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NONLINEAR ANALYSIS OF R.C. BEAMS STRENGTHENED USING PRE-STRESSED NSM CFRP RODS

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Abstract

A nonlinear finite element study has been conducted to investigate the effectiveness of strengthening RC beams with pre-stressed NSM CFRP rods in enhancing the flexural strength of the beams. Four beams have been considered. One acted as control to simulate an existing structural member. The other beams were strengthened with non-pre-stressed CFRP rod (0% pre-stressed), and pre-stressed CFRP rod (40%, or 60% pre-stressed of the CFRP rod tensile strength).

The results of analysis showed that strengthening of the RC beams with non pre-stressed NSM CFRP rods improved both of the flexural capacity and deflection response. Increases in the cracking, yield and ultimate load of 17%, 20% and 48% were achieved. With a pre-stress level of 40%, an enhancement of 117%, 53% and 80% respectively over that of the control beam. For the beam strengthened with pre-stress level of 60%, more enhancements in response were achieved in term of the cracking and yield loads, but a slight reduction in ultimate loads (compared with lower level of pre-stress). Increments up to 160% and 67% in the cracking and yield load respectively. While 77% in ultimate load over that of the control beam were obtained.

Keywords: RC Beams, NSM CFRP Rods, Strengthening, Non-Pre-Stress, Pre-Stress, Cracking, Yielding, Ultimate Strength.

التحليل اللاخطي للعتبات الخرسانية المسلحة المقواة باستخدام قضبان الياف الكاربون والمزروعة قرب سطح الخرسانة

د.قاسم محمد شاكر/جامعة الكوفة /كلية الهندسة/القسم المدني

الخلاصة

تضمن البحث إجراء دراسة نظرية لاخطية الهدف منها التعرف على مدى فاعلية أسلوب تقوية العتبات الخرسانية المسلحة باستخدام قضبان CFRP المسبقة الجهد و المزروعة قرب سطح الخرسانة في تحسين مقاومة الانثناء فيها. ولأجل ذلك تم تحليل أربعة عتبات ،احدها لم يتم تقويته ليمثل الحالة الواقعية للعتبات المزمع تقويتها وتم اعتباره المرجعي لغرض المقارنة .أما الثلاث الأخرى فقد تم تقويتها بقضبان CFRP المسبقة الجهد وبمستويات إجهاد (كنسبة من تحمل الشد لها) هي 0% و 40% و60%على التوالي. لقد بينت نتائج الدراسة أن أسلوب التقوية بدون إجهاد مسبق أدى إلى تحسن ملحوظ في أداء العتبة .إذ لوحظ زيادة أحمال التشقق و الخضوع والحمل الأقصى بنسب 17% و 20% و 48% على التوالي مقارنة مع النتائج المستحصلة من تحليل العتبة المرجعية. أما عندما كان مستوى الإجهاد المسلط على عناصر CFRP بمقدار 40% فقد وجد أن هناك تحسن اكبر في أداء العتبة الخرسانية المسلحة, حيث وجد أن النسب أعلاه قد ازدادت إلى 117%و 80% و53% على التوالي.أما العتبة التي تم تقويتها باستخدام عناصر CFRP المسبقة الجهد بمستوى جهد 60% فقد لوحظ زيادة أعلى في كل من حمل التشقق و الخضوع مع نقصان بسيط في التحمل الأقصى فكانت النتائج بمقدار 160% و67% و77% على التوالي .

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1. Introduction

Rehabilitation and strengthening of reinforced concrete structures are major challenges facing structural engineers. In general, reinforced concrete (and steel) structure may need to be strengthened due to change in use, demand for increases in load carrying capacity, corrosion in existing structures, environmental degradation or to improve fatigue performance**[*Rosenboom* et al,2006]**. Strengthening structural elements with Fiber Reinforced Polymer (FRP) plates has become a popular alternative to traditional techniques such as installing steel plates or external pre-stressing cables. They are relatively cheap, easy to handle and work with, effective and not subjected to corrosion **[*Badawi,2007*].**

The use of the FRP can be either as internal reinforcement for a new construction, or as surface or near surface mounted (NSM) reinforcement (rod/strip) for strengthening an existing reinforced concrete (RC) structure. Recently, using FRP as NSM is considered to be a promising method for strengthening and rehabilitating RC structures **[*Taljsten,2006,Badawi,2007*].**

The advantages of using NSM FRP strengthening compared to other FRP strengthening techniques are numerous. In the NSM technique, the FRP is typically embedded in a pre-cut groove in the concrete structural member and bonded by epoxy, which protects the FRP material from any physical impact or vandalism. In strengthening of the negative moment region of a continuous slab, for example where the surface may be exposed to physical and environmental damage, the NSM technique does not require a protection for the FRP because it is embedded in epoxy, whereas for externally bonded FRP, a protection is needed **[*Nordin*,2003,*EI-Hacha* et al, *Emadi,J.* et al,2014]**.

Pre-stressing of the strengthening FRP materials has many advantages. It provides a better utilization of the FRP reinforcement, reduces the stress in the internal steel reinforcement, and increases the yield load of a beam. It also decreases the crack width size and the mean crack spacing resulting in more durable structures. Several studies have shown an increase in the yield load of 50% compared to un-strengthened beams and up to 25% compared to beams strengthened with non-pre-stressed strengthening materials **[*Nordin* et al, 2001 , *Yue-lin* et al, 2005,*Omran* et al ,*2012*]**.

2. Material Constitutive Relations

2.1 Concrete in Compression

The behavior of concrete in compression can be simulated in ANSYS program by an elasto-plastic work hardening model followed by a perfectly plastic response that is terminated at the onset of crushing.

The compressive uniaxial stress-strain curve, **Figure 1**, for concrete model was obtained using the following equations to compute the multi-linear isotropic stress-strain curve for the concrete.

$$f_c = \varepsilon \ E_c \quad \text{for } 0 \le \varepsilon \le \varepsilon_1 \text{.................................eq.(1)}$$

$$f_c = \frac{\varepsilon \, E_c}{1+\left(\varepsilon/\varepsilon_0\right)^2} \quad \text{for } \varepsilon_1 \le \varepsilon \le \varepsilon_\circ \text{.........................eq.(2)}$$

$$f_c = f_c^{'} \quad \text{for } \varepsilon_\circ \le \varepsilon \le \varepsilon_{cu} \text{..................................eq.(3)}$$

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and

$\varepsilon_1 = 0.3 f_c^{'} / (E_c)$ (Hooke' s law) ....eq.(4)

$\varepsilon_{\circ} = 2 f_c^{'} / (E_c)$ ....eq.(5)

2.2 Tensile Behavior of Concrete

Until the crack, initial tangent modulus E_c is used to find the maximum positive (tensile) stress. After concrete cracking takes place, a smeared model is used to represent the discontinuous macro crack behavior. This cracked concrete can still carry some tensile stress perpendicular to the crack, which is termed tension stiffening. The tension stiffening factor (α_m), **Figure 2**, was assumed 0.6. In this work, a simple descending line is used to model this tension stiffening phenomenon.

2.3 Modeling of Crack

In concrete, when the tensile stress in the principal direction exceeds the tensile strength, $f`_t$, of concrete, the tensile failure would occur **[*Desai* et al,*2002*]**. After the crack forms, both normal and shear stiffness are reduced.

2.4 Steel Reinforcing Bars

The stress-strain curve of the reinforcing bars is assumed to be elastic up to the steel yield stress (f_y) followed by linear hardening up to the steel ultimate strength (f_u) as shown in Figure 3. The dowel action of the reinforcing steel is neglected and the bond between steel and concrete is assumed to remain perfect.

3. Specimen Configurations

Four beams were tested by **Badawi**, one non strengthened while the three others were strengthened with prestressed NSM CFRP rods with 0%,40% and 60% of the tensile strength of the rod respectively **[*Badawi,2007*]**. All beams had a total length of 3500 mm with a clear span of 3300 mm as shown in Figure 4. The dimensions of the cross section were 152 mm wide by 254 mm deep. They were reinforced with 2 ϕ 15 rebars as bottom reinforcement and 2 ϕ 10 as top reinforcement. ϕ8@ 75 mm stirrups and a typical concrete cover of 30 mm were used. For the strengthened beams, a groove of 15×25 mm was cut into the bottom face of beams to allow the placement of the CFRP rod in the beams as near surface mounted (NSM) reinforcement. The CFRP strengthening reinforcement was in all cases, 1ϕ 9.5 mm CFRP rod placed at the centre of the NSM groove. The properties of concrete and steel obtained from the experimental tests conducted by **Badawi [*Badawi,2007*]**, are summarized in **Table 1**. Also, prestress force details are listed in **Table 2**.

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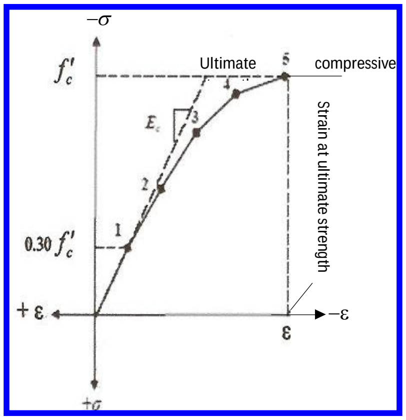

Fig. 1 Uniaxial Stress-Strain Curve for Concrete

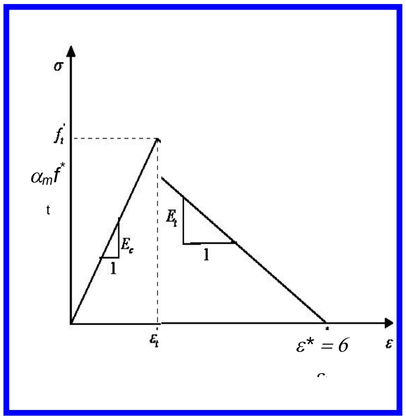

Fig. 2 Tension Stiffening Model

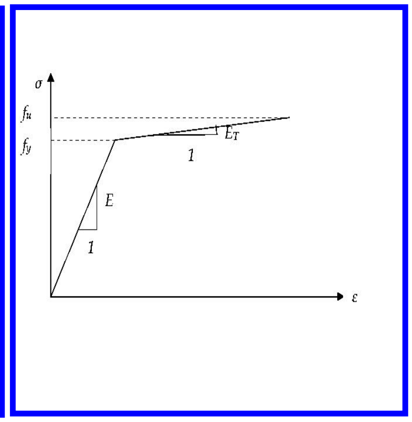

Fig. 3 Uniaxial Stress-Strain Relation for Steel

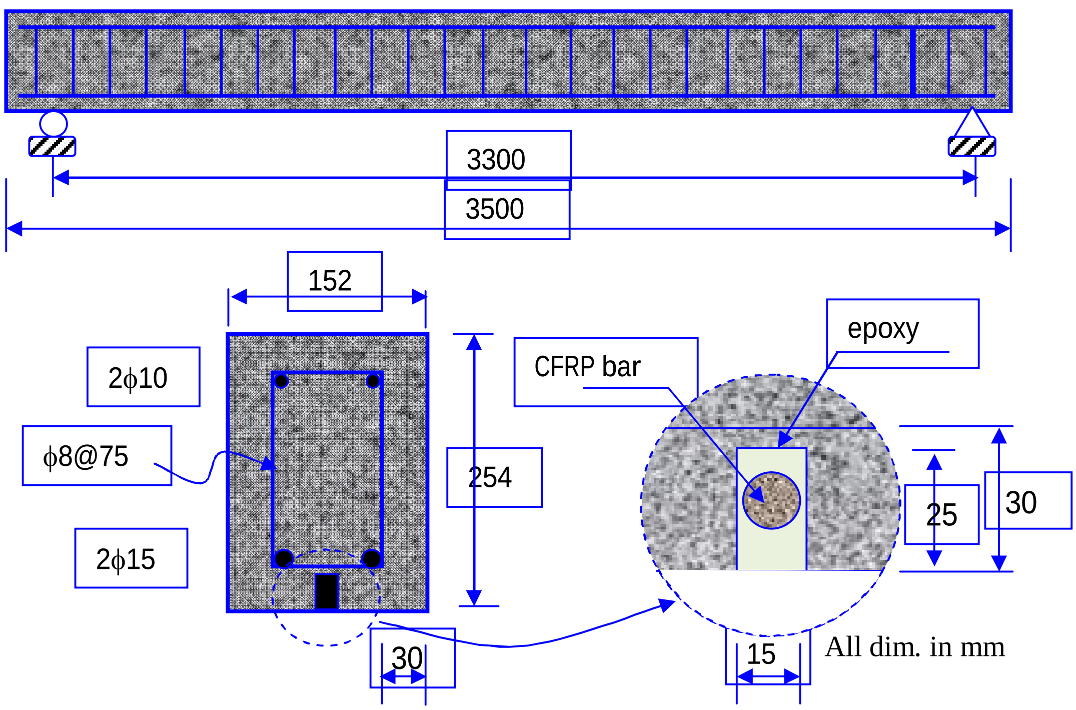

Fig. 4 Specimen Details

Table 1 Material Properties [Badawi,2007]

Material	Strength (MPa)	Modulus of Elasticity (GPa)
CFRP rods tensile strength	1970	136
All steel bars tensile strength	440	190
Epoxy	24.8	2.690
Concrete cylinder compressive strength	40	29.700

Table 2 Pre-Stress Force Details

Beam Designation	B_0	B_1	B_2	B_3
Pre-stress level	Non strengthened	0%	40%	60%
Pre-stress force (kN)	Control	0	55	80

4. Ansys Finite Element Model

The FEA study included modeling a RC beam strengthened with NSM CFRP rods with different level of pre-stressing force with the dimensions and properties as previously mentioned.

4.1 Element Types

A solid element, SOLID 65 **(Figure 5)** is used to model the concrete and the epoxy groove (when considered) in ANSYS. The solid element has eight nodes with three degrees of freedom at each node, translations in the nodal *x*, *y*, and *z* directions. The element is capable of plastic deformation, and cracking in three orthogonal directions. A LINK8 **(Figure 6)** element is used to model the steel reinforcement and NSM CFRP rods. Two nodes are required for this element. At each node, degrees of freedom are identical to those for the SOLID 65. The element is also capable of plastic deformation.

A solid element, SOLID 45 **(Figure 7)** is used to model the loading and support plates.

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also eight nodes with three degrees of freedom at each node, translations in the nodal x, y, and z directions.

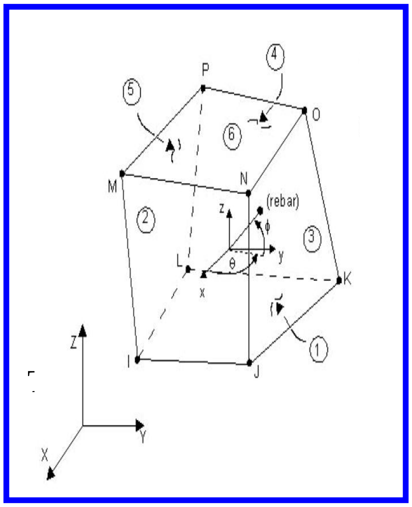

Fig. 5 Solid65 Element [SAS ANSYS 11,2005]

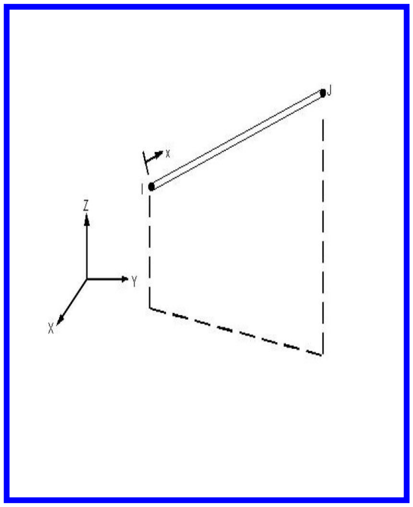

Fig. 6 Solid8 Element[SAS ANSYS 11,2005]

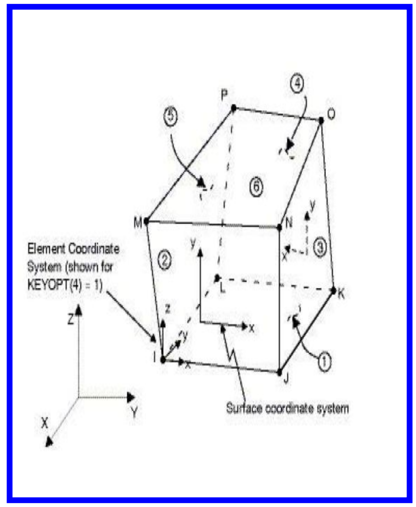

Fig. 7 Solid 45 Element [SAS ANSYS 11.2005]

4.2 *Real Constants*

The real constants needed for this model are shown in **Table 3**. It can be noted that the individual elements may contain different real constants. No real constant set exists for the Solid 45 element. All initial strain values are taken to be zero and pre-stressing effects are applied as axial forces subjected on the CFRP elements.

Table 3 Real Constant Typical Model.

Real constant set	*Element type*		*Real Constants*		
1 (concrete)	Solid 65	Material number ,Volume Ratios, and Orientation Angles	For Rebar1	for Rebar 2	for Rebar 3
6 (Epoxy Groove)			0	0	0
2(bottom reinforcement)	Link 8	Cross-sec.area (mm^2)	176.71		
3 (top reinforcement)	Link 8	Cross-sec.area (mm^2)	78.54		
4 (stirrups)	Link 8	Cross-sec.area (mm^2)	50.265		
5(CFRP rods)	Link 8	Cross-sec.area (mm^2)	35.44		

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4.3 Material Properties

Parameters needed to define the material models are listed in **Table 4**. The first material model refers to the Solid 65 element. It requires linear isotropic and multi-linear isotropic material properties to properly model concrete.

Table 4 Material Models for Typical Model

Material Model Number	*Element Type*	*Material Properties*						
1	Solid65 (Concrete)	*Linear Isotropic*		*Multilinear Isotropic*			*Concrete*	
		EX	30041 MPa	Point	Strain	Stress (MPa)	ShrCf-Op	0.6
		PRXY	0.2	Point1	0.000444	13.33	ShrCf-Cl	0.9
				Point2	0.000888	24	UnTensS	3.8
				Point3	0.001331	32	UnComp	40
				Point4	0.00775	36.92	BiCompS	0
				Point5	0.002219	39.34	HydroPrs	0
				Point6	0.002663	40	BiCompS	0
				ft=(0.09-0.12)f'c= taken in ANSYS as 0.095f'c,not given by **Badawi			nTensrFac	0
							St TenC	0.6
2	Link8 (Steel Bars)	Linear Isotropic		Bilinear Isotropic				
		EX	190E5 MPa	Yield Stress	440 MPa			
		PRXY	0.3	Tang Mod	11000 MPa			
3	Solid 45 (Loading Plate/SuP port)	Linear Isotropic						
		EX	190E5 MPa					
		PRXY	0.3					
4	Link8 (CFRP bars)	Linear Isotropic		Bilinear Isotropic				
		EX	1360E 5	Yield Stress	1970 Mpa			
		PRXY	0.2	Tang Mod	500MPa			
5	Solid65 (epoxy)	Linear Isotropic		Bilinear Isotropic				
		EX	2690 MPa	Yield Stress	24.8 MPa			
		PRXY	0.3	Tang Mod	10 MPa			

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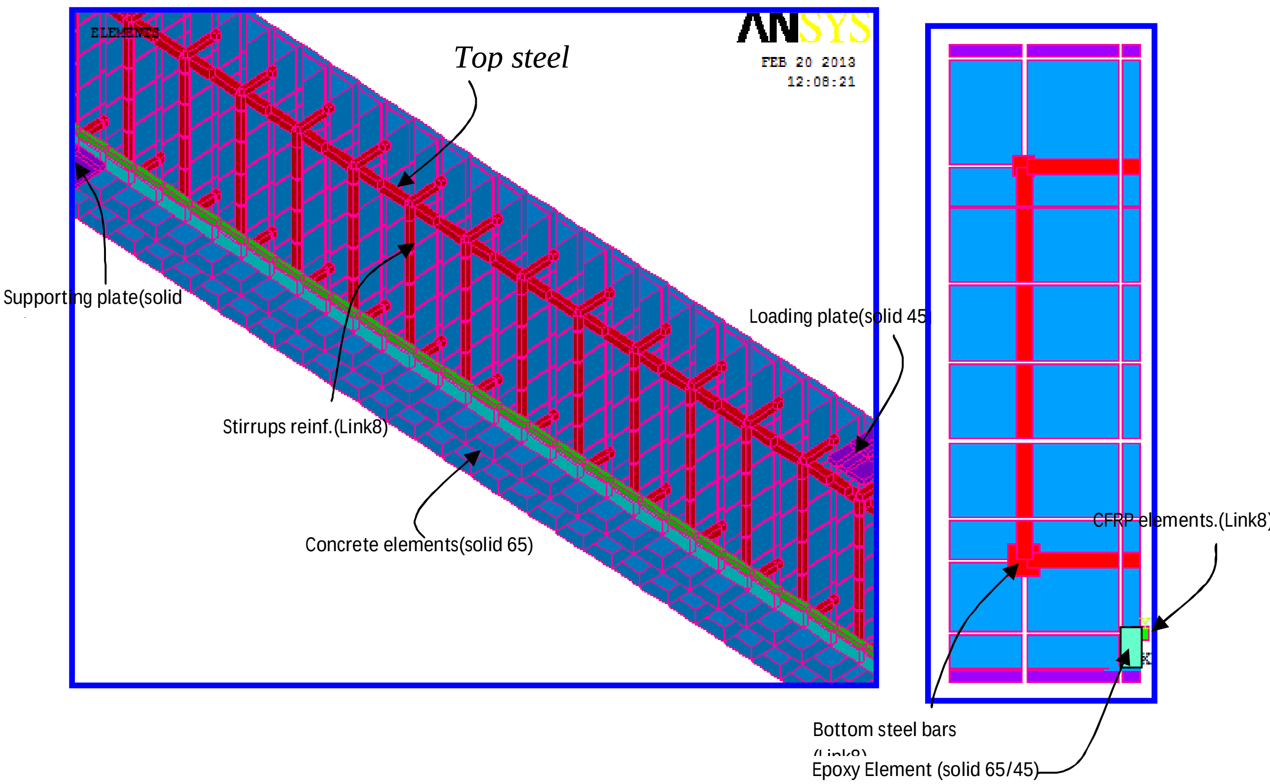
The multi-linear isotropic material uses the Von Mises failure criterion along with the **William and Warnke [William, Wranke,1974]** model to define the failure of the concrete. The compressive uniaxial stress-strain relationship for the concrete model was obtained using the equations (1) to (5) to compute the multilinear isotropic stress-strain curve for concrete.

Material Model Number 2 and 4 refer to the Link8 assumed to be bilinear isotropic material based on the Von-Mises failure criteria. Material Model Number 3 refers to the solid45 that is modeled as a linear isotropic element. While, Material Model Number 5 refer to the solid65 that is assumed to be bilinear isotropic material.

4.4 Modeling Methodology

By taking advantage of the symmetry of the beams, a quarter of the full beams are used for modeling with proper boundary conditions, **(Figure 8)**. However, in this study, perfect bond between materials is assumed. The boundary conditions of symmetry are shown in **Figure 9**.



a) Isometric View **b) Cross Section**

Fig. 8 Mesh of Concrete, Steel Plate, and Steel *Support.*

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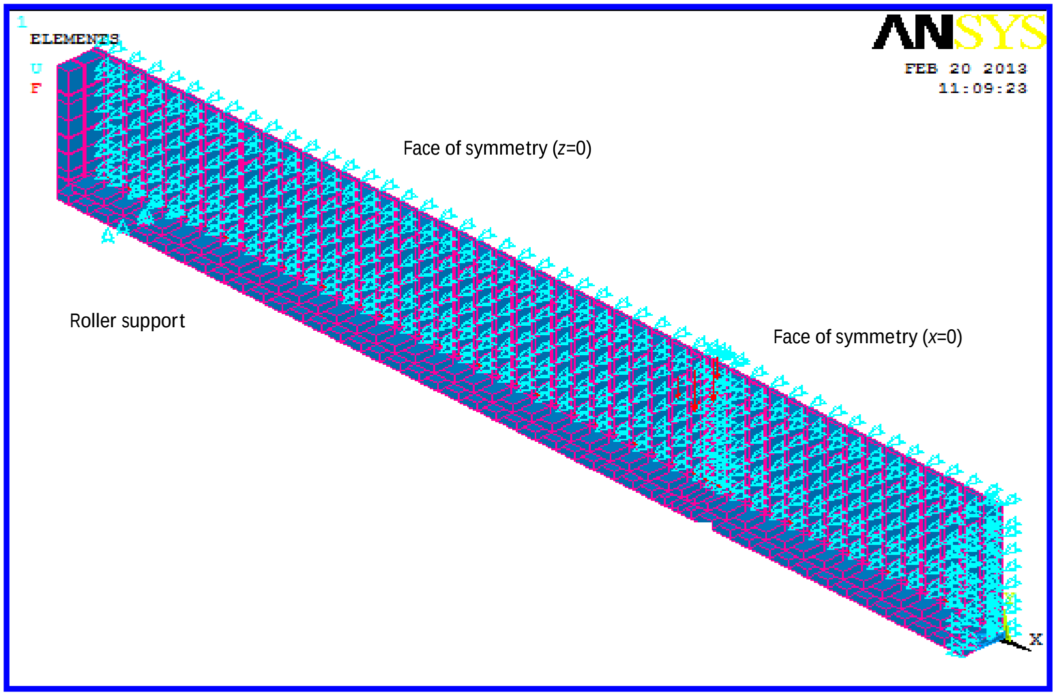

Fig. 9 Boundary Conditions.

5. Load-Deflection Relationship

The load-deflection curve for the control beam considered in the present work are shown in **Figure 10.** It is obvious that there is a good agreement with the experimental results done by **Badawi [Badawi,2007]**. The beams exhibited three regimes (pre-cracking, pre-yielding, and post-yielding stages) typical of RC beam.

Figure 11 and **Figure 12** show the load-deflection curves for the non-pre-stressed strengthened beam. It is obvious that there is a good agreement with the experimental results. The effect of including of epoxy groove in the nonlinear analysis is shown in **Figure 11**. It is clear that there is no significant effect of this parameter when the beam is non-pre-stressed. The effect of the type of element adopted to simulate the epoxy groove is shown in **Figure 12**. It can be released easily that solid65 element yields good results compared with solid 45 element.

Figures 13 and 14 shows the response of the CFRP strengthened beam with pre-stressing level of 40%. Generally, a good agreement with experimental results can be seen.The effect of considering the epoxy groove when simulation can be seen obviously in **Figure 13**. Also, the effect of the brick element type adopted to represent the epoxy groove is shown in **Figure 14** Result insures that solid 65 element is more efficient than solid 45 elements is such simulation.

Figure15 shows the load-deflection curves for the beam that strengthened with 60% pre stressed CFRP rod. Again a good agreement (more than 90%) can be seen between the finite element analysis and experimental tests.

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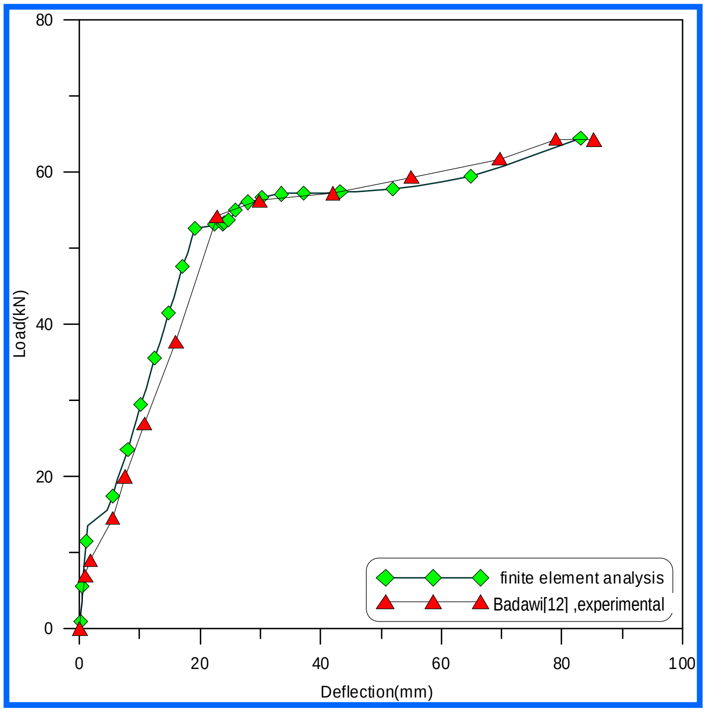

Fig. 10 Load–Deflection Curve for the Control Beam

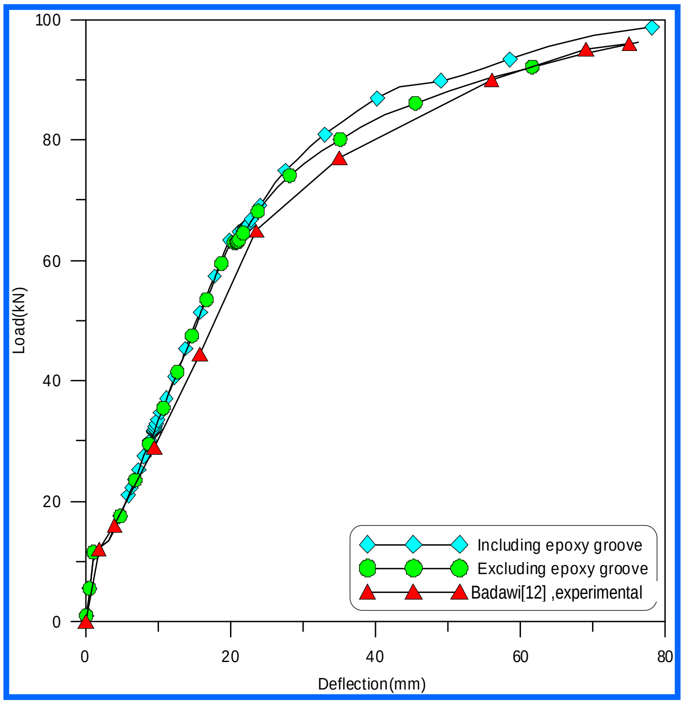

Fig. 11 Effect of Epoxy Groove on the Response for Beam B_1

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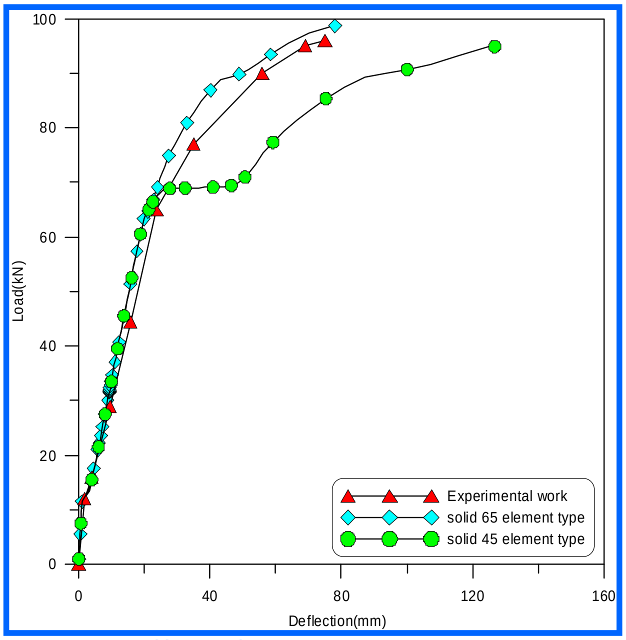

Fig.12 Effect of Epoxy Groove Simulation on the Response for Beam B_1.

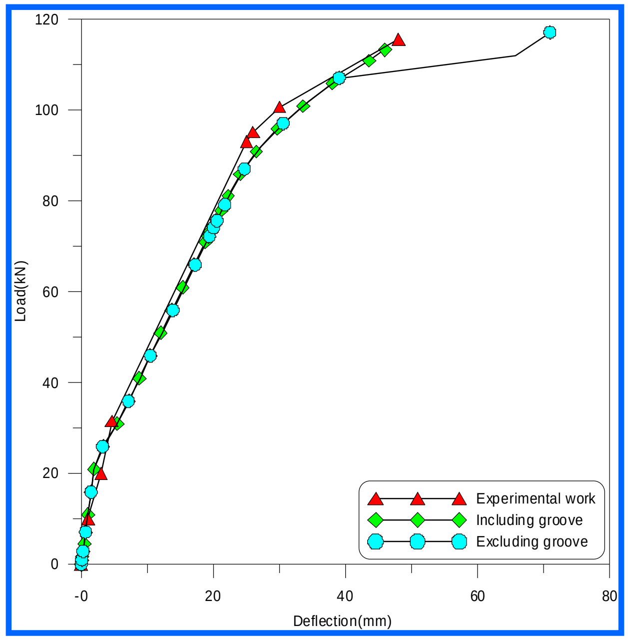

Fig.13 Effect of Epoxy Groove on the Response for Beam B_2.

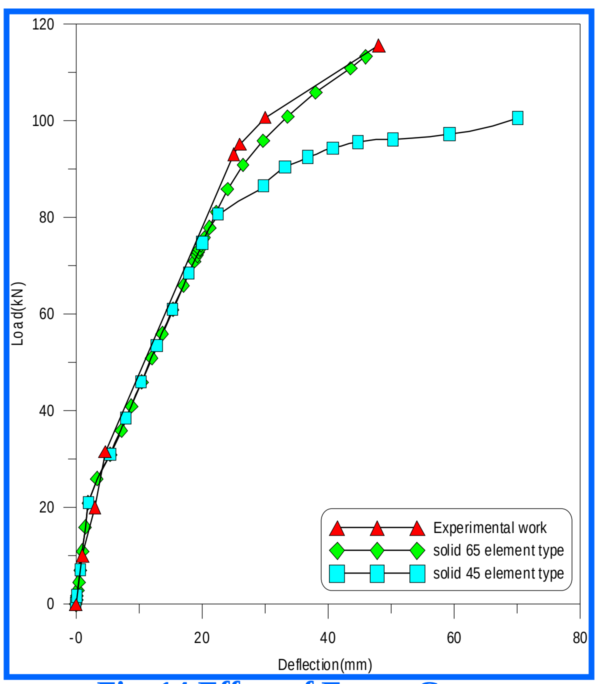

Fig. 14 Effect of Epoxy Groove Simulation on the Response for Beam B

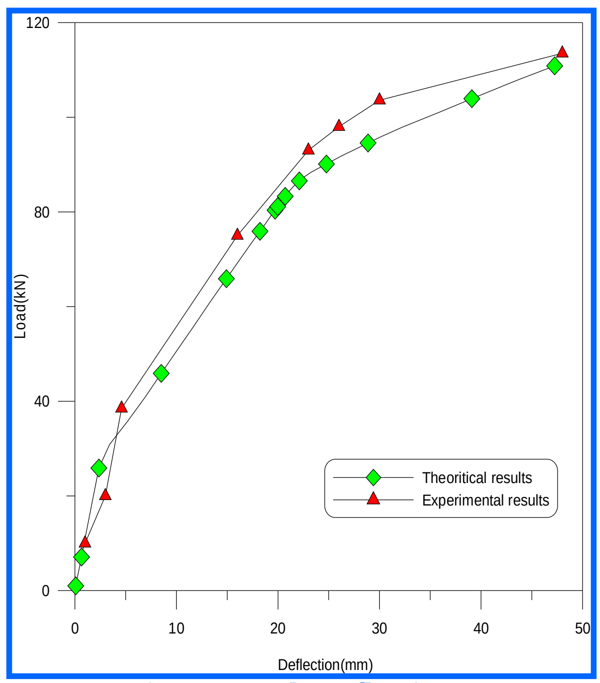

Fig. 15 Load- Deflection Curves for the Beam B_3.

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6. Cracking Load

Figure16 shows the cracking load variation with the different levels of pre-stressing. The results show a reasonably good agreement with experimental data. It is obvious, that when the beam was strengthened with non-prestressed (0% Pre-stressed) CFRP rod, an increase in the cracking load was obtained together with a small decrease in the corresponding deflection. The changes were about 17% and -1.5% for cracking load and deflection, respectively. A similar results (less value) for cracking load had been obtained by **Orman and El-Hacha [*Omran* et al,*2012*]** A considerable increase in the cracking load was obtained when the beam was strengthened with a pre-stressed CFRP rod. The increases were obtained to be 2.1 to 2.6 times for the cracking load of the control beam for the 40% and 60% pre-stress levels respectively. Pre-stressing of a CFRP strengthening rod increases the cracking loads, and gives narrower flexural crack widths and smaller deflection, which is advantageous in terms of the serviceability and durability of a structural element. The maximum allowable pre-stressing level is restricted to a 60% of the ultimate static capacity of the rod **[ACI Committee 440R,2007].** The main purpose of this limitation is to prevent a creep rupture in the CFRP rod. It is expected that, the combination of a higher cracking load and smaller crack widths would enhance the durability of the structure.

7. Yield Load

The yield load of a RC beam is the load causing yielding of the tension steel reinforcement. The control beam had a yield load of 55.1 kN at a mid-span deflection of 23.5 mm. When the beam was strengthened with non-pre-stressed CFRP rod, an increase of 26% in the yield load and an increased deflection of 26.03 mm at the mid-span section were observed. Increases of 72.4% and 90.6% of the yield load of the control beam were obtained for the 40% and 60% pre-stressed strengthened beams, respectively. **Figure 17** shows a good agreement between the experimental results and the FEA model. Also, it should be noted that the increase in the yield load is approximately linear with the applied pre-stress level **[Badawi,2007].**

8. Ultimate Load

The results of the ultimate load versus the level of prestressing is shown in **Figure18**. It can be seen that as pre-stress level increases, the ultimate load also increases. Up to a stress level of 60% that there a slight reduction in ultimate load ,this may be due to the severe spreading of cracks in the concrete elements around the epoxy groove leading to some reduction in prestress effect. Therefore it is expected that for high strength concrete, this reduction may be diminished.

9. Ductility Index

The use of externally bonded FRP reinforcement for flexural strengthening will reduce the ductility of the original member **[ACI Committee 440,2000,2002**] Also,as noted previously, a high pre-stressing level reduces the deflection at which the CFRP rod ruptures **[Badawi,2007]**. **Figure 19** shows the ductility index versus pre-stress level for the beams considered in study. The ductility index decreases continuously as the pre-stressing level increases. It can be seen that there is approximately linear relationship between the reduction of the ductility index of the strengthened beams and the pre-stressing level. Using the extrapolation technique, the ductility

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index will equal unity at the pre-stressing level of about 77% at which the yielding and ultimate loads are the same.

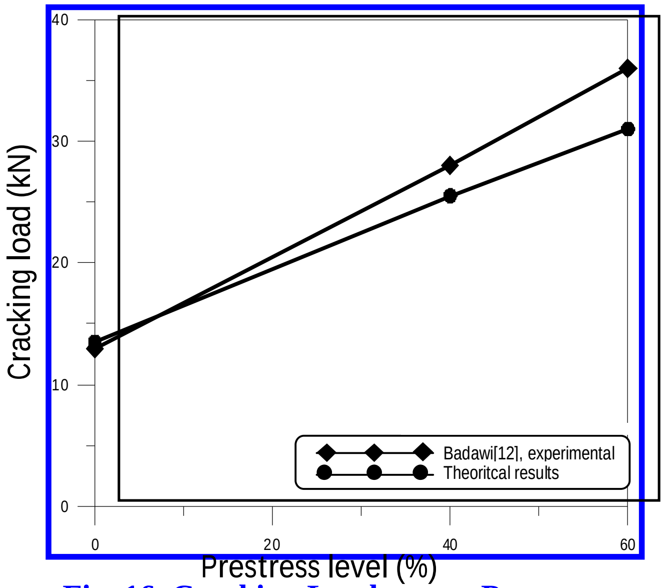

Fig. 16 Cracking Load versus Prestress.

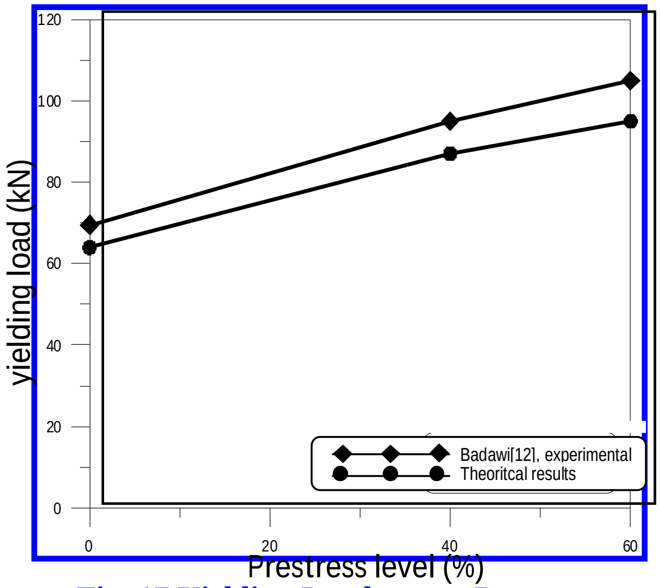

Fig. 17 Yielding Load versus Pestress.

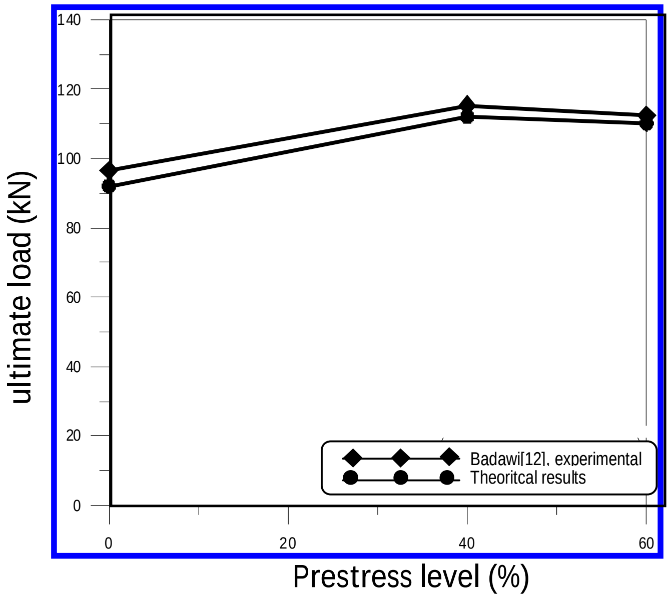

Fig 18 Ultimate Load versus Prestress

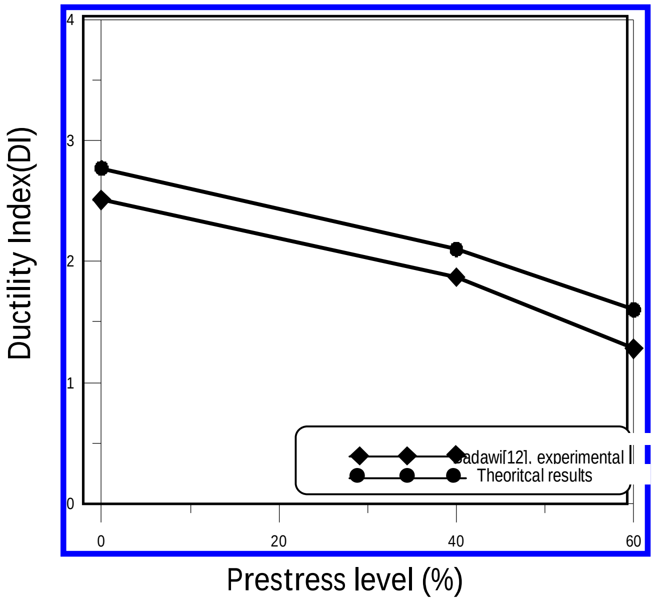

Fig.19 Ductility Index versus Prestress Level

10. Conclusions

Based on the finite element results, the main findings are:

1. The NSM technique is very effective in increasing the flexural capacity of a RC beam. With non-pre-stressed strengthened RC beam, a reasonable reduction in the ductility is obtained with respect to that of the control beam;i.e. for a prestress levels of 40% and 60% ,reduction of 18% and 38% were obtained.
2. Excluding the epoxy groove has no significant effect in low levels of pre-stressing, but this effect seems to be increased with increasing pre-stressing force. This conclusion can be accepted if one but in mind that epoxy has more tensile strength than concrete and neglecting it in high stress level results in more ductile behavior than the actual situation.
3. It is found that the solid65 element is more efficient in representing the epoxy groove than the solid45 element. This may be due to the similarity of behavior of solid epoxy to that of concrete more than metals as steel.
4. Pre-stressing the CFRP rod up to 40% of its capacity as NSM greatly increases the flexural performance in terms of cracking, yielding, and ultimate load. It also shows about 5% higher flexural strength than the 60% pre-stressed strengthened beam.
5. Pre-stressing the NSM CFRP rod up to 60% gives the greatest enhancement in the flexural behavior over all the other beams, but a small reduction in the ultimate capacity compared to that of 40% pre-stressed beam.
6. Using NSM CFRP rod for strengthening results in a reduction in the ductility index of the flexural element and pre-stressing further lowers this parameter.

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Notation

f_c :	stress at any strain ε, (MPa)
ε :	strain at stress f
ε_o :	strain at ultimate compressive stress $f`_c$
$E_c = 4750\sqrt{f'_c}$: ,	Concrete elastic modulus, taken as (MPa)
α_m:	The tension stiffening factor
$f`_t$:	tensile strength of concrete
f_y:	steel yield stress.
f_u:	steel ultimate strength.
FEA:	finite element analysis.
CFRP:	carbon fiber reinforced polymer.
NSM:	near surface mounted.
DI:	ductility index= deflection at the ultimate load/deflection yielding load
R.C.	reinforced concrete