A Novel Experimental Work and Study on Flow and Energy Dissipation over Stepped Spillways

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Abstract

In this study, a novel experimental work was used to study the flow over the stepped spillways and energy dissipation. This novel work was an attempt to increase energy dissipation on stepped spillways since it not include stilling basin and have less cost compared with traditional spillways. So, in this paper, to investigate the increasing of energy dissipation of flow over stepped spillways of different step shapes, a twelve physical plywood models have been built. Experiments have been carried out for different types of step shapes: - plain steps, steps have half cut, inclined end sill steps normal to downstream slopes of stepped face, and steps have a rough surface using crushed gravel. Three downstream slopes of stepped face (è = 30 ^{o}, 50 ^{o}, and 70 ^{o}) were tested with spillway having constant number of steps and constant height. In total, 144 experiments were done, the hydraulic parameters of flow over the models were measured and the energy dissipation of flow was calculated. Experimental results showed that the energy dissipation of flow on inclined end sill, and rough steps are more than the plain one. While the experiments results on the cutting steps showed that the energy dissipation are less than the plain one. With decreasing the downstream slopes of the stepped spillway face, the energy dissipation is increased. In addition, when compared with traditional spillways the case of rough steps and steps with inclined end sill can be used to increase the relative energy dissipation on plain steps by about 19 % and about 18 % respectively for small angle of spillway face since the stepped spillways more effective than traditional spillways by 10 %. Therefore, double that means that the affectivity of stepped spillway was approximately. On the other side, the energy dissipation increased at low discharges by 0.1 % when using the cutting steps but decreased by 13 % with larger discharges. In summary, the flow energy dissipation decreases with increasing the flow rate, and the roughed step spillway surface is more effective compared to the other spillway surfaces at low or high flow rate. According to the flow types, the range of Nappe, Transition, and Skimming will increase for all the forms of steps that used compare with plain step.

Keywords: Stepped spillways, energy dissipation, flow types.

الخلاصة

هذا البحث هو عبارة عن تقديم لدراسة عملية جديدة استخدمت لتحليل الجريان فوق المطافح المدرجة وتشتيت الطاقة هذه الملحقات الجديدة هي محاولة لزيادة تشتيت الطاقة كون المطافح المدرجة لا تحتاج احواض تشتيت الطاقة وبالتالي تكون اقل كلفة مقارنة بالمطافح التقليدية. ولذلك في هذه الدراسة وللتحري في زيادة تشتيت طاقة الجريان على المطافح المدرجة التي تملك عدة اشكال لدرجاتها تم عمل اثنا عشر نوذجا فيزيائيا من الخشب المقاوم للماء. التجارب تم اجرائها على عدة اشكال من الدرجات :- مستوية، نصف مقطوعة، عوارض مائلة عمودية على وجه المطفح المدرج و درجات مخشنة باستخدام الحصى المكسر. تم استخدام ثلاث ميول لاوجه المطافح المدرجة (è = 30^{o} و è = 50^{o} و è= 70^{o}) مع ثبوت عدد درجات المطافح مع ثبوت ارتفاع تلك المطافح. بصورة عامه تم تشغيل 144 تشغيلة من خالها تم تسجيل المتغيرات الهيدروليكية للجريان ومن ثم تم حساب تشتيت طاقة الجريان. النتائج بينت بان تشتيت طاقة الجريان على الدرجات ذات العوارض المائلة والعوارض المخشنة هي اكثر من العوارض المستوية عدا الدرجات المقطوعة كانت اقل. كذلك ان النتائج بينت انه عندما يقل ميلان وجه المطفح سيؤدي الى زيادة تشتيت الطاقة وعند استخدام الحصى المكسر والعوارض المائلة سيؤدي الى زيادة تشتيت الطاقة بنسبة 19% و 18 % على التوالي بالمقارنة مع المطافح التقليدية غير المدرجة مع العلم ان المطافح المدرجة هي افضل من تلك التقليدية بنسبة 10 %. أي بمعنى ان كفاءة المطافح المدرجة قد تم زيادتها بنسبة مقاربة للضعف. من جانب اخر وعند استخدام الدرجات المقطوعة فان تشتيت الطاقة سيزداد عند المطافح ذات الميول القليلة وعن التصاريف القليلة وبنسبة 0.1 % اما عند زيادة تلك التصاريف فان تشتيت الطاقة سيقل وبنسبة 13% . وبصورة عامه فان تشتيت الطاقة سيقل بزيادة التصاريف. ويتلخص من ذلك فان المطافح المخشنة تبدي تشتيتا اكثر للطاقة لكل التصاريف مقارنة مع بقية الاشكال للدرجات. بالنسبة لانواع الجريان فان مدى جريان الـــ nappe و transition و skimming سوف يزداد لكل اشكال الدرجات المستخمة مقارنة مع الدرجات المستوية .

الكلمات المفتاحية: المطافح المدرجة، تشتيت الطاقة، انواع الجريان

1. Introduction

Spillways are hydraulic structures used to release excessive water in the reservoir. Analyses of the flow and the energy dissipation over the spillways are two of the major problems in hydraulic engineering. It becomes usual to protect the spillway surface from cavitation erosion by introducing air next to the spillway surface using aeration devices located on the spillway bottom and sometimes on the sidewalls [Chanson,1997].Since the high discharge of flow over spillway, their design and construction was more complicated [Novak,1990]. Since the flow is supercritical and has a very high velocity and hence erosive power. Therefore, this energy should be dissipated to protect agenized scouring of the downstream river bed and undermining of the foundations. Stepped spillways allow to increase energy dissipation over spillway and that lead to lower the risk of cavitation on spillway surface. Also, the cavitation risk along the spillway decreases significantly due to smaller flow velocities and the large air entertainment rate [Rajaratnam, 1990].

Barani *et al.*[1999] investigated energy dissipation of flow over different step shapes. Experimental work on many stepped such as (plain steps, steps with bottom adverse slope of 15^o,26^o,36^o, and 45°, and end sill steps with thickness of 1, 2, 3 and 4 cm and). Overall, the hydraulic parameters were measured and the energy dissipation for the flow were calculated. The energy dissipation was increased by increasing the thickness of end sill or the adverse slope size.[Al-Talib, 2007] found that stepped weirs are more efficient than flat sloped weirs and the energy dissipation was improved by the ratio approximately 10 %.[Chen *et. al.*2002] analyzed the flow of the stepped spillway; he applied the finite volume technique and used the k-e model for the determination of the flow turbulence. [Alghazali and Jasim,2014] present an Experimental work using twelve stepped spillway models. These models were manufactured with three downstream slope angles which are: 25, 35 and 45°, and four numbers of steps which are: 5,10, 15 and 20. Five cases of steps were tested, which are traditional Flat, porous end sills, pooled, pooled with gabions and porous end sills with gabions. The results revealed that the end sills highly affect flow regime type; this effect is primarily for the lower limits of skimming flow. Using end sills that lead to increases the range of transition flow regime (by increasing the lower limit of skimming flow) also that lead to increases the instabilities that occur in this flow regime. Gabions reduce the effects of end sills on the lower limit of skimming flow. New empirical equations were established based on the experimental work.

El- Jumaily and Al- Lami [2009] take a suggested design of Bastora stepped spillway as a prototype to build a physical wooden model with scale of 1:20 (Lm/Lp). Experiments have been carried out on the model with slope of upward inclined steps of 42^0,28^0,14^0 and 0^0.For every slope of the steps, experiments were conducted in three flow regimes, nappe, transition, and skimming. As observed in experiments, the increase in the slope of steps has no significant effect on the flow behavior over stepped spillway. The hydraulic depths of flow over the model were measured and the energy dissipation rate was calculated. Results show that the energy dissipation decreases with increasing the discharge, and more than on the horizontal stepped spillways compared with upward inclined steps is, it increases with increasing the adverse slope of steps. [Khalaf *et. al.*,2014] studied the flow characteristics and energy dissipation over traditional and stepped spillway with semicircular crest. His experimental results show that when increasing the ratio between upstream water depth to the water depth at crest and Froud No. at spillway toe lead to increase energy dissipation on traditional and stepped spillway.

The main objectives of this paper are to study the flow characteristics, energy dissipation and increasing it on stepped spillway. In this study, the affectivity of stepped spillway was increased using more than one case.

2. Theoretical Work

The general relationships for the flow energy dissipation can be verified. Applying energy equations between Upstream (U/S) and Downstream (D/S) of stepped spillway, and that lead to the following equations [Henderson, 1966]:

$$E_o = y_o + \frac{\alpha v o^2}{2g} \quad \ldots(1)$$

$$E_1 = y_1 + \frac{\alpha v 1^2}{2g} \quad \ldots(2)$$

$$\frac{\Delta E}{Eo}\% = \frac{(Eo - E1)}{Eo}\% \quad \ldots(3)$$

Where:
E_o = U/S energy (m),
E_1 = D/S energy (m),
V_o = velocity at U/S (m/sec),
V_1 = velocity at D/S at toe of spillway (m/sec),
y_o: depth of water at U/S,
y_1: depth of water at toe of spillway,
$\frac{\Delta E}{Eo}$ % = Relative energy dissipation between upstream U/S and downstream D/S of stepped and flat sloped spillway,
α = kinetic correction coefficient, for turbulent flow, generally equal to 1.1, [Chow, 1959],
g = acceleration due to gravity (m/s^2).

3. Laboratory Work

The experimental work using ARMFIELD flume was performed at the hydraulics laboratories in the college of Engineering at Al-Qadisiyah University. The flume is rectangular with a width of 7.5 cm, length of 2.45 m and height of 0.25 m as shown in Fig. 1. A sharp crested weir with a height of 2.5 cm was installed at the end of flume to create a hydraulic jump.

The upstream flow heads were started to measure at a location more than (9 y_c) upstream of the spillway model, where y_c is the depth of water over the spillway crest. Discharges were measured by a calibrated flow meter installed at the channel outlet and the maximum discharge of the flume is 2.64 ℓ/sec. Using an accurate point gauge reading to 0.1 mm, the water surface levels were measured at different locations.

The length of all crests in all models and radius of curvature of upstream face are calculated from the following formulas [Henderson,1966] [Chow, 1959], respectively.

$$\frac{L\ crest}{HT - he} > (1.5 - 3) \quad \ldots(4)$$

$$R = 0.2\,(HT - he) \quad \ldots(5)$$

Where:
L_{crest}: is the length of the broad crested weir in the flow direction,
HT : U/S total head above the channel bed (from maximum discharge);
he : weir height above the channel bed,
R: radius of curvature of upstream face.

Using the equations (4) and (5) with characteristics shown in Table (1), twelve models were designed from plywood to calculate the length of crest and radius of curvature, respectively. In this study, the height of all models used was constant and equal to 60 % of the flume height i.e.15 cm to avoid overflow. All spillways were installed at distance of 0.75 m from the flume entrance to eliminate turbulence. Three angles (è = 30 ^{o}, 50 ^{o}, and 70 ^{o}) of the spillways faces are randomly used for each groups of the models. These angles are equal or greater than the critical value defined by Chanson (1994) which is è = 27°. The installation allows mainly observing the different flow regimes.

Fig.(1): The ARMFIELD flume used for the experiment.

Figure (2) represents the schematic presentation of flume used.

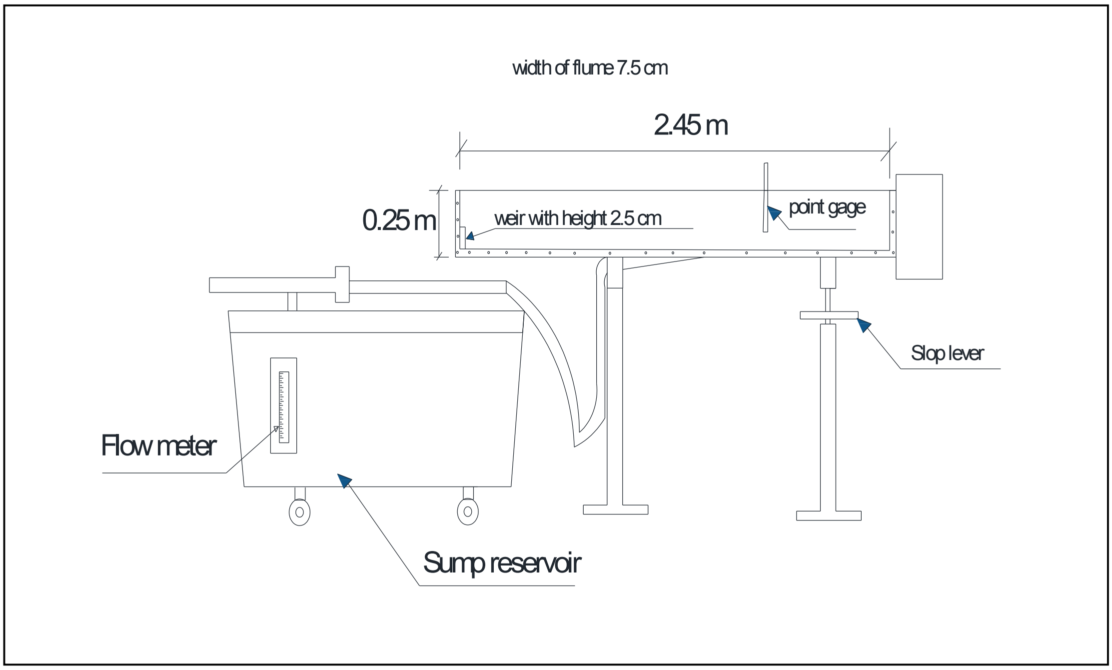

Fig.(2): The schematic of flume used.

The maximum U/S head above bed channel was 21 cm from previous observations, and the height of spillways that was considered in this work was 15 cm. So, the length of crest was about 10 cm using equation 4 and the U/S edge radius of all spillways was 1.2 cm using equation 5 to prevent separation of stream flow from the crest. The twelve models were designed and made from plywood and numbered as in Table (1) below.

Table (1): Characteristics and the name of the twelve models.

Step case	Model Number (No.) or description		
	Face slope = 30 ^{o}	Face slope = 50 ^{o}	Face slope = 70 ^{o}
plain	1	2	3
Half cut	4	5	6
Rough	7	8	9
Inclined end sill	10	11	12

For each model twelve runs are used including measuring the discharge (Q), U/S flow depth (y_o), critical flow depth over spillway (y_c), and the D/S flow depth (y_1) for each run. Overall, 144 experiments were done, the hydraulic parameters of flow over the models were measured and the energy dissipation of flow were calculated. In this experimental work, the twelve models were used – which were explained in Table (1)- to study increasing energy dissipation and regimes of flow on stepped spillway. Figure (3) represents the dimensions of plain models.

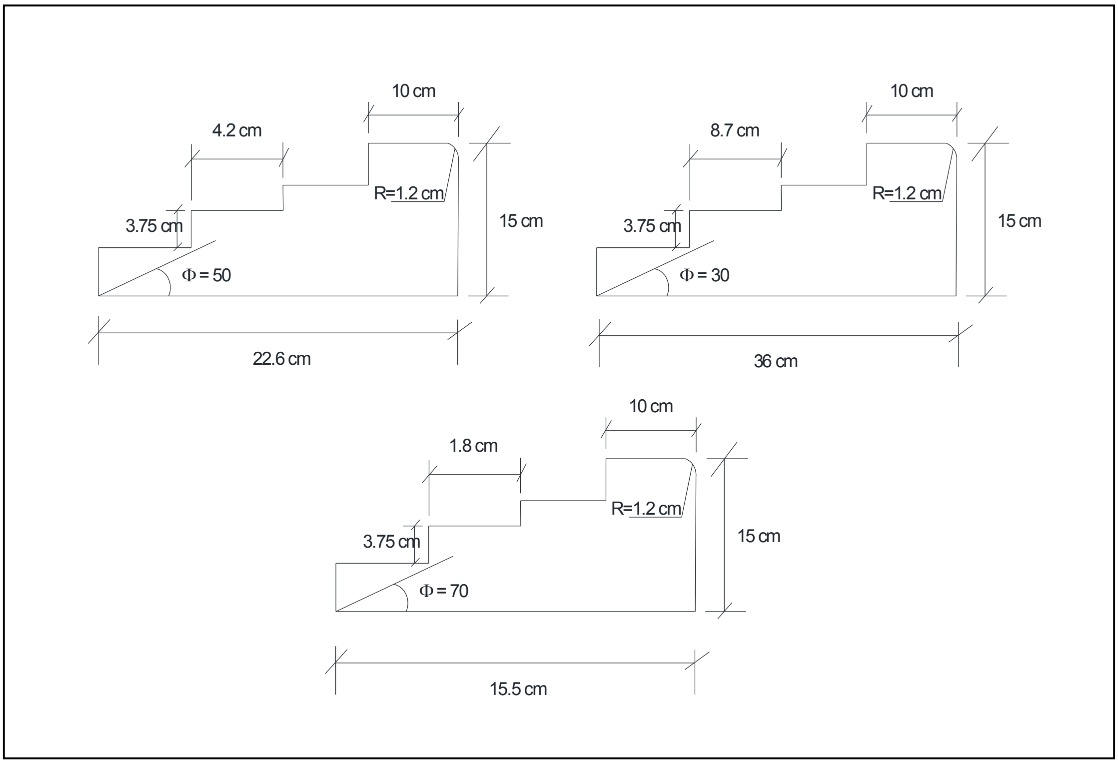

Fig. (3): The dimensions of plain models used.

4. The Regimes of Flow

Generally, the flows on stepped spillway - and which was also occurred in this experiment - are three flow regimes. These regimes are the Nappe, the transition, and the skimming flow. The study of these experimental results showed that the Nappe flow appeared for discharge less than 0.46 ℓ / s. The skimming flow was observed for discharge larger than 1.66 ℓ / s. Surely, the shape characteristic and the step surface quality affect the flow regime change. Table(2) shows the limitation of Nappe, transition, and skimming flow for the twelve models. Figures (4) to (6) show the three types of flow which were established in this study.

Table (2): Limitations of Flow Regimes.

Step case	**Limits and types of flow**		
	Model face slope = 30 °	**Model face slope = 50 °**	**Model face slope = 70 °**
plain	1)- Nappe ≤ 0.46 ℓ/s - transition (0.47 ≤ Q ≤ 1.56) ℓ/s - skimming (Q ≥ 1.57) ℓ/s	2) -Nappe ≤ 0.38 ℓ/s - transition (0.39 ≤ Q ≤ 1.31) ℓ/s - skimming (Q ≥ 1.32) ℓ/s	3)- Nappe ≤ 0.20 ℓ/s - transition (0.21 ≤ Q ≤ 1.20) ℓ/s - skimming (Q ≥ 1.21) ℓ/s
Half cut	4) - Nappe ≤ 0.62 ℓ/s - transition (0.63 ≤ Q ≤ 1.96) ℓ/s - skimming (Q ≥ 1.97) ℓ/s	5) -Nappe ≤ 0.44 ℓ/s - transition (0.45 ≤ Q ≤ 1.76) ℓ/s - skimming (Q ≥ 1.77) ℓ/s	6) -Nappe ≤ 0.26 ℓ/s - transition (0.27 ≤ Q ≤ 1.60) ℓ/s - skimming (Q ≥ 1.61) ℓ/s
Rough	7) - Nappe ≤ 0.82 ℓ/s - transition (0.83 ≤ Q ≤ 2.20) ℓ/s - skimming (Q ≥ 2.21) ℓ/s	8) - Nappe ≤ 0.52 ℓ/s - transition (0.53 ≤ Q ≤ 1.82) ℓ/s - skimming (Q ≥ 1.83) ℓ/s	9) - Nappe ≤ 0.36 ℓ/s - transition (0.37 ≤ Q ≤ 1.65) ℓ/s - skimming (Q ≥ 1.66) ℓ/s
With inclined end sill	10) -Nappe ≤ 0.65 ℓ/s - transition (0.66 ≤ Q ≤ 2.18) ℓ/s - skimming (Q ≥ 2.19) ℓ/s	11) -Nappe ≤ 0.46 ℓ/s - transition (0.47 ≤ Q ≤ 1.78) ℓ/s - skimming (Q ≥ 1.79) ℓ/s	12) -Nappe ≤ 0.30 ℓ/s - transition (0.31 ≤ Q ≤ 1.62) ℓ/s - skimming (Q ≥ 1.63) ℓ/s

Fig.(4): Nappe flow.

Fig.(5): Transition flow.

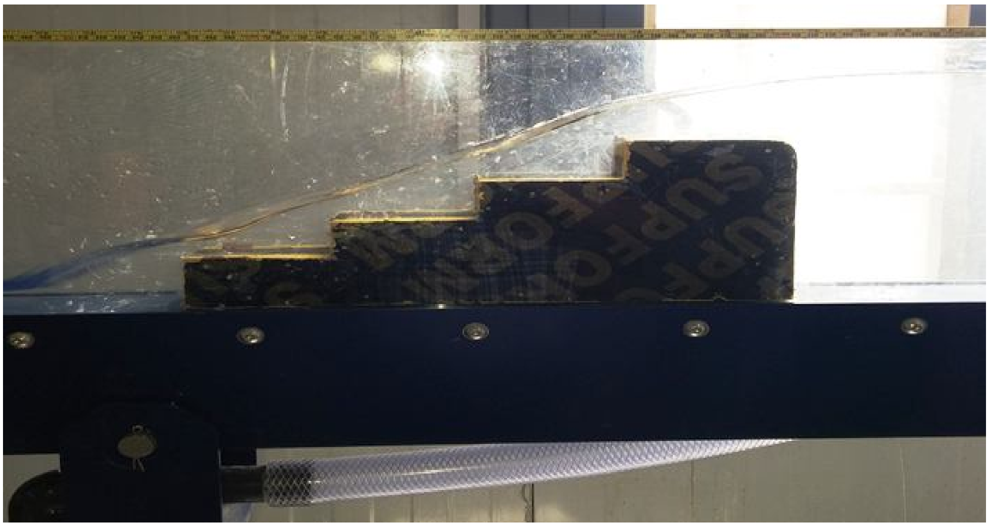

Fig.(6): Skimming flow.

5- The range between Nappe and Skimming flow

The ranges for the twelve spillways in this experimental work were observed. To quantify this range, the parameters which have been considered are the critical depth (y_c) (depth of water at spillway crest) and the spillway shape, which are the non-dimensional ratio y_c / h where h is the height of step. Table (3) represents the values of y_c / h for all models.

Table (3): y_c / h Values.

Step case	Model No.) Values of y_c / h		
	Face slope = 30 ^{o}	**Face slope = 50 ^{o}**	**Face slope = 70 ^{o}**
plain	1) 0.55-0.90	2) 0.53-0.88	3) 0.50-0.80
Half cut	4) 0.25-0.60	5) 0.19- 0.59	6) 0.12-0.50
Rough	7) 0.67- 0.98	8) 0.58-0.95	9) 0.52-0.93
With inclined end sill	10) 0.60-0.95	11) 0.55-0.93	12) 0.50-0.90

Table (4) indicated several authors who have taken this proposition into account in their researches as follows.

Table (4): y_c / h Values for some Authors for comparison.

Chafi (2010) [Chafi,2010]	Matos et. al. (1995) [Matos,1995]	Kells (1995) [Kells, 1995]	Chanson (1994) [Chanson,1994]	Degoutte (1992) [Degoutte , 1992]	Rajaratnam (1990) [Rajaratnam , 1990]	Essery et.al. (1978) [Essery,1978]	Author
0.67	0.83	0.50	0.80	0.69	0.8	0.81	y_c / h

The maximum of the ratio value (y_1/y_c) which corresponds to the structures of this experiment is equal to 32.2. This value is remarkably less than the limit value of ($y_1/y_c < 35$) [Chafi, 2010].

6- Analyze the Energy Dissipation

The results of this experiment indicate that the maximum energy dissipation value is observed for a specific value of a normal depth of the flow y_1 to the depth of flow at crest y_c which is in the model No. 7.

$$(y_1/y_c) = 26.50 \qquad \ldots (6)$$

Figure (7) indicates that the energy dissipation will increase when decrease the slope of spillway faces which special to models No.1, No.2, and No.3. These three models considered as references models according to other models in this study. Also, this Figure indicates that the energy dissipation will decrease when the discharge increased.

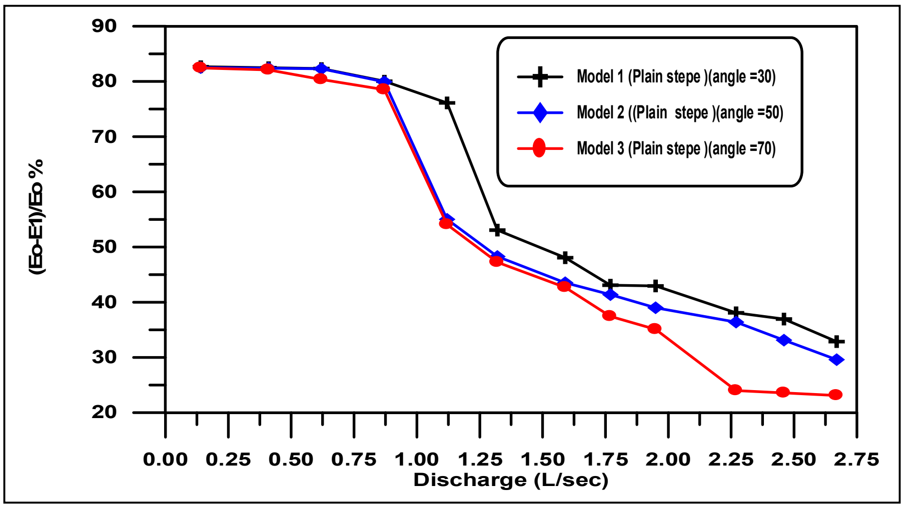

Fig.(7): Relation between relative energy dissipation and discharge for the models No. 1, No. 2, and No. 3 .

7- Study of increasing energy dissipation

The main aim of this paper is to study the increasing the energy dissipation on stepped spillway. So, and to carry out that, three cases were used which are the half cut steps, rough steps, and steps with inclined end sill as a novel study using and using three angles as the references models for each case. The details of each case as follows.

7-1 Half cut steps

A half cut steps were used for all steps of the Models No. 4, No.5, and No.6. Figure (8) represents a general sketch of these models and to show the half cut steps. Figures (9) and (10) show the application and results for the half cut models, respectively.

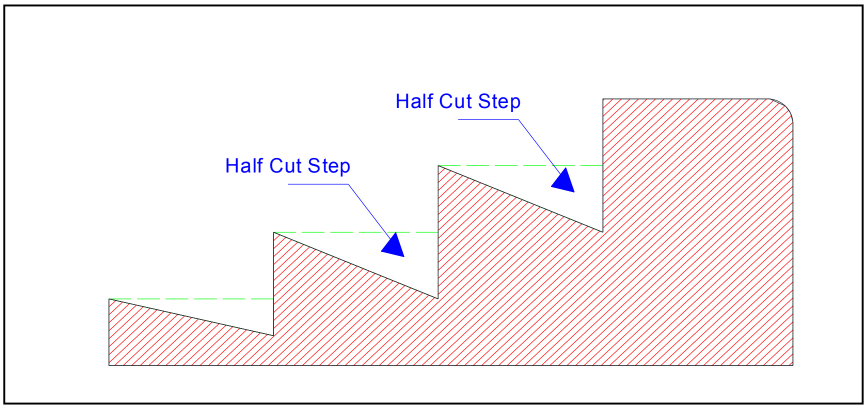

Fig.(8): General Sketch for the models No. 4, No. 5, and No. 6 .

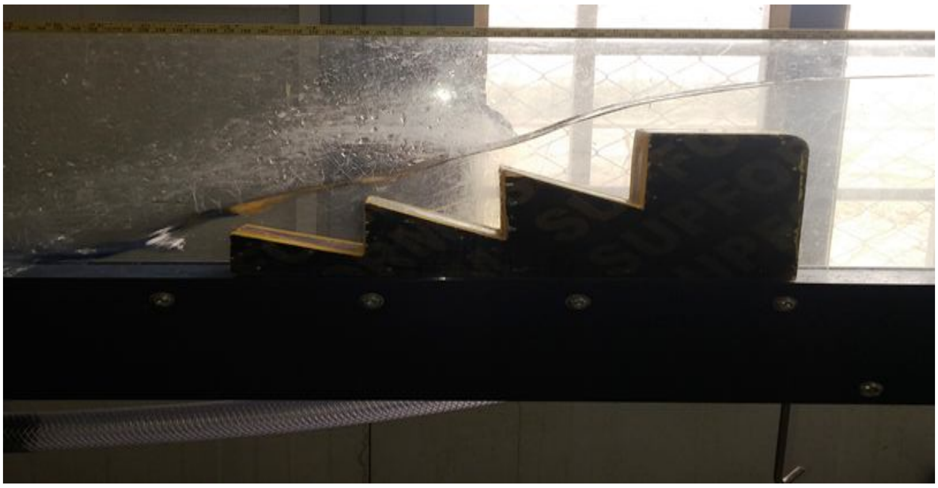

Fig.(9): Application of half cut steps for the models No. 4, No.5, and No.6.

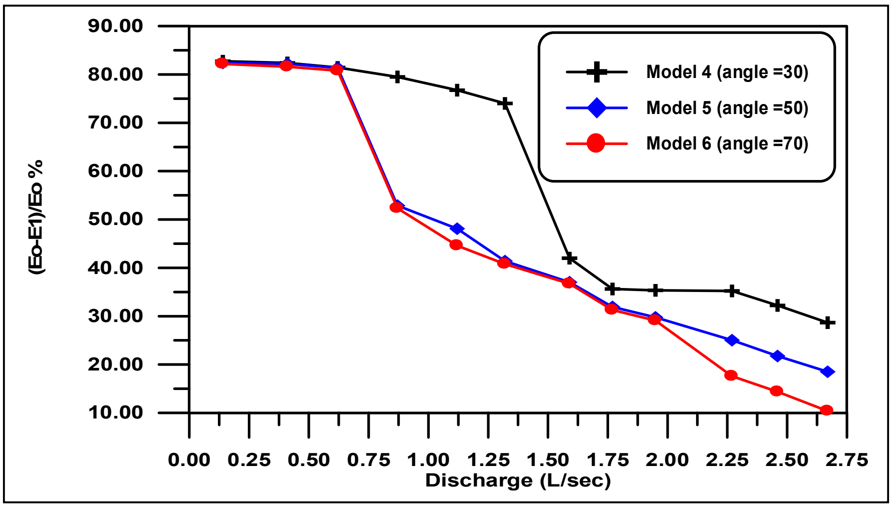

Fig.(10): Relation between relative energy dissipation and discharge for the models No. 4, No. 5, and No. 6 .

7-2 Rough Steps

The steps were roughed using crashed gravel stocked on surface of each step (rest and rise) by glue that has porosity 41% in the recommended range (38-42) % which used in the gabions by Alghazali [Alghazali,2014] and d_{50} = 13.8 mm . Figures (11) and (12) represent an example of the application of roughed steps in the models

No.7,No.8,and No.9 and the relation between relative energy dissipation and discharge, respectively.

Fig.(11): Application of rough steps for the Models No. 7, No.8, and No.9.

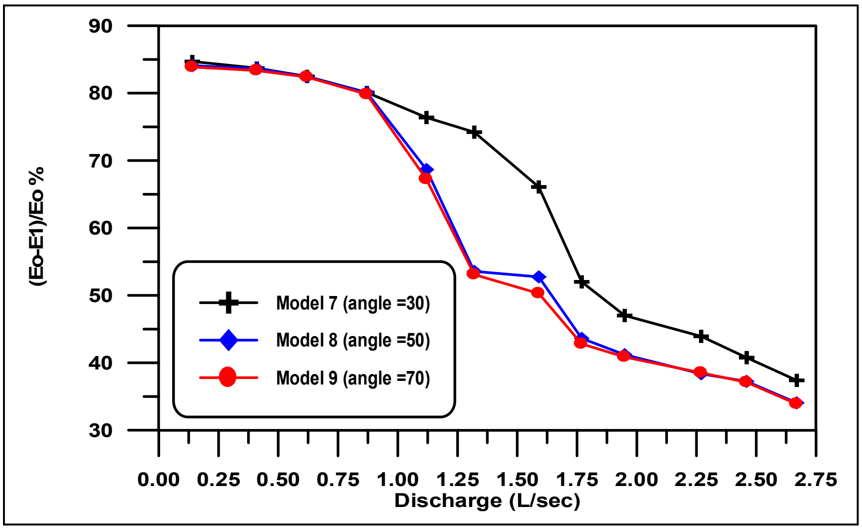

Fig.(12): Relation between relative energy dissipation and discharge for the models No. 7, No. 8, and No. 9.

7-3 Steps with inclined end sill

The steps were provided with inclined end sill which have angle normal to the slope of spillway face (α = 60^o, 40 ^{o},20^o) for models No. 10, No.11, and No.12, respectively (where α is the angle of end sill with horizontal plan). The vertical height of these end sills are equal to 0.5 *h* (*h* is the height of step) which lead to the end sill length are 6 cm, 2.5 cm, and 2 cm for models No.10, No.11, and No.12, respectively. Figures (13) and (14) represent an example of application of inclined end sill to the steps for the models No. 10, No.11, and No.12 and the relation between the relative energy dissipation and discharge, respectively.

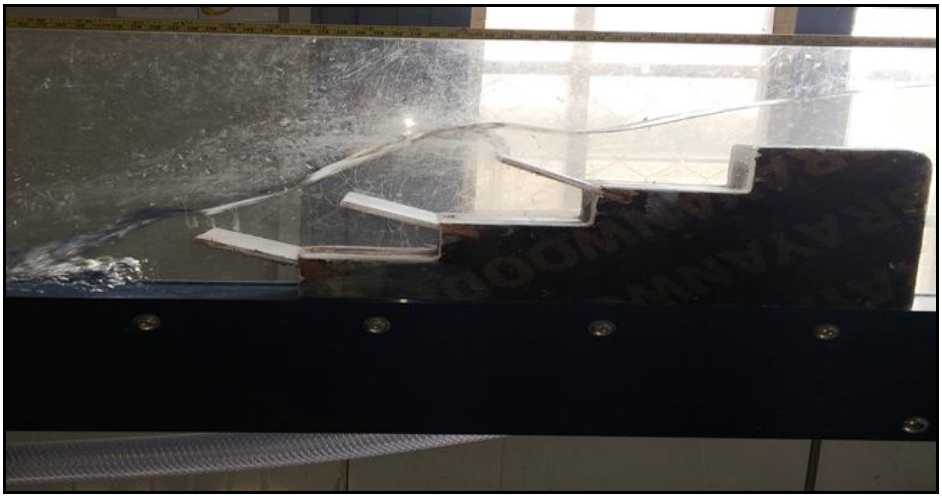

Fig. 13: Application of the inclined end sill steps for the Models No. 10, No.11, and No.12.

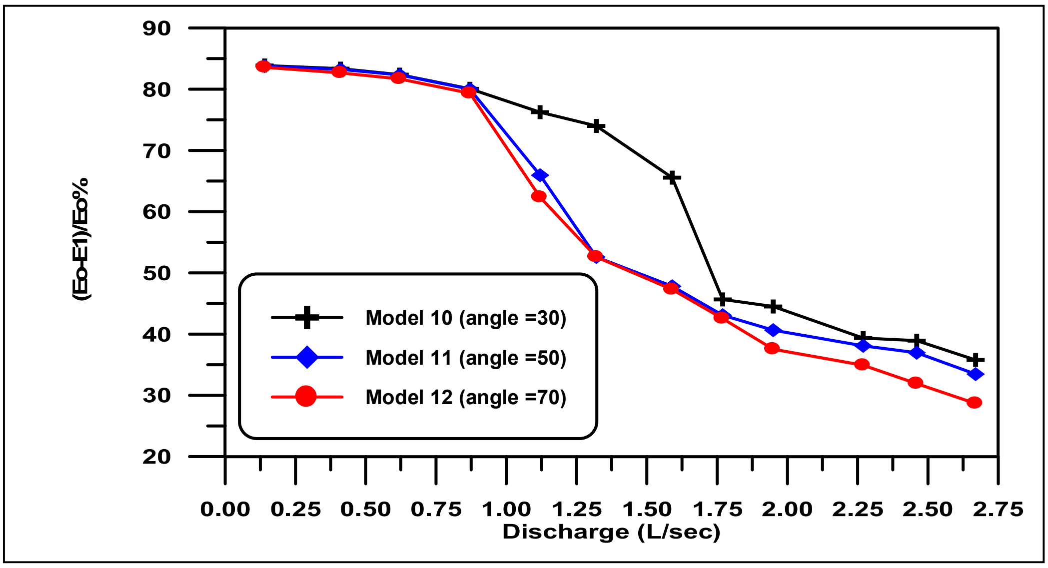

Fig.(14): The relative energy dissipation and the discharge of the models No. 10, No. 11, and No. 12.

8- Comparison between references models and the other models

As mentioned previously, Al-Talib [Al-Talib,2007]found that stepped spillways are more efficient than traditional spillways and the maximum energy dissipation ratio in stepped spillways were approximately 10% higher than in traditional spillways. In this study an attempt was used to increase this ratio using three cases which are half cut steps, rough steps, and steps with inclined end sill and compared with the references stepped spillway models. Figures (15) through (17) show the comparison between the reference models and the other cases.

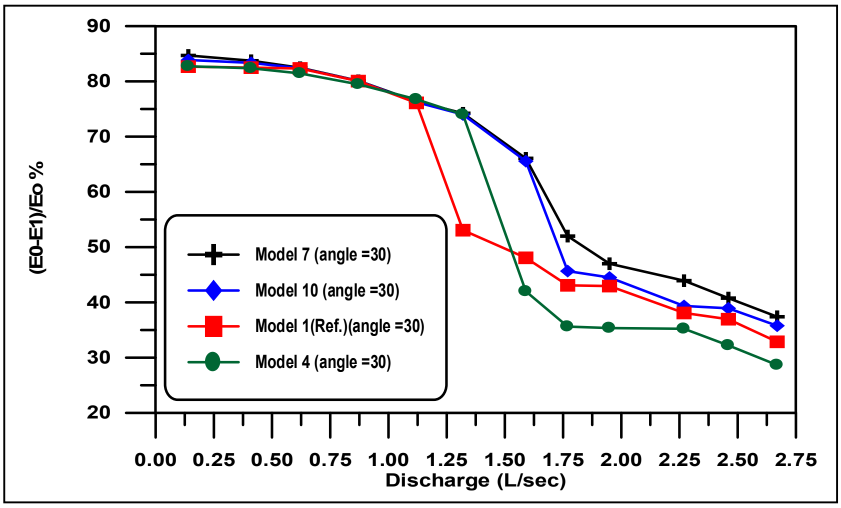

Fig. (15): Comparison between the reference model No.1 and model No. 4, No. 7, and No. 10.

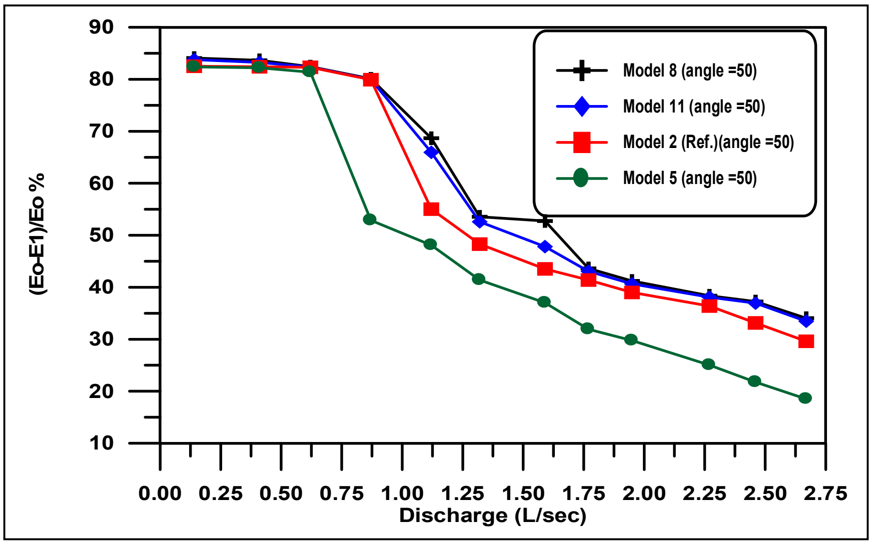

Fig. (16): Comparison between the reference model No.2 and model No. 5, No. 8, and No. 11.

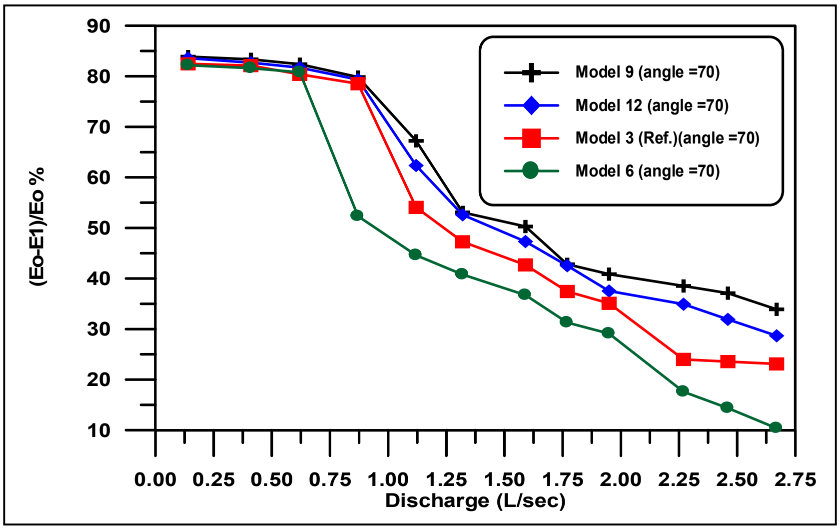

Fig. (17): Comparison between the reference model No.3 and model No. 6, No. 9, and No. 12.

The results indicate that the maximum relative energy dissipation was occurred on model No.7 (i.e. the rough steps) by ratio 14% at the maximum discharge compared with reference model No.1. In general, the maximum energy dissipations were on rough steps (model No. 7, No. 8, and No.9) compared with all other models that have the same slope of spillway for each one. The next model that has the maximum relative energy dissipation was model No.11 (i.e. steps have inclined end sill) which increased by ratio 9 % at maximum discharge compared with reference model No.1. The next maximum energy dissipations were on steps that have inclined end sill (model No. 7, No. 8, and No.9) compared with all other models that have the same slope of spillway for each one. While the cut steps gave relative energy dissipation very slightly higher than the relative energy at the smaller discharges by ratio 0.1% and have the least relative energy dissipation which is less than the reference model No.1 by 13% at the maximum discharge, but overall the half cut steps will decrease the relative energy dissipation compared with plain steps. Table (5) shows the comparison between the reference models and the others using the Root Mean Square Error (RMSE). The positive value of RMSE was considered here as the experimental values were greater than the references values while the negative value was considered as the experimental values were lower than the reference values. So, here the maximum value of RMSE indicates the larger relative energy dissipation.

Table (5): Values of RMSE for the twelve models.

Values of RMSE			**Step case**
Model face slope = 70 ^{o}	**Model face slope = 50 ^{o}**	**Model face slope = 30 ^{o}**	
3) 0	2) 0	1) 0	Plain (Reference model)
6) -10.07	5) -10.91	4) -7.29	Half cut
9) +8.40	8) +5.43	7) +8.86	Rough
12) +5.59	11) +4.08	10) +8.01	With inclined end sill

From Table (5) the case of rough steps gave the maximum RMSE and the next case is the steps with inclined end sill for all angle of spillway face.

9- Conclusion

In this experimental work, an attempt to increase the energy dissipation using novel step shapes which was; half cut steps, rough steps, and inclined end sill steps. Studying the flow types and the effect of changing angles of the spillway face was also considered. Results show that the energy dissipation of the flow on the inclined end sill and the rough steps are more effective, while the cutting steps is less effective compares with plain one. Also, this energy dissipation was increased by 14 % and 9 % with implementing the crushed gravels step face and the inclined end sill face, respectively. On the other side, the energy dissipation increased at low discharges by 0.1 % when using the cutting steps but decreased by 13 % with larger discharges. In summary, the flow energy dissipation decreases with increasing the flow rate, and the roughed step spillway surface is more effective compared to the other spillway surfaces at low or high flow rate.So, the case of rough steps and steps with inclined end sill can be used to increase the relative energy dissipation on plain steps by about 9 % and about 8% respectively for small angle of spillway face. Also, when compared with traditional spillways the case of rough steps and steps with inclined end sill can be used to increase the relative energy dissipation on plain steps by about 19 % and about 18% respectively for small angle of spillway face since the stepped spillways more effective than traditional spillways by 10 %. According to the flow types, the range of nappe, transition, and skimming will increase for all the forms of steps that used compare with plain step.

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