Calculation of the potential energy surface and the reduced of electric transition probability of $Yb^{170-176}$ Isotopes by the Model(IBM-1)

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Abstract:

In this paper $Yb^{170-176}$ even isotopes have been studies by the interacting boson model (IBM-1) to determine the nucleus shape by studying the surface potential energy using the equations of Hamilton function . In addition, the program IBMT-1 was used for evaluating the reduced transition probability B(E2). The obtained results for $^{170-176}$ Yb were reasonably in good agreement with the previous experimental and theoretical values . According to the IBM , It was found that these isotopes belong to the rotational region SU(3).

حساب طاقة جهد السطح و احتمالية الانتقال الكهربائية المختزلة لنظائر ($Yb^{170-176}$) باستخدام نموذج (IBM-1)

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الخلاصة :

في هذا البحث درست نظائر $Yb^{170-176}$ باستعمال نموذج البوزونات المتفاعلة (IBM-1) لتحديد شكل النواة باستعمال دراسة طاقة جهد السطح حسب معادلات دالة هاملتون ، بالأضافة إلى استعمال البرنامج (IBMT-1) لتحديد احتمالية الانتقال الكهربائية المختزلة . وأظهرت نتائج نظائر $Yb^{170-176}$ تطابقا جيدا للقيم العملية مع النظرية السابقة , ومن خلال نموذج البوزونات المتفاعلة IBM وجد بان هذه النظائر تنتمي إلى المنطقة الدورانية SU(3) .

1-Introduction :

Since the discovery of the discrete line superdeformed energy bands (denoted as SD band hereafter) [1]

Extensive experimental and theoretical studies have been launched in this fieldThere are several striking features for these SD bands. The quadrupole moments are quite large. In a given mass region the dynamic moments of inertia deduced from the energy spectra data are sometimes quite similar for different SD bands. Also, many SD bands for different superdeformed nuclei have nearly the same transition energies. It was conjectured that quantized pseudo-spin alignments are responsible for the occurrence of identical SD bands [2].. The preliminary success of the geometrical model suggests that in the superdeformed nuclei the most important excitation degree of freedom is the collective rotation .Based on the dominant role of the $J^{\pi} = O^{+}$ and 2^{+} nuclear pairs in the Nilsson wave functions, Otsuka and Honma proposed an algebraic super deformed interacting boson model [3]. In this model the inert core is smaller as compared with the normal interacting boson model and the number of valence bosons is increased by a factor of 2 to 3. The model was analyzed in the SU(3) limit which provides analytical expressions for energy eigen values. The normal interacting boson model has been quite successful in correlating collective nuclear properties in both dynamic symmetry limits and transitional regions. The important feature of the superdeformed interacting boson model is the smaller core and large numbers of valence bosons. Therefore, it will be very helpful if we can find a way to deal with large number of active bosons. If this technical problem can be solved, we can apply the model to superdeformed nuclei without any restriction to a specific dynamic symmetry limit. This work is an effort toward this direction.

In (1990) D. S. Chuu et al[4] studied asimple procedure to optimize the interaction parameters in IBA-1 as used to calculate the energy levels of strongly deformed nuclei $^{154\text{-}158}$Sm, $^{154\text{-}160}$Gd, $^{156\text{-}164}$Dy, $^{160\text{-}168}$Er, $^{162\text{-}172}$Yb and $^{168\text{-}176}$Hf. It was found that the variation in the interaction parameters for each isotope. The B(E2) values were also calculated, and compared with the experimental data and previous work. Satisfactory agreement was obtained. In(1999)[5] N. Minkov and etal studied derive analytic expressions for the energies and B(E2)-transition probabilities in the states of the ground and γ bands of heavily deformed nuclei (including Yb Isotopes) within a collective vector-boson model with SU(3) dynamical symmetry. E Biémont ,etal[6] in (2001) The analysis of the spectrum of Yb III has been extended allowing us to establish 11 new energy level values. The good agreement between experimental results and semi-empirical calculations performed with the relativistic Hartree-Fock method including core-polarization effects allows the determination of transition probabilities for 15 lines. In (2003) A. K. Rath, Stevenson, Regan, XU, and Walkr[7] were studied Ground-state deformations, binding energies, and potential energy surfaces as calculated for even-even dysprosium isotopes between ^{160}Dy and ^{180}Dy in the framework of density-dependent Hartree-Fock calculations with BCS pairing correlations. In2008 R. Rodríguez-Guzmán and etal[8] were studied the evolution of shapes with the number of nucleons in various chains of Yb, Hf, W, Os, and Pt isotopes from neutron number N=110 up to N=122

In 2011 The low-lying quadrupole collective states in neutron-rich even-even Yb, Hf, W, Os, and Pt isotopes were studied in a systematic way. Spectroscopic calculations are performed in terms of the Interacting Boson Model Hamiltonian, which is determined from the Hartree-Fock-Bogoliubov (HFB) approach with Gogny Energy Density Functionals (EDFs)[9].

2- IBM model :

In the scheme of interacting boson model-I (IBM-I) which does not distinguish between the neutron- and proton- boson there are 31 valence bosons. To carry out a calculation with so many bosons an efficient
method of calculating the boson coefficients of fractional parentage (cfp) is needed. The Hamiltonian for IBM-I is given by [10] :

$$H = \varepsilon + a_0 \hat{P}^{\dagger} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad \text{.....(1)}$$

where $\hat{P}, \hat{L}, \hat{Q}, \hat{T}_3, \hat{T}_4$ are the pairing, angular momentum, quadrupole, octopole and hexadecapole operators respectively.

The general Hamilton operator function formula for this isotopes is[11]

$$H^1 = a_1 L^2 + a_2 Q^2 \text{.....(2)}$$

and the equation of eigen value to Hamilton is given by [12] :

$$E|N,(\lambda,\mu),K,L,M\rangle = \frac{a_2}{2}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)+(a_1-\frac{3a_2}{8}).L(L-1)\text{.....(3)}$$

where :
{(λ,μ),K,L,M} the quantum numbers, but (λ,μ) determined the rotational limit SU(3) state.
The potential energy surface function of total number of bosons and deformation factors of (β,γ) were calculated from the equation (9).[11,12]

$$V(N,\beta,\gamma) = \frac{\langle N,\beta,\gamma \left| \overset{\Lambda}{H} \right| N,\beta,\gamma\rangle}{\langle N,\beta,\gamma | N,\beta,\gamma\rangle} \text{.....(4)}$$

By differentiating the equation (9) with respect (α,β) we obtain :[11,12]

$$\varepsilon_d\beta^2) + \frac{N(N-1)}{(1+\beta^2)}(A_1\beta^2 + A_2\beta^3\cos 3\gamma + A_3\beta^2 + A_4) \text{.....(5)}$$

N : total number of bosons.

β : magnitude of nuclear deformation ,take the value (0 to 2.4).

γ : asymmetry angle , take the value from 0^o to 60^o.

A_1, A_2, A_3, A_4 : Parameters of potential surface .

The Deformed Nuclei depended on (β,γ) factors .when β=0 the Nuclei is spherical and when β>0 the Nuclei is Deformed , otherwise when $\gamma = 0^o$ the Deformed nuclei are spherical of Prelate Shape and when $\gamma = 60^o$ the Deformed nuclei are spherical of Oblate Shape **.** The equation which is used for calculating the reduced transition probability B(E2) is [13] :

$$B(E_2, L+2 \to L) = \alpha_2^2 \frac{3}{4}\frac{(L+2)(L+1)}{(2L+3)(2L+5)}(2N-L)(2N+L+3)\text{...(6)}$$

or

$$B(E_2, 2_1^+ \to 0_1^+) = \alpha_2 \frac{N}{5}(2N+3)\text{.....(7)}$$

The formula for determining the electric quadruple moment (Q) to these limits is :[13]

$$QL = -\alpha_2 \frac{\sqrt{16\pi}}{40}\frac{L}{2L+3}(4N+3)\text{.....(8)} \text{ or}$$

$$Q_{2_1^+} = -\alpha_2 \frac{\sqrt{16\pi}}{40}\frac{2}{7}(4N+3)\text{.....(9)}$$

The relation between (α_2) and (β_2) for these limits is :[12]

$$\beta_2 = -\frac{\sqrt{7}}{2}\alpha_2 \text{.....(10)}$$

Where ((α_2) and (β_2)) parameters used for determining the reduced transition probability.

3-**Calculations**

When we studied the surface potential energy of $Yb^{170-176}$ isotopes, it was found that these isotopes determine rotational limit ,Depending on the results which were calculated by using interacting boson model -1 (IBM-1)program ,we compared our results with experimental results such as the shape of surface potential energy .

3.1. Calculation potential energy surface :

The potential energy surface was calculated after determining the parameters of Hamilton function operator that specialized for $Yb^{170-176}$ isotopes . Table (1) shows the parameters which are used in (IBMP-1) program for calculating potential energy surface V(β,γ)

Table1. parameters used in this program for calculating potential energy surface for $^{170\text{-}176}$ Yb in MeV

Isotopes	a_o	a_1	a_2	a_3	E2SD	E2DD
^{170}Yb	-0.004	-0.029	-0.055	0.000	-0.068	0.026
^{172}Yb	-0.004	-0.027	-0.050	0.000	-0.062	0.026
^{174}Yb	-0.005	-0.036	-0.067	0.000	-0.083	0.006
^{176}Yb	-0.005	-0.034	-0.060	0.000	-0.075	0.017

The surface potential energy gives the last shape of nucleus .Its agreed with Hamilton function [15] of two parameters (β,γ).Fig (1) shows the rotational limit SU(3) for $Yb^{170\text{-}176}$ isotopes as compared of the papers [12].

4- Calculation the square of rotational energy and the moment of inertia.:

Depending on the values of parameter (β_2, α_2) which were calculated from equations(5,8). In this paper we studied the determination of these parameters depending on the experimental values for transitions B(E2,21^+ 0_2^+) the parameters which were used in(IBMT-1)program (E_2SD & E_2DD) could be calculated as follow: [16]

$$E_2DD = \sqrt{5}\beta_2 \ldots\ldots\ldots\ldots\ldots (10)$$

In addition to ,The value of (α_2)can be found after calculated transition B(E_2) from the following equation .[17]

$$B(E_2)\downarrow = \frac{56.57}{E_\gamma^5 t_{\frac{1}{2}}[1+\alpha_{tot}]}(e^2b^2) \ldots (11)$$

Where :

E_γ : gamma ray transition energy in (KeV) , $t_{1/2}$: half life for the level (2_2^+)

α_{tot} : Total internal conversion coefficient which can be found from the table [18].

Table (2) show the parameters (E_2SD & E_2DD) used in (IBMT-1) program for calculation reduced transition probability B(E_2)of $Yb^{170-176}$ while table (4) shows the comparison between the previous experimental and theoretical values.

Table 2. Parameters for calculation reduced transition probability B(E_2)

Isotopes	*Transition Energy*	
	E2SD	E2DD
Yb^{170}	0.10299	-0.30463
Yb^{172}	0.10564	-0.31250
Yb^{174}	0.09767	-0.28890
Yb^{176}	0.14015	**-0.41458**

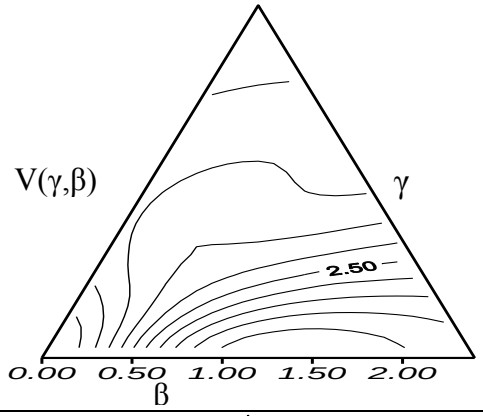
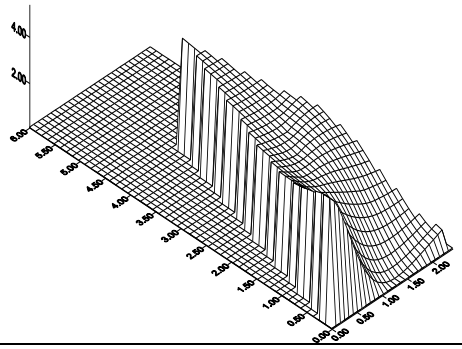
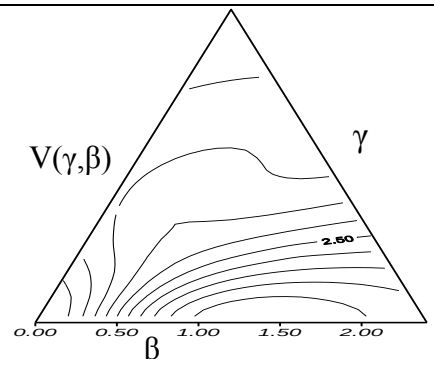
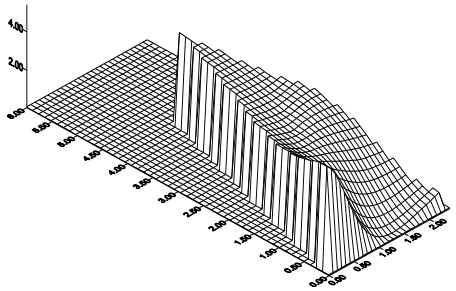
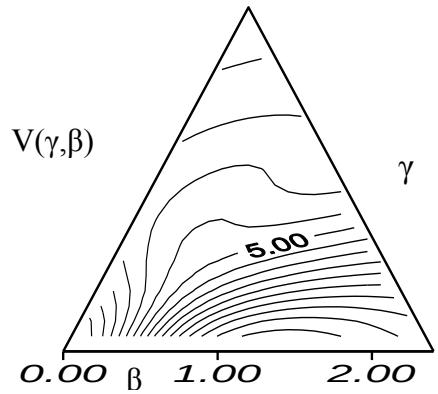
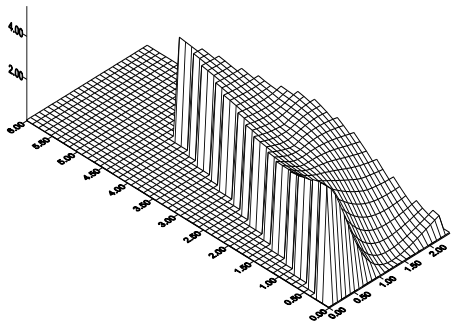
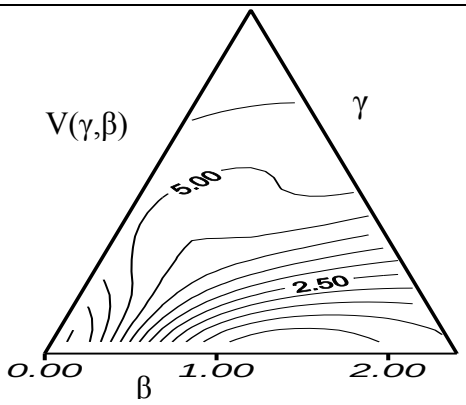
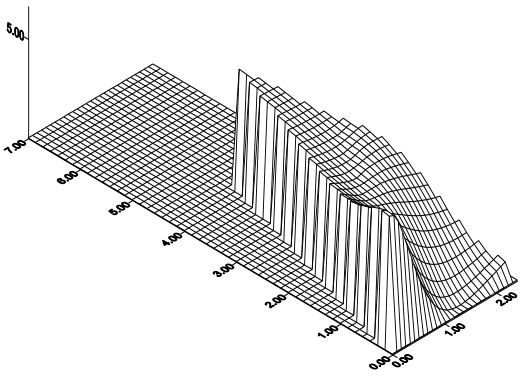
Isotopes	Contour Figures	Surface Figures
Yb^{170}		
Yb^{172}		
Yb^{174}		
Yb^{176}		

Figure (1): Show the surface potential energy for $Yb^{170\text{-}176}$ Isotopes

Table 3.shows the experimental and theoretical reduced transition probability **B(E_2)** of $Yb^{170-176}$

Isotopes	*Spin Parity (I_i^+---- I_f^+)*	*Transition Energy* ***B(E_2)↓e^2b^2***	
		This Work	Experimental.[15,16]
Yb^{170}	$2_1^+ \rightarrow 0_1^+$	1.0503110	1.05
	$2_1^+ \rightarrow 0_2^+$	0.041315	----------
	$2_2^+ \rightarrow 0_1^+$	0.1034659	----------
	$2_2^+ \rightarrow 0_2^+$	0.0028200	----------
	$2_4^+ \rightarrow 0_2^+$	0.0008898	----------
	$2_4^+ \rightarrow 0_3^+$	0.5435797	----------
	$4_1^+ \rightarrow 2_1^+$	1.4106880	----------
	$4_1^+ \rightarrow 2_2^+$	0.0016532	----------
	$4_1^+ \rightarrow 2_3^+$	0.0794709	----------
	$4_2^+ \rightarrow 2_1^+$	0.0531275	----------
	$4_2^+ \rightarrow 2_2^+$	0.4163464	----------
	$4_2^+ \rightarrow 2_3^+$	0.0046899	----------
Yb^{172}	$2_1^+ \rightarrow 0_1^+$	1.2577230	1.25
	$2_1^+ \rightarrow 0_2^+$	0.0545284	----------
	$2_2^+ \rightarrow 0_1^+$	0.1159457	----------
	$2_2^+ \rightarrow 0_2^+$	0.0029227	----------
	$2_4^+ \rightarrow 0_2^+$	0.0007817	----------
	$2_4^+ \rightarrow 0_3^+$	0.6817595	----------
	$4_1^+ \rightarrow 2_1^+$	1.6972970	----------
	$4_1^+ \rightarrow 2_2^+$	0.0017935	----------
	$4_1^+ \rightarrow 2_3^+$	0.0876087	----------
	$4_2^+ \rightarrow 2_1^+$	0.0601390	----------
	$4_2^+ \rightarrow 2_2^+$	0.5143330	----------
	$4_2^+ \rightarrow 2_3^+$	0.0045977	----------
Yb^{174}	$2_1^+ \rightarrow 0_1^+$	1.2136070	1.2
	$2_1^+ \rightarrow 0_2^+$	0.0490262	----------
	$2_2^+ \rightarrow 0_1^+$	0.1051390	----------
	$2_2^+ \rightarrow 0_2^+$	0.0024652	----------
	$2_4^+ \rightarrow 0_2^+$	0.0005743	----------
	$2_4^+ \rightarrow 0_3^+$	0.6849377	----------
	$4_1^+ \rightarrow 2_1^+$	0.0000012	----------
	$4_1^+ \rightarrow 2_2^+$	0.1334128	----------
	$4_1^+ \rightarrow 2_3^+$	0.0003342	----------
	$4_2^+ \rightarrow 2_1^+$	1.6444730	----------
	$4_2^+ \rightarrow 2_2^+$	0.0015820	----------
	$4_2^+ \rightarrow 2_3^+$	0.0782926	----------
Yb^{176}	$2_1^+ \rightarrow 0_1^+$	0.9724975	0.97
	$2_1^+ \rightarrow 0_2^+$	0.0403781	----------
	$2_2^+ \rightarrow 0_1^+$	0.0880412	----------
	$2_2^+ \rightarrow 0_2^+$	0.0024950	----------
	$2_4^+ \rightarrow 0_2^+$	0.0005344	----------
	$2_4^+ \rightarrow 0_3^+$	0.5415317	----------
	$4_1^+ \rightarrow 2_1^+$	1.3137060	----------

$4_1^+ \rightarrow 2_2^+$	0.0013633	----------
$4_1^+ \rightarrow 2_3^+$	0.0647619	----------
$4_2^+ \rightarrow 2_1^+$	0.0455563	----------
$4_2^+ \rightarrow 2_2^+$	0.4011564	----------
$4_2^+ \rightarrow 2_3^+$	0.0036679	----------

5- Result and discussion :

Our calculated values of the surface potential energy and reduced transition probability **B(E_2)** of $Yb^{170-176}$ had reasonable agreements with experimental data . The obtained results for transition energies of $Yb^{170-176}$ of transition ($2_1^+ \rightarrow 0_1^+$) for the (0.08323 , 0.07819 ,0.07603 and 0.08211)MeV were (1.0503110 , 1.2577230 , 1.2136070 and 0.9724975) e^2b^2 on succession so they were in good agreement with the experimental values. The figures in uppermost refer to the $Yb^{170-176}$ isotopes rotational limits SU(3).

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