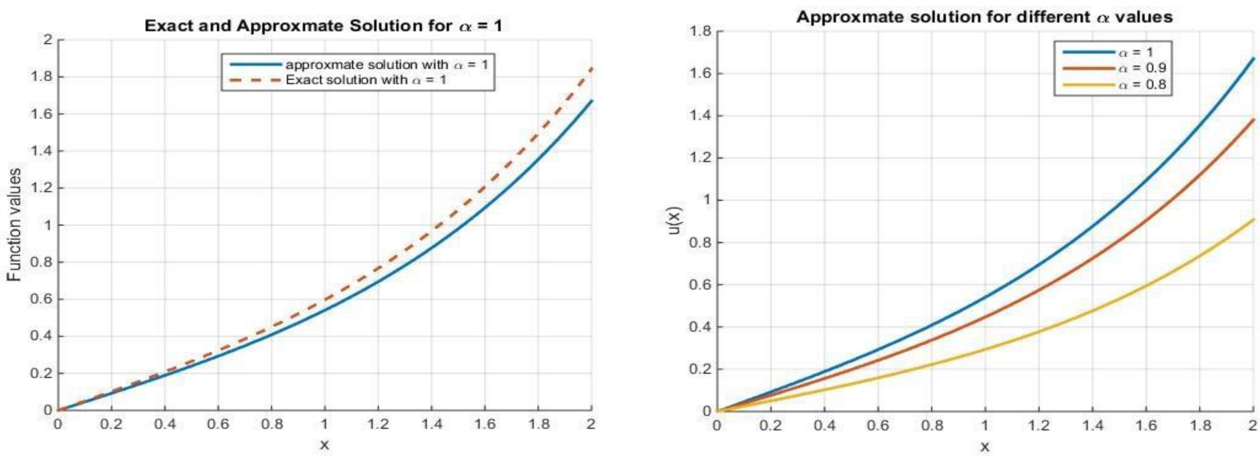

Fig. 3. Graphs of the approximate solution $u\,(x, \mathcal{J}, z, t)$ for various values of $\propto$ $alpha\alpha$ while keeping x constant.

Table 2. Quantitative results of the approximate and exact solutions for various values of x and t when $\ltimes = 0.8, 0.9, 1$.

x	$\ltimes = 1$	$\ltimes = 0.9$	$\ltimes = 0.8$	$\ltimes = 0.7$
0	0	0	0	0
0.22222	1.523e-07	0.0060198	0.012712	0.020036
0.44444	3.1215e-07	0.012338	0.026055	0.041066
0.66667	4.8748e-07	0.019268	0.04069	0.064132
0.88889	6.8698e-07	0.027154	0.057342	0.090378
1.1111	9.2055e-07	0.036386	0.076837	0.12111
1.3333	1.1998e-06	0.047422	0.10014	0.15784
1.5556	1.5385e-06	0.06081	0.12841	0.2024
1.7778	1.9535e-0.6	0.077213	0.16305	0.25699
2	2.4653e-0.6	0.097445	0.20578	0.32433

Solution 3. *Applying the Y_aT transform and differentiating both sides of Eq. (37) with respect to $\mathfrak{x}$, we obtain*

$$\frac{Y_a\{u\,(x, \mathcal{J}, t)\}}{v^{2\ltimes}} - \frac{u\,(\mathfrak{x}, \mathcal{J}, 0)}{v^{2\ltimes-1}} - \frac{u_t\,(\mathfrak{x}, \mathcal{J}, 0)}{v^{2\ltimes-2}} = Y_a\left[\frac{\partial^2 u}{\partial \mathfrak{x}^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 2\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x+\mathcal{J})-4t} - 2e^{(x+\mathcal{J})-2t}\right].$$

$$Y_a\{u\,(\mathfrak{x}, \mathcal{J}, t)\} = vu\,(x, \mathcal{J}, 0) + v^2 u_t\,(\mathfrak{x}, \mathcal{J}, 0) + v^{2\ltimes} Y_a\left[\frac{\partial^2 u}{\partial \mathfrak{x}^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 2\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x+\mathcal{J})-4t} - 2e^{(x+\mathcal{J})-2t}\right]. \tag{38}$$

Taking the invers Y_aT of (38) yields

$$u\,(x, \mathcal{J}, t) = e^{x+\mathcal{J}} - 2e^{x+\mathcal{J}}\frac{t^{\ltimes}}{\Gamma(\ltimes+1)} + Y_a^{-1}\left\{v^{2\ltimes} Y_a\left[\frac{\partial^2 u}{\partial \mathfrak{x}^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 2\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x+\mathcal{J})-4t} - 2e^{(x+\mathcal{J})-2t}\right]\right\}. \tag{39}$$

Applying an algorithm (HJ) Equation (39) It can be articulated as follows

$$\sum_i^{\infty} u_i = \sum_i^{\infty} \frac{D_t^{i\ltimes}\left\{e^{x+\mathcal{J}} - 2\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\left\{v^{2\ltimes} Y_a\left[\frac{\partial^2 u}{\partial \mathfrak{x}^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} - 2\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x+\mathcal{J})-4t} - 2e^{(x+\mathcal{J})-2t}\right]\right\}\right\} t=0}{\Gamma(i\ltimes+1)} t^{i\ltimes}.$$

$$u_0\,(\mathfrak{x}, \mathcal{J}, t) = u\,(x, \mathcal{J}, 0) = e^{x+\mathcal{J}}.$$

$$u_1\,(\mathfrak{x}, \mathcal{J}, t) = u_t\,(\mathfrak{x}, \mathcal{J}, 0) = -2e^{x+\mathcal{J}}\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}.$$

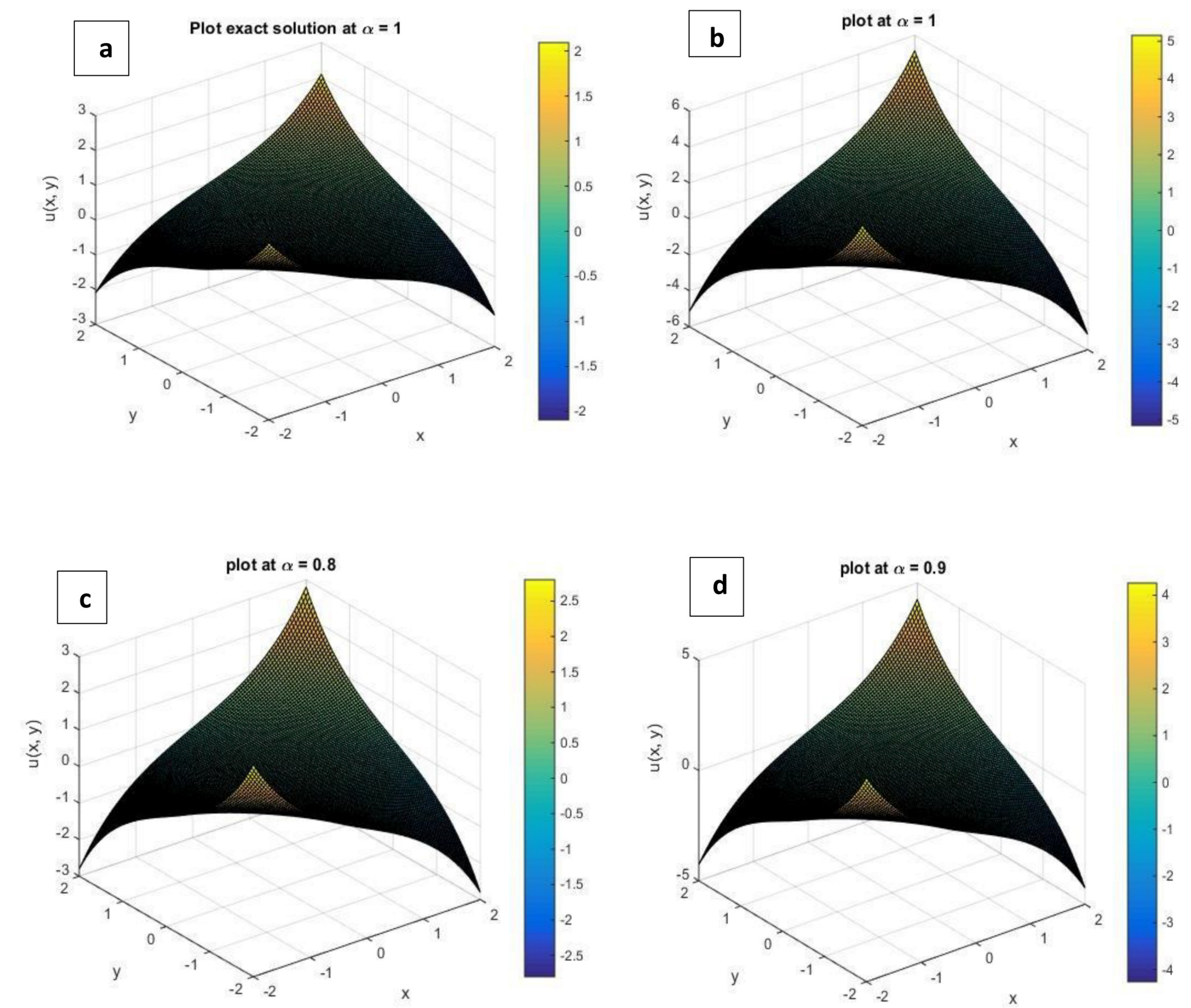

Fig. 4. The surface graph of the approximate solution $u(x, \mathcal{J}, z, t)$ of Eq. (36); (a) $u(x, \mathcal{J}, z, t)$ when exact solution, (b) $u(x, \mathcal{J}, z, t)$ when $\alpha = 1$, (c) $u(x, \mathcal{J}, z, t)$ when $\alpha = 0.9$ (d) $u(x, \mathcal{J}, z, t)$ when = 0.8.

$$u_2(\mathfrak{x}, \mathcal{J}, t) = \frac{D_t^{2\ltimes}\left\{e^{x+\mathcal{J}} - 2\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2(u_0+u_1)}{\partial \mathfrak{x}^2} + \frac{\partial^2(u_0+u_1)}{\partial \mathcal{J}^2} - 2\frac{\partial^{\ltimes}(u_0+u_1)}{\partial t^{\ltimes}} - (u_0+u_1)^2 + e^{2(x+\mathcal{J})-4t} - 2e^{(x+\mathcal{J})-2t} - \right]\right\}\right\}_{t=0}}{\Gamma(2\ltimes+1)} t^{2\ltimes}$$

$$u_2(\mathfrak{x}, \mathcal{J}, t) = 4e^{x+\mathcal{J}}\frac{t^{2\ltimes}}{\Gamma(2\ltimes+1)} = e^{x+\mathcal{J}}\frac{(2t^{\ltimes})^2}{\Gamma(2\ltimes+1)}.$$

$$u_3(\mathfrak{x}, \mathcal{J}, t) = \frac{D_t^{3\ltimes}\left\{e^{x+\mathcal{J}} - 3\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x+\mathcal{J}} + Y_a^{-1}\left\{v^{2\ltimes}Y_a\left[\frac{\partial^2(u_0+u_1+u_2)}{\partial \mathfrak{x}^2} + \frac{\partial^2(u_0+u_1+u_2)}{\partial \mathcal{J}^2} - 2\frac{\partial^{\ltimes}(u_0+u_1+u_2)}{\partial t^{\ltimes}} - (u_0+u_1+u_2)^2 + e^{2(x+\mathcal{J})-4t} - 2e^{(x+\mathcal{J})-2t}\right]\right\}\right\}_{t=0}}{\Gamma(3\ltimes+1)} t^{3\ltimes}$$

$$u_3(\mathfrak{x}, \mathcal{J}, t) = -8e^{x+\mathcal{J}}\frac{t^{3\ltimes}}{\Gamma(3\ltimes+1)} = -e^{x+\mathcal{J}}\frac{(2t^{\ltimes})^3}{\Gamma(3\ltimes+1)}.$$

$$u(x, \mathcal{J}, t) = \sum_{i}^{\infty} u_i = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots\cdots.$$

$$u(x, \mathcal{J}, t) = e^{x+\mathcal{J}} - e^{x+\mathcal{J}}\frac{2t^{\ltimes}}{\Gamma(\ltimes+1)} + e^{x+\mathcal{J}}\frac{(2t^{\ltimes})^2}{\Gamma(2\ltimes+1)} - e^{x+\mathcal{J}}\frac{(2t^{\ltimes})^3}{\Gamma(3\ltimes+1)} + \cdots\cdots$$

$$u(x, \mathcal{J}, t) = e^{x+\mathcal{J}}\left(1 - \frac{2t^{\ltimes}}{\Gamma(\ltimes+1)} + \frac{(2t^{\ltimes})^2}{\Gamma(2\ltimes+1)} - \frac{(2t^{\ltimes})^3}{\Gamma(3\ltimes+1)} + \cdots\cdots\right). \tag{40}$$

Table 3. Quantitative results of the approximate and exact solutions for various values of x and t when $\ltimes = 0.8, 0.9, 1$.

x	$\ltimes = 1$	$\ltimes = 0.9$	$\ltimes = 0.8$	$\ltimes = 0.7$
−2	5.5856e-07	0.022078	0.046622	0.073482
−1.7778	6.9755e-07	0.027572	0.058224	0.091768
−1.5556	8.7114e-07	0.034433	0.072713	0.1146
−1.3333	1.0879e-06	0.043001	0.090807	0.14312
−1.1111	1.3586e-06	0.053702	0.1134	0.17874
−0.88889	1.6967e-06	0.067066	0.14163	0.22322
−0.66667	2.119e-06	0.083755	0.17687	0.27877
−0.44444	2.6463e-06	0.1046	0.22088	0.34814
−0.22222	3.3048e-0.6	0.13063	0.27585	0.43477
0	4.1272e-0.6	0.16313	0.34449	0.54296

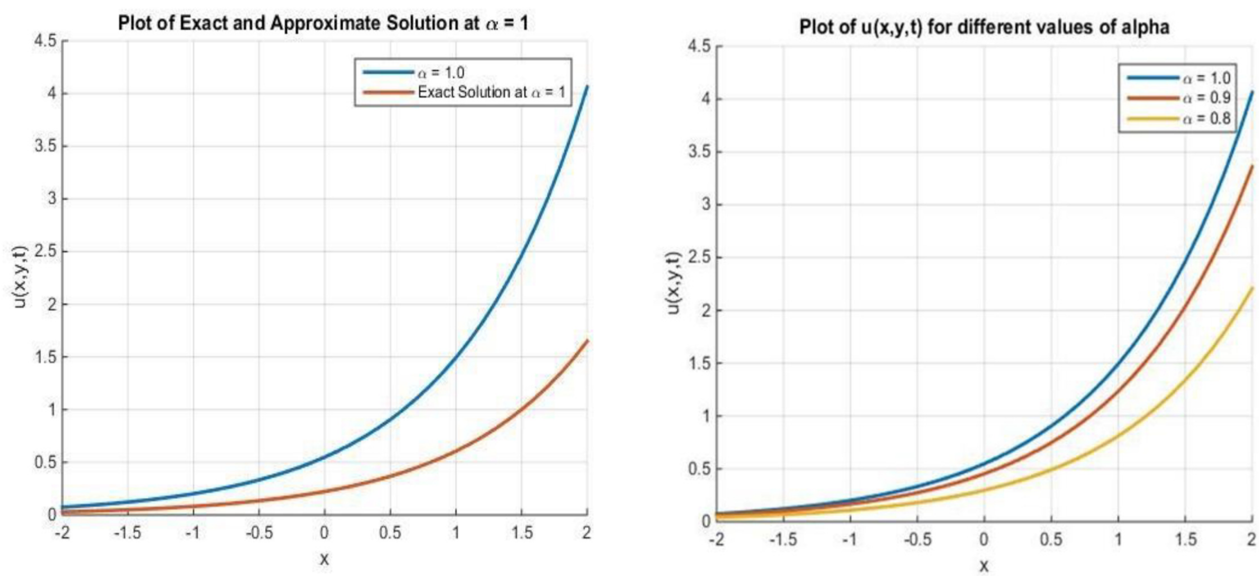

Fig. 5. Graphs of the approximate solution $u(x, \mathcal{J}, t)$ for various values of $\propto alpha\alpha$ while keeping x constant.

When $\ltimes = 1$

$$\begin{aligned} u(x, \mathcal{J}, t) &= e^{x+\mathcal{J}}\left(1 - 2t + \frac{(2t)^2}{2!} - \frac{(2t)^3}{3!} + \cdots\cdots\right) \\ u(x, \mathcal{J}, t) &= e^{x+\mathcal{J}}.e^{-2t} \\ u(x, \mathcal{J}, t) &= e^{x+\mathcal{J}-2t}. \end{aligned} \tag{41}$$

In Fig. 5, we plot the approximate solution curve for the two Eq. (37) when ($\ltimes = 0.8, 0.9, 1$) In Fig. 6, the surface solution for the two Eq. (37) and is represented when ($\ltimes = 0.8, 0.9, 1$). In Table 3, we have evaluated the numerical values of the approximate solution $u(x, \mathcal{J}, t)$ for the problem (37) obtained using the (YHJM) method and compared them with the exact solution given in Eq. (41) through different values of $(x, \mathcal{J}, t)$ when ($\ltimes = 0.8, 0.9, 1$).

Instance 4. *Consider the time – fractional 3D nonlinear telegraph equation*

$$\frac{\partial^{2\ltimes} u}{\partial t^{2\ltimes}} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} + \frac{\partial^2 u}{\partial \mathbb{Z}^2} + \frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}, \qquad 0 < \ltimes \leq 1 \tag{42}$$

With, $u(x, \mathcal{J}, \mathbb{Z}, 0) = e^{x-\mathcal{J}-\mathbb{Z}}$, $u_t(x, \mathcal{J}, \mathbb{Z}, 0) = -e^{x-\mathcal{J}-\mathbb{Z}}$.

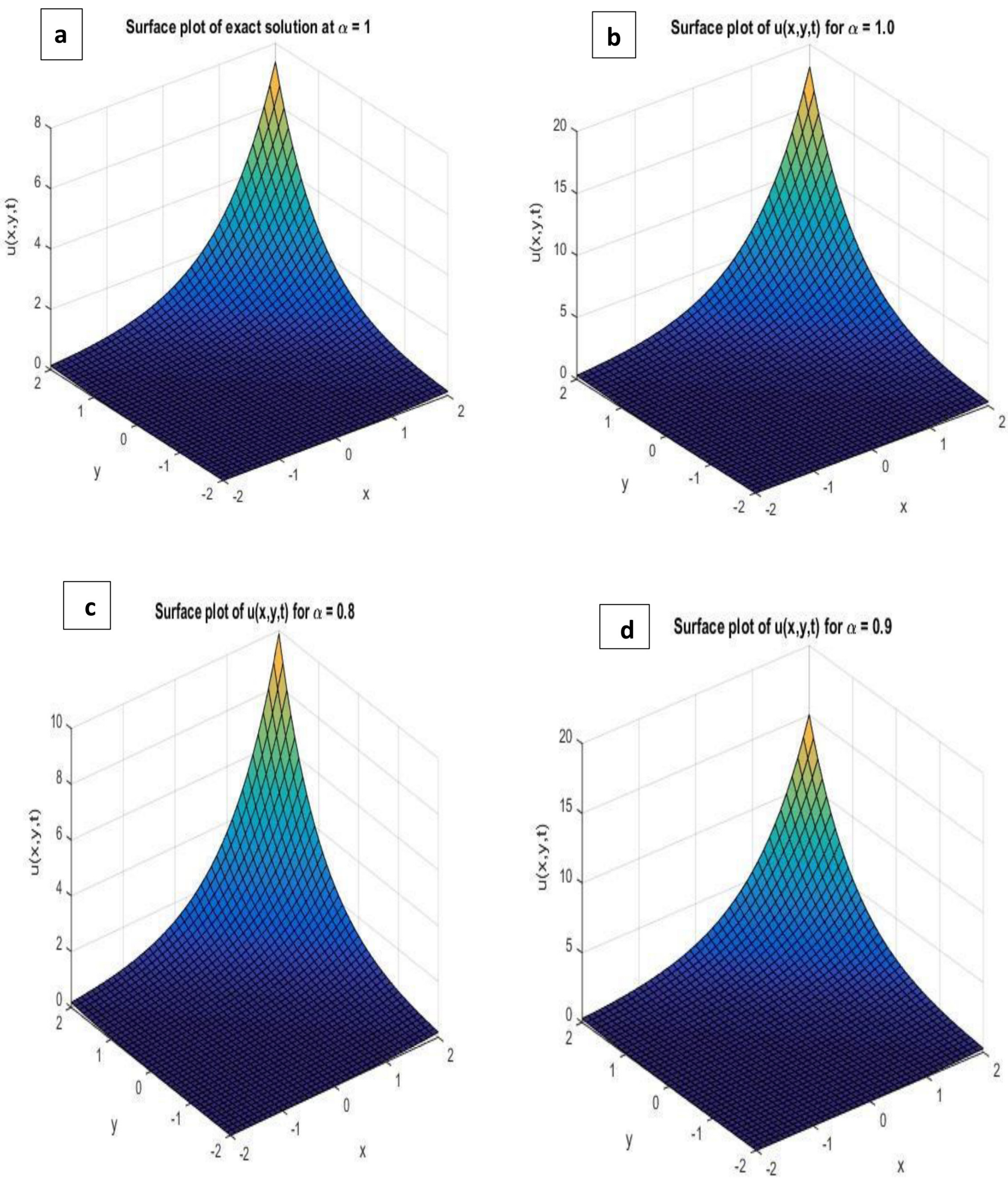

Fig. 6. The surface graph of the approximate solution $u\,(x, \mathcal{J}, t)$ of Eq. (41); (a) $u\,(x, \mathcal{J}, t)$ when exact solution, (b) $u\,(x, \mathcal{J}, t)$ when $\alpha = 1$, (c) $u\,(x, \mathcal{J}, t)$ when $\alpha = 0.8$ (d) $u\,(x, \mathcal{J}, t)$ when $\alpha = 0.9$.

Solution 4. *Applying the $Y_a T$ transform and Differentiating Both Sides of Eq. (42) with respect to $\mathfrak{x}$, we obtain*

$$\frac{Y_a\{u\,(x, \mathcal{J}, \mathbb{Z}, t)\}}{v^{2\ltimes}} - \frac{u\,(\mathfrak{x}, \mathcal{J}, \mathbb{Z}, 0)}{v^{2\ltimes-1}} - \frac{u_t\,(\mathfrak{x}, \mathcal{J}, \mathbb{Z}, 0)}{v^{2\ltimes-2}} = Y_a\left[\frac{\partial^2 u}{\partial \mathfrak{x}^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} + \frac{\partial^2 u}{\partial \mathbb{Z}^2} + \frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}\right]. \tag{43}$$

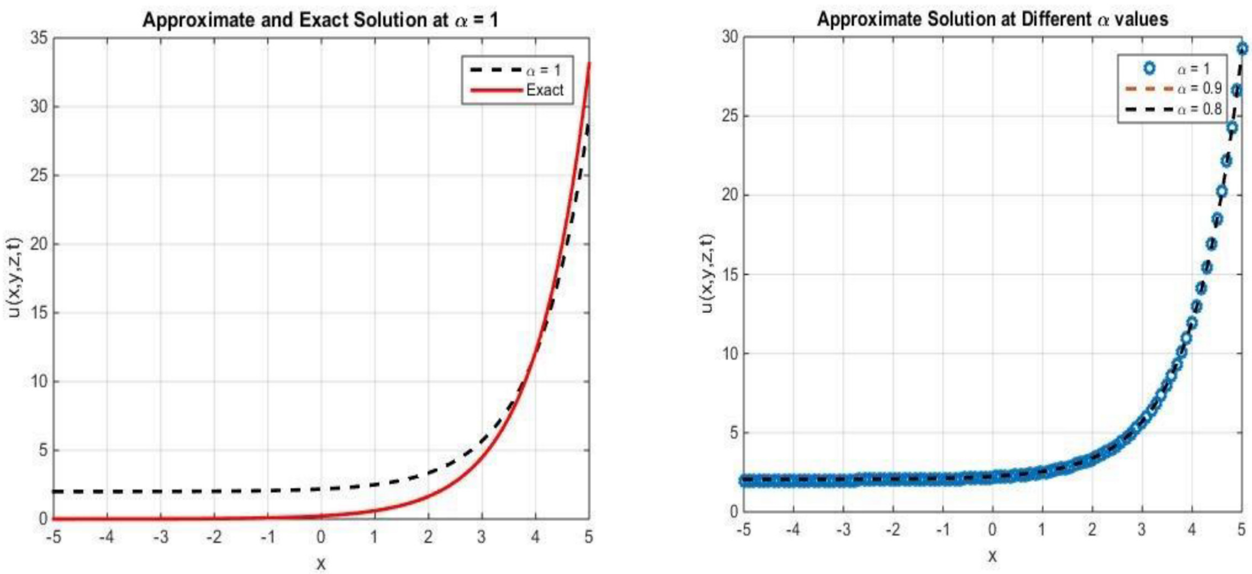

Fig. 7. Graphs of the approximate solution $u\,(x, \mathcal{J}, t)$ for various values of $\propto$ $alpha\alpha$ while keeping x constant.

$$\Upsilon_a\left\{u\,(x,\mathcal{J},\mathbb{Z},t)\right\} = v\;u\,(x,\mathcal{J},\mathbb{Z},0) + v^2\;u_t\,(x,\mathcal{J},\mathbb{Z},0) + v^{2\ltimes}\Upsilon_a\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} + \frac{\partial^2 u}{\partial \mathbb{Z}^2} - u^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}\right]. \tag{44}$$

Taking the invers $\Upsilon_a T$ of (44) yields

$$u\,(x,\mathcal{J},\mathbb{Z},t) = e^{x-\mathcal{J}-\mathbb{Z}} - \frac{t^{\ltimes}}{\Gamma(\ltimes+1)}\,e^{x-\mathcal{J}-\mathbb{Z}} + \Upsilon_a^{-1}\left\{v^{2\ltimes}\Upsilon_a\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \mathcal{J}^2} + \frac{\partial^2 u}{\partial \mathbb{Z}^2} + \frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}\right]\right\}. \tag{45}$$

Applying an algorithm (HJ) (45) It can be articulated as follows

$$\sum_i^{\infty} u_i = \sum_i^{\infty} \frac{D_t^{i\ltimes}\left\{e^{x-\mathcal{J}-\mathbb{Z}} - \frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x-\mathcal{J}-\mathbb{Z}} + \Upsilon_a^{-1}\left\{v^{2\ltimes}\Upsilon_a\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial \mathcal{J}^2}+\frac{\partial^2 u}{\partial \mathbb{Z}^2}+\frac{\partial^{\ltimes} u}{\partial t^{\ltimes}} - u^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}\right]\right\}\right\}t=0}{\Gamma(i\ltimes+1)}\,t^{i\ltimes}.$$

$$u_0\,(x,\mathcal{J},\mathbb{Z},t) = u\,(x,\mathcal{J},\mathbb{Z},0) = e^{x-\mathcal{J}-\mathbb{Z}}.$$

$$u_1\,(x,\mathcal{J},\mathbb{Z},t) = \frac{t^{\ltimes}}{\Gamma(\ltimes+1)}u_t\,(x,\mathcal{J},\mathbb{Z},0) = -\frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x-\mathcal{J}-\mathbb{Z}}.$$

$$u_2\,(x,\mathcal{J},\mathbb{Z},t) = \frac{D_t^{2\ltimes}\left\{e^{x-\mathcal{J}-\mathbb{Z}} - \frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x-\mathcal{J}-\mathbb{Z}} + \Upsilon_a^{-1}\left\{v^{2\ltimes}\Upsilon_a\left[\frac{\partial^2(u_0+u_1)}{\partial x^2}+\frac{\partial^2(u_0+u_1)}{\partial \mathcal{J}^2}+\frac{\partial^2(u_0+u_1)}{\partial \mathbb{Z}^2} - 2\frac{\partial^{\ltimes}(u_0+u_1)}{\partial t^{\ltimes}} - (u_0+u_1)^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}\right]\right\}\right\}t=0}{\Gamma(2\ltimes+1)}\,t^{2\ltimes}$$

$$u_2\,(x,\mathcal{J},\mathbb{Z},t) = \frac{t^{2\ltimes}}{\Gamma(2\ltimes+1)}e^{x-\mathcal{J}-\mathbb{Z}}.$$

$$u_3\,(x,\mathcal{J},\mathbb{Z},t) = \frac{D_t^{3\ltimes}\{e^{x-\mathcal{J}-\mathbb{Z}} - \frac{t^{\ltimes}}{\Gamma(\ltimes+1)}e^{x-\mathcal{J}-\mathbb{Z}} + \Upsilon_a^{-1}\{v^{2\ltimes}\Upsilon_a[\frac{\partial^2(u_0+u_1+u_2)}{\partial x^2}+\frac{\partial^2(u_0+u_1+u_2)}{\partial \mathcal{J}^2}+\frac{\partial^2(u_0+u_1+u_2)}{\partial \mathbb{Z}^2} - 2\frac{\partial^{\ltimes}(u_0+u_1+u_2)}{\partial t^{\ltimes}} - (u_0+u_1+u_2)^2 + e^{2(x-\mathcal{J}-\mathbb{Z})-2t} - e^{(x-\mathcal{J}-\mathbb{Z})-t}]\}\}t=0}{\Gamma(3\ltimes+1)} \times t^{3\ltimes}$$

$$u_3\,(x,\mathcal{J},\mathbb{Z},t) = -\frac{t^{3\ltimes}}{\Gamma(3\ltimes+1)}e^{x-\mathcal{J}-\mathbb{Z}}.$$

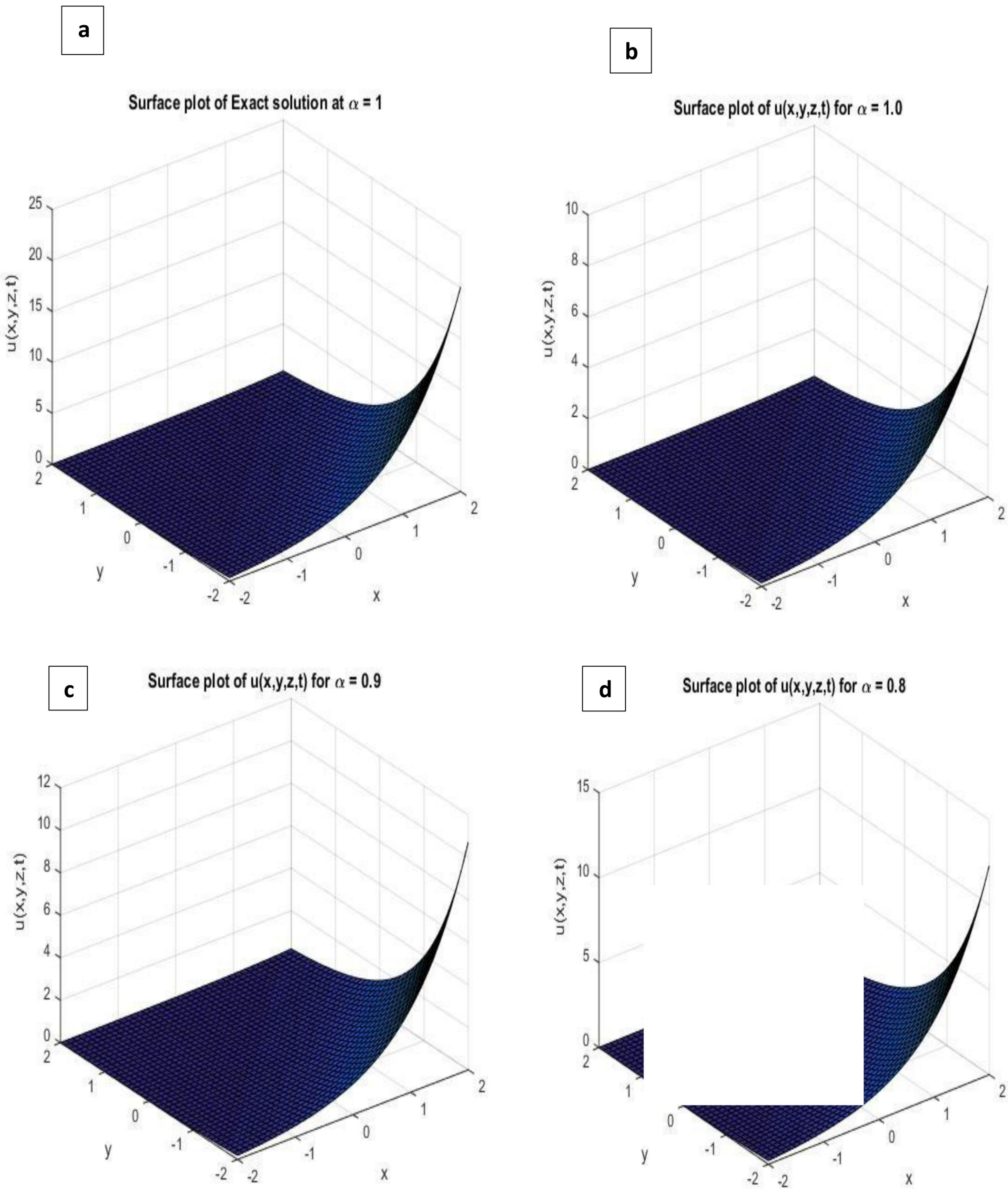

Fig. 8. The surface graph of the approximate solution $u\,(x, \mathcal{J}, t)$ of Eq. (47); (a) $u\,(x, \mathcal{J}, t)$ when exact solution, (b) $u\,(x, \mathcal{J}, t)$ when $\alpha = 1$, (c) $u\,(x, \mathcal{J}, t)$ when $\alpha = 0.8$ (d) $u\,(x, \mathcal{J}, t)$ when $\alpha = 0.9$.

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$$u\,(x, \mathcal{J}, \mathbb{Z}, t) = \sum_{i}^{\infty} u_i = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots\cdots . \tag{46}$$

$$u\,(x, \mathcal{J}, \mathbb{Z}, t) = e^{x - \mathcal{J} - \mathbb{Z}}\left[1 - \frac{t^{\varkappa}}{\Gamma(\varkappa+1)} + \frac{t^{2\varkappa}}{\Gamma(2\varkappa+1)} - \frac{t^{3\varkappa}}{\Gamma(3\varkappa+1)} + \cdots\cdots\cdots\right].$$

Table 4. Quantitative results of the approximate and exact solutions for various values of x and t when $\ltimes = 0.8, 0.9, 1$.

x	$\ltimes = 1$	$\ltimes = 0.9$	$\ltimes = 0.8$	$\ltimes = 0.7$
−2	1.8854e-09	0.012241	0.026654	0.042958
−1.7778	2.3545e-09	0.015287	0.033286	0.053648
−1.5556	2.9404e-09	0.019091	0.04157	0.066998
−1.3333	3.6722e-09	0.023841	0.051914	0.08367
−1.1111	4.586e-09	0.029774	0.064833	0.10449
−0.88889	5.7272e-09	0.037184	0.080967	0.13049
−0.66667	7.1524e-09	0.046437	0.10112	0.16297
−0.44444	8.9323e-09	0.057992	0.12628	0.20352
−0.22222	1.1155e-0.8	0.072424	0.1577	0.25417
0	1.3931e-0.8	0.090446	0.19695	0.31742

When $\ltimes = 1$

$$u(x, \mathcal{J}, \mathbb{Z}, t) = e^{x-\mathcal{J}-\mathbb{Z}}\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right)$$
$$u(x, \mathcal{J}, \mathbb{Z}, t) = e^{x-\mathcal{J}-\mathbb{Z}-t}. \tag{47}$$

In Fig. 7, we plot the approximate solution curve for the two Eq. (42) when ($\ltimes = 0.8, 0.9, 1$) In Fig. 8, the surface solution for the two Eq. (42) and is represented when ($\ltimes = 0.8, 0.9, 1$). In Table 4, we have evaluated the numerical values of the approximate solution $u(x, \mathcal{J}, Z, t)$ for the problem (42) obtained using the (YHJM) method and compared them with the exact solution given in Eq. (47) through different values of $(x, \mathcal{J}, Z, t)$ when ($\ltimes = 0.8, 0.9, 1$).

7. Conclusion

This paper introduces a new method, named Yang Hussain Jassim method, for solving fractional time-telegraph equations in the half-space domain. The Caputo derivative was used along with the Yang transform for both the time and space components. The solutions were expressed in the form of a series that converges rapidly to a closed-form solution with straightforward and easy-to-calculate terms. The calculations were direct and uncomplicated. The methodology was validated using six distinct examples, including one-dimensional linear and nonlinear examples, as well as two-dimensional and three-dimensional examples under various conditions. The approach has proven to be effective, reliable, and efficient, and can be adapted to address both linear and nonlinear fractional problems in applied sciences. This method contributed to: Accuracy of solutions: The Hussain Jassim method allows for finding accurate and effective solutions to linear and nonlinear differential equations, enhancing the ability of researchers and engineers to tackle complex problems. Reducing computational complexity: This method helps reduce computational effort compared to traditional methods, allowing solutions to be obtained in a shorter amount of time. Various applications: The Hussain Jassim method can be applied in numerous scientific and engineering fields, such as mechanics, physics, electricity, and other applications requiring the solving of differential equations. The Hussain Jassim method is part of ongoing developments in the mathematical field aimed at improving methods used to address challenging mathematical problems, and continues to provide promising solutions across a wide range of applications.

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Ethical approval

This study was conducted in accordance with the general ethical principles established in the field of scientific research, and all recognized ethical standards have been adhered to in this context.

Conflict of interest

The authors declare no conflict of interest.

Data availability

The data that support the findings of this study are available on request from the corresponding author.

Author contribution

All authors contributed equally to the design and execution of the study, data analysis, and writing of the final draft of the manuscript. All authors reviewed and edited the text and approved the final version.

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