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NEURAL NETWORK PREDICTION OF CONFINED PEAK STRESSES OF RC COLUMNS

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ABSTRACT

The research presents ANN ("Artificial Neural Networks") estimation of confined peak strength for R.C columns. The modeling of the strength of reinforced concrete columns by uses of the (FEM) finite element method gets many difficulties, starting in geometric representation down to nonlinearities due to loads. The use of neural networks trained well can give us a model that can be utilized as an alternative and successful model for those columns. Experimental sets of data for concrete of square and circular concrete columns were gathered from many researches to develop an Artificial Neural Network formula as input data set parameters consist of ultimate strengths, size of mainly longitudinal and ties reinforcements, compressive concrete strength, thickness of concrete cover for reinforcement, specimen geometric dimension, and stirrup bars spacing. Confined Peaking Compressive Strength (CPCS) of square and circular concrete columns is predicted by neural networks technique and sorted with analytical models and found that they are scientifically accepted. The prediction was performed by package program (Mat Lap).

KEYWORDS: Artificial neural network; Compressive strength; Confined concrete strength; Concrete columns

التنبؤ بواسطة الشبكة العصبية لإجهادات الانضغاط المقيدة القصوى لعمود من الخرسانة المسلحة

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جامعة الكوفة، كلية الهندسة، قسم المنشآت والموارد المائية

الخلاصة

يعرض هذا البحث طريقة التنبؤ باستخدام تقنية الشبكة العصبية لإجهادات الانضغاط القصوى لعمود من الخرسانة المسلحة , وتكمن الفائدة العلمية من هذه الطريقة بانها تعالج صعوبات رياضية معقدة التي تحل بطريقة العناصر المحددة وذلك بسبب تعقيد الصعوبات الرياضية لوجود عدم الخطية في المعادلات واحتساب الاجهادات المترتبة من الاحمال. لذلك تم استخدام شبكات عصبية يتم تدريبها بشكل كفوء وسليم عن طريق الاعتماد على نتائج بحوث علمية تجريبية موثوقة تم ادخالها مسبقا لبناء الموديل الرياضي لهذه الشبكة واستخدام قسم من نتائج البحوث في تأكيد سلامة المخرجات وتقاربها من المتوقع. وتم استخدام هذه التقنية لاحقا لبيان تأثير العديد من المتغيرات التي ستجرى عن قصد لبيان معرفة تأثيرها العلمي على سلوك العمود الخرساني إنشائيا وبطريقة مبسطة واكثر وضوحا. وهذه المتغيرات شملت أقصى انفعال في حديد التسليح وكمية حديد التسليح الرئيسي والعرضي وقابلية الانضغاط العظمى للخرسانة وكذلك متغيرات تتعلق بالأبعاد الهندسية لنموذج العمود. وتم استخدام برنامج (MAT LAP) لإدارة الشبكة العصبية.

1. INTRODUCTION

The effectiveness of various parameters was studied on the CPCS. In many studies these analytical and empirical studies have been investigating by many researchers. Therefore many analytical approaches have been presented for predicting CPCS for columns. These analytical approaches were presented in the tables and references by means of stress-strain relationship models. These analytical models can be listed in the literature by Table 1:

Table 1. Various Analytical Models have been Suggested to Predict the CPCS for Columns Considering the Various Parameters Stated Below

- **Square column : Some Analytical Models for Confined Compressive Strength of Square Concrete Columns**

Researchers	Formula - strength equation
Sargin, 1971	$f_{cc} = \left\{1 + 0.05468\left[1 - 0.245\frac{s}{b_e}\right]\frac{\rho_s}{\sqrt{f'c}}fy\right\}f'c$
Yong et al., 1988	$f_{cc} = \{K\ f'c\} = \left\{1 + 0.11\left\{1 - \frac{0.245\ s}{h}\right\}\{\rho_s + \frac{nd"}{8sd}\rho\right\}\frac{fyt}{\sqrt{f'c}}\}f'c$

- **Circular column : Some Analytical Models for CPCS of Circular Concrete Columns**

Mander et al., 1988	*"($f_{cc}=f'_{co}(-1.254+2.254(1+7.94f'_l/f'_{co}-2f'_l/f'_{co})^{0.5}$* *where, $f'_l= k_e\ \rho_s\ f_{yh}/2$, $k_e=(1-s/2d)^n/(1-\rho_{cc})$ and* *$n=2$ for circular hoops)"*
Sakai et al., 2000	*"($f'_{cc}=f'_{co}+3.83\rho_s f_{yh}$* *$f'_{cc}=f'_{co}(0.94+4.7C)$* *where, $C=K_s[\rho_s f_{yh}/(2f'_{co})]$ and $K_s=[1-s/(d\tan 30)]\geq 0$)"*

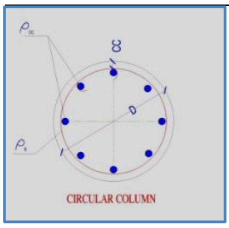
2. EXPERIMENTAL DATABASE FOR NN MODEL

Table 2 as shown below presents the data sets used from the experimental studies presented by Mander et al. (1988b); Sakai et al. (200); and Sakai (2001). The values of the parameters are drawn:

1. Compressive strength test of unconfined concrete for cylinder, f'c.
2. Compressive strength test of unconfined concrete specimen (similar in size and configuration of geometry), f'co.
3. Diameter of confined concrete of circular column, d.
4. Column specimen height, H.
5. Yield strength of ties reinforcement, f yh .
6. Ratio of volume of ties reinforcement to volume of concrete surrounding by tie, ρs.
7. Spacing between two tie bars or spiral pitch, s.
8. Ratio of main reinforcement to area of concrete surrounding by tie, ρcc, and.
9. Peak compressive stress of concrete specimen (confined), f 'cc.

14 circular columns used in this research. The 6 specimens' that got from Mander et al. (1988) are 500 millimeter in diameter and 1,500 millimeter in rise. The effective diameters of the confined concrete area are 438 mm. All column specimens get ties (lateral) and main (longitudinal) bars with changing diameter of bars and the spacing between tie bars. The 4 specimens presented by Sakai et al. (2000) are of 180 millimeter in diameter and having 600 millimeter in rise, and having ties and main reinforcement (ten bars having 6.35 millimeter in diameter). The 4 tested specimens of Sakai (2001) have 300 millimeters in diameter (280 millimeter enclosed core diameters) and 900 millimeter specimen in rise. 16 main bars having approximated diameter of 10 millimeter were tested. Column's series C1 have one to three layers of tie reinforcement. 6 sets of experimental data tended to (square) RC column specimens have been collected from Yong et al. (1988). Geometric and mechanical properties of confined core of column specimens, tie reinforcement, and main bars were taken as different values for this research. These changed values represent concrete strength obtained from compressive strength from experimental tests, f'c , edge dimension of a cross section of square column, b, and specimen highness, H, concrete reinforcement cover, cc, yield strength of tie reinforcement, fyh, and main reinforcement, fyl, tie bar size, Dt, the bar diameter of main reinforcement, Dl, tie reinforcement spacing, s. The types of tie reinforcement configuration are given in Table 3.

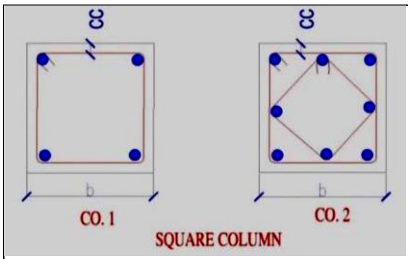
Table 2. Intervals Values of Parameters in Experimental, Circular Concrete Column



Specimens	*f'c (MPa)*	*f'co (MPa)*	*D mm*	*H mm*	*fyh (MPa)*	*ρs %*	*S, mm*	*ρcc %*	*f'cc (MPa)*
Mander et al. Training set for NN									
M-a	28	24	438	1500	310	2.0	52	1.60	38
M-b	31	30	438	1500	340	2.0	52	1.60	48
M-1	28	29	438	1500	340	2.5	41	1.60	51
M-2	28	29	438	1500	340	1.5	69	1.60	46
M-7	31	32	438	1500	340	2.0	52	3.27	52
M-8	27	30	438	1500	340	2.0	52	3.30	49
Mander et al. (1988b) Testing set for NN									
N 1-7-2	28	29	438	1500	320	2.01	50	1.72	47.56
Sikia et al. (2000). Training set for NN									
N-1a	29.8	24.6	185	600	376	0.57	120	1.18	29.6
N-2a	29.8	24.6	185	600	376	1.14	60	1.18	29.7
N-3	29.8	24.6	185	600	376	1.71	40	1.18	35.9
D-1	29.8	24.6	185	600	376	0.57	240	1.18	31.1
Sikia et al. (2000). Testing set for NN									
N 1-7-2	29.8	24.6	185	600	376	0.99	152	1.18	31.55
Sikia et al. (2001). Training set for NN									
C1-20	19.4	21	280	900	363	2.26	20	1.85	35.4
C1-30	19.4	21	280	900	363	1.51	30	1.85	29.7
C1-40	19.4	21	280	900	363	1.31	40	1.85	27.0
C1-60	19.4	21	280	900	363	0.75	60	1.85	24.0
Sikia et al. (2001). Testing set for NN									
N 1-7-2	19.4	21	280	900	363	1.41	38	1.85	29.10

(*) Mean that values will be constant in parametric study.

Table 3. Intervals Values of Parameters in Experimental, Square Concrete Columns



Parameters	Range
Compressive Strength of concrete from test, f'c, (MPa)	[83.64-93.5](*) (*) [High Strength]
Edges length of cross section of square column b, (mm)	134-152
Specimen length, H, (mm)	457 [constant] (**) (**) the value of parameter is constant for all study
Concrete cover thickness, cc, (mm)	0-1.27
Center-to-center of tie reinforcement, s, (mm)	25.4-152
Yield strength of tie reinforcement, fyt, (MPa)	496 [constant]
Yield strength of main reinforcement, fyl, (MPa)	424 [constant]
Diameter of transverse reinforcement, Dt	3,2 [constant]
Diameter of longitudinal reinforcement, Dl	10 [constant]
Longitudinal reinforcement number at the corners, Nc	4 [constant]
Longitudinal reinforcement number at the sides, Ns	0 or 4
Transverse reinforcement types, CO.1-2	0 or 1

Specimens	*f'c (MPa)*	*b, (mm)*	*H, (mm)*	*cc, (mm)*	*S (mm)*	*fyt, (MPa)*	*fyl, (MPa)*	*Dt,Dl (mm)*	*Nc, Ns*	*Output f'_{cc}, (MPa)*
Yong et al. Training set for NN										
A	88.6	152	457	1.27	25.4	496	424	3.2,10	4,4	99.00
B	93.50	152	457	1.27	50.8	496	424	3.2,10	4,4	101.6
C	88.46	152	457	1.27	76.2	496	424	3.2,10	4,4	90.90
D	84.46	152	457	1.27	152	496	424	3.2,10	4,4	83.10
N	83.64	134	457	0.00	50.8	496	424	3.2,10	4,4	90.90
L	89.77	152	457	1.27	76.2	496	424	3.2,10	4,0	89.00
Yong et al. Testing set for NN										
A-7-1-2	88.1	141	457	1.02	50	496	424	3.2,10	4,4	92.68

3. NEURAL NETWORK ARCHITECTURE

Artificial neural networks ANNs are computer models of simulation of what happens between nerve cells (neurons) in the nervous system of human. The neurological unit called neuron is the main unit in the processing of information related to the neural network model. The nerve cell (neuron) consists of four parts as main parts can be shown in Fig. 1. The dendrites will collect the input data as signals from other neurons and get these data to another neuron (Ertekin Öztekin, 2012).

The process of replacing the transfer of signals using mathematical simulations, such as replacing input paths by connection weights, important activation functions, and output paths instead of the dendrite wires. A mathematical formulation in neuron calculates the sum of weighted of its input sets signals by using Equation 1, and it will give output signals by using activation function. A standard function of activation defined by Equation 2 was used in this research. These formulas will produce outputs sets by neurons that are either used as input data sets for next layer of neurons or used as results for final output (Andres et al., 2003 and Ertekin Öztekin, 2012).

$$u_j = \sum_{i=1}^{n} w_{ij} x_i \tag{1}$$

$$f(u_j) = \frac{1}{1 + e^{-\beta(u_j - b_j)}} \tag{2}$$

Where: wij weight between neurons i and j,

xi input for unit of neuron,

uj summation of multi - inputs,

bj the bias, and

f(uj) output of neuron

In this research can be use of (feed forward) multi-layer of NNs and (back propagation feed). A feed forward of ANNs and an artificial (neuron) can be seen in Fig. 1-a and in Fig. 1-b, respectively.

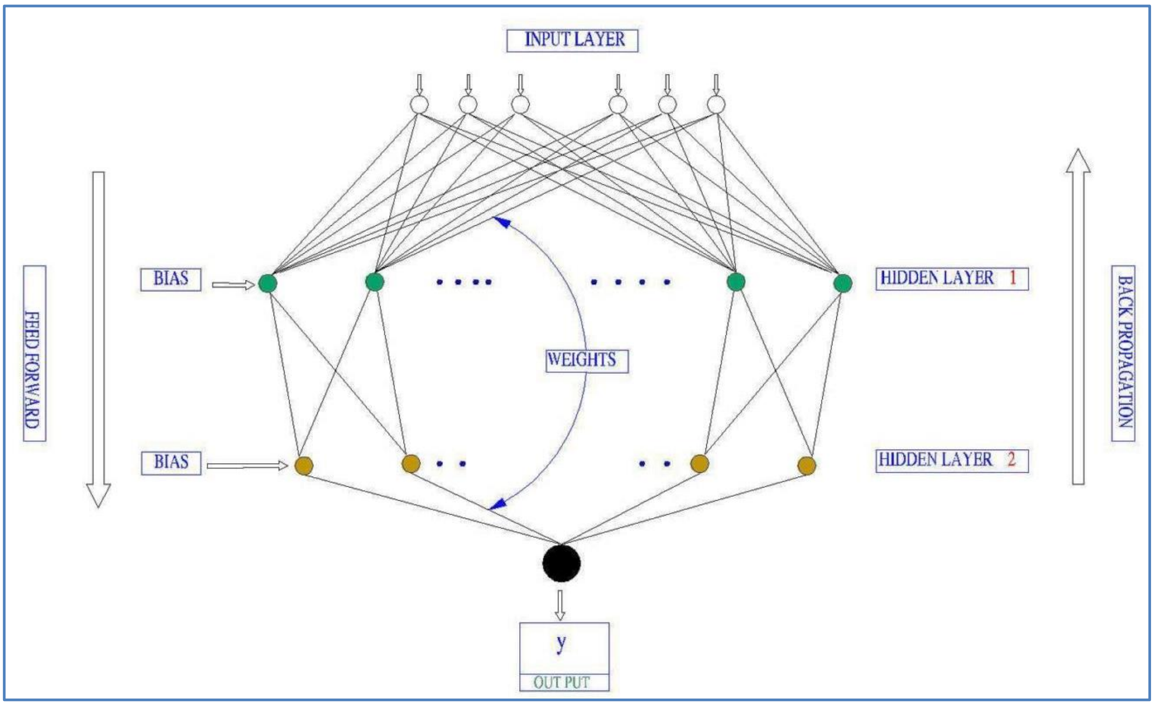


Fig. 1-a. Feed forward artificial neural network.

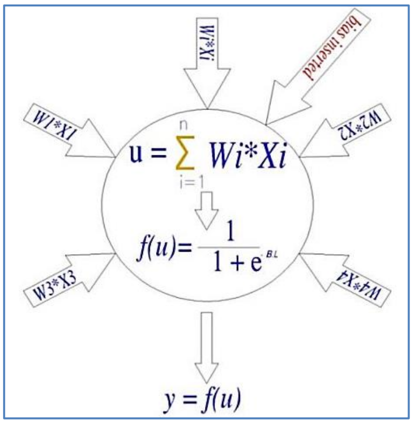
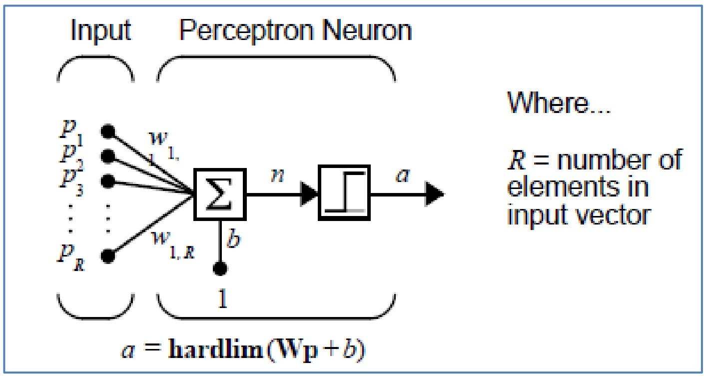


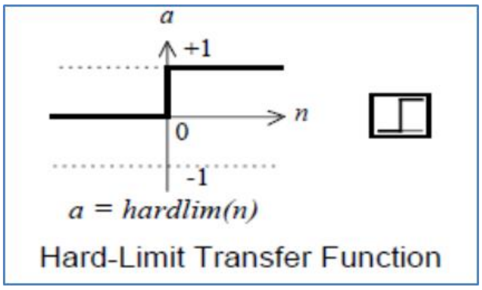
Fig. 1-b. Node function of ANN.

A feed forward NN with multi- layer consist of an input data layer with one or multi- hidden layers and a targeted output layer. The input and output layers having the same numbers in neurons of variables and outputs case, respectively. There is a real difficulty in determining numbers of neurons and the quintets of layers in the hidden layer, and it's clear and visible in feed forward NN studies. This determination of the number of layers and neurons would be through the use of trial and error sequences of approach and depending on type of the problem. Bias values and Synaptic weights values can be fed at the beginning of the training phase of NN randomly with use of back propagation technic. After the neural network outputs provide the NN will determine the errors by comparing the outputs obtained with the desired outputs and returned account configuration process in the neural network to get to the outputs of convincing scientifically. Through the back propagation of network in the process of producing the Synaptic weights will be recalculated with new values (Ertekin Öztekin, 2012).

4. NEURON MODEL

A perceptron neuron, which uses in MATLAB Version 4.0 is hard-limit transfer function hardlim, is shown below.





w1j variable represents of the weight of each external input. The transfer function will collect sum of the input weights.

The transfer function (hard-limit) gives the ability to classify and evaluate the input vectors to isolate the input space available to the two regions. For example, outputs will be 0 if the net value of input n is less than (zero), or one if the net value of input n is (zero) or more in value. The hard limit neuron unit input space has the weights value (Howard and M. Beale):

W1,1= -1 , W1,2= 1 and a bias = 1.

5. NETWORK VALIDATION AND ERROR ANALYSIS

The use of statistical measurement equations to estimate the level of the error in the outputs makes use of neural network model is valid and acquires accepted scientific confidence. So, it can be accomplished by the use of mean absolute error (MAE), root mean squared error (RMSE), and mean squared error (MSE). Mean absolute percent error (MAPE) equations are listed below.

$$MSE = Mean\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\left(t_{ij} - y_{ij}\right)^2\right) \tag{3}$$

$$MAPE = Mean\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left|t_{ij} - y_{ij}\right|}{y_{ij}}\right) \tag{4}$$

$$MAE = Mean\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\left|t_{ij} - y_{ij}\right|\right) \tag{5}$$

$$RMSE = \sqrt{Mean\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\left(t_{ij} - y_{ij}\right)^2\right)} \tag{6}$$

The validation and evaluation of neural networks model prediction can be achieved by using error metrics like (MAE) or (RMSE) .The equations in the above definition of MAE and RMSE (Andres and Kawashima, 2003 and Tsoukalas and Uhrig, 1997).

Where: n No. of patterns in the validation set

m No. of components in the output vector

o The output of a single neuron j; and

y The target output j

6. NETWORK DATA PREPARATION

Neural networks will be affected by the absolute magnitudes of the inputs and outputs because of its high sensitivity, so it's better to minimize this effect to control numerical overflow. Therefore all inputs data and outputs data to a NN were scaled; as shown in Table 4. Because of the sigmoidal function characteristic which is nearly to values (0 and 1), the derivative equal or near to values (0 and 1) will get a zero value in magnitude, and this will get to slow learning as a result of very small signal. Therefore, it's better to avoid the slow rate of NN learning close to the end points (output range); it is submitted to give range of the data between (0.1 and 0.9) as interval of scaled range (Teh, 1997). A submitted scaling equation presented by Tsoukalas and Uhrig (1997) for a variable limited to minimum (xmin) and maximum (xmax) values listed in Table 4 was used in this NN model and it's written as;

$$y = (0.8/\Delta)\, x + [0.9 - 0.8(x_{max}/\Delta)] \tag{7}$$

Where: $\Delta = (x_{max} - x_{min})$

- For circular columns, the ANN have six input nodes, one output nodes, and the hidden layers have variable number of nodes selected by experimentation. The input variables are: f'c, H, d, ρs, f yh , s, and ρcc .The choose of unconfined compressive strength of concrete f'c, as the input variable instead of the CPCS, f'co, gets good indications for this characteristic that can be determined distinctly by an experimental work. The one output value corresponds to f 'cc. In the error calculations of the (RMSE) and (MAE), the NN output with range of values between (1 and 0), and the linear submitted equation gives (scaled) values of the desired output were adopted. It obtained the desired convergence model in the training phase depending on the reduction of error of tolerance by (MSE) error during the cycles of training phase and monitoring the performance of the NNs by comparing the outputs. See Fig. 2-a.

- For square columns as shown in Fig. 2-b, the ANN models have eleven input nodes, one output nodes, and the hidden layer nodes will be varied in number and notified by experimentation. Connection weights between (0 and 1) were selected randomly by computer program in Mat lab software. Learning rate was 1. MAE, RMSE, MSE, and MAPE were employed for the checking of computation assessments of different NNs architecture. The trail code (9-1-2-1) architecture was chosen as the preferable ANNs architecture. The Selection of ANN architecture has 9-7-7-2 configuration as shown in Fig. 3.

Table 4. Scaling down Data for NN, (Circular Concrete Columns)

Parameter	**Range of data**	
	Minimum	**Maximum**
f'c or *f'co (MPa)*	19.4	32.00
D (mm)	280	438.0
H (mm)	600	1500
f_{yh} *(MPa)*	310	376.0
ρ_s *%*	0.75	2.260
S mm	20.0	69.00
ρcc %	1.18	3.300
f'cc (MPa)-Experimental	24.0	52.00

Table 5. Scaling down Data for NN, (Square Concrete Columns)

Parameter	**Range of data**	
	Minimum	**Maximum**
Yong et al. [ ]		
f'c (MPa)	88.6 (*)	93.5 (*)
b, (mm)	134	152
H, (mm)	457	457
cc, (mm)	0	1.27
S (mm)	25.4	152
fyt, (MPa)	496	496
fyl, (MPa)	424	424
Dt,Dl (mm)	3.2,10	3.2,10
Nc,Ns	4,0	4,4
f'_{cc} *(MPa)*	83.1	101.6

(*) high strength concrete

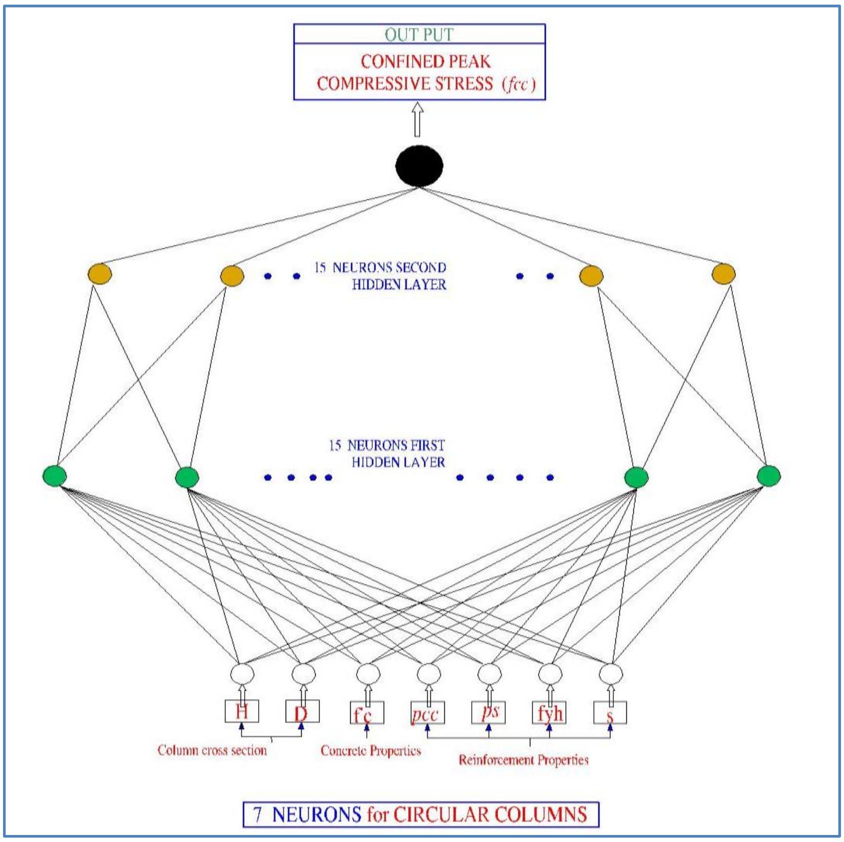


Fig. 2-a. Determined artificial neural network architecture for circular columns.

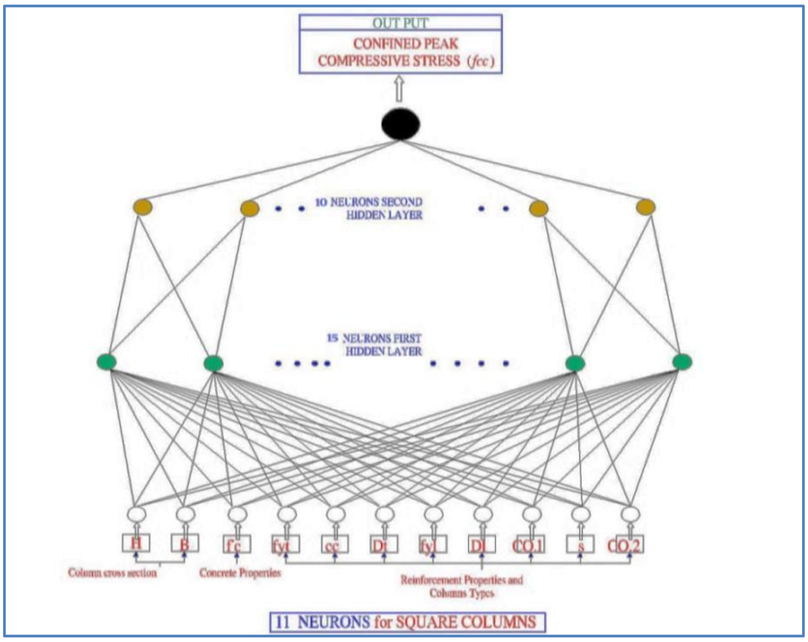


Fig. 2-b. Determined artificial neural network architecture for square columns.

7. TRAINING AND TESTING OF ANN MODEL

- 14 data sets 4 confined materials of circular RC column data were chosen randomly for testing. Remaining 10 data sets can be used for training data sets. Training phase of the NN model showed Fig. 2-a was completed at the few seconds the epoch with 0.269 % error.

- 6 data sets 2 confined materials of square RC column data were chosen randomly for testing. Remaining 4 data sets can be used as training data sets. Training phase of the NN model showed Fig. 2-b was completed at the few seconds the epoch with 0.395 % error, with a personal computer dell core I-7 for this NN architecture. The error of output was evaluated by use the mean squared error for each of the 5 seconds epochs during process of the training phase, and the NN outputs error graphic shown in Fig. 4 was evaluated at the end of training phase process. When the training phase processes were completed, the artificial neural networks model was tested and evaluated, and learning will be achieved with desired accuracy, see Table 3.

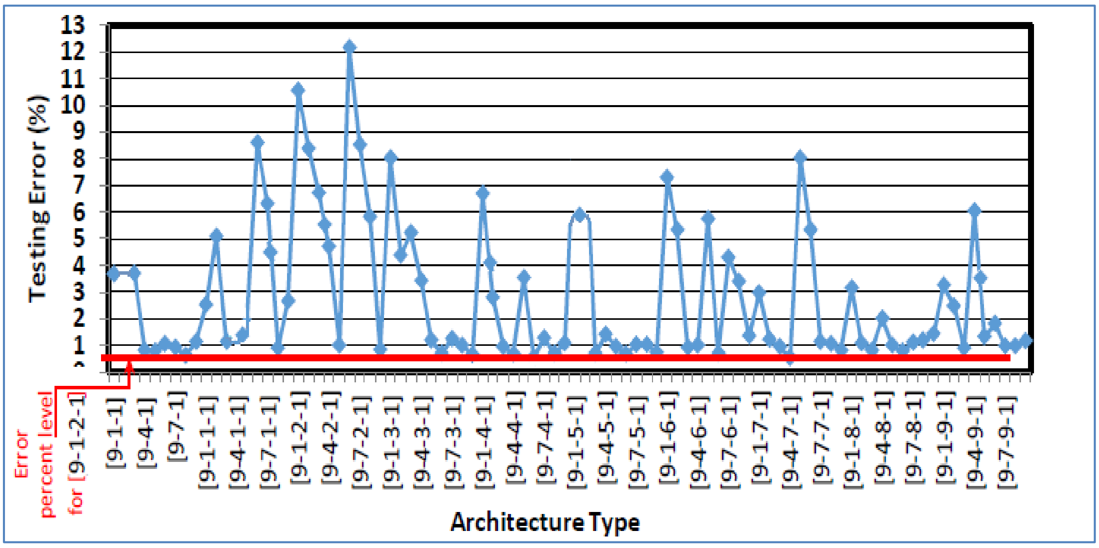


Fig. 3. Illustration the errors of many trail architectures.

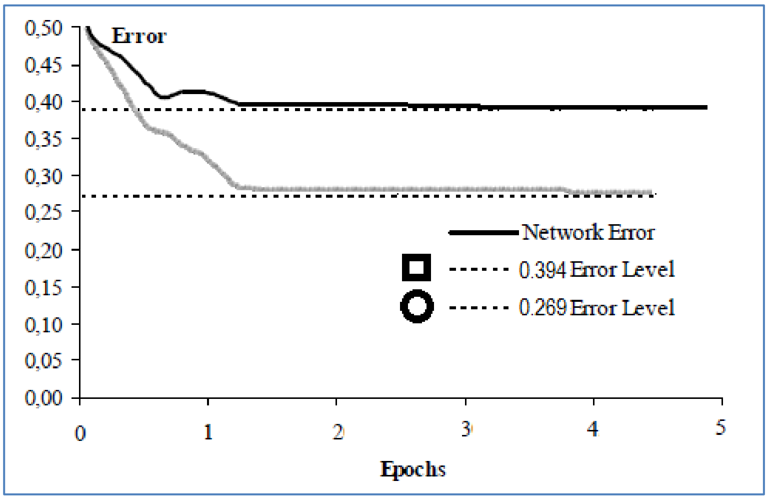


Fig. 4. Error occurs at the end of training process after the training process for NN.

8. COMPARISON OF ANN MODEL WITH ANALYTICAL MODELS

8.1. Square Concrete Columns:

Evaluation the values of confined concrete strength, fcc (model) by artificial neural networks model was contrasted with analytical experimental models offered by Yong et al. 1988. The comparison between the prediction and analytical results was achieved under the ratio of fcc(model) / fcc (by experiment) and the statistical characteristic values, which are defined previously (MAE, RMSE, MSE, MAPE, and R2). The ratio of fcc (model) / fcc(experimental) for artificial neural networks model is shown in Fig. 5-a. As seen from this figure, the prediction of output values obtained by artificial neural networks model is mostly closer to the desired results (experimental). Computed for ANNs predictions output are lowest in the outcome than analytical models and the result can be corrected by improving the performance of the artificial neural networks model.

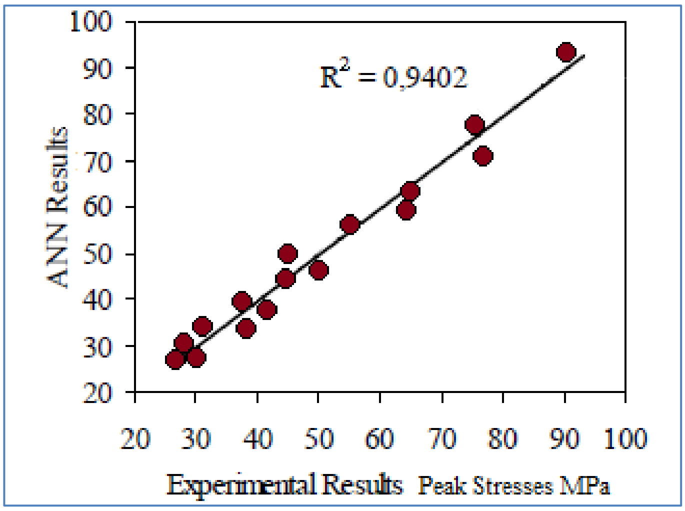


Fig. 5-a. The relation between evaluated values by ANNs model and experimental results (Square Columns).

Table 6. MAE, RMSE, MSE, MAPE, and R^2 Values for the NNs

Yong et al.				
MAE	MSE	RMSE	MAPE	R^2
0.000148	0.002582	0.050744	0.113334	0.940212

8.2. Circular Concrete Columns:

Predicted confined concrete strength values for circular concrete columns by ANNs were compared with analytical formulas given by Mander et al., 1988b; Sakai et al., 2000; and Sakai, 2001).

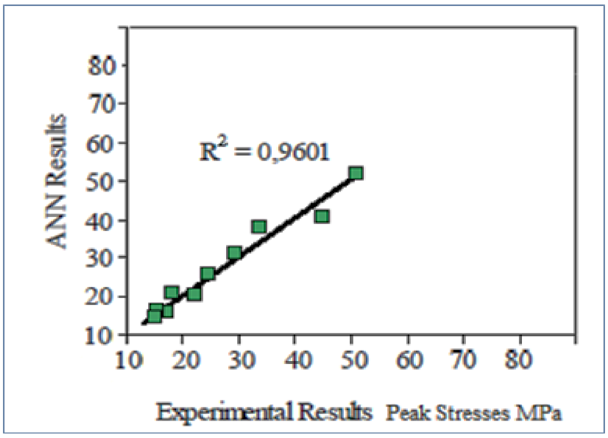


Fig. 5-b. The relation between evaluated values by ANNs model and experimental results (circular columns).

Table 7. MAE, RMSE, MSE, MAPE, and R^2 Values for the NNs

Mander et al				
MAE	MSE	RMSE	MAPE	R^2
0.000107	0.001540	0.031152	0.132646	0.960155

- **Contrasting of the Volumetric Reinforcement Ratio (ρ_s) and Fixed Tie Spacing (s).**

For a given transverse reinforcement (tie) spacing, s, the amount of reinforcement ratio, ρs , corresponding to tie reinforcement bars in (1 to 3) layer may be selected. As a result, an artificial neural model may be examined its demeanor due to changing of amount of reinforcement ratio, for fixed tie spacing (s). Fig. 6 shows the curves of peak confined concrete stresses for constant (tie) spacing, s, of the N 1-7-2 model for the circular columns. It is noticed that the peak confined concrete stress raises as the amount of tie reinforcement bars is increased for a fixed transvers reinforcement (tie) spacing, (for ρs = 2.5) the increase in f'cc =17.7 MPa, 87.2%).

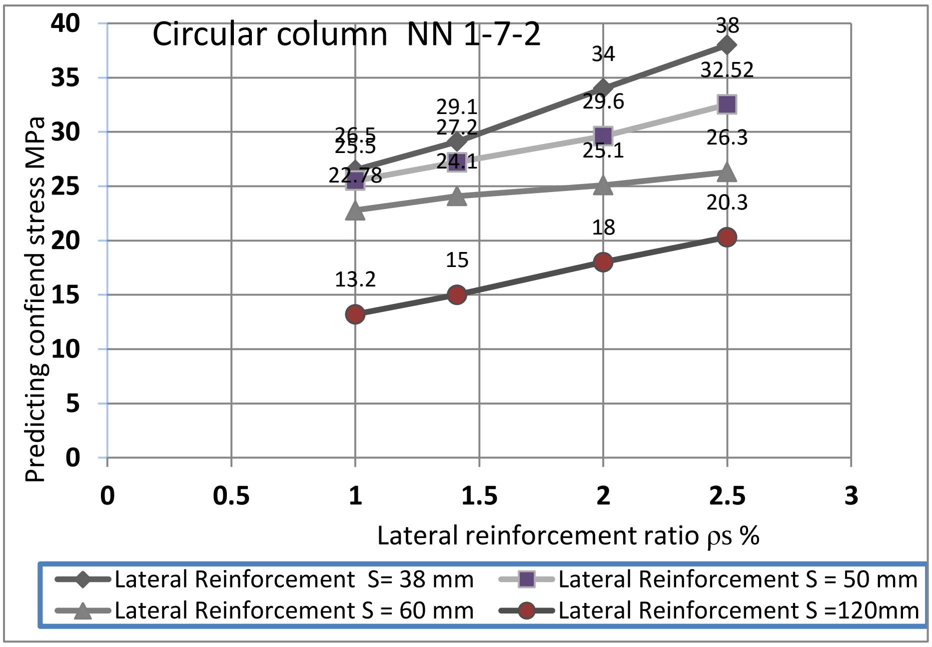


Fig. 6. The curves of confined stresses for fixed spacing for ties, *S*.

- **Varying Main Bars Steel Ratio**

The both of circular and square columns spaceman's were examined for the NN 1-7-2 model when the main steel bars ratio, ρcc , is varied from 1.2% to 3%. All other needed input parameters were identified. In Fig. 7, the percentage value of f'cc was increased from (28.3 to 35) MPa, 4.37 % for (C1-20 to C1-60) columns. The number of longitudinal bars has a minor effect on confined stress. This fact was also noticed by Mander et al. (1988) in their works.

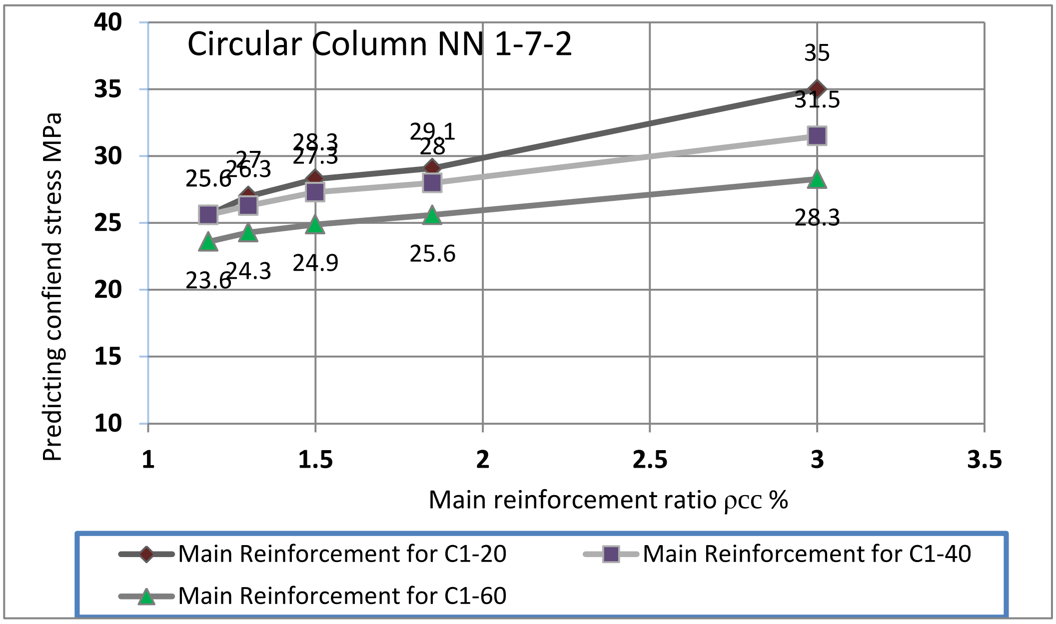


Fig. 7. The effect of main reinforcement ratio on the f'cc.

9. CONCLUSION

An artificial neural networks application technology used in this research to predict the confined concrete compressive strength of circular and square concrete column sections. The best result was obtained for many trail and evaluation of these results. These results were compared with both of the analytical and experimental works. The amendment artificial neural model estimated closer outputs to the many experimental and analytical model results. This conclusion comes to light the ability to use the developed NN model to estimate the confined compressive strength of reinforced concrete columns for high strength (f'c = 88.6 – 93.5) MPa for square concrete columns and normal strength (f'c = 19.4 - 31 MPa) for circular concrete columns. Many different types of confinement configurations were illustrated in this research. The percentage of errors given by (MAPE) of testing output sets was obtained 0.113334 and 0.132646, respectively. The final errors were computed below 12 % for testing sets.

Future study could develop a NN model to include other variables are set to study the effect of these variables in clear and them, like hooks in tie angle , modulus of elasticity of materials. Where it is not used in this research.

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