# THE EFFECT OF ELASTIC MODULUS RATIO BETWEEN FIBER/MATRIX ON THE BEHAVIOUR OF UNIDIRECTIONAL LAMINATE

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## Abstract

Effect of elastic modulus ratio between fiber and matrix on the behaviour of unidirectional laminate was studying by micromechanical finite element techniques. This study was carried out for two values of H/b ratios (H is the fiber spacing and b is the length of the model). Low value when H/b =0.2 and high value when H/b =1.0. A constant strain finite element was formulated. The results that obtained from this finite element for three values of elastic modulus ratio between fiber and matrix ($E_f/E_m$ = 1,10,100) show that whenever $E_f/E_m$ increased the fiber-matrix shear stress decreased and this reduction is depended on fiber size (H/b).

### Nomenclature

| | |
|---|---|
| A | Cross section area of element |
| b | Length of the model |
| $[B]^{(e)}$ | Element strain matrix |
| $D_f$ | The flexural stiffness |
| $E_f$ | Modulus of elasticity of fiber |
| $E_m$ | Modulus of elasticity of matrix |
| $h_f$ | Width of fiber |
| $h_m$ | Width of [illegible] |
| [K] | Global [illegible] |
| $U_m$ | Strain energy [illegible] |
| [illegible] | Strain energy [illegible] |
| Uc | Strain energy [illegible] |
| $\mu_f, \lambda_f$ | Lame's mater[illegible] |
| $\mu_m, \lambda_m$ | Lame's materi[illegible] |
| $\mu_{121}$ | Couple stress |
| $\sigma$ | Normal stress |
| $\varepsilon$ | Normal strain |
| $\tau$ | Shear stress |
| $\gamma$ | Shear strain |
| $\Psi_{21}$ | Measure of the she[illegible] direction |
| $\Psi_{22}$ | Measure of the shear [illegible] direction of the fiber |
| $\chi_{121}$ | The derivative of $\Psi_2$ [illegible] |
| $\{\Phi\}$ | Global displacement |

## 1. Introduction

Modern technology has found extensive use for unidirectionally fiber-reinforced composite materials. To make effective use of these materials, knowledge of their properties and performances when subjected to loads is essential. Many aspects of their behaviour are directly associated with the microscopic structure of these materials. The desire to understand these materials drives the research in this field into the micromechanics of these types of materials, [1,3]. However, when the micromechanical behaviour of such a composite material is of interest, for example, the initiation and development of a damage process well before a macroscopic failure, one has to take account of the heterogeneity between fibers and matrix.

The behaviour of composites depends on variety parameters such as the properties of the components, bonding between the components, alignment of fibers and so on. Thus, the micromechanical finite element analysis, which has been employed increasingly for fiber-reinforced composites in the past decade, has become an important means of understanding this behaviour of composites. The theoretical and experimental methods usually applied in micromechanical analysis in order to tackle the problems that caused by the above parameters.

So the present paper studies the effect of elastic modules ratio ($E_f/E_m$) on the behaviour of unidirectional laminate or fiber - reinforced materials when to the mechanical loading based upon two dimensional plane elasticity theory and two dimensional finite element method using displacement formulation by using the computer program (SAOUDI) in Ref.[2].

## 2. Unidirectional laminate.

In this approach the unidirectional laminate can be examined microscopically by considering a unidirectional composite with reinforcing layers (fibers) in ($x_1$ direction) as shown in Fig.(1). The representative volume element of this composite is chosen to be two fibers embedded in a matrix plate. The fibers are assumed homogeneous and have a rectangular cross section with the same thickness as the matrix plate. Therefore, the width ratio $h_f$ / ( $h_f$ + $h_m$ ) is chosen to be the same as the fiber volume fraction of the composite itself. The H is the fiber spacing and the $\bar{u}_i$ represent displacements within the fibers and these displacements are independent of displacement, $u_i$, which define the deformation of a classical continuum. Both displacement are related to Cartesian co-ordinates $x_i$.

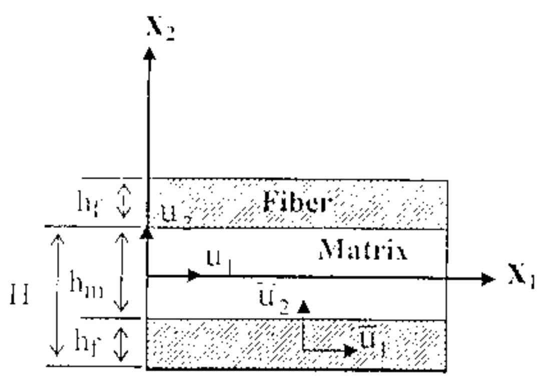



**Fig. (1) Arrangement of fiber and matrix**

The strain energy [4.6] of **n** discrete reinforcing layers (fiber) can be supposed by a weighted integral over the entire region. Therefore :-

$$\sum_{k=1}^{n} U_f^k \approx \frac{V_f}{h_f}\int U_f \, dx_2 \qquad (1)$$

where $V_f = h_f/H$ indicates the fiber volume fraction or density of the fibers and $U_f^k$ is the strain energy density per unit length of the $k_{th}$ fiber.

by assuming that longitudinal stress in the fiber varies linearly across its width and that transverse normal stress is uniform across the fiber. So the strain energy density of the fiber may be written as :-

$$U_f^k = \frac{1}{2}D_f\left(\frac{\partial \psi_{21}}{\partial x_1}\right)^2 + \frac{1}{2}\mu_f h_f\left(\psi_{21}+\frac{\partial u_2}{\partial x_1}\right)^2 + \frac{1}{2}\lambda_f h_f\left(\frac{\partial u_1}{\partial x_1}+\psi_{22}\right)^2 + \mu_f h_f\left[\left(\frac{\partial u_1}{\partial x_1}\right)^2+\psi_{22}^2\right] \qquad \ldots(2)$$

Where, $\lambda_f$, $\mu_f$ Lame's materials constant for fibers layer and

$$\psi_{21} = \frac{\partial u_1}{\partial x_2}, \quad \psi_{22} = \frac{\partial u_2}{\partial x_2} \qquad \ldots(3)$$

Within the matrix the normal strains in the $x_1$ directions are assumed identical to those in the fiber; the transverse normal and shear strains within the matrix, however, are not dependent solely on the overall displacement field $u_i$, but are also influenced by the strains within, and the thickness of the fiber the strain energy per unit volume of the matrix may be shown to be :-

$$U_m = \frac{1}{2}\lambda_m\left[\varepsilon_{11}+\left(\varepsilon_{22}+\frac{V_f}{1-V_f}\gamma_{22}\right)\right]^2 + \mu_m\left[\varepsilon_{11}^2+\left(\varepsilon_{22}+\frac{V_f}{1-V_f}\gamma_{22}\right)^2+2\left(\varepsilon_{12}+\frac{1}{2}\frac{V_f}{1-V_f}\gamma_{21}\right)^2\right] \qquad \ldots(4)$$

Where, $\lambda_m$ and $\mu_m$ are Lame's material constants for the matrix. Thus the total strain energy per unit volume of the composite may be written as :-

$$U_c = \frac{V_f}{h_f}U_f + (1-V_f)U_m \qquad \ldots(5)$$

by substituted Eqn. (2) and (4) in Eqn. (5) and compared with the strain energy of an isotropic material with microstructure and then assuming plane stress instead of plane strain [2] the relation between strees and strain of unidirectional laminate can be written as:

$$
\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \\ \sigma_{21} \\ \sigma_{22} \\ \mu_{121} \end{Bmatrix} = \begin{bmatrix} d_1 & d_2 & 0 & 0 & -d_8 & 0 \\ d_2 & d_1 & 0 & 0 & -d_9 & 0 \\ 0 & 0 & d_3 & d_5 & 0 & 0 \\ 0 & 0 & d_5 & d_4 & 0 & 0 \\ d_8 & d_9 & 0 & 0 & d_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_7 \end{bmatrix} . \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \\ \gamma_{21} \\ \gamma_{22} \\ \chi_{121} \end{Bmatrix} \quad \ldots(6)
$$

where:-

$$
\begin{aligned}
d_1 &= 4\mu(\mu+\lambda)/(\lambda+2\mu) \\
d_2 &= 2\mu\lambda/(\lambda+2\mu) \\
d_3 &= \mu \\
d_4 &= V_f\left(\mu_f + \frac{V_f}{1-V_f}\mu_m\right) \\
d_5 &= V_f\left(\mu_m - \mu_f\right) \\
d_6 &= V_f\left[\lambda_f + 2\mu_f + \frac{V_f}{1-V_f}(\lambda_m + 2\mu_m) - V_f\left(\lambda_m - \lambda_f\right)^2/(\lambda+2\mu)\right] \\
d_7 &= V_f\,\mu_f\,h_f^2/6(1-v_f) \\
d_8 &= V_f\,\mu(\lambda_m-\lambda_f)/(\lambda+2\mu) \\
d_9 &= d_8 + 2V_f\left(\mu_m - \mu_f\right)
\end{aligned} \quad \ldots(7)
$$

## 3. Finite Element Formulation

The element that depending on the present paper is a triangular element which has straight sides and three nodes, one at each corner as shown in Fig.(2), like wise, with three degree of freedom per node. . The local shear deformation $\phi_3$ that denoted by $(\phi_{3i}, \phi_{3j}, \text{and}\, \phi_{3k})$ at each node is equal to $\psi_{21}$, so by using an expansion of the displacements $\phi_1, \phi_2, \text{and}\, \phi_3$ the strain matrix [B] can be derived as:-

$$
\begin{aligned}
\phi_1 &= c_1 + c_2 x_1 + c_3 x_2 \\
\phi_2 &= c_4 + c_5 x_1 + c_6 x_2 \\
\phi_3 &= c_7 + c_8 x_1 + c_9 x_2 = \psi_{21}
\end{aligned} \quad \ldots(8)
$$

Where $c_1$ to $c_9$ are constants. So the element strain-displacement relations can be written as:-

$$
\left\{\varepsilon^{(e)}\right\} = \left[B^{(e)}\right]\left\{\phi^{(e)}\right\} \quad \ldots(9)
$$

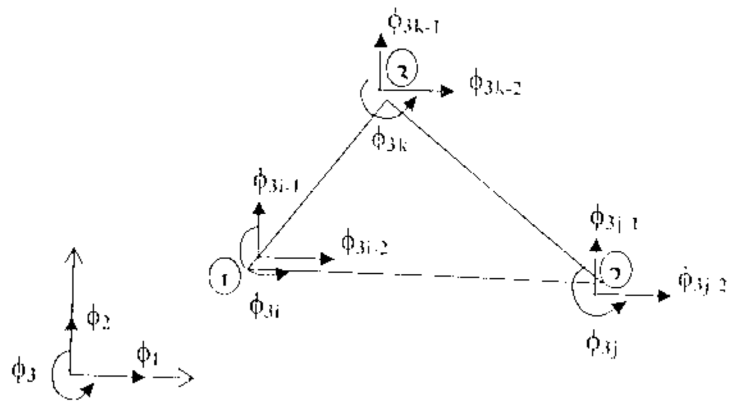



**Fig.( 2):** Numbering of finite element node and degree of freedom.

The stiffness matrix [k] is then formed by the following expression

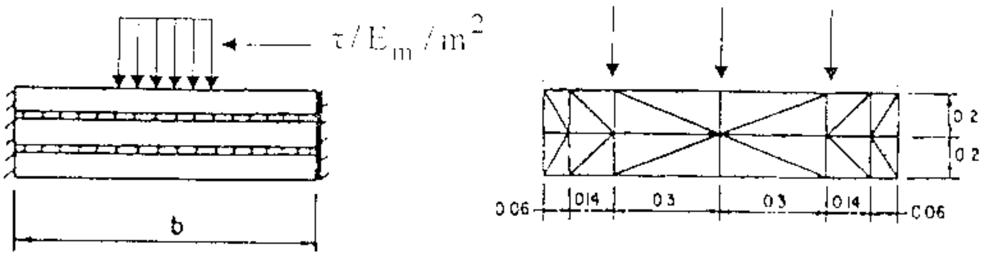
$$[K] = [B]^T [D] [B] \, t \, . \, A \qquad \ldots (10)$$

Where [D] is the elasticity matrix for unidirectional laminate given in Eqn.(6). The term t and A are the thickness of composite and area of the element respectively. And [B] is given as:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x_1} & 0 & 0 & \frac{\partial N_2}{\partial x_1} & 0 & 0 & \frac{\partial N_3}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial N_1}{\partial x_2} & 0 & 0 & \frac{\partial N_2}{\partial x_2} & 0 & 0 & \frac{\partial N_3}{\partial x_2} & 0 \\ \frac{\partial N_1}{\partial x_2} & \frac{\partial N_1}{\partial x_1} & 0 & \frac{\partial N_2}{\partial x_2} & \frac{\partial N_2}{\partial x_1} & 0 & \frac{\partial N_3}{\partial x_2} & \frac{\partial N_3}{\partial x_1} & 0 \\ \frac{\partial N_1}{\partial x_2} & 0 & N_1 & \frac{\partial N_2}{\partial x_2} & 0 & N_2 & \frac{\partial N_3}{\partial x_2} & 0 & N_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_1}{\partial x_1} & 0 & 0 & \frac{\partial N_2}{\partial x_1} & 0 & 0 & \frac{\partial N_3}{\partial x_1} \end{bmatrix} \ldots (11)$$

## 3. Distributed Load and Unidirectional laminate

This example represent a plate has rigid supports at both ends so the local shear deformation $\phi_3$ are zero in these ends. This plate composed from unidirectional laminate which can be studied the effect of elastic modules ratio $E_f/E_m$ by assuming constant fiber volume fraction $V_f = 0.25$. and constant fiber and matrix poissn's ($\nu_m = \nu_f = 0.2$) and also assuming three values of ( $E_f/E_m$=1. 10. 100) as shown in Fig. (3-a) and Fig. (3-b) shows the finite element arrangement which apply and the finite element program (SAUDOL) in Ref. [2].




( a ) ( b )

**Fig.(3):** Plate under distributed load

**(a)** unidirectional laminate **(b)** Finite element

## 4.Results.

By using two values of fiber size first low (H/b = 0.2) and second high (H/b = 1.0). Fig.(4) shows that the $\phi_2$ increased with increasing the elastic modulus ratio ($E_f/E_m$) and this increment increased when fiber size has high value (H/b =1.0) as shown in Fig.(8). All the figures of $\phi_2$ start from zero and decreased until it has a min. value at the position of max. load and then begins to increase until reaching zero value again at the end of plate because the rigid supports in both end given a zero value of $\phi_2$ and $\phi_3$.

From Fig.(5) and (9) which show the effect of $E_f/E_m$ on $\phi_3$ its clear that $\phi_3$ has a little change when $E_f/E_m$ increases at low H/b (see Fig.(5)). While at high value of H/b, $\phi_3$ has a sensitive change or sensitive reduction when $E_f/E_m$ increases (see Fig.(9)).

Thus, the elastic modules ratio $E_f/E_m$ has a large effect of $\phi_3$ because the changing on them causes changing in B.C. for the problem.

Fig.(6) and (10) show the effect of $E_f/E_m$ on $\mu_{121}$ which has a large changing when $E_f/E_m$ increases and also this changing increased at high fiber size as shown in Fig. (10).

Finally Fig.(7) and (11) show the effect of $E_f/E_m$ on FMSS which is increased when $E_f/E_m$ increased and has a max. value at the end of plate also this increment depended on fiber size thus it is higher at low fiber size than high fiber size.

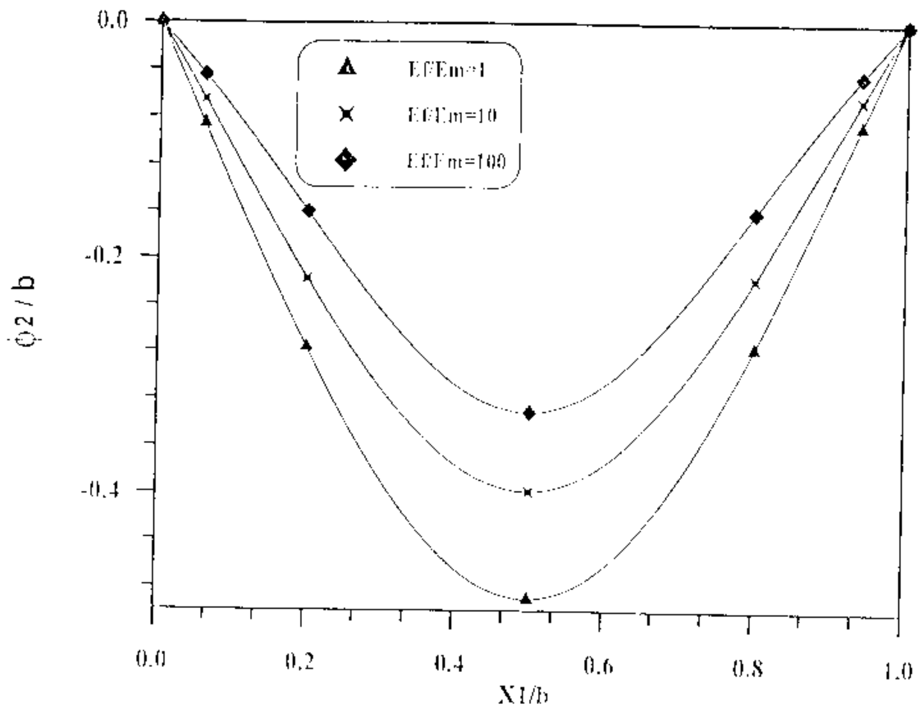
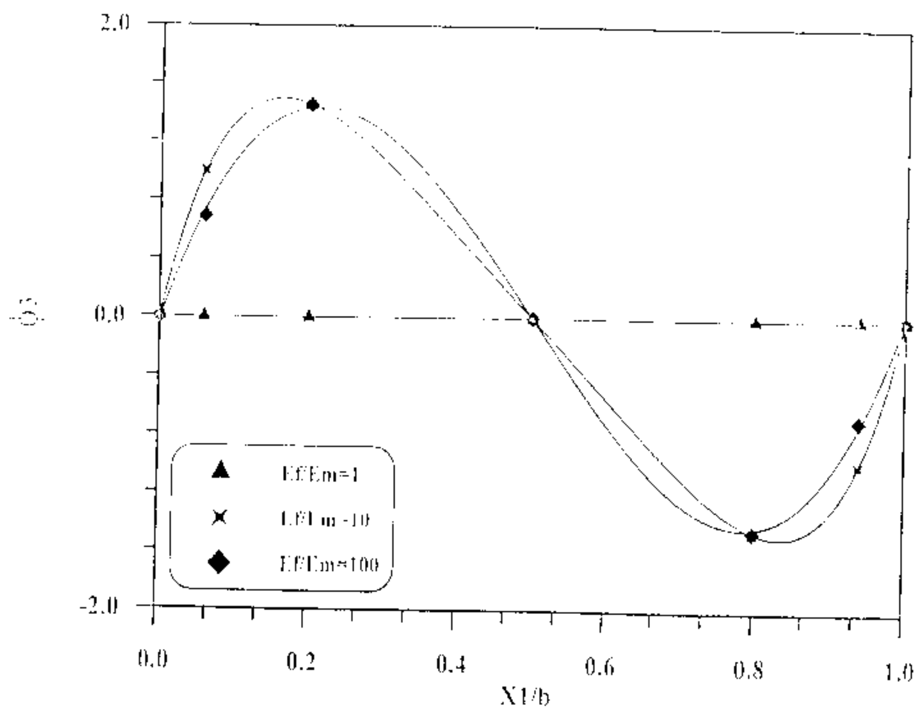



Fig.(4):Effect of elastic modulus ratio between fiber and matrix on vertical dislacement at low fiber size (H/ b=0.2) .




Fig(5)-Effect of elastic modulus ratio between fiber and matrix on local shear deformation at low fiber size (H/ b=0.2) .

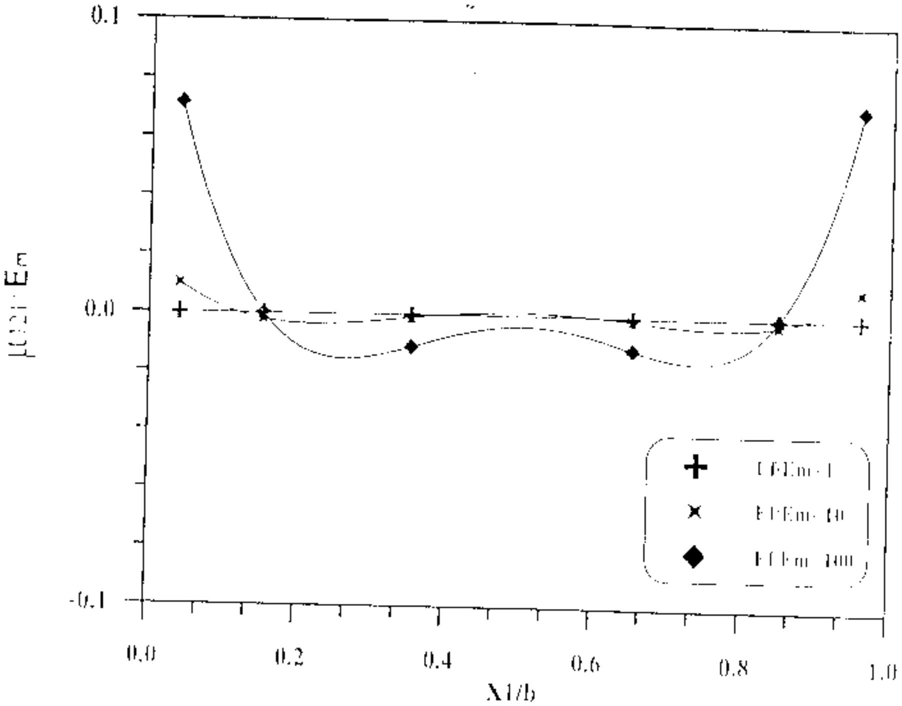



Fig.(6):Effect of elastic modulus ratio
between fiber and matrix on couple stress
at low fiber size (H/ b -0.2) .

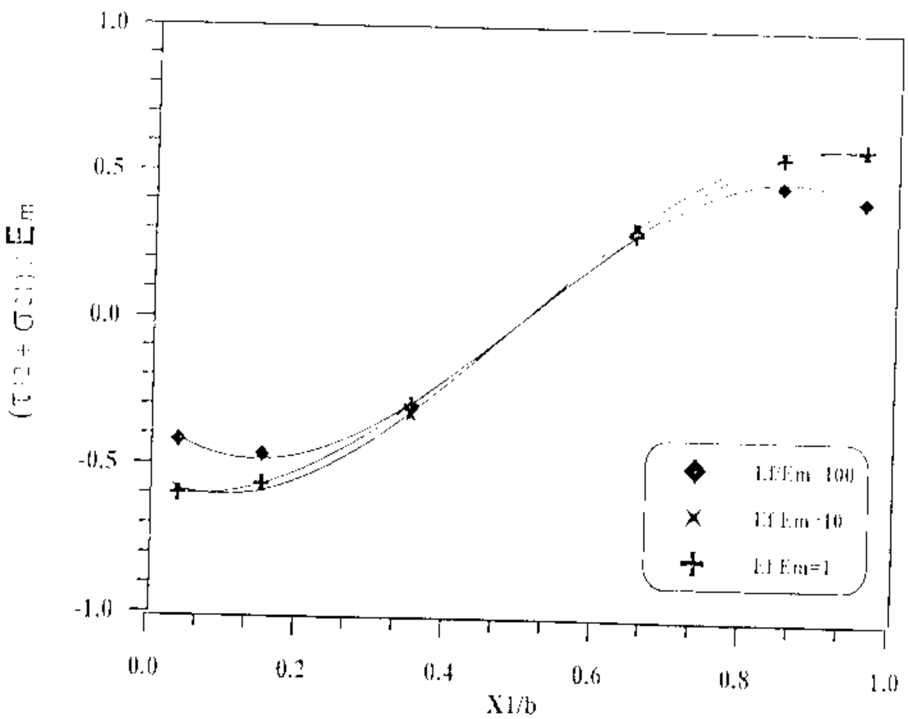



Fig.(7):Effect of elastic modulus ratio
between fiber and matrix on fiber-matrix shear stress
at low fiber size (H/ b=0.2) .

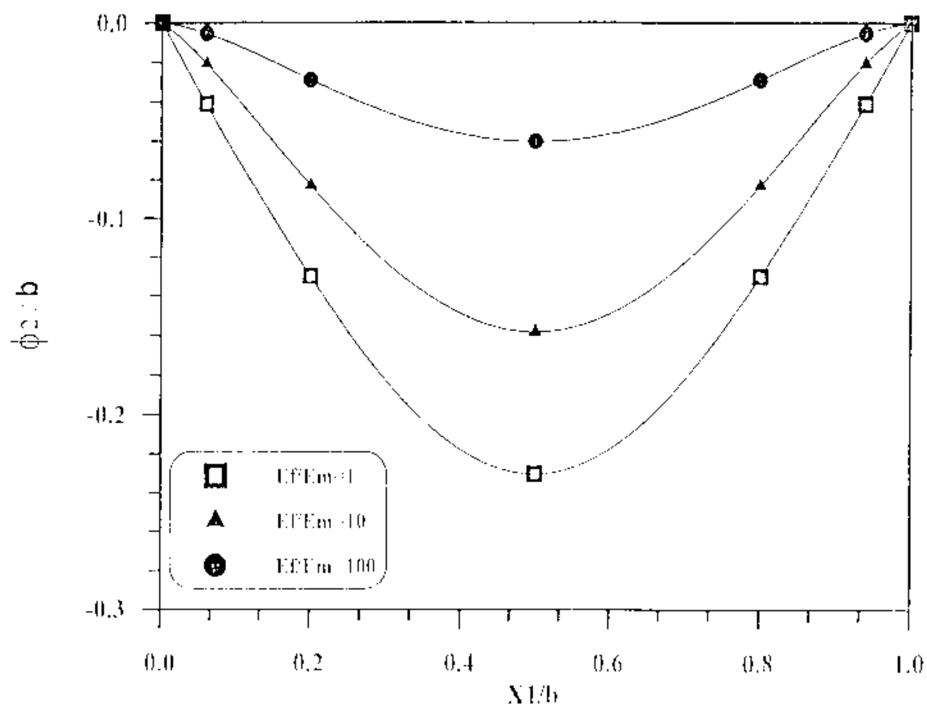



Fig.(8):Effect of elastic modulus ratio
between fiber and matrix on vertical displacement
at high fiber size (H/ b=1.0) .

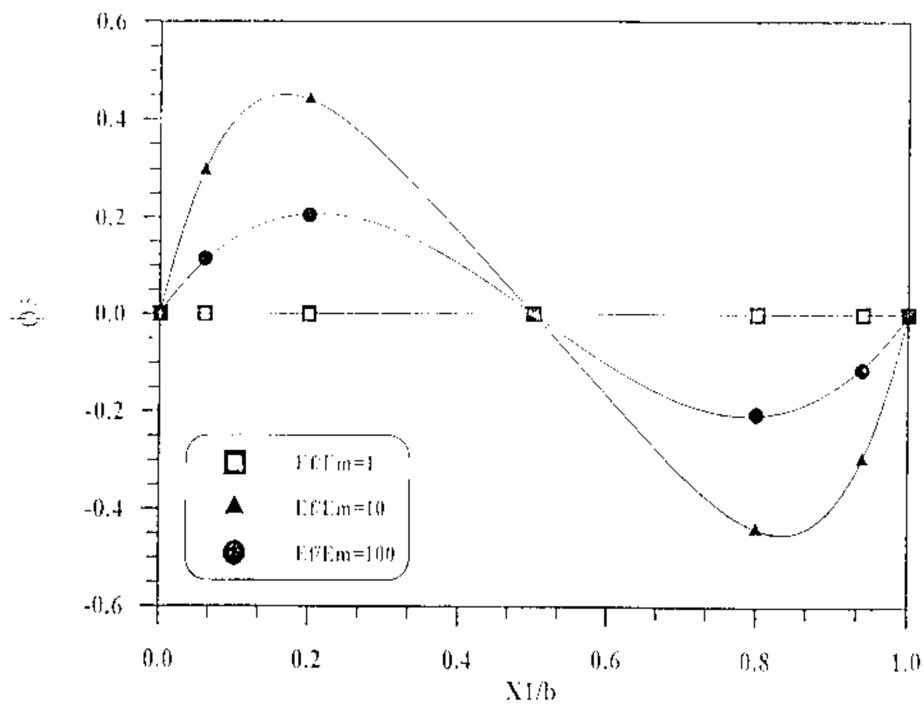



Fig.(9)-Effect of elastic modulus ratio
between fiber and matrix on local shear deformation
at high fiber size (H/ b=1.0) .

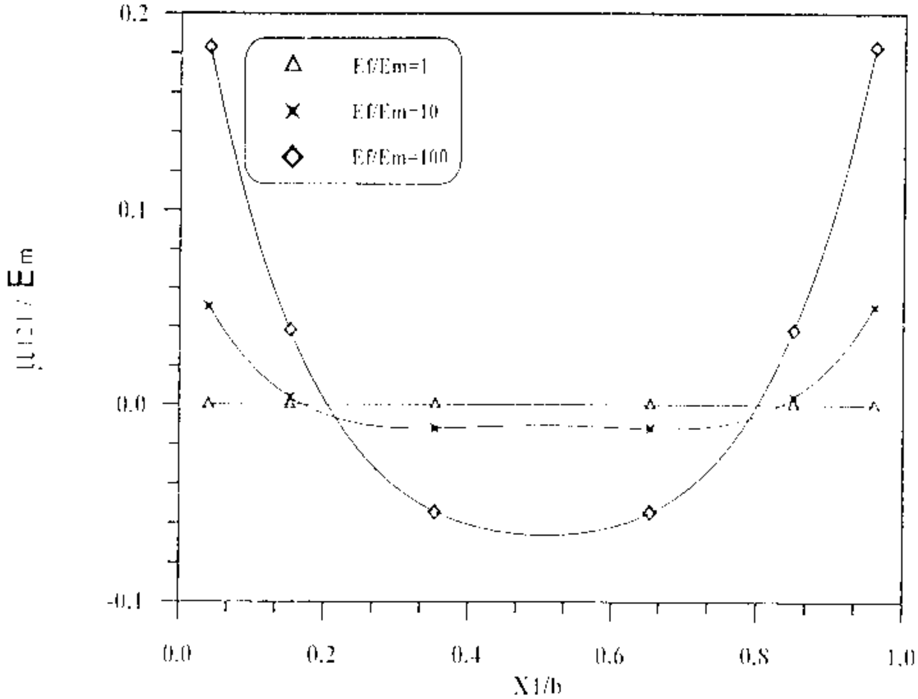



Fig.(10):Effect of elastic modulus ratio between fiber and matrix on couple stress at high fiber size (H/ b=1.0) .

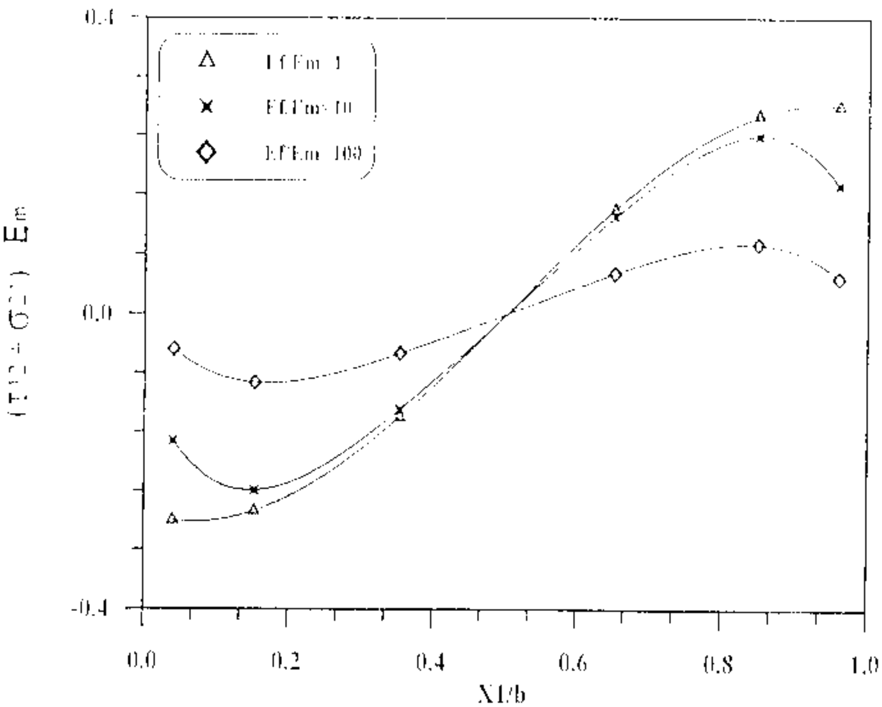



Fig.(11):Effect of elastic modulus ratio between fiber and matrix on fiber-matrix shear stress at high fiber size (H/ b=1.0) .

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## تأثير نسبة معامل المرونة بين الليف/ المادة الاساس على سلوك الصفائح المقوات باتجاه موحد


الخلاصة

تم دراسة تأثير نسبة معامل المرونة لليف ($E_f$) ومعامل المرونة للمادة الاساس ($E_m$) على سلوك الصفائح المقوات باتجاه موحد باعتماد التحليل المايكرو ميكانيكي وبواسطة تقنية العناصر المنتهية. اجريت الدراسة لقيمتين من النسبة بين فراغ الالياف (H) وطول النموذج (b)، قيمة واطئة (H/b=0.2) وقيمة عالية (H/b=1). اظهرت النتائج ولثلاث قيم من نسبة معامل المرونة ($E_f/E_m$ = 1, 10, 100) بانه عندما تزداد النسبة ($E_f/E_m$) فان اجهاد القص للمادة الاساس ينخفض، وهذا الانخفاض يعتمد على حجم الليف (نسبة H/b).