Building a Reliability Model to Improve the System Performance

Saad Talib Hasson
Saad_aljebori@yahoo.com

Head of Computer Science Department- College of Sciences- University of Babylon- Iraq

Abstract

Reliability is to make the product more reliable. The reliable systems or devices are those with minimum possible number of faults or defects during their designed operation time. Reliability helps in maximizing the system accurate life time. Improving the system design process will helps in exceeding many difficulties during the practical operation life. The modeling process is often used to analyze, control and optimize the system or any of its components. One of the suitable and possible models to be built is a mathematical model which used to represent, model and analyze the system operation. A Mathematical model is the main fundamental tool that can be used to understand, analyze, measure and evaluate complex systems performance. It describes each system by a set of variables, a set of inequalities and a set of equations to establish relationships among these variables. In this paper a stochastic mathematical model was built using stochastic transitions to represent the process of modeling, analysis to estimate the system reliability. This model can be used to indicate the critical weak points in the suggested system and the available alternatives to achieve certain developing and improvement corrections.

Keywords: reliability, Modeling, Markov Models, Queuing. Failure rates, performance measure.

الخلاصة

تعني المعولية (الموثوقية) أن تجعل المنتج أكثر وثوقية. كل منظومة ذات معولية عالية هي المنظومة التي تحدث فيها اقل ما يمكن من الأعطال والإخفاقات خلال مراحل عملها ضمن عمرها التصميمي. تساعد المعولية في تحسين وزيادة عمر المنظومة. أي تحسين في عمليات التصميم الأساسية قد تساعد في تجاوز الكثير من الصعوبات والتحديات خلال مراحل تشغيل المنظومة لاحقا. عملية النمذجة غالبا ما تستخدم للتحليل – السيطرة وإيجاد البديل الامثل للمنظومة ككل أو أي من مكوناتها. احد هذه النماذج التي يمكن بناؤها وتداولها هي النماذج الرياضية والتي تستخدم لتمثيل ونمذجة وتحليل عملية تشغيل أية منظومة. تعتبر النماذج الرياضية من الأدوات الأساسية والتي يمكن استخدامها لفهم وتحليل وتقييم وقياس أداء المنظومات المعقدة. يمكن وصف كل منظومة من خلال استخدام مجموعة من المتغيرات و/او – مجموعة من المتراجحات و مجموعة من المعادلات بإيجاد علاقات تربط بعضها بالبعض الأخر. في هذا البحث تم بناء نموذج رياضي تصادفي باستخدام عمليات انتقال تصادفي لتمثيل عملية النمذجة والتحليل لتقدير معولية منظومة معينة. هذا النموذج يمكن استخدامه لتحديد نقاط الضعف ألحرجه في المنظومة المقترحة وإيجاد البدائل المتاحة لتحقيق التصحيح التحسيني والتطويري الممكن.

1. Introduction

Reliability means the ability of a system to meet its requirements according to the specifications. It gives a specific measure of the essential characteristic of any device or system. Reliability indicates the system ability to perform its assigned intended function. It was directed towards the determination of the practical possibilities of realizing the aims of the system [Blischke, W. R., et al , 2000]. System always means any collection of interacting and interdependent components, organized in a manner to provide one or more functions. Reliability in general is the Probability that a system will operate successfully for a given period of time and under given operating conditions [Salfner, F., et al, 2006]. Its crucial measurement tool is a set of mathematical techniques that can be used to estimate

and predict the reliability behavior of system during its development and operation. The primary goal of the reliability modeling is to estimate the probability that the system will fail in a given time interval, or, to find the expected duration between successive failures [Zahid Raza and Deo Prakash Vidyarthi, 2009]. The main definition to the failure is the loss of the ability to perform a required operation in its specific environment. The fault means a defect in a system or in one of its component that causes failure. The mean time between failures (MTBF) and the Mean time to failure (MTTF) are widely used as a measure of the system reliability and its performance evaluation. The MTBF value is often calculated by dividing the total operating time of the devices by the total number of failures encountered. MTTF for non-repairable systems means the time between failures for repairable systems or "inter failure times".

2. Failure

Failure is the system ability termination to perform its required operation during its working specified conditions. The failure means that the component being unable or unsafe to continue its operation. It can be defined depending on the cause, timing, degree of the failure and other factors [R. E. Brown, et al, 2004]**.** The rate in which that failure happens is called the failure rate (λ). Failure rate is the most important indicator about the system reliability. There are many differences between Reliability and the quality which represents the degree of conformity to applicable specifications and standards [Vodă, V. Gh., 2009]. Quality is not a function of time and environment. Reliability ensures the product good design while quality insists on a product of good manufacturing [Dr. Nasser N. Khmiss and Ziad Sabah Abir, 2009]. If the quality of certain product is defined by the time of failures during its operation time, then f(t) represents its probability density function. This probability density function is used to measure the quality of any system depending on the average time-to-failure values.

3. Reliability Modeling

Modeling is the process of building a model to represent the actual system with some simplifying assumptions. This process is to allow the designers or the users to experiment, test and change the system parameters to indicate their effects and relations to each others. Such process can be done without any effecting on the real system as an economical scientific manner. It can save costs in system development if the modeling process is properly handled.

Reliability can not able to predict when an item will fail. Certain statistical methods are used widely to discuss the systems reliability. Reliability is measured by the system's mean time to failure (MTTF). MTTF of a system is the average expected time until the occurrence of the first system failure. Sets of mathematical techniques can be used to predict the reliability behavior of a system during its development and operation. It based on tests done on a large number of items[Blischke, W. R., et al , 2000].

Let "T" be a random variable representing the lifetime of a certain system. The probability that it will fail by time "t" is [Allan, R. and Billinton, R, 2000].

$$F(t) = p[T \le t] = \int_0^t f(x)dx \qquad \ldots\ldots\ldots\ldots \quad (1)$$

The probability that the system will survives until time "t" is:

$$R(t) = P[T > t] = 1 - F(t) = \int_t^{\infty} f(x)dx \qquad \text{……..(2)}$$

$$R(t) + F(t) = 1 \qquad \text{…….(3)}$$

$$MTTF = E(t) = \int_0^{\infty} tf(t) = \int_0^{\infty} R(t)dt \qquad \text{……..(4)}$$

t = value of the random variable and f(t) dt = the frequency of recurrence of t.

$$MTBF = \frac{1}{MTTF} \qquad \text{……..(5)}$$

The failure rate function for a certain component can be estimated at time t as follows:

$$H(t) = \lim_{\Delta t \to \infty} \frac{F(t+\Delta t) - F(t)}{R(t).\Delta t} = \frac{f(t)}{R(t)} \qquad \text{……(6)}$$

When Δt is small then:

$$P(t < T \le t + \Delta t / T > t) \approx \lambda(t)\Delta t$$

In this paper we supposed that f(t) is exponentially distributed, then [Hee – Cheul, et al, 2009]:

$$f(t) = \lambda e^{-\lambda t} \qquad ; \qquad F(t) = 1 - e^{-\lambda t}$$

$$E(t) = \int_0^{\infty} t(\lambda e^{-\lambda t})dt = \frac{1}{\lambda} \qquad \text{……(7)}$$

$$E(t) = Mean = MTTF = \frac{1}{MTBF} \; ; \qquad \text{.....(8)}$$

$$Variance = \frac{1}{\lambda^2}$$

$$R(t) = e^{-\lambda t} \qquad \text{……..(9)}$$

The failure rate (λ) is the probability that a failure will occurs in the interval (t, t+ Δt) given that a failure has not occurred before time t [Hee – Cheul, et al, 2009].

$$\frac{P[t \le T \le t + \Delta t) / T > t]}{\Delta t} = \frac{P[t \le T \le t + \Delta t]}{\Delta t.P(T > t)} \qquad \text{……..(10)}$$

The limit of the failure rate as the length of the interval will approaches to zero, it will be denoted by the term "Hazard rate" H(t). H(t) represents the conditional probability density function of failure during the period (t, t + Δt) , given no-failure will occur up to time *t*:

H(t)= Prob. of failure in (t, t +Δt)/ Prob. of no failure will occur up to t.

$$H(t) = \frac{f(t)}{1 - F(t)} \qquad \text{……(11)}$$

$$H(t) = \lim_{\Delta t \to \infty} \frac{F(t+\Delta t) - F(t)}{R(t).\Delta t} = \frac{f(t)}{R(t)} \qquad \text{……..(12)}$$

$$f(t) = \frac{dF(t)}{dt} = \frac{d(1 - R(t))}{dt} = -R'(t) \qquad \text{……(13)}$$

$$f(t) = H(t)R(t) = H(t)\exp[-\int_0^t H(s)ds] \qquad \text{.. ……(14)}$$

$$MTTF = \int_0^{\infty} R(t)dt \qquad \text{……..(15)}$$

Hazard means the instantaneous failure rate at time t, given that the system survived until time t. The terms hazard rate and failure rate are often used interchangeably.

4. Performance Measurements

A performance measure is an indicator that aims to obtain certain required data by observing the events and the activities of an existing system. While Performance modeling means representing the system by a model and imitates this model to obtain information about its behavior and performance. The performance of the system can be estimated either directly or by characterizing the system workload. There is a strong relation between the system performance and its reliability. In this work we will try to model both effects and relations. The system measures are, its response time, the total service time, the workflow, the numbers of completed or aborted service requests, the total waiting time, the queue length, the number of transactions completed per unit time, the ratio of blocked connection requests and the rollback completion time are the most important performance measures [S. Sonntag, M. and Gries, C. Sauer, 2007]. The studied system must be divided into single items during its analyzing step. Every queuing system can be used to represent and analyze the most systems required performances measures.

5. Markov Model

A Markov model consists of a list of the possible states of any given system. The possible transition paths between the states and the transition rate parameters must be indicated. In reliability analysis the transitions usually consist of failures and repairs. A stochastic process is a random process which evolves with time. Its basic suitable model is the Markov chain (MC). MC is a set of "states" together with transition probabilities. The transitions some time represents the probability of going from certain state n to another state m or the probability that the system was in state n now and will be in state m after short time (Δt) [J. Rajgopal and M. Mazumdar,2002]. No more than one event allows happening in this very short time duration. The most powerful analytic techniques for evaluating complex system performance are based on the theory of Markov chains. Stochastic processes and Markov chains are probability models suitable to deal with such systems. A random process is called a Markov Process if it will be conditional on the current state of the process and its future is independent of its past. A Markov chain is a mathematical model for stochastic systems whose states are either discrete or continuous. The transition probability is the main tool that governing each process. Since all the states of the real systems changes randomly, it is impossible to predict the exact state of the system in future. The future statistical properties of the system can be predicted which are important in many applications. Transitions mean the change of the state of the system, and the probabilities associated with various state-changes are called transition probabilities. The collection of the states and transition probabilities form a Markov chain [Bolch G. et al, 2006]. Many queuing models are in fact representing Markov processes. There are a large number of real engineering, industrial, physical, biological, economics, and social phenomena that can be described and analyzed as Markov models [Isaic-Maniu, et al, 2009]. If the system changed from state n at time t, to certain state (n+1) after certain short duration (Δt) (then we can call this process as a birth process). If it changed from state (n+1) to state n then this process is a death process, while staying in the same state means (no birth and no death process). The following four cases can be used to develop the equations that can be used to represent a mathematical model for the birth death process.

Case1.

The probability that the system will stay in the same state (En) after certain time (Δt) without any change; this means no birth or death will happens:

$$(1-\mu_n \Delta t)(1-\lambda_n \Delta t)$$

Case2.

The probability that the system will be increased by 1, its state (E_{n-1}) will changes after certain time (Δt) to state (E_n); this means that a birth will happens without any death.

$$\lambda_{n-1}.\Delta t\,(1-\mu_{n-1}.\Delta t) \ldots$$

Case3.

The probability that the system will decrease by 1, its state (E_{n+1}) will changes after certain time (Δt) to state (E_n); this means that a death will happens without any birth.

$$\mu_{n+1}.\Delta t\,(1-\lambda_{n+1}.\Delta t)$$

Case4.

The probability that there is more than one event will happen after certain time (Δt) is $0(\Delta t)$.

Now one can add all these possible cases to find the probability of finding n failures in the system during the time period (t, t+ Δt):

$$P_n(t,t+\Delta t) = P_n(t)(1-\mu_n\Delta t)(1-\lambda_n\Delta t) +$$
$$P_{n-1}(t).\lambda_{n-1}\Delta t(1-\mu_n\Delta t) + P_{n+1}(t).\mu_{n+1}\Delta t(1-\lambda_{n+1}\Delta t) + o(\Delta t)$$
$$= P_n(t).[1-(\lambda_n+\mu_n)\Delta t] + \lambda_{n-1}P_{n-1}(t)\Delta t + \mu_{n+1}P_{n+1}(t)\Delta t + o(\Delta t) \quad \ldots\ldots\ldots(16)$$

And, after arrangement and division on Δt we obtain:

$$\frac{P_n(t,t+\Delta t)-P_n(t)}{\Delta t} = -(\lambda_n+\mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t)$$

$$\frac{dP_n(t)}{dt} = -(\lambda_n+\mu_n)P_n(t) + \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t)$$

When n = 0:

$$\frac{dP_n(t)}{dt} = -\lambda_0 P_0(t) + \mu_1 P_1(t)$$

At steady state …

$$0 = -(\lambda_n+\mu_n)P_n + \lambda_{n-1}P_{n-1} + \mu_{n+1}P_{n+1}$$
$$O = -\lambda_0 P_0 + \mu_1 P_1$$
$$P_n = \frac{\lambda_{n-1}}{\mu_n}P_{n-1} \quad \ldots\ldots\ldots(17)$$
$$P_n = \frac{\lambda_0\lambda_1\ldots\lambda_{n-1}}{\mu_1\mu_2\ldots\mu_n}P_0 = P_0\prod_{i=0}^{n-1}\left(\frac{\lambda_i}{\mu_{i+1}}\right)$$
$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \quad \ldots\ldots\ldots(18)$$

6. Single Component

When the system is composed of one component (called single component system), one can build its transition states as shown in figure 1. In such system λi equals λ and μ_i will be μ.

$$\sum_{n=0}^{\infty} P_n = 1$$
$$\sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^n P_0 = 1$$

$$\rho = \frac{\lambda}{\mu} < 1$$

$$P_0 = 1 - \rho$$

$$P_n = \rho^n P_0 = (1-\rho)\rho^n \qquad n = 0, 1, \ldots$$

To find the expected number of failures in the system, called (L):

$$L = E(N) = \sum_{n=0}^{\infty} n\, P^n = \sum_{n=0}^{\infty} n(1-\rho)\,\rho^n$$

$$L = \frac{(1-\rho)\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \qquad \text{……(19)}$$

So the expected number of failures in each component can be estimated. It will be depending on the failure rate (λ). To estimated the mean time to failure (MTTF) which is known as the average waiting time (W) in Queuing theory:

$$E(T) = \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} = W \qquad \text{……..(20)}$$

And we can use the following little formula**:**

L = λ. w

If we suppose that λ is to be failure rate and μ be the repair rate, then:

$$\lambda = \frac{1}{MTBF}, \quad \Rightarrow MTBF = \frac{1}{\lambda},$$

$$L = \frac{\lambda}{\mu-\lambda} \Rightarrow \lambda = \frac{L.\mu}{L+1}, \quad \Rightarrow MTBF = \frac{L+1}{L.\mu}$$

$$MTTF = W = \frac{1}{\mu-\lambda} \qquad \text{……(21)}$$

These parameters are used to estimate the system reliability either after its production (during operation phase) or during its design using the suitable simulation techniques.

To develop a simulation experiment, one must indicate the following related relations:

$$\mu(t) = \lambda t, \triangleright\triangleright\triangleright\triangleright MTTF = \frac{1}{\lambda} \triangleright\triangleright\triangleright R(t) = e^{-\mu(t)}$$

$$MTTF = \frac{\sum_{i=1}^{N}(t_i - t_0)}{N} = \frac{Total\ \ Operational\ \ time}{Number\ \ of\ \ failures\ \ in\ \ that\ \ time}$$

$$MTTR = \frac{Total\ \ \ failure\ \ times}{Number\ \ of\ \ \ failures\ \ in\ \ that\ \ time} \qquad \text{……..(22)}$$

Figure 1, shows a simple representation of the operating and failure times of certain system with the suitable calculation process. B1, B2 and B3 are Operating times, while A1, A2 and A3 are the failure times.

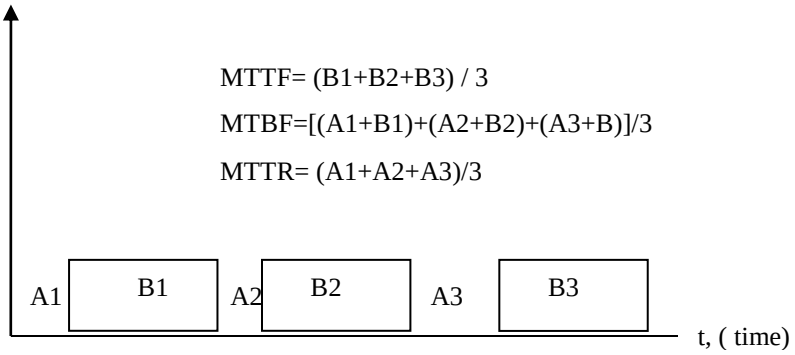

Figure 1: Operating and failure times

Figure **2**, shows a simple markov representation of the system in figure 1. The system has two states operating (state 0) or fail (state 1). The transition rate from failure state to operating state is μ which is the repair rate and the transition from operating state to the failure state is λ (the failure rate).

$\lambda = 1 / MTTF = 3 / (B1 + B2 + B3)$ and $\mu = 1 / MTTR = 3 / (A1 + A2 + A3)$

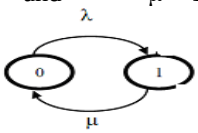

Figure **2**: state space representation.

(Failure Rate) = 1 / MTBF, while (Maintenance Rate) = 1 / MTTR

7. Series Components

If the failure of any single component in certain system causes the system itself to fail, this system is a series system. Figure 3, represents this situation as a block diagram in which the components are connected in series.

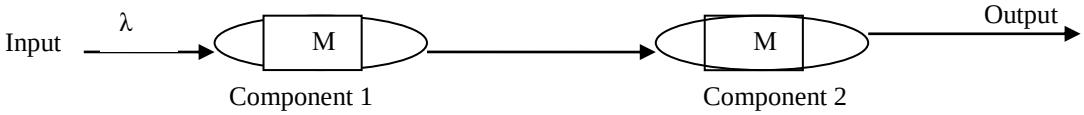

Figure **3**: Series system configuration.

If we suggest the following system with three states, state 0 being the state where both components are working. State 1 is the state where one component has failed and the rate of transition from state 0 to state 1 is λ_1, where in the duration Δt the probability of either component failing is $\lambda_1 \Delta t$. And finally, state 2 is the failed state (for both components) and the rate of transition from state 1 to state 2 is λ_2. The reliability diagram for this situation is shown in figure 4.

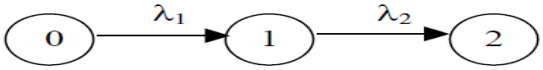

Figure **4**: Three states system reliability diagram.

The state equations are obtained from

$$P_0(t+\Delta t) = P_0(t)(1-\lambda_1 \Delta t)$$

$$P_1(t+\Delta t) = \lambda_1 \Delta t P_0(t) + P_1(t)(1-\lambda_2 \Delta t)$$

$$P_2(t+\Delta t) = \lambda_2 \Delta t P_1(t) + P_2(t).$$

Where the third equation includes the fact that if the system reaches state 2 it will stay there, it is an absorbing state. The matrix E that was required to calculate MTTF is given by:

$$E = \begin{bmatrix} -\lambda_1 & 0 \\ \lambda_1 & -\lambda_2 \end{bmatrix}$$

It means the sum of the mean time to go from state 0 to state 1 and the mean time to go from state 1 to state 2. While the repair rate (μ) is the inverse of the MTTR.

$$MTTF = \frac{\lambda_2 + \lambda_1}{\lambda_1 \lambda_2} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

8. Conclusions

This paper focused on system reliability estimation and calculation using analytical procedure based on the Queuing and Markov process models by employing the system states and transitions. Each state can be defined as an operating or failed. Transitions are defined by the transition probability, and transition time determined by the component behavior probability density function. Markov model can be used and implemented to the system and components which can serve for complex failure and repair behaviors. System with on (operating) and off (failed) state is modeled with the queuing and Markov fundamentals to estimate its reliability and the other important required evaluation factors. Failure domains may used to describe dependencies between failures in a probability manner. Future work will focus on applying simulation and analytical procedures to develop and model real specific complex systems.

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