

Thermal Analysis of Laminar Free Convection in Horizontal Annulus with triangular Radial Fins attached to inner cylinder

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1-Abstract

This study is used to construct a mathematical model to analyze flow through air gap inside in horizontal annulus with triangular radial fins attached to inner cylinder, the cylinders has constant temperature. This study used when the inner cylinder more hotter than outer cylinder and contrarily for to Rayleigh number 3×10^7 , This study illustrate effect the length, number of fins and its arrangements on the inner cylinder on convection heat transfer in air gap. This study include using the case of free convection through horizontal annulus, the governing equations of flow and heat transfer across the annulus which were simplified from the general equations (conservation of mass, momentum and energy equations) using polar coordinates. The governing equations were transformed to the vorticity-stream function formula as for momentum equations and to the temperature and stream function for energy equation.

The construct result used program (ANSYS 5.4) to find temperature, velocity and pressure distribution also the vector of velocity inside the air gap for several cases. Firstly the inner cylinder has eight triangular fins has length 0.01 m, 0.015 m the inner cylinder more hotter than outer cylinder and contrarily. Secondly the inner cylinder has four triangular fins has length 0.01m, 0.015m in x,y axis, the inner cylinder more heater than outer cylinder and contrarily. Thirdly the inner cylinder has four triangular fins has length 0.01, 0.015 in inclined on x,y axis, the inner cylinder more hotter than outer cylinder and contrarily, and forth the cylinder has no fins.

Keywords /Heat transfer, Numerical, Laminar, convection, Fins, Forced convection, cylinder, Annulus

الخلاصة

أُجريت في هذا البحث دراسة نظرية لانتقال الحرارة بالحمل الحر خلال فجوة حلقيّة تقع بين اسطوانتين مُتخذتي المركز بثبوت درجة حرارة سطحها، الاسطوانة الداخلية مزعنة بزعانف مثلثة الشكل حلقيّة، إذ بينت الدراسة تأثير كل من طول الزعانف المثلثة وعددها وكيفية ترتيبها على انتقال الحرارة بالحمل الحر خلال الفجوة حيث تمت دراسة عندما تكون الاسطوانة الداخلية أكثر سخونة وبالعكس لعدد رالي 3×10^7 ، إذ تم تبسيط المعادلات الحاكمة للجريان وانتقال الحرارة خلال الفجوة من المعادلات الأساسية (الاستمرارية، الزخم والطاقة) باستعمال الإحداثيات القطبية، حولت المعادلات الحاكمة بدلالة الدوامية -دالة الجريان (Vorticity-Stream Function) بالنسبة لمعادلات الزخم ودالة الجريان ودرجة الحرارة بالنسبة لمعادلة الطاقة. تم استخدام برنامج متطور (ANSYS 5.4) لإيجاد توزيع درجات الحرارة والسرعة وكذلك متجهات السرعة داخل الفجوة الحلقيّة وتوزيع الضغط لعدة حالات أولاً عندما تكون الاسطوانة الداخلية مزعنة بثمانية زعانف بطول 0.01, 0.015 mm الاسطوانة الداخلية أكثر سخونة وبالعكس. ثانياً عندما تكون الاسطوانة الداخلية مزعنة بأربعة على طول الإحداثيات x,y زعانف بطول 0.01, 0.015 mm الاسطوانة الداخلية أكثر سخونة وبالعكس. ثالثاً عندما تكون الاسطوانة الداخلية مزعنة بأربعة زعانف مائلة بطول 0.01, 0.015 mm الاسطوانة الداخلية أكثر سخونة وبالعكس. رابعاً عندما تكون الاسطوانة الداخلية غير مزعنة، الأسطوانة الداخلية أكثر سخونة وبالعكس.

2-Nomenclature

C_p : specific heat at constant pressure, J /Kg. k

g : gravitational acceleration. m /sec²

Gr : Grashof number, $g \beta (T_o - T_i) r_i^3 / \nu^2$

k : thermal conductivity w/m²k

Nu : local Nusselt number.-

Pr : Prandtl number, ν/α
 r : radial distance ,m
 R : dimensionless radial coordinate, r/r_i
 Ra : Rayleigh number, $Ra = Gr.Pr$
 T :Temperature , k
 U : dimensionless radial velocity, uri/ν
 u : radial velocity ,m/sec
 V :dimensionless tangential velocity, vri/ν
 v : tangential velocity, m/sec
 w : Vorticity
 x :

Greek symbols

ψ dimensionless stream function
 ρ fluid density, Kg/m³
 α :thermal diffusivity,($k/r\ cp$)
 β : thermal expansion coefficient of fluid (1/k)
 θ : dimensionless temperature, $(T-T_0)/(T_i-T_0)$
 ν : kinematics viscosity ,m²/sec
 Ω :dimensionless vorticity
 ϕ : radial distance

Subscripts

i,o inner and outer, respectively
 m: mean

3-INTRODUCTION

In recent years, free convection heat transfer in fluid flows through a cylindrical annulus has attracted much attention with relation to thermal storage systems, solar collectors, spent nuclear air fuel cooling, nuclear reactors, air craft fuselages insulation, cooling of electrical equipments. Also often encountered in engineering applications such as turbo machinery, tube heat exchangers, and the drilling of oil wells.

Vafai and Ettfagh [1991] investigated the study theoretical three dimension heat transfer across an closed horizontal annulus include air and study the effect of ends on heat transfer. The result represent as couture charts for stream function and temperature also put a curves the change Nusselt number and time .

Farinas et al. [1999] investigated laminar natural convection in horizontal and finned annulus for different fin numbers and lengths and Rayleigh numbers ranging from 10^3 to 10^7 . They concluded that the heat transfer is maximized for a narrow cavity with two longer fins.

Rahnama et al. [1999] studied the effect of fins attached to the inner and outer cylinders of annuli on flow and temperature fields in a horizontal annulus. They studied the effect of fin orientation and height attached to the cylinders for Rayleigh numbers less than 10^6 .

Al-Mahroom, F.[2000] studied theoretical and analytical for heat transfer by free convection from horizontal inside covered square section has ports high and lower ends for Rayleigh numbers ranging from 1×10^5 to 6×10^5

The purpose of this study is to investigate numerically the free convection heat transfer in horizontal annuli where the inner tube has triangular radial fins. Effects of fin height and its number, and inner and outer annulus radii will be investigated. **Fig 1**

represent flow inside the gap between two cylinders the inner has triangular radial fins and the effect of variation of Prandtl number and diameter ratio.

4- Mathematical model

4.1- The analysis is based on the following assumptions:

1. Two dimension and steady state for fluid motion and temperature distribution.
2. All the physical properties are assumed to be constant except for the density variation with temperature.
3. The fluid in an annulus bounded by two isothermal surfaces is viscous and compressible.
4. The fluid is considered viscous and compressible
5. frictional heating is negligible
6. There are no heat sources within the fin itself.

For the flow and heat transfer analysis in circular concentric annuli, the cylindrical coordinate system is the best coordinates to adopt so as to simplify the form of the governing equations and easily impose the boundary conditions. Assuming that the flow and heat transfer are steady, pressure change only in the stream-wise direction, no axial diffusion of heat and momentum and axially symmetric flow and heat transfer, the boundary layer governing equations in cylindrical coordinates expressing the conservation equations of mass, momentum and energy for free convection in horizontal circular and concentric ducts respectively are:

The continuity equation in r and ϕ directions is:

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{1}{r} \frac{\partial v}{\partial \phi} = 0 \quad (1)$$

The momentum equations in r and ϕ directions are:

$$\rho_o \left[u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \phi} - \frac{v^2}{r} \right] = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(ru) \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} - \frac{2}{r^2} \frac{\partial v}{\partial \phi} \right] - g \rho_o [1 - \beta(T - T_o)] \cos \phi \quad (2)$$

$$\rho_o \left[u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \phi} + \frac{uv}{r} \right] = -\frac{1}{r} \frac{\partial p}{\partial \phi} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r}(rv) \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \phi^2} + \frac{2}{r^2} \frac{\partial u}{\partial \phi} \right] + g \rho_o [1 - \beta(T - T_o)] \sin \phi \quad (3)$$

The energy equation in r and ϕ directions is:

$$u \frac{\partial T}{\partial r} + \frac{v}{r} \frac{\partial T}{\partial \phi} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} \right] \quad (4)$$

The weight conform in two component in r and ϕ directions as:

$$g_r = -g \cos \phi \cos \phi \quad (5)$$

$$g_\phi = g \sin \phi \sin \phi \quad (6)$$

Bejan, (1984) for natural convection flow, the change in density is responsible for the flow and the ideal gas state equation was provided as an input to estimate the fluid density,

$$\rho_f = \rho_m [1 - \beta(T - T_m)] \quad (7)$$

$$T_m = -\frac{T_o - T_i}{2} \quad (8)$$

Ozoe & Shibata [1981] get equation for vorticity (ω) in coordinate (r, φ)

$$\omega = \frac{1}{r} \frac{\partial}{\partial r} (rv) - \frac{1}{r} \frac{\partial u}{\partial \varphi} \quad (9)$$

$$v = -\frac{\partial \Psi}{\partial r} \quad \text{and} \quad u = \frac{1}{r} \frac{\partial \Psi}{\partial \varphi}$$

The coordinates r , are measured from the center of the system, and θ , measured counterclockwise from the left ward horizontal line. The radial velocity u is positive radials out wards, and the tangential (angular) velocity v positive in the counterclockwise direction for $0 \leq \theta \leq \pi$ the following dimensionless variables and parameters are used for redial coordinate $R = \frac{r}{r_i}$ and stream function $\Psi = \frac{\psi}{\alpha}$ and

vorticity $\Omega = \frac{r_i^2}{\alpha} \omega$ and dimensionless temperature $\theta = \frac{T - T_o}{T_i - T_o}$ and substitutes these

parameters in Eq. (1), Eq.(2), Eq. (3), Eq.(4) gives

$$\frac{1}{Pr} \frac{1}{R} \left[\frac{\partial \Psi}{\partial \varphi} \frac{\partial \Omega}{\partial R} - \frac{\partial \Psi}{\partial R} \frac{\partial \Omega}{\partial \varphi} \right] = \nabla^2 \Omega - Ra \left[\frac{\cos \varphi}{R} \frac{\partial \theta}{\partial \varphi} + \sin \varphi \frac{\partial \theta}{\partial R} \right] \quad (10)$$

$$\frac{1}{R} \left[\frac{\partial \Psi}{\partial \varphi} \frac{\partial \theta}{\partial R} - \frac{\partial \Psi}{\partial R} \frac{\partial \theta}{\partial \varphi} \right] = \nabla^2 \theta \quad (11)$$

$$\Omega = -\frac{\partial^2 \Psi(R, \varphi)}{\partial R^2} \quad (12)$$

4.2-The boundary conditions

the boundary conditions of the system are :

$$\text{At } R = R_i \text{ and } R = R_o, \quad \Omega = -\nabla^2 \Psi = 0$$

$$\Psi(R, \varphi) = \frac{1}{R} \frac{\partial \Psi}{\partial \varphi} = \frac{\partial \Psi}{\partial R} = 0 \quad (13)$$

At $R = R_o$ the temperature $= T_o$ and $V_{\text{air}} = 0$

At $R = R_i$ [along fins and inner cylinder] the temperature $= T_i$ and $V_{\text{air}} = 0$

5- Procedure of Solution:

In order to verify our results obtained using **ANSYS 5.4** based finite element. In order to be consistent with what we have for **ANSYS 5.4** based on finite element model, the length of the annulus in z direction is used adjacent to the symmetry line. The grid points are not distributed uniformly over the computational domain shown in **Figure 2** is generated using **ANSYS 5.4** because they have greater density near

surfaces of the inner and outer surfaces of the annulus and the surface of the triangular fins.. The triangular Radial Fins attached to inner cylinder is designed to be decrease the temperature variations across the thickness of the inner and outer cylinders. The spacing expansion and contraction factors of grid distribution along the radial cylinders selected 45 ,outer cylinder 90 and the length of triangular 20 increments.

6. Result and Discussion.

Table 1 shows the input data employed in the present investigation. Free convection in enclosures inside two cylinder ,Used several fins 0, 4 , 4 inclined with 45° , and 8 .also used two length 0.015 m and 0.02 m from the center of annulus. Typical results of stream lines , temperature distribution , pressure drop and velocity distributions are shown in figures 3 to 15.

6.1 Effect number of the triangular fins

The effects number of fin on stream line and temperature distribution for temperature difference of 200 K, are shown in figures 3 to 15.Used number of fins 0 ,4 , 4 inclined with 45° ,and 8 fins. The stream line and temperature transfer to the areas that lower pressure therefore the pressure in the top of cylinder always high more than the lower of cylinder .always when the number of fins increase the surface area of inner cylinder increase therefore the heat transfer increase velocity distribution increase for 4 fins there are two symmetric circulation loops and for 4 inclined fins there are two symmetric circulation loops and two loops no symmetric in the top of cylinder and for 8 fins there are two symmetric circulation loops beside the inner cylinder and two loops symmetric in the top of cylinders because the area division increase where the temperature comes from the side and the surface of inner cylinder ,also here the researcher is agree with (Alshahrani & Zeitoun, 2002). This mean the number of loop increase the temperature and velocity distribution increase. As shown in figures 3 to 15

6.2 Effects of fins length

The effect of fin height on velocity, stream line and temperature distribution for temperature difference of 200 k. are shown in figures 3 to 15.Used number of fins 0 ,4 , 4 inclined with 45° ,and 8 fins. For annulus without fins, as shown in the figure, there are two circulation loops. when the length of fins increase the circulation loop become thin in the level of fins until they are divided into four loops as shown in figure 3 and figure 5 , and divided from four loop to eight loop as shown in figure and 12, this mean the number of loop increase the temperature and velocity distribution increase .The figures shown also indicates that the loop beside the inner cylinder are stronger than the upper two loops. The existence of fins as shown in the figures to 15 increases the thermal boundary layer along the bottom part of the inner cylinder and attached fins. For the outer cylinder, the heat transfer intensifies along the upper parts. The existence of fins decreases the boundary layer along the outer cylinder surface. These results indicate that the heat transfer will increase as fin length increases, this will be discussed later. increasing fin height increases heat transfer but at lower rate due to resistance of fins to the free circulation flow here the researcher is agree with (Han *et al.*, 1999).

6.3Effects of Temperature variation

The isothermal regions in the annulus without fins show that the heat transfer concentrated around the bottom of the inner cylinder if $[T_i > T_o]$ and contrarily ,here the researcher is agree with (Yeh, 2002). The isothermal regions in the annulus with eight fins show that the heat transfer concentrated around near fins of the inner cylinder if $[T_i > T_o]$ and symmetric in y axis the heat is transfer from the upper fins

more than in the lower because number of loop in the upper but $[T_o > T_i]$ the temperature near the fins in the lower half of annulus is less than that in the upper because the temperature always travels to the high also in annulus four fins .

6.4 Effects fins arrangements on the inner cylinder

The effect the fins arrangements on velocity, stream line and temperature distribution for temperature difference of 200 k. are shown in figures 3 to 9. For inner cylinder has 4 fins has length 0.02 m if arrangements at X,Y axis the number of loops is four distribute two in the upper and two in the lower. The loop in the upper the inner cylinder are more stronger than the lower two loops, but for inner cylinder has 4 inclined with 45° fins as shown in figure 5 and 6 has length 0.02 m the loop beside the inner cylinder are stronger than the lower two loops.

For inner cylinder has 4 fins has length 0.015 m if arrangements at X,Y axis the number of loops is two distribute in the beside the inner cylinder. for inner cylinder has 4 inclined fins as shown in figure 3 and 4 has length 0.015 m the loop beside the inner cylinder are stronger than the loops if the four fins arrangements at X,Y axis.

7. Conclusions

Free convection heat transfer between two horizontal Annulus with triangular Radial Fins attached to inner cylinder was analysis numerically using finite element technique. Laminar conditions up to Rayleigh number Rai of 3×10^7 were studding. Effects of number of the triangular fins, fins length, Temperature variation and fins arrangements on the inner cylinder. Some conclusions are summarized as follows:

- a-The thermal resistance of finned annuli was obtained using the conduction analysis from the inner cylinder and fins to air gap.
- b-The thermal resistance decreases as fin length increases this mean as the fin is extended within the annulus gap the area of annular gap decreases.
- c-Fins arrangements in the inner cylinder effect on temperature and velocity distribution, when fins length increase the area between the fins near inner cylinder has velocity is zero.

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Table 1: The dimensions and temperatures of inner and outer cylinder and fins

0.06	0.02	0	-	500	300
0.06	0.02	0	-	300	500
0.06	0.02	4	0.02	500	300
0.06	0.02	4	0.015	500	300
0.06	0.02	4	0.015	300	500
0.06	0.02	4	0.02	300	500
0.06	0.02	4(inclined with 45°)	0.02	500	300
0.06	0.02	4(inclined with 45°)	0.015	500	300
0.06	0.02	4(inclined with 45°)	0.02	300	500
0.06	0.02	4(inclined with 45°)	0.015	300	500
0.06	0.02	8	0.015	500	300
0.06	0.02	8	0.02	300	500
0.06	0.02	8	0.015	300	500
0.06	0.02	8	0.02	500	300

8-Figures:

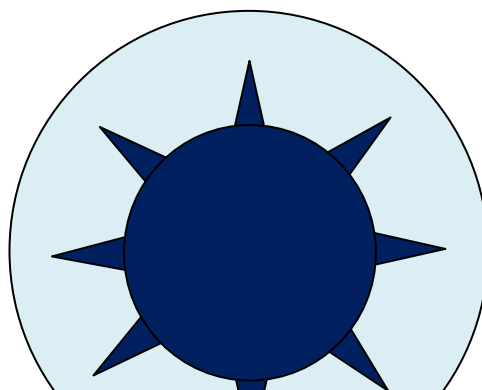


Figure 1:Horizontal annulars with eight triangular radial fins attached to inner cylinder

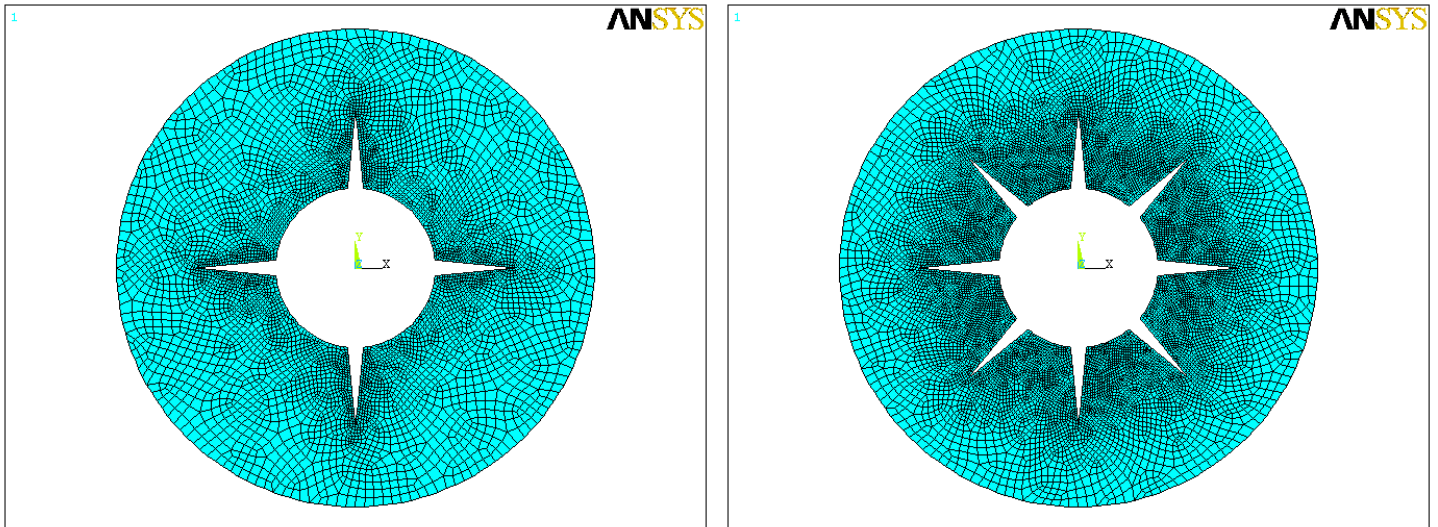


Figure 2:Grid system of Horizontal annulars with four and eight triangular radial fins attached to inner cylinder

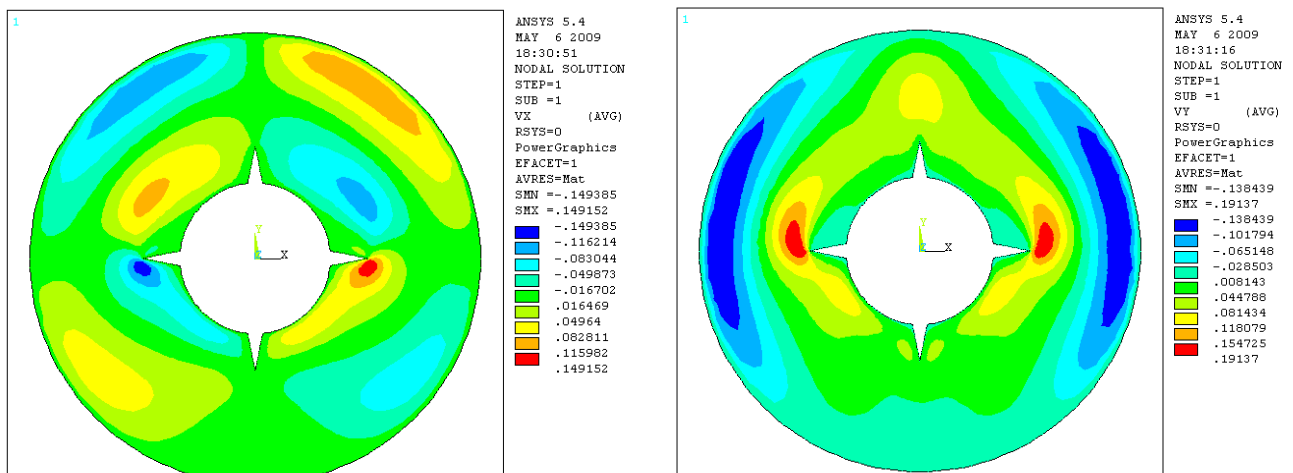


Figure 3a: Velocity distribution profiles in x-direction (v_x) for four triangular radial fins. $T_i > T_o$

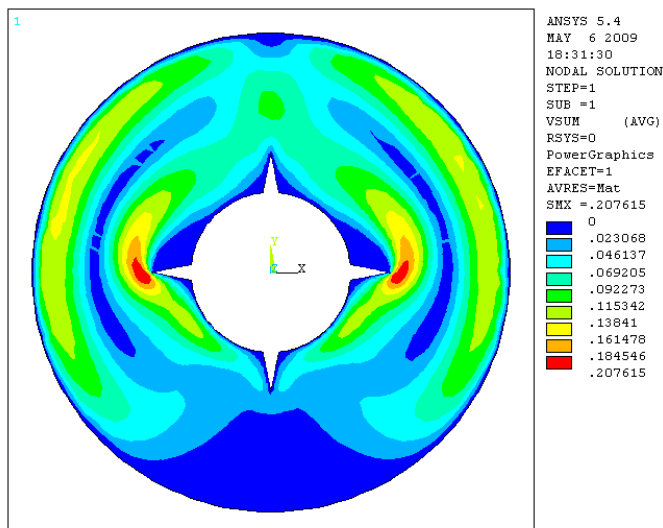


Figure 3b: Velocity distribution profiles in y-direction (v_y) for four triangular radial fins. $T_i > T_o$

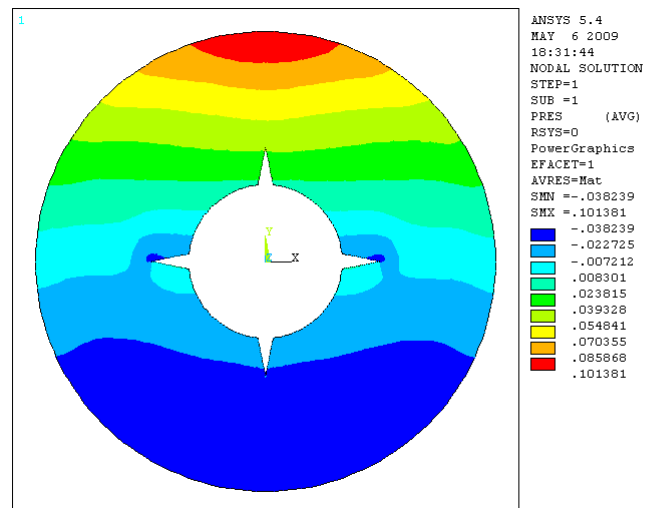


Figure 3c: Velocity distribution profiles (v_{sum}) with four triangular radial fins. $T_i > T_o$

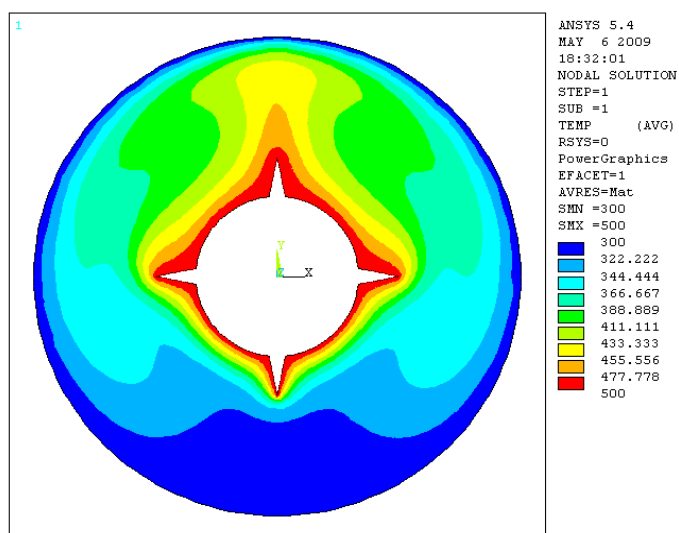


Figure 3d: Pressure distribution profiles with four triangular radial fins. $T_i > T_o$

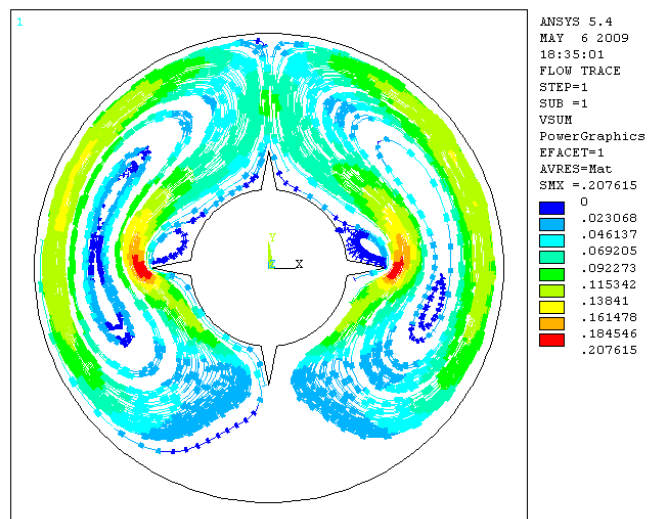
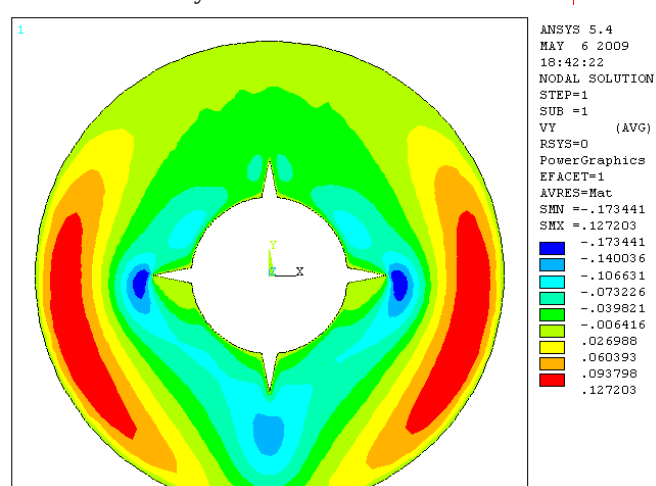
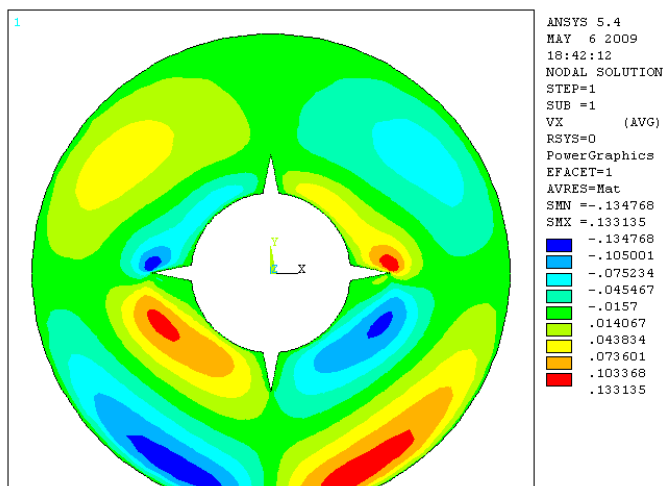


Figure 3f: velocity distribution vectors with four



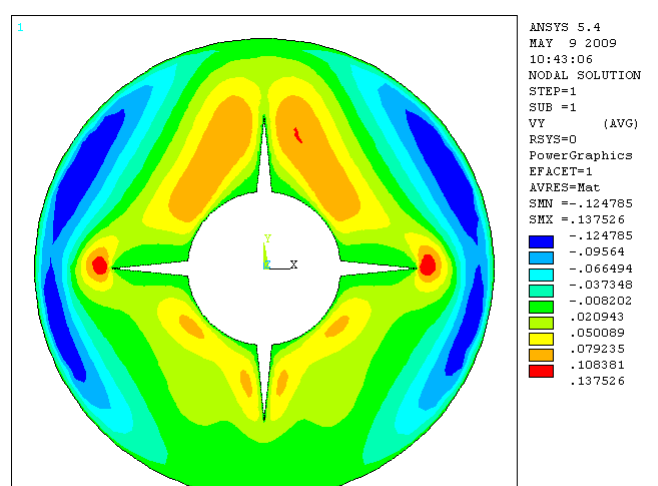
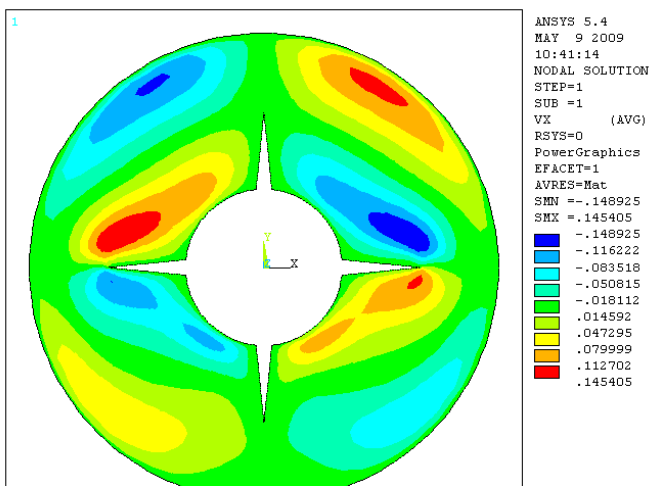


Figure 5a: velocity distribution in x direction with four triangular radial fins. $T_o > T_i$

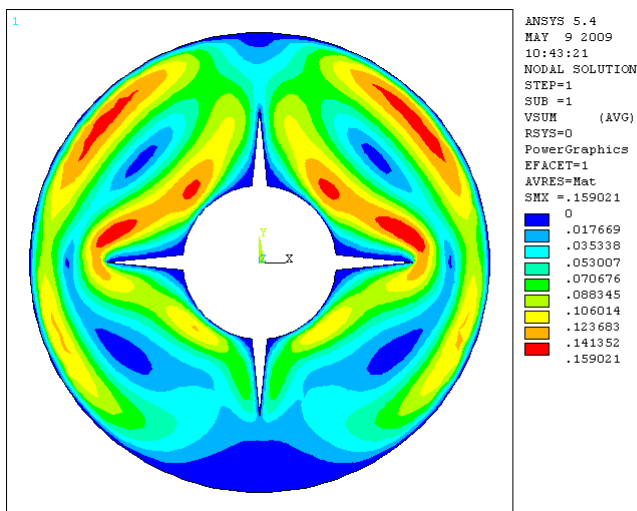


Figure 5b: velocity distribution in y direction with four triangular radial fins. $T_o > T_i$

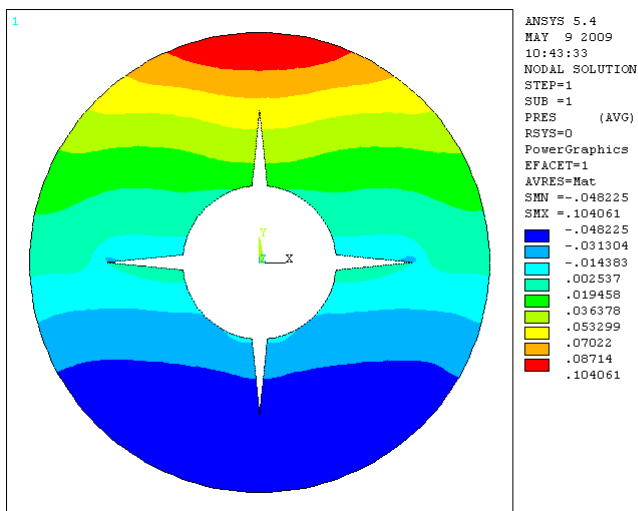


Figure 5c: velocity distribution (V_{sum}) with four triangular radial fins. $T_o > T_i$

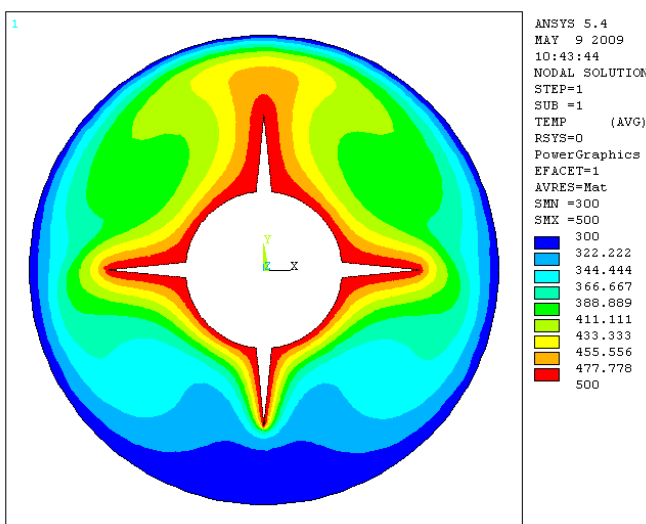


Figure 5d: pressure distribution with four triangular radial fins. $T_o > T_i$

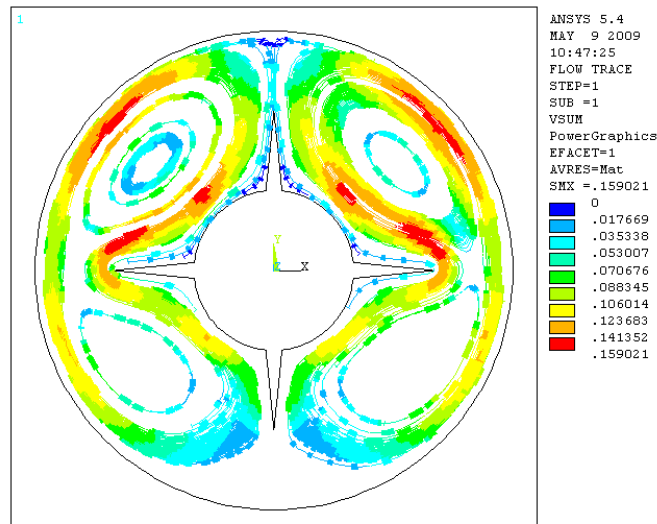


Figure 5e: Temperature distribution with four triangular radial fins. $T_o > T_i$

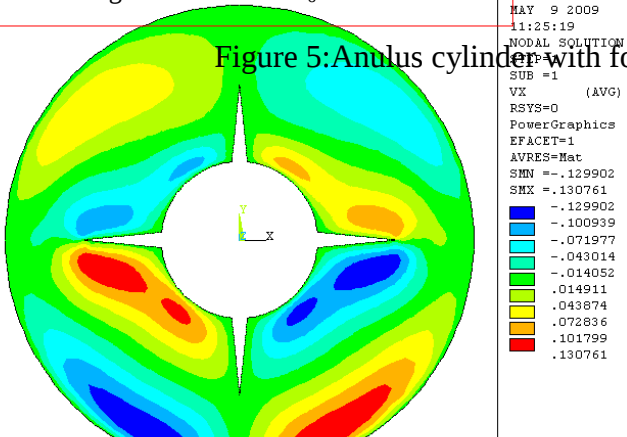


Figure 5f: velocity distribution vector with four triangular radial fins. $T_o > T_i$

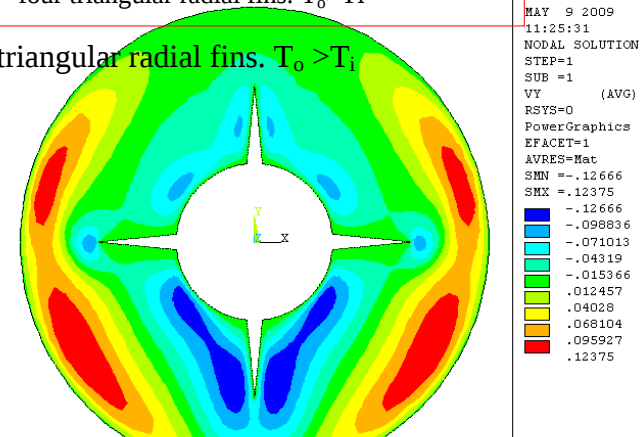
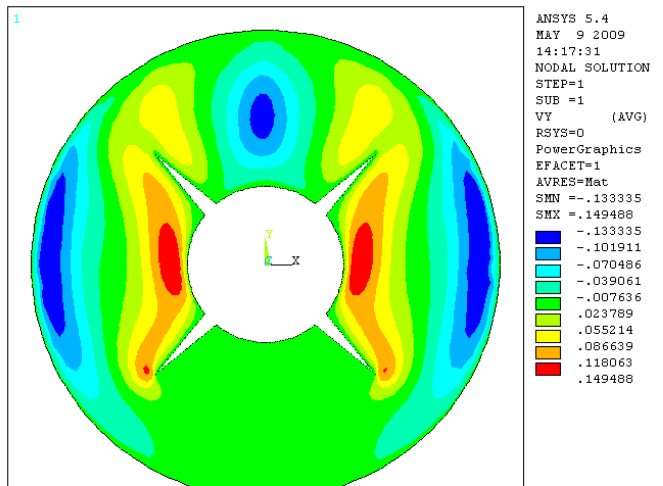
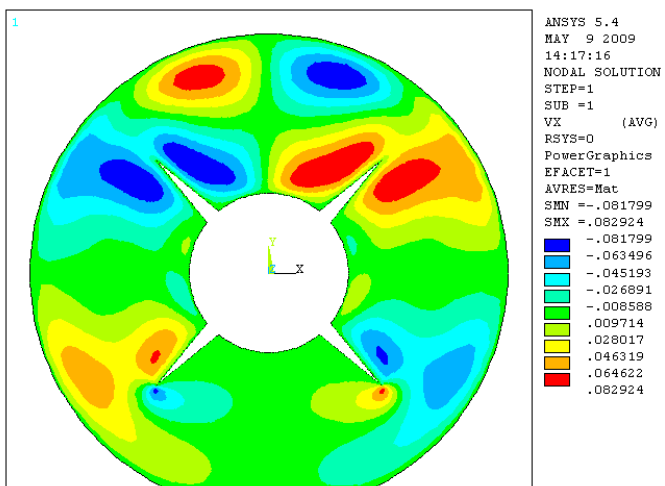
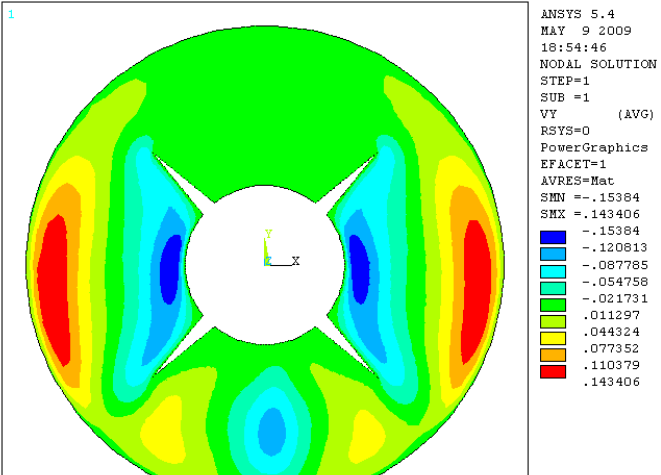
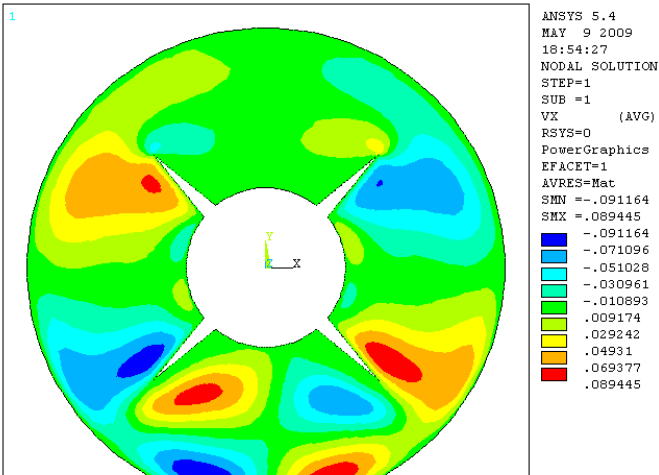
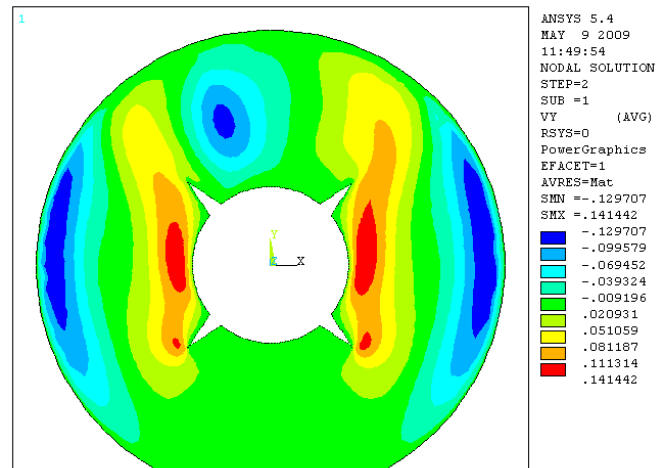
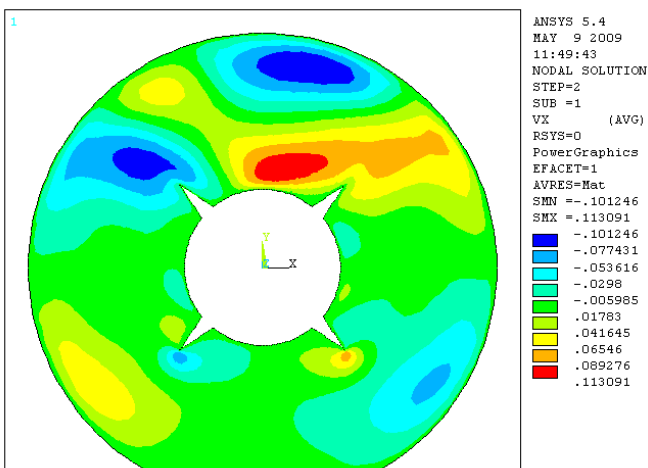
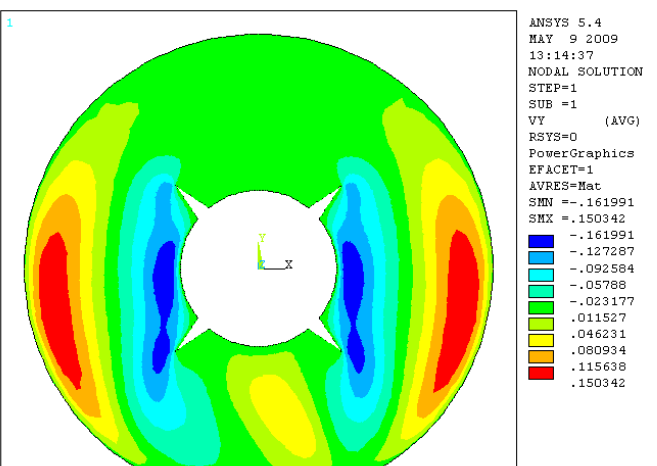
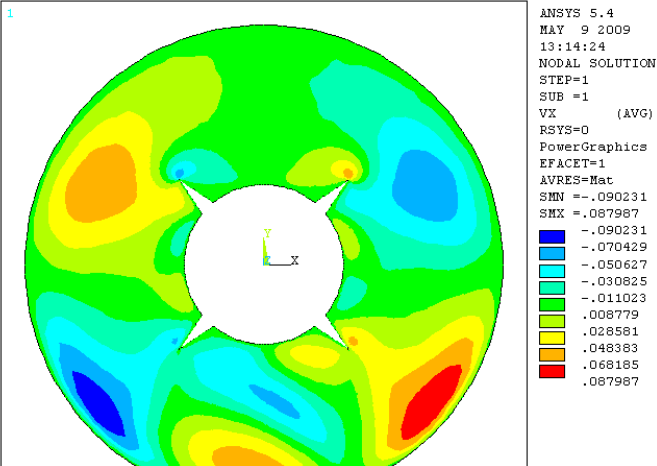


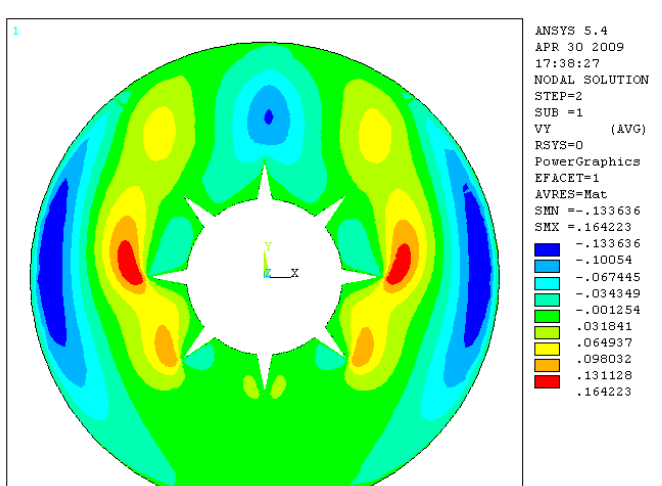
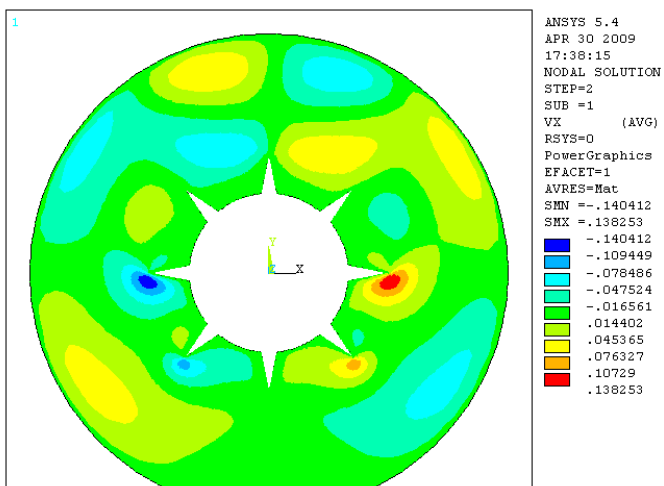
Figure 5: Annulus cylinder with four triangular radial fins. $T_o > T_i$

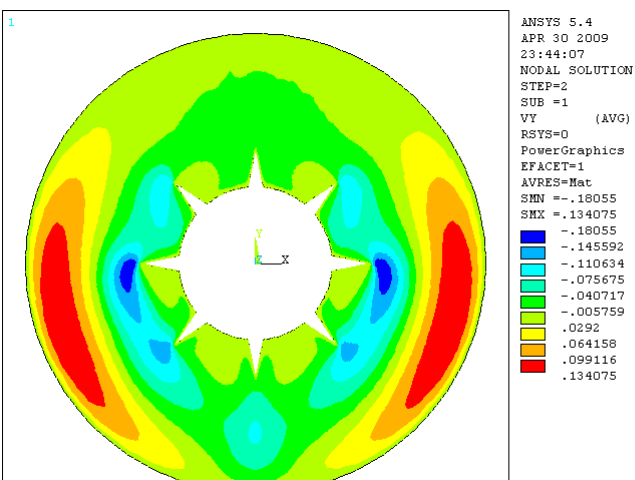
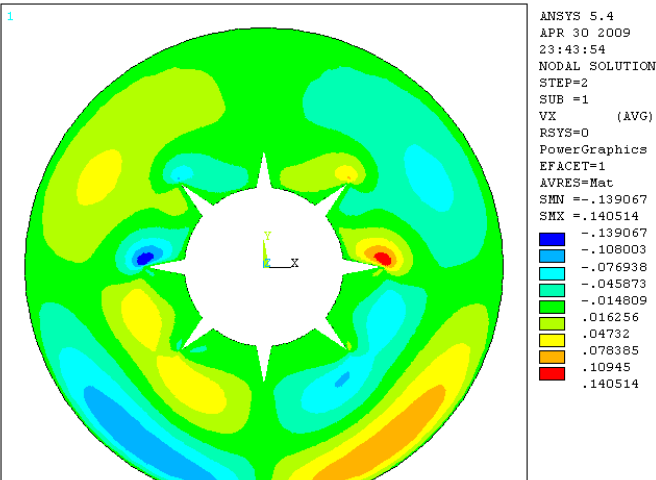


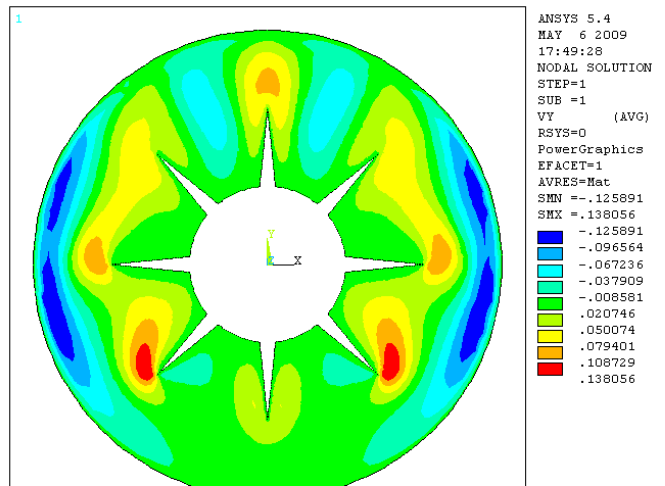
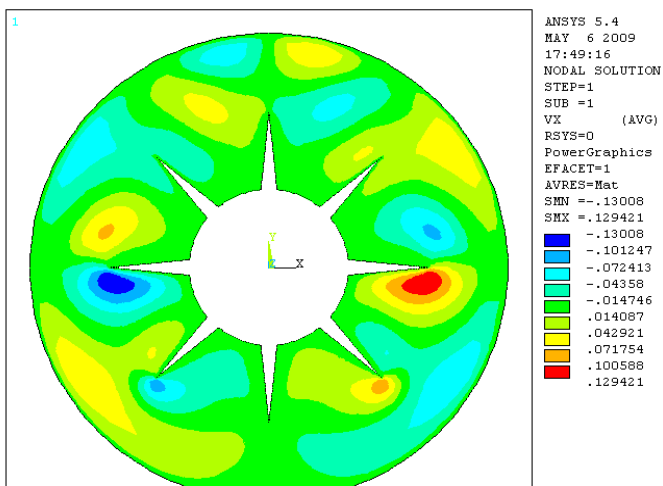












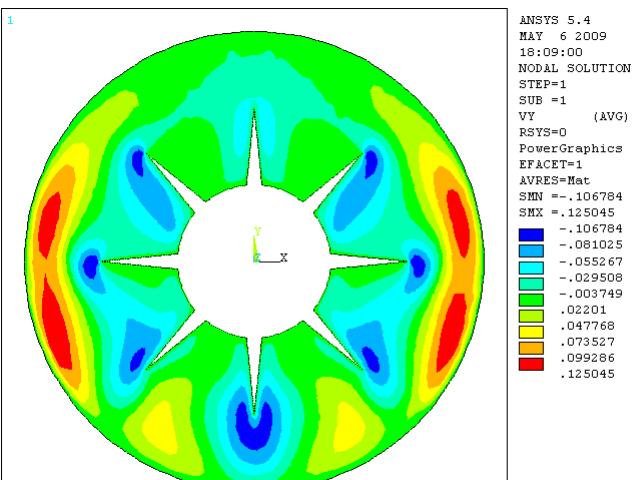
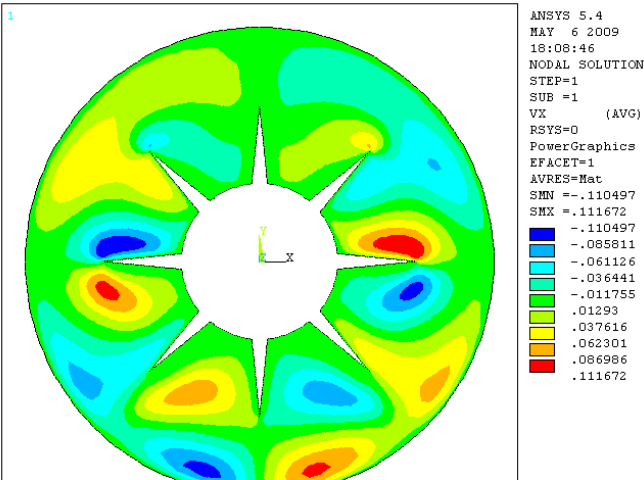


Figure 14a: Velocity distribution in x direction with eight triangular radial fins. $T_o > T_i$

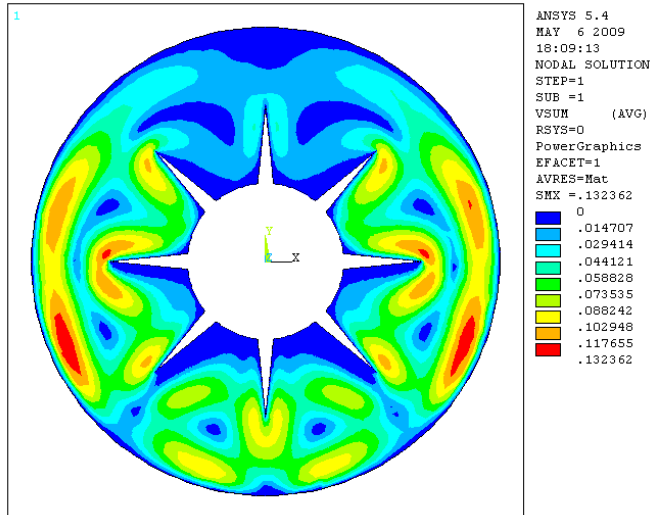


Figure 14b: velocity distribution in y direction with eight triangular radial fins. $T_o > T_i$

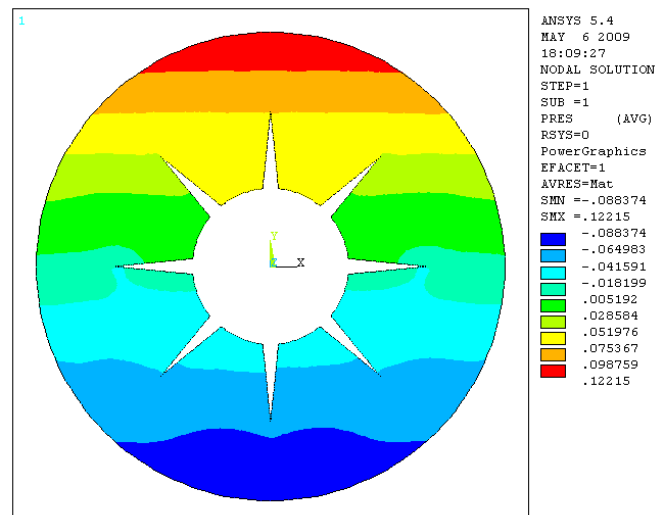


Figure 14c: Velocity distribution (V_{sum}) with eight triangular radial fins. $T_o > T_i$

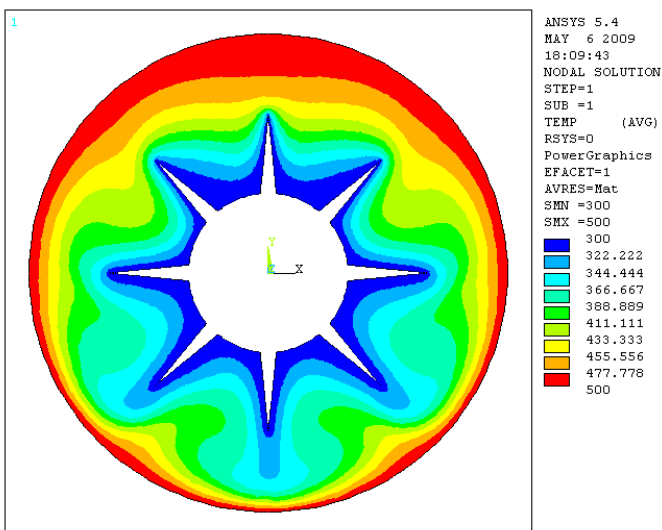


Figure 14d: Pressure distribution with eight triangular radial fins. $T_o > T_i$

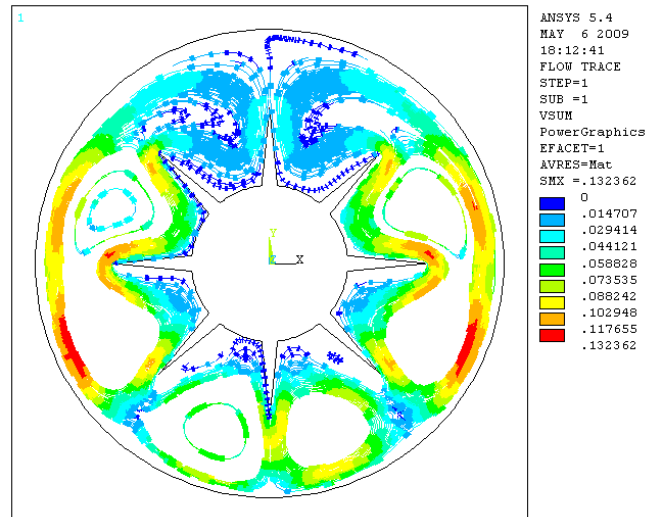


Figure 14e: Temperature distribution with eight triangular radial fins. $T_o > T_i$

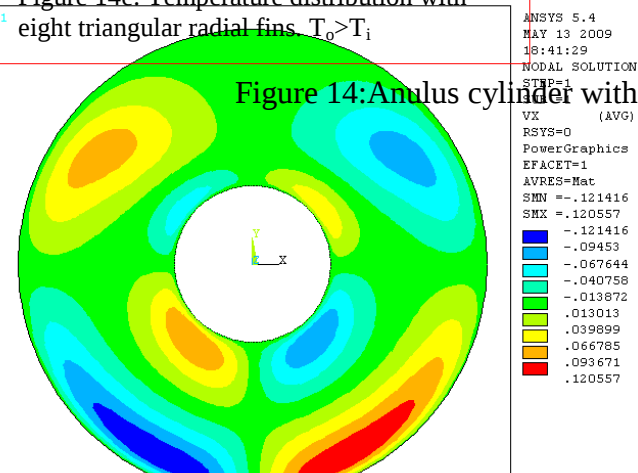


Figure 14f: velocity distribution vectors with eight triangular radial fins. $T_o > T_i$

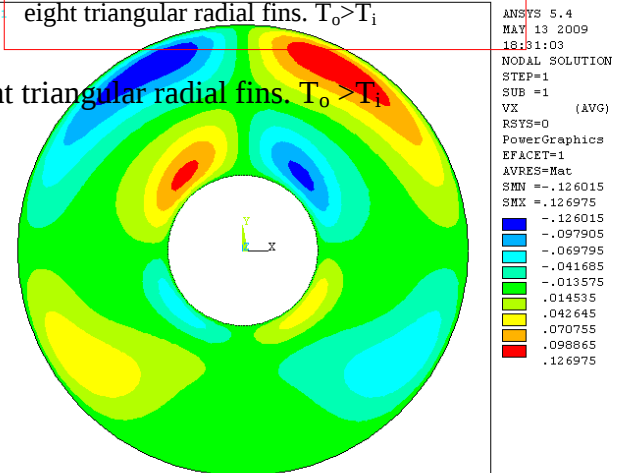


Figure 14: Annulus cylinder with eight triangular radial fins. $T_o > T_i$

Figure 15a: Velocity distribution in x direction without fins. $T_o > T_i$

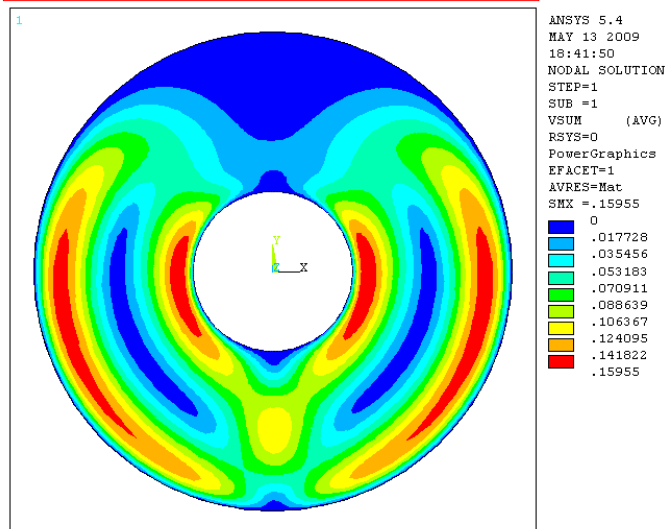


Figure 15b: velocity distribution in y direction without fins. $T_i > T_o$

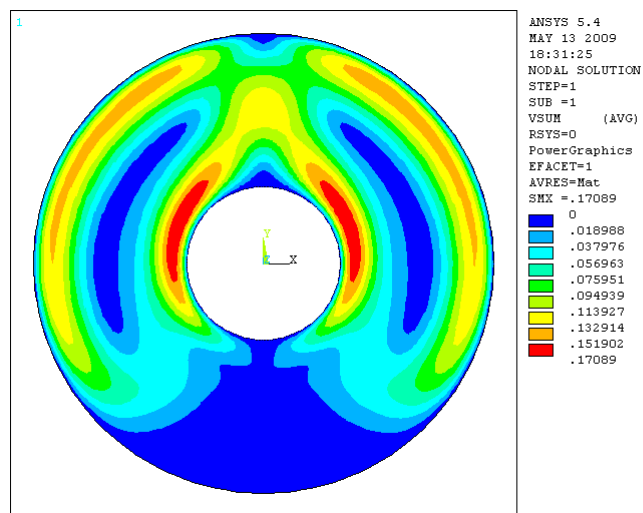


Figure 15c: velocity distribution (V_{sum}) without fins. $T_o > T_i$

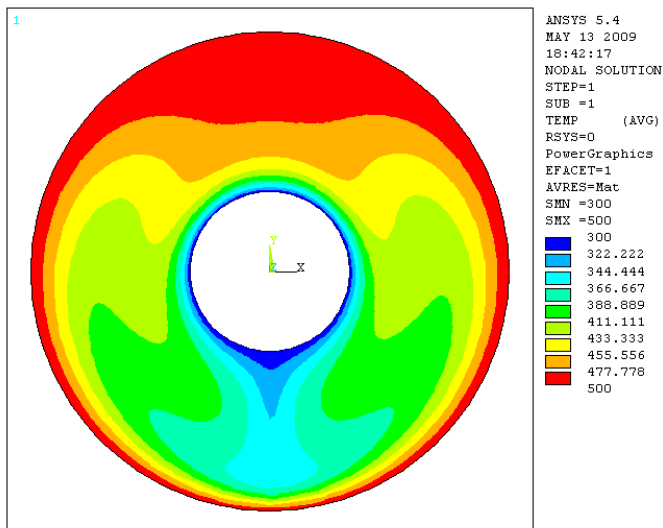


Figure 15c: Temperature distribution without. $T_o > T_i$

Figure 15:Anulus cylinder without fins