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Tournaments (-1) –Chain and Connexity

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Abstract

In this paper , we study the connexity in the (-1) chain tournament , also the cycle of length 3 contained in these tournaments , and we prove that the (-1) chain is (-1) reconstructible .

الخلاصة

يتناول البحث دراسة في الاتصال لـ (1-) لعلاقة السلسلة الدورية و علاقة الدورة ذات الطول 3 الموجودة في العلاقة أعلاه . ثم برهننا بان (1-) علاقة السلسلة الدورية هي (1-) علاقة قابلة لإعادة بناءها .

Introduction

The tournament (-1) – chain is the tournament of order n . which contain a restriction isomorphic to the chain of n-1 elements . we study the connexity in the (-1) - chain tournament, and the cycle of length 3 which contained in these tournament , and we prove that the (-1) chain is (-1) - reconstructible .

Definitions:

A digraph T is said to be relation of tournament or simply tournament if : •

a) T is complete : for each x and y in E (E is the basis of T) , (x,y) or (y,x) is in T .

b) T is anti symmetric : (x,y) in T implies (y,x) is not in T .

c) T is irreflexive : (x,x) is not in T .

- A (-1) chain tournament is a tournament of order n and admits a restriction isomorphic a chain of (n-1) elements .
- Tow tournament T and T' with the same basis E are said to be (-1) hypomorphic if , for each x belongs to E, $T\backslash_{E-x}$ is isomorph to $T'\backslash_{E-x}$.
- A tournament is connex if any two of his vertices are connected by an elementary ordinary path .
- A digraph D is strongly connex , or (S-connex) if for any two vertices u and v in D , there exist an oriented elementary path form u to v and oriented elementary path form v to u .
- Let T be a tournament of basis E ={x_1,……x_n }, we say that x_i in E is a head , if every three circuit of T passes through x_i, that is equivalently to T – x_i is a chain .
- The following properties are well known for the tournaments whose the basis contains at lest 5 elements .

a) If T' is (-1) hypomorph to a chain T , then T' is also a chain .

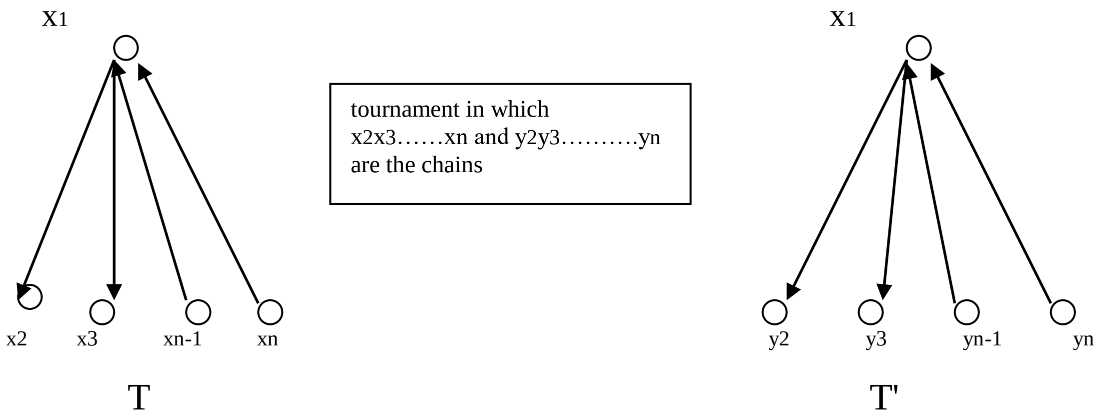
b) If T is an S-connex tournament and (-1) chain , then every tournament T' (-1) that hypomorphic to T, is S-connex .

we prove :

Proposition 1 :

Let T be a (-1) chain tournament and S-connex ,T' be a tournament (-1) hypomorph to T, such T – x_1 and T' – x_1 are respectively, the chain x_2,.......x_n and the chain y_2,.......yn , then we have the following cases :

(1) If {y_2,yn} ∩ {x_2,xn}= Ø the T contains the arrows x_1x_2......$x_{n-1}x_1$, and xnx_1, T' contains the arrows x_1y_2,x_1y_3,......$y_{n-1}x_1$ and ynx_1



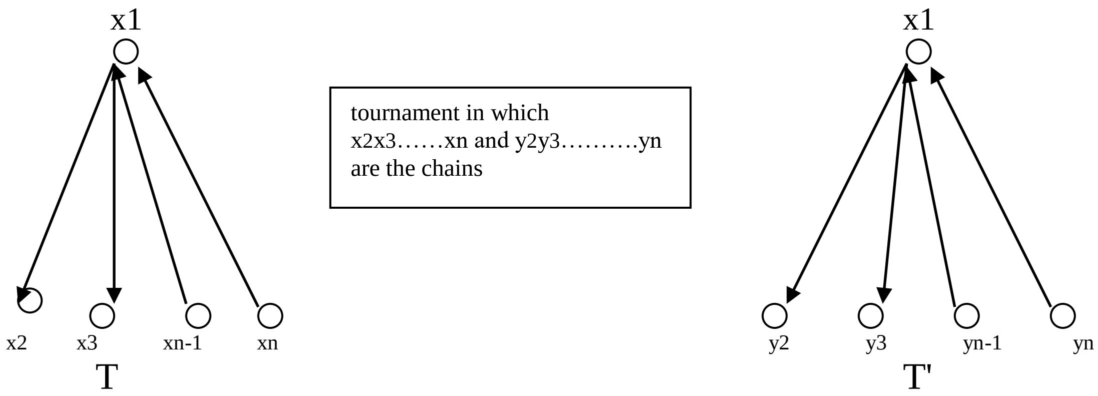
(2) If $x_2 = y_n$ and if y_n is not in { x_2,x_n } then T and T' have the same forms as in **(1)** except for the arrows x_1,x_3 and x_1y_3 which are not required .
(3) If y_2 is not in {x_2,x_n} and $y_n = x_n$ then T and T' have the same forms as in **(1)** except for the arrows x_{n-1},x_1 which are not imposed .
(4) If y_2 is not in { x_2,x_n } and $x_2=y_n$ then T has the same forms as in **(2)** and T' is the same form as in **(3)** with $y_n = x_2$.
(5) If $x_2 = y_2$ and y_n is not in { x_2,x_n } then T has the same forms as in **(3)** and T' is the same form as in **(2)** with $x_n = y_2$.

<u>Proof</u>

If y_2 is not in { x_2 , x_n } T – y_2 is S-connex , so T – y_2 is S-connex and thus x_1y_3 in T .

Generally , if yn is not in { x_2,x_n } then y_{n-1} is in T . x_2 not in {y_2,y_n } implies that x_1x_3 is in T and not in {y_2,y_n} implies that x_{n-1} is in T . (Bermond , Thomassen, 1981; Bondy and Hemminger, 1977) .

The illustrations below show without difficulty the truth of properties mentioned above .



<u>Propositions 2 :</u>

In a tournament which is not in chain , there is at most three head every tournament which is not a chain and has 3 head with only 3-cycle and is (-1) chain .

<u>Proof :</u>

Let T be a 3-cycle abc, if T has a 3 head the circuit abc passes through these three head so{a, b, c} is the set of these 3 head , and every other 3-circuit passes through a, b, and c. Also if a is a head, T –a is a chain .

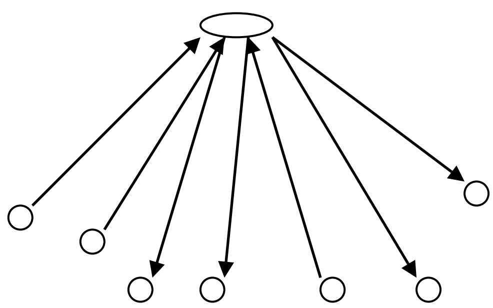
Corollary 1 :

If T is a tournament having two head , a and b, such that the arrow is in T , then either T is a chain or the chain T – a may be partitioned in three consectives parts A, B and C such that b is the end of B, a is dominated by all the points of A and by b, a domains all the other points with .

$$d^+_T(a) > d^+_T(b)$$

Proof :

Suppose that T is not a chain , let a and b the two head and assume that b a is in T , so T – a is chain . Let x be the first point of T – a such that axb is a 3 – cycle of T , we have ax and xb are in T , and for y ≠ a and y ≠ b , if by is in T . Thus gives the form of T with $d^+_T(b) > d^+_T(a)$. (see the following figure) (Fraisse, 398p.).



T

Moreover , since T has not three head there exist x' ≠ x , such that a x' b is a 3-cycle of T , ax' is in T and x'b is in T , so $d^+_T(a) > d^+_T(b)$.

Main Theorem :

If T is a tournament with 3-head , such that card (T) > 5, then t is (-1) reconstructible .

Proof :

If T is a chain , this is proved . So suppose that T is not a chain , and then T is (-1) chain with the unique 3-circuit (a, b, c) such that in T – a , b and c are consecutives, T is not S-connex , then (-1) reconstructible .(Bolobas, 1990; Harary and Palmer, 1967).

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