NOLINEAR DYNAMIC ANALYYSIS OF REIN-FORCED CONCRETE ARCH STRUCTRE BY METHOD OF FINITE EMENT

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Abstract

The present study is concerned with three dimensional nonlinear analysis of reinforced concrete arch structures subjected to dynamic loading .

Material nonlinearity as a result of tension cracking, strain softening after cracking, nonlinear response of concrete in compression, crushing of concrete and yielding of the reinforcement are considered .

The three dimensional computational model is adopted in the present study. The eight and twenty noded hexahedral isoparametric elements are used for the spatial discretization of concrete , while the steel reinforcement is assumed to have uniaxial properties in the direction of the bars and it is incorporated in the concrete brick element by assuming a perfect bond .

Concrete is considered as a linear elastic strain softening material in tension and as an elasto–viscoplastic material in compression. A classical elasto – viscoplastic model is used in the present study to model the steel reinforcement .

الخلاصة

يتعلق البحث الحالي بموضوع التحليل الاخطي الثلاثي الأبعاد للأقواس الخرسانية المسلحة و المعرضة للأحمال الديناميكية .

و لقد تم إستخدام التمثيل اللاخطي للمادة في التحليل المجرى ضمن هذه الدراسة كما أخذ بنظر الاعتبار دراسة العوامل اللاخطية للمادة و تشمل تشقق الخرسانة بالسحب و انحدار الانفعال بعد التشقق و السلوك اللاخطي للخرسانة تحت الضغط مع تهشم الخرسانة و مطيلية حديد التسليح .

المنشأ الخرساني مثل بموديل ثلاثي الأبعاد . و استخدام العنصر الفضائي الواحد للكونكريت من ثمانية أو عشرين عقده و هو من نوع ال (isoparametric)،بينما يفترض أن حديد التسليح له مواصفات أحادية الاتجاه و باتجاه قضبان التسليح و لتمثيل المنشات الخرسانية المسلحة تم فرض وقوع حديد التسليح ضمني داخل عنصر الخرسانة الطابوقي الشكل و كذلك بافتراض وجود رابطة محكمة بينهما .

تتميز الخرسانة بكونها مادة مرنة خطية و لها هبوط بالانفعال (strain softening) بعد التشقق عند خضوعها لإجهاد الشد، و بكونه مادة مرنة – لزجة– لدنة عند خضوعها لإجهاد الضغط. و قد تم اعتماد مادة مرنة – لزجة اللدونه لتمثيل حديد التسليح.

Symbol	Description
[B]	Strain – nodal displacement matrix .
[B_s]	Strain – nodal displacement matrix of steel .
c	Damping coefficient.
[C]	damping matrix.
[D]	Elasticity matrix
[D_s]	Stresses – strain matrix of steel .
E_O	Elastic Young's modulus .
E_c	Young's modulus of concrete .
E_s	Young's modulus of steel .
f'_c	Ultimate compressive strength for concrete .

f_t	Tensile strength for concrete .
f_y	Yield stress for steel .
f_1 ,f_2 , f_3	x , y and z components of gravity direction vector .
F_{xi} ,F_{yi} ,F_{zi}	External nodal forces in x , y and z – direction respectively .
g	Gravity constant .
G_f	Fracture energy .
[J]	Jacobian matrix .
$[J_s]$	Jacobian matrix for steel member .
[K]	Total stiffness matrix .
$[K]_c$, $[K]_s$	Concrete and steel stiffness matrices respectively .
[M]	Mass matrix .
$[N_i]$	Shape function .
{R}	Vector of internal resisting forces .
{U}	Vectors of nodal displacements .
$\{\dot{U}\}\{\ddot{U}\}$	Vectors of nodal velocities and accelerations respectively .
U ,V ,W	Displacement components in x , y and z – direction respectively .
x_i , y_i , z_i	Global coordinates of ith node .
X , Y , Z	Global or Cartisian coordinates .
α_c	Yield surface function .
□□ , □1	Failure surface function .
□ , □	Parameters of Newmark method .
□cu	Ultimate concrete strain .
□c , □s	Mass density for steel and concrete respectively .
□	Stresses .
{□}	Stress vector .
□	Poisson's ratio .
ξ , ŋ , τ	Natural local coordinate system .

Introduction

In the past , the arch represents one of the few structure systems which make it possible to cover large spans, AL-Dahash(2006) . The earliest inhabitants developed the arch as an important element of their architectural objects as expressed by old bridges, aqueducts and large public buildings .

Today, the same importance is presented especially in construction of bridges and arched structures which are constructed in different shapes and from various materials such as brick, steel, reinforced concrete, ferrocement and timber. The main aim of the arch is to enhance the load carrying capacity more than that b y straight beam. This may be attributed to the stiffening behavior due to the membrane action which leads to reduce the bending moment , shear forces and axial forces. Arching beam, not only reduces the bending moments in the arch in comparison with a straight member of same properties and loading patterns, but even reduces the shear forces as discussed by Winter(1972) On the other hand, an axial compressive force is introduced due to the arch action. This state of action is compatible with the concrete material, which is relatively weak in carrying tension and shear stresses but adequate in carrying compressive stresses. With the advent of modern computers and sophisticated analytical techniques of concrete structures, an intense research effort has been used recently to model the concrete under short– time loading for one, two or three–dimensional stresses states there Morris (1968) studied the effect of the axial deformation in the strain energy expression and derived a three dimensional exact stiffness matrix for curved beam, but Noor(1979) used the explicit central difference method in addition to Newmark's average acceleration and Park's stiffly stable method to study the nonlinear dynamic analysis of curved beam . Prathap(1982) proposed the concept of (membrane locking) phenomenon to explain the very poor behavior of exactly integrated low order independently interpolated polynomial fields . AL-Naimi(1996) studied stresses analyses of the three dimensional reinforced concrete structures subjected to static or dynamic loadings. Nonlinear material and time dependent effects are included in the analysis. This research is concerned with creep and shrinkage as a time dependent behavior of concrete . The material nonlinearities as a results of tension cracking, strain softening after cracking , the nonlinear response of concrete in compression, crushing of concrete and the yielding of the reinforcement are considered. AL- Daami(2000) developed a space curved beam element and used the exact strain energy expression. In the normal strain, the influence of the axial force, bending moment, bimoment, direct shear forces and torsion moment are included . This research includes warping deformations for that the stiffness matrix can be used for thin– walled section. Besides its application to solid of revolution , the semi– analytical method can be applied to prismatic solids. Chen (2005) discussed the (1/2) sub harmonic bifurcation and universal unfolding problems for an arch structure with parametric and forced excitation in their research. The amplitude frequency curved and some dynamical behavior have been shown for this class of problems by Liu *et al*. .

The reinforced concrete is a composite material made up of concrete and steel and it is efficient in the construction of some arched structures such as gable frames and arched bridges, AL-Dahash(2006) studied stresses analysis of the three dimensional reinforced concrete arch structures subjected to dynamic loading .

Concrete Stiffness Matrices

The elastic stresses is used to check for cracking and to modify these stresses to real viscoplastic stresses at each Gaussian points by AL-Dahash(2006) . Using the Gaussian product rules , the stiffness matrix linking node (i) and (j) is obtained :

Defining $[T_{ij}]_c = [B_i]^T \; [D] \; [B_j] \; |J|$

where ; $|J|$ is the determinant of Jacobian matrix ,

$[D]$ is the elasticity matrix ,

$[B]$ is the strain – nodal displacement matrix .

then $$[K_{ij}]_c = \sum_{p=1}^{NGAUSS} \sum_{q=1}^{NGAUSS} \sum_{L=1}^{NGAUSS} [T]_c\,(\xi_p , \eta_q , \zeta_L)_{ij} \; w_p . w_q . w_L \qquad (1)$$

where ;

w_p , w_q and w_L are the weighted values of the numerical integration ,

NGAUSS is the number of Gaussian points in (X , Y and Z) respetively .

In the 8 – noded element the sampling points are located at the centers of the six faces used by Hinton(1988).

Formulation of Load Matrix

External nodal forces have three components in x , y and z - directions which are F_{xi} , F_{yi} and F_{zi} respectively . Gravity load is treated as consistent nodal forces . For node i of an element , the forces are ;

$$\begin{Bmatrix} F_{xi} \\ F_{yi} \\ F_{zi} \end{Bmatrix} = \int_{V} [Ni] \; \square_c . \, g \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} . \, d_v \qquad (2)$$

where ; v is the element volume , [Ni] is the shape function matrix , g is the gravity constant ; f_1 , f_2 and f_3 are x , y and z - components of gravity direction vector , which are usually 0 , 0 and -1 respectively .

Steel Stiffness Matrix

Using the Gauss – Legendre quadrature numerical integration scheme , the steel membrane matrix linking node (i) and (j) is obtained :

Defining $[T_{ij}]_s = [B_{si}]^T \; [D_s] \; [B_{sj}] \; |J_s|$

then $$[K_{ij}]_s = \sum_{p=1}^{NGASS} \sum_{q=1}^{NGASS} T_s(\xi_p , \eta_q)_{ij} \; w_p . w_q \qquad (3)$$

where NGASS is the number of Gaussian points of steel membrane . $[D_s]$, $[B_s]$, $|J_s|$, are the stress – strain relation matrix, strain – nodal displacement matrix , and the determinant of Jacobian matrix for steel membrane respectively . P

$$[K] = [K]_c + [K]_s \qquad (4)$$

Where ; [K] is the total stiffness matrix of the element , $[K]_c$ and $[K]_s$ are the concrete and steel element stiffness matrices , respectively .

Dynamic Equilibrium Equation

The nonlinear dynamic equilibrium equations can be written in semi – discrete form as :

$$[M] \{\ddot{U}\} + [C] \{\dot{U}\} + [K] \{U\} = \{F\} - \{R\} \qquad (5)$$

where ; $\{U\}$, $\{\dot{U}\}$ and $\{\ddot{U}\}$ are vectors of nodal displacements , velocities and accelerations respectively ,

[M] and [C] are the mass and damping matrices , {F} is the vector of external applied forces, and{R} is the vector of internal resisting forces . The global mass and damping matrices are defined as :

$$[M] = \int_{V} [N]^T \; \square \; [N] \; d_v \qquad (6a)$$

Defining $[T_{ij}] = \square_c \, [Ni]^T \; [N_j] \; |J|$

then $$[M_{ij}] = \sum_{p=1}^{NGAUSS} \sum_{q=1}^{NGAUSS} \sum_{L=}^{NGAUSS} [T] (\xi_p , \eta_q , \square_L)_{ij} \; w_p . w_q . w_L \qquad (6b)$$

$$[C] = \int_{V} [N]^T c \, [N] \; d_v \qquad (7a)$$

Defining $[T_{ij}] = c \, [Ni]^T \; [N_j] \; |J|$

then $$[C_{ij}] = \sum_{p=1}^{NGAUSS} \sum_{q=1}^{NGAUSS} \sum_{L=1}^{NGAUSS} [T] \; (\xi_p , \eta_q , \square_L)_{ij} \; w_p . w_q . w_L \qquad (7b)$$

where $T(\xi_p, \eta_q, \square_L)_{ij}\ w_p . w_q . w_L$ is the transformation matrix , and (c) is the damping coefficient . The vector of internal resisting forces {R} is given by the expression :

$$\{R\} = \int_{PP^V} [B]^T \{\sigma\}\, d_v. \tag{8}$$

where $\{\sigma\}$ is the vector of total stresses

Nonlinear Solution Technique

Nonlinear problems is solid mechanics are classified into two forms , first nonlinearity due to strain – displacement relationship which is geometric nonlinearity , and secondly nonlinearity due to nonlinear stresses – strain relationship which is material nonlinearity . In the present study , only the second form is taken into consideration . The solution of nonlinear problems by the finite element method is usually attempted by one of three basic technique.

General

a – Incremental Procedures

The basis of the incremental or piecewise linear procedure is the subdivision of the load into many small partial loads or incremental . The stiffness matrix may take different values during differential load increments . The displacement increments are accumulated to give the total displacement at any stage of loading , and the incremental process is repeated until the total load has been reached .

b – Iterative Procedures

The iterative procedure is a sequence of calculations in which the body or the structure is fully loaded in each iteration . Different approaches are updated , the variable stiffness method , while in others a constant linear matrix is used throughout requiring only a single matrix inversion .

c – Mixed Procedures

The mixed procedures utilize a combination of the incremental and iterative schemes . The load is applied incrementally , but after each increment successive iterations are performed . This method yields higher accuracy but with a large cost of computational effort .This procedure is adopted in the present study .

The Newmark Method

The Newmark method is an extension of the linear acceleration that used by Hinton, E. (1988) .This method adopted in this work , method . The dynamic equilibrium equation is linedrized and written at time (t_{n+1}) as :

$$[M]\ \{\ddot{U}_{n+1}\} + [C]\ \{\dot{U}_{n+1}\} + [K]\ \{U_{n+1}\} = \{F_{n+1}\} \tag{9}$$

where the internal force {R} is not included .

The following assumption on the variation of displacement and velocities are made within a typical time step ;

$$\{U_{n+1}\} = \{U_n\} + \Delta t\,\{\dot{U}_n\} + ((1-2\square)\{\ddot{U}_n\} + 2\square\{\ddot{U}_{n+1}\})\,\Delta t^2/2 \tag{10a}$$

$$\{\dot{U}_{n+1}\} = \{\dot{U}_n\} + \Delta t\,((1-\square)\{\ddot{U}_n\} + \square\{\ddot{U}_{n+1}\}) \tag{10b}$$

where ; $\{U_n\}$, $\{\dot{U}_n\}$ and $\{\ddot{U}_n\}$ are values of displacement , velocities and accelerations known at time (t) . Parameters $\square$ and $\square$ control the stability and accuracy of the method . When $\square$ is equal to (1/6) and $\square$ equal to (1/2) , relation (10) corresponds to the linear acceleration method . Newmark method was originally proposed as an unconditional stable scheme . The constant – average acceleration method (also called trapezoidal rule) , in which case □ is equal (1/2) and □ equal to (1/4) is adopted in the present study .

Computer Program for Dynamic Analysis

As apart of this work , a computer program DARC3 (three dimensional , nonlinear, dynamic analysis) from Hinton(1988)used to solved the examples in this research .

The program is coded in FORTRAN–77 and has been tested on the personal computer, of the civil engineering department. This program is used to solve two types of structural material: steel and reinforced concrete ; consists of a main program and fifty one subroutines .In the present study, the Fortran Power Station 4.0 compiler produced by Microsoft incorporation was used to operate the program under PC Pentium IIII with Intel MMX 1.80 GHZ processor and 254 MB RAM

Application (1):

A clamped circular arch which having cross section and reinforcement steel as in Figure (1) subjected to uniformly distributed normal load ($1kg/cm^2$) with triangular load – time function . The material properties of concrete and steel and addition parameters are shown in Table (1).

Tene(1975) analyzed this problem by linear theory to find the elastic stress and deflection for this problem. The effects of transverse shear deformation and rotary inertia were first introduced in the theory of straight beam by Timoshenko , but they didn't take the effective of damping in their studying . The numerical solution is obtained by Houbolt´s method and by the finite differences and they used the time interval equals to (0.01) . The number of location differences for half the arch was.

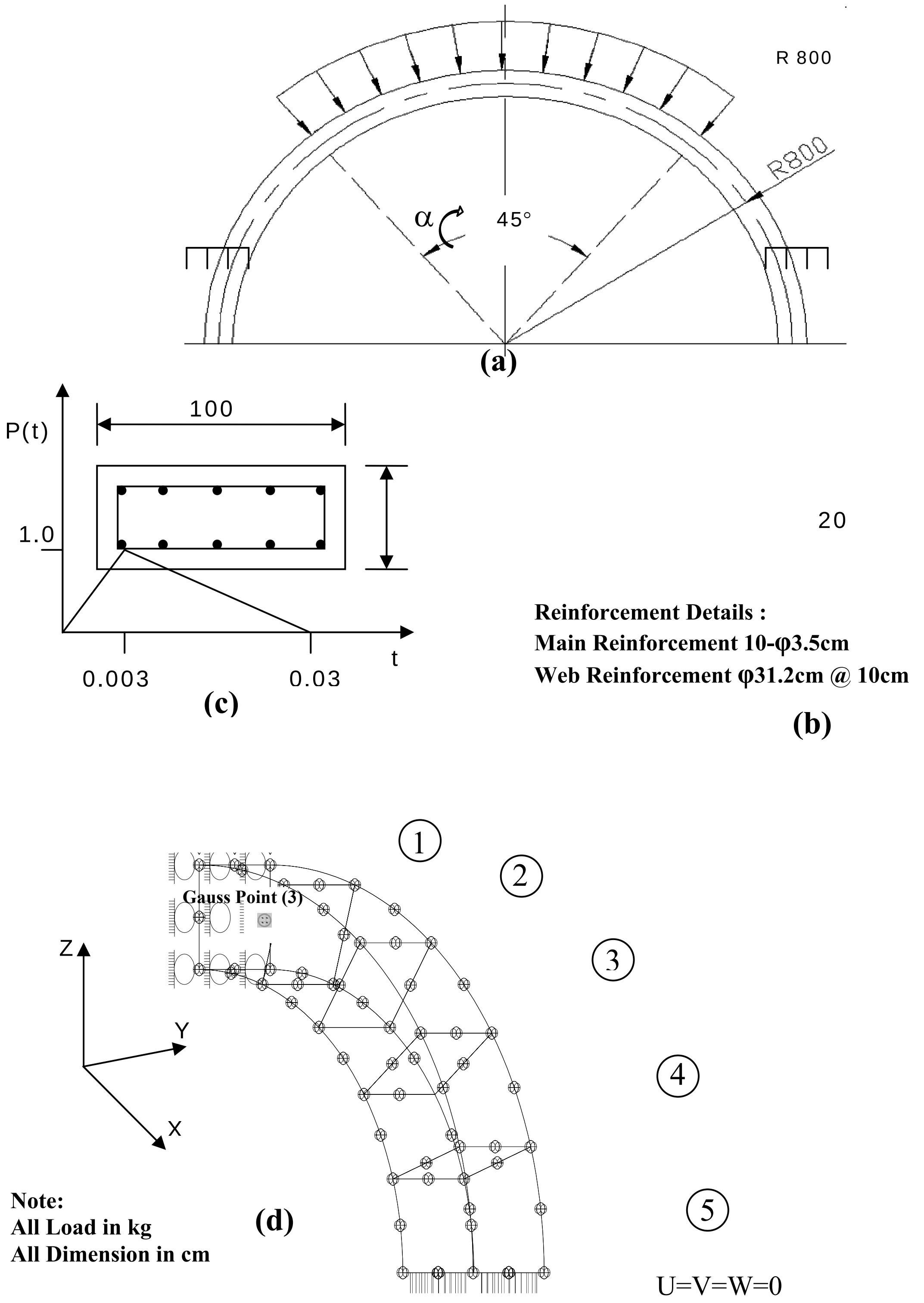
AL–Maroof(1987) analyzed this problem by three dimensional elements based on the linear theory by the finite element method to get the variation of displacements and forces with time. No damping and damping ratios (□$_0$=2.6,□$_1$=0.003846) is considered in the arch . In both cases the arch is discritized by 10 curved elements . Also the numerical results obtained by the method of characteristic by AL-Daami(1992) are compared with those obtained by Tene(1975) the agreement between the two solution is encouraging .

In the present study , due to symmetry , half of the arch with five of twenty nodded brick elements , and the center of arch is fixed in x and y direction but it is free in z – direction , for represented this we used the roller at the center of arch, as shown in Figure(1)(d). The steel reinforcement by a four layers two of them represent a longitudinal top and bottom reinforcement with thickness equal to (0.161)cm for each and the others represent the lateral ties with thickness equal to (0.0565)cm for each and rotational angle equal to (90º) from the x-axis . For dynamic analysis , a constant time step of (0.01)sec. is used , a number of time steps is (60) , a number of iteration for nonlinear solution is taken equal to (500) .

The example is solved twice No damping is considered and with damping ratios (□$_0$=2.6,□$_1$ =0.003846) and the results are compared with those obtained by Tene9) and Al-maroof() when solved the same example . Good agreement is found with these solutions . From Figures (2) , (3) and (4) , for central deflection without damping , the maximum different found equal to (30% and 10%) when the results compared with Tene(1975) and AL–Maroof(1987) respectively. But for central deflection with damping the maximum different is equal to (20.6%) when the results compared with AL–Maroof(1987). The maximum different between central deflection with and without damping that obtained from the present study equal to (60.29%) .

Table (1) Material Properties.

	Material Properties and Additional Parameter	Symbol , unit	Value
Concrete	Young's Modulus	E_C (kg/cmPP^2)	200000
	Compressive Strength	F_c (kg/cmPP^2)	300
	Tensile Strength	F_t (kg/cmPP^2)	48
	Poisson's Ratio	□	0.16
	Ultimate Compressive Strain	□cu	0.0035
	Fracture Energy	G_f (kg/cm)	0.153
	Yield Surface Function	□c	10
	Mass Density	□c (kg.secPP^2/cm^4)	0.24E-05
	Fluidity Parameter	$_0$a	0.3055
		$_1$a	0.76
	Failure Surface Function	□$_0$	1.84
		□$_1$	1.09
Steel	Young's Modulus	E_S (kg/cmPP^2)	2000000
	Yield Stresses	F_y (kg/cmPP^2)	3500
	Mass Density	□$_s$ (kg.secPP^2/cm^4)	14
	Poisson's Ratio	□	0.2
Newmark's Parameters		□	0.5
		□	0.25



(a) geometry and loading of arch (b) cross section (c) load – time relation
(d) finite element idealization and numbering of half of the arch

Figure (1) Circular Arch Under Triangular Normal Load

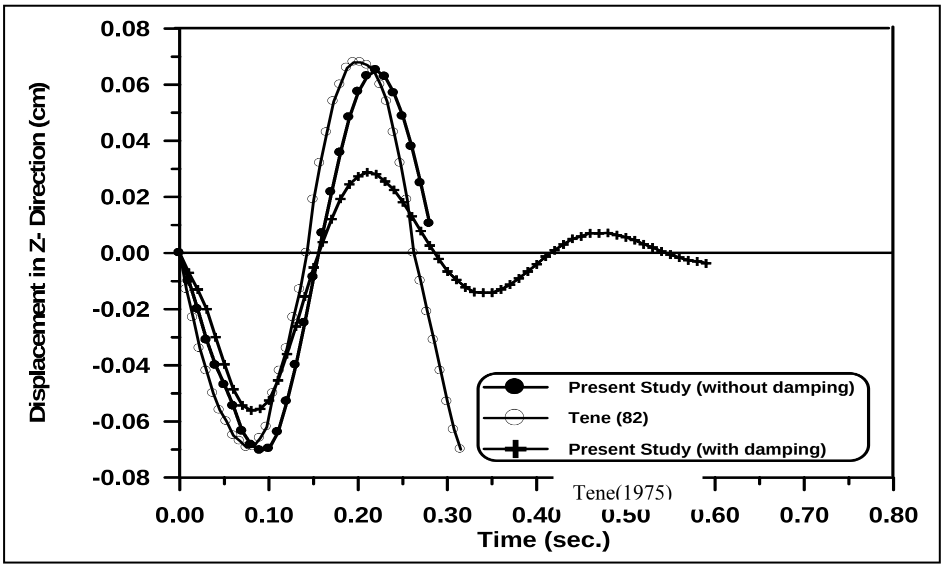


Figure (2) Time – Displacement in Z - Direction Response for Dynamic Response of the Circular Arch at ($\theta_{\circ}$ = 90°).

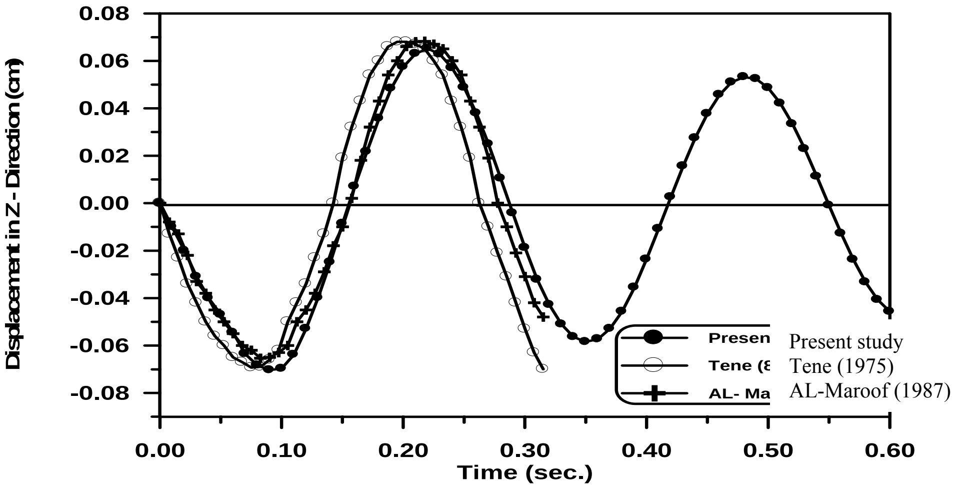


Figure (3) Time – Displacement in Z - Direction Response for Dynamic Response of the Circular Arch Without Damping at ($\theta_{\circ}$ = 90°).

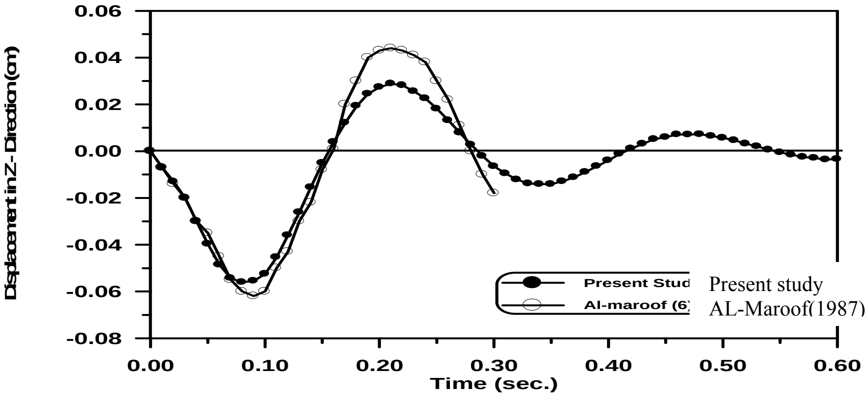


Figure (4) Time – Displacement in Z - Direction Response for Dynamic Response of the Circular Arch With Damping at (θ = 90°).

Application (2):

In this example , the effect of thickness of reinforced layers , time step and initial displacement are studied on a simply supported arch having cross section and reinforcement steel shown in Figure (5) are subjected to a concentrated load with harmonic excitation load – time function p(t)=1+sin0.1t . The material properties of concrete and steel are given in Table (2), AL-Dahash(2006) .

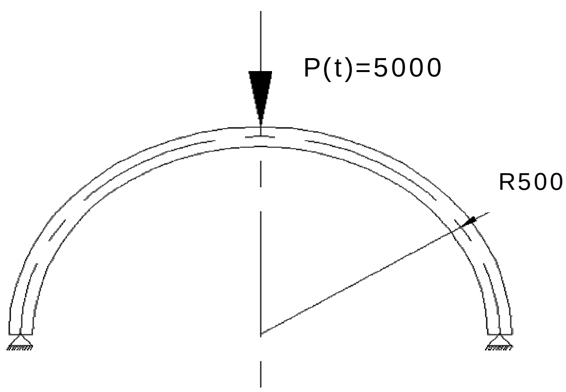
Figure (5(d)) shows the details of the finite element mesh for the reinforced concrete arch with seven eight noded brick element . The steel reinforcement represented by four layers , two of them represent the longitudal top and bottom reinforcement with thickness equal to (0.524)cm for each layer and the others represent the lateral ties with thickness (0.0094)cm for each layer and rotational angle equal to (90°) from x – axis. For dynamic analysis , a constant time step of (0.01sec.) is used, (150) numbers of time steps , a number of iteration for nonlinear solution is taken equal to (500) .

In this example Figure (6) shows the effect of change time step on central deflection of arch, if the time step decreases, the central deflection converges to exact solution . the maximum difference percent between time step(0.01 and 0.005)sec. is (29.23%) . The effect of thickness layers of main reinforcement on central deflection of this arch is studied, three cases from reinforcement were analyzed . The first case has main reinforcement with thickness of layers equals to (0.51)cm and (0.21)cm for the second case with maximum difference percent equal to (14.5%) . Finally , the third case is without main reinforcement layers, with the maximum difference percent equal to (12.7% and 25.4%) with thickness of main reinforcement equals to (0.21 and 0.51)cm respectively . Figure (7) shows that ,the central deflection increases if the thickness of main reinforcement layers decreases .

The effect of initial displacements at hinge in x – direction on central deflection is studied and the result explained the initial displacement on vibration in deflection . Figures (8) and (9) show the difference between the central deflection without the initial displacement and with the initial displacement equal to (0.2,-0.2)cm with maximum deflection percent equal to (29.6% and 27.5%) respectively . When the initial displacement equal to (0.2)cm , the maximum stress at Gauss point Numder (6) (at the center of arch) is equal to (-123)kg/cm^2 but it is equal to (110)kg/cm^2 when the initial displacement is equal to (-0.2)cm , the maximum percent difference is equal to (10.6%) . But when no initial displacement, the maximum stress at Gauss point Number (6) is equals to (-8.5)kg/cm^2 as shown in Figures (10) and (11) .

Table (2) Material Properties.

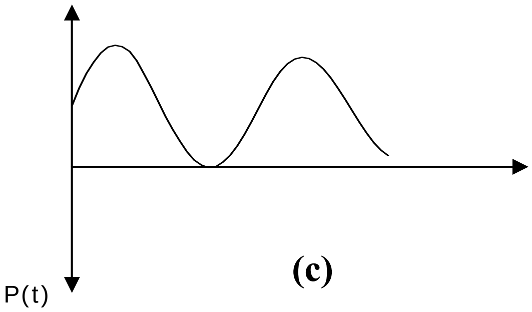
	Material Properties and Additional Parameter	Symbol	Value
Concrete	Young's Modulus	E_C (kg/cm^2)	300000
	Compressive Strength	f'_c (kg/cm^2)	420
	Tensile Strength	f_t (kg/cm^2)	28
	Poisson's Ratio	□	0.15
	Ultimate Compressive Strain	□$_{cu}$	0.003
	Fracture Energy	G_f (kg/cm)	0.153
	Yield Surface Function	□c	10
	Mass Density	□$_c$ (kg.sec^2/cm^4)	0.24E-05
	Fluidity Parameter	$_0$a	0.3055
		$_1$a	0.76
	Failure Surface Function	□$_0$	1.84
		□$_1$	1.09
Steel	Young's Modulus	ES (kg/cm^2)	2000000
	Yield Stresses	f_y (kg/cm^2)	3840
	Poisson's Ratio	□	0.2
Newmark's Parameters		□	0.5
		□	0.25



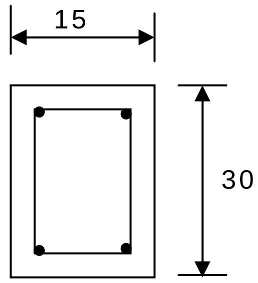
P(t) =1+sin0.1t

(a)

(a)

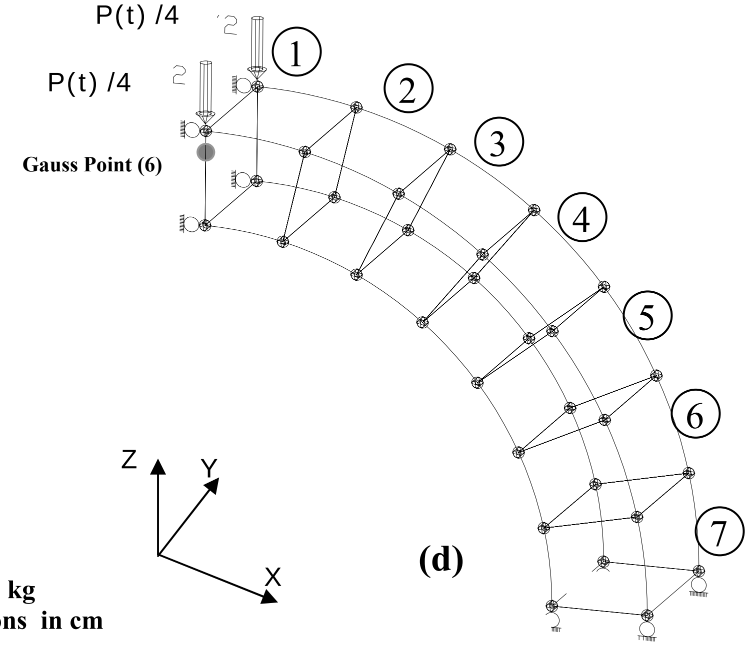


(c)



(b)

Reinforcement Details :
Main Reinforcement 4-φ2.2cm
Web Reinforcement φ0.6cm @8cm



Note:
All Loads in kg
All Dimensions in cm

(d)

(a) geometry and loading of arch (b) cross section (c) load – time relation
(d) finite element idealization and numbering of half of the arch

Figure (5) Simply Supported Arch Under Concentrated Load with Harmonic Excitation Load – Time Function

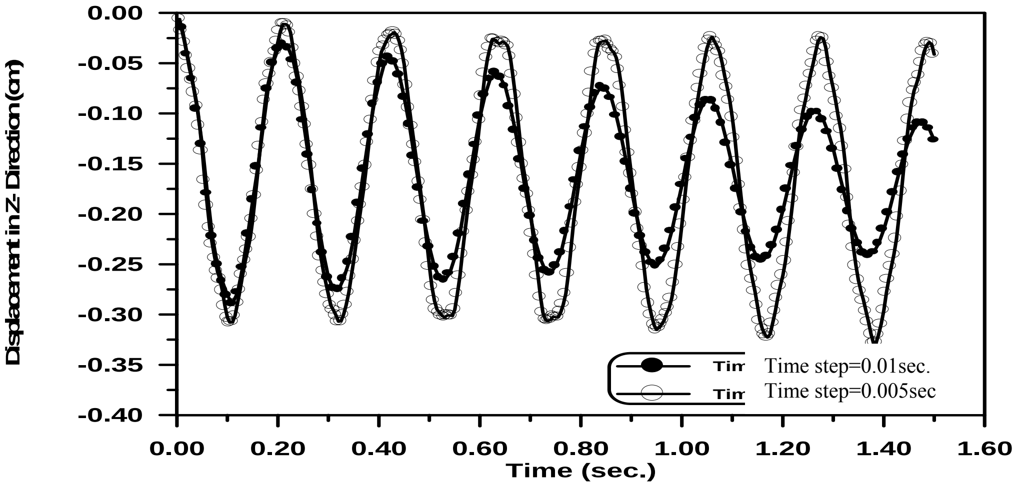


Figure (6) The Effect of Time Step on Time – Displacement in Z - Direction Response at Center of Arch .

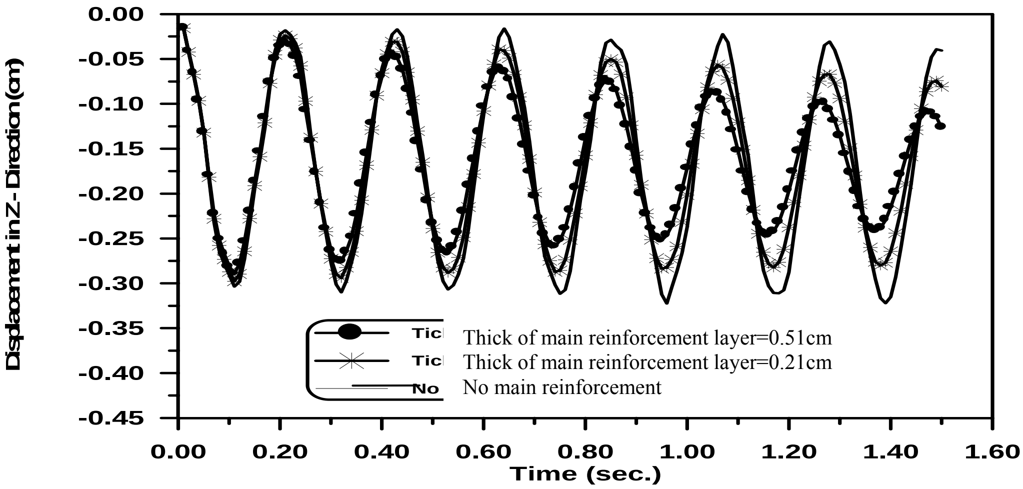


Figure (7) Effect the Thick of Main Reinforcement Layers on Time – Displacement in Z - Direction Response at Center of Arch .

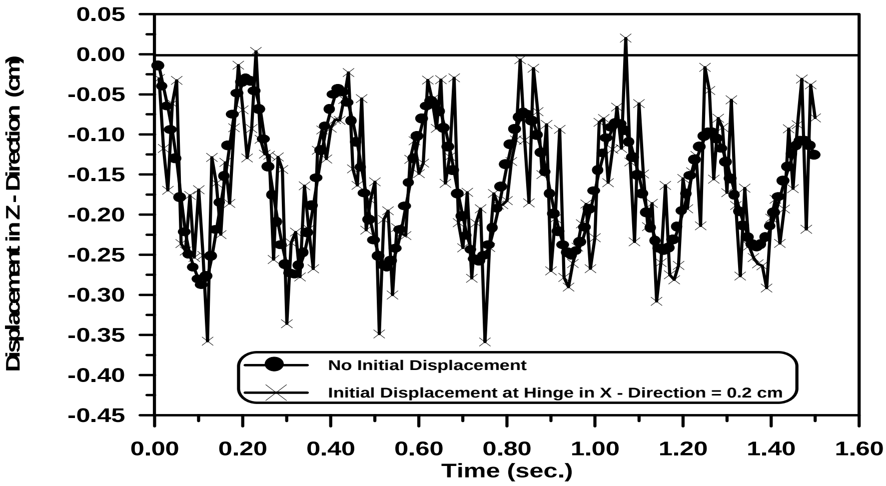


Figure (8) The Effect of Initial Displacement in X - Direction on Time – Displacement in Z - Direction Response at Center of Arch .

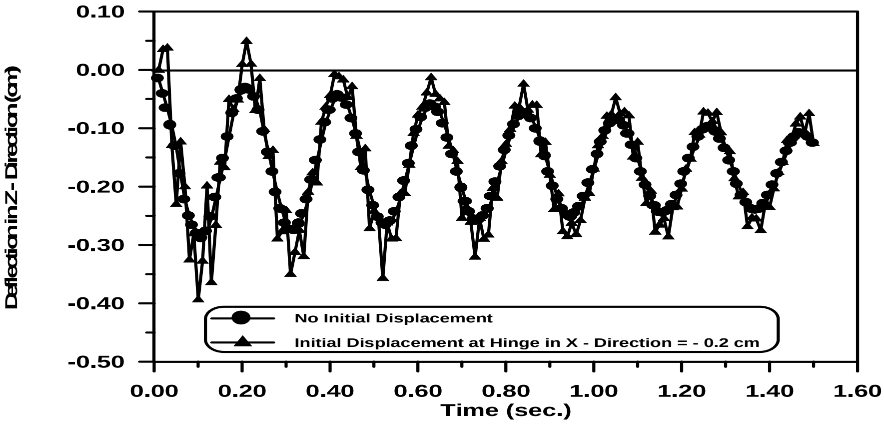


Figure (9) Effect the Initial Displacement opposite X – Direction on Time – Displacement in Z - Direction Response at Center of Arch .

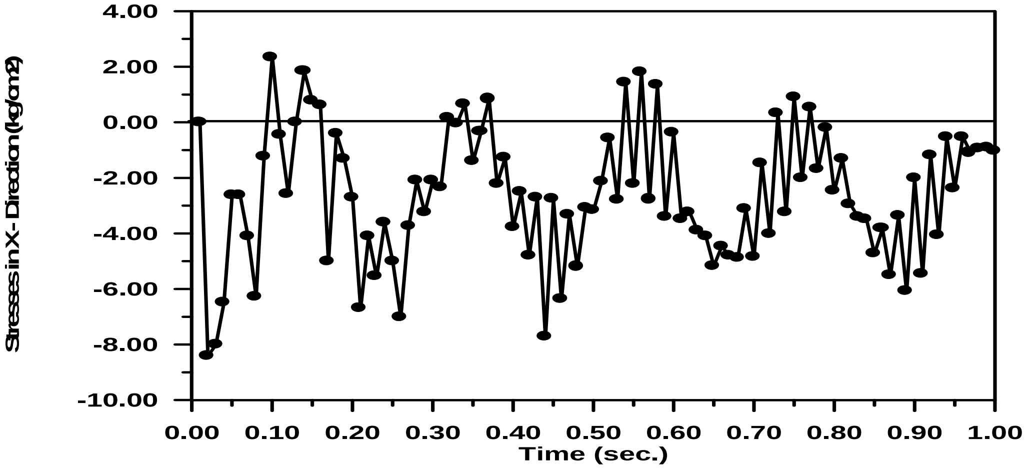


Figure (10) Time – Stress in X - Direction Relation at Gauss Point NO.6 (at center of arch) .

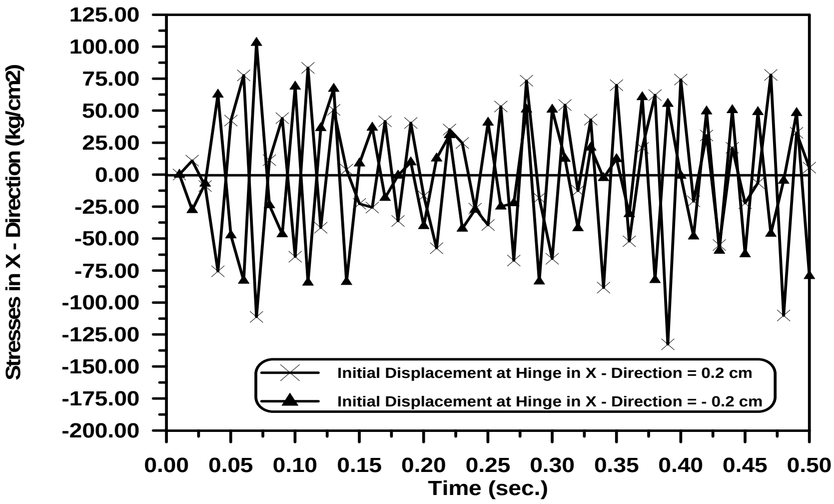


Figure (11) Effect of Initial Displacement on Time – Stress Response at Gauss Point NO.6 (at center of arch) .

Conclusions: from the present study , can noticed that

1- If initial displacement happened in the supports of the arch , the maximum central deflection increases , when this displacement is in the direction of X – axis with magnitude equal to (0.2,-0.2)cm, the maximum difference percent of central deflection by (57.5%) .

2- The maximum central deflection for arch structure plain concrete is greater than itis of reinforced arch with (0.236)cm thick for main layers and (0.127)cm thick for web layers reinforcement by about (7.4%) . But the maximum stresses in arch's center for plain concrete is less than that for this reinforced arch with main and web reinforcement by about (30%) in elastic stage .

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