Calculation of the Energy levels, the Square of rotational energy and The moment of inertia of $^{170\text{-}176}$Yb Isotopes by the Interacting Boson Model-1

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Abstract :

In this paper , $^{170\text{-}176}$Yb isotopes have been studied by using interacting boson model (IBM-1) to determine energy levels and it was classified in two bands the ground state and bata band addition, the square of rotational energy and the moment of inertia values for $^{170\text{-}176}$Yb were calculated and compared with the experimental data. The obtained results for $^{170\text{-}176}$Yb were reasonably in good agreement with the experimental data .The attestations refer to this isotopes belong to the rotational limit SU(3).

حساب مستويات الطاقة ومربع الطاقة الدورانية وعزم القصور الذاتي لنظائر ($Yb^{170\text{-}176}$) باستخدام نموذج البوزونات المتفاعلة – 1 (IBM-1)

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الخلاصة :

في هذا البحث درست نظائر $^{170\text{-}176}$Yb باستعمال نموذج البوزونات المتفاعلة -1 لتحديد مستويات الطاقة , وصنفت إلى صنفين الحالة الأرضية و نطاق بيتا, بالإضافة إلى حساب مربع الطاقة الدورانية وعزم القصور الذاتي gنظائر $^{170\text{-}176}$Yb ومقارنتها مع النتائج العملية .وأظهرت النتائج تطابق جيد للقيم النظرية و القيم العملية وتشير الدلائل إن هذه النظائر تنتمي إلى المنطقة الدورانية SU(3).

Introduction :

The most fames of models is the Interacting Boson Model of the atomic nucleus, introduced in 1974 by Arima and Iachello, in which the fundamental constituents were correlated pairs of protons and neutrons treated as bosons [1]. The algebraic structure of this model is that of the unitary group in six dimensions, U(6). This model, together with the Nuclear Shell Model and the Liquid Drop Model, form the basis for the description of all nuclear phenomena. Several aspects of nuclear structure physics were being investigated at the moment, including the nature of shape phase transitions [2], the origin of anharmonicites, and the occurrence of collective states in which protons and neutrons move out of phase. The concept of symmetry was enlarged in the 70's to include a new type, called supersymmetry. One of the most important models which exploits the concept of supersymmetry is the Interacting Boson-Fermion Model of the nucleus, in which the fundamental constituents were correlated pairs (bosons) together with unpaired protons and neutrons (fermions). This model, introduced in 1980 by Iachello, forms the basis for the description of nuclei with an odd number of particles [3]. The IBM is a model for the structure of even-even collective nuclei which assumes that the monopole and quadruple degrees of freedom were the most important. It also assumes that all excitations were boson because of the existence of pairing interactions which were dominant at low energies. It is suitable for describing intermediate and heavy atomic nuclei. Adjusting a small number of parameters, it reproduces the majority of the low-lying states [4]. In (1990) D. S. Chuu etal[5] studied asimple procedure to optimize the interaction parameters in IBM-1 was used to calculate the energy levels of strongly deformed nuclei $^{154-158}$Sm, $^{154-160}$Gd, $^{156-164}$Dy, $^{160-168}$Er, $^{162-172}$Yb and $^{168-176}$Hf. It is found that there was variation in the interaction parameters for each isotope. The B(E2) values were also calculated.

In(1999)[6] N. Minkov and etal studied and derove analytic expressions for the energies and B(E2)-transition probabilities in the states of the ground and γ bands of heavy deformed nuclei (including Yb Isotopes) within a collective vector-boson model with SU(3) dynamical symmetry. E Biémont etal [7] in (2001) the analysis of the spectrum of Yb III which has been extended allowing us to establish 11 new energy level values. The good agreement between experimental results and semi-empirical calculations performed with the relativistic Hartree-Fock method including core-polarization effects allows the determination of transition probabilities for 15 lines. In2008 R. Rodríguez-Guzmán and etal[9] were studied the evolution of shapes with the number of nucleons in various chains of Yb, Hf, W, Os, and Pt isotopes from neutron number N=110 up to N=122

In 2011 the low-lying quadrupole collective states in neutron-rich even-even Yb, Hf, W, Os, and Pt isotopes were studied in a systematic way. Spectroscopic calculations were performed in terms of the Interacting Boson Model Hamiltonian, which is determined from the Hartree-Fock-Bogoliubov (HFB) approach with Gogny Energy Density Functionals (EDFs)[9].

Theory of IBM-1 model :

The (IBM-1) was described for low lying collective state of energy levels in (even – even) nucleus which can be described by bosons (s) bosons when ($J^{\Pi} = 0^{+}$) and (d) bosons when ($J^{\Pi} = 2^{+}$) The general Hamilton operator function formula for this isotopes is[10]

$$\hat{H} = a_1 \hat{L}^{1} + a_2 \hat{Q}^{2} \text{ ....}(1)$$

and the equation of eigen value to Hamilton is given by [12] :

$$E|(N,(\lambda,\mu),K,L,M\rangle = \frac{a_2}{2}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)+(a_1-\frac{3a_2}{8}).L(L+1)\text{............}(2)$$

where :

$\{(\lambda,\mu),K,L,M\}$ the quantum numbers, but (λ,μ) determined the Rotational limit SU(3) state.

The transition operator $T_m^{(E_2)}$ for this limits were given by following formula [10]:

While The formulas for calculating all the square of rotational energy and the moment of inertia are : [15]

$$\hbar^2\omega^2 = (L^2-L+1)\left[\frac{E(L\to L-2)}{2L-1}\right]^2 \text{..............}(3)$$

$$\frac{2\upsilon}{\hbar^2} = \frac{4L-2}{E(L\to L-2)} \text{.......................}(4)$$

Method of calculation :

The isotopes $Yb^{170-176}$ have $N_{\Pi} = 6$ and N_{ν} varies from 9 , 10 ,11 and 12, while the parameters L.L, Q.Q and CH1 as below in table (1) which take the energy levels a good agreement with the previous experimental data shown in table (2) . the square of rotational energy and the moment of inertia can be calculated from equations (4,5) after found the energy levels by using (IBM-1) program and angular moment to all energy levels were found by using(IBM-1) program . Table (3) shows comparison between the theoretical and experimental values ,for all the square of rotational energy and the moments of inertia

Table 1. Hamiltonian parameters

Isotopes	*For calculation Energy Levels*		
	L.L	Q.Q	CH1
Yb^{170}	0.0089	-0.0136	-1.000
Yb^{172}	0.0085	-0.0125	-1.000
Yb^{174}	0.0066	-0.0167	-1.000
Yb^{176}	0.0082	-0.0150	**-1.000**

Table 2. Calculated and experimental energy levels of $Yb^{170-176}$

Isotopes	*Energy Band K^+*	*Spin Parity I^+*	*Energy Levels (MeV)* This Work	Experimental.[13]
Yb^{170}	Ground state Band $K^+=0^+$	0^+	0.0000	0.0000
		2^+	0.08323	0.08426
		4^+	0.27742	0.27740
		6^+	0.58247	0.5736
		8^+	0.99830	0.9636
		10^+	1.52475	1.4379
	Bata Band $K^+=2^+$	2^+	1.06708	1.06935
		3^+	1.01575	1.1390
		4^+	1.21181	(1.2943)
		5^+	1.51959	1.5216
		6^+	1.93873	1.8037
		7^+	2.46873	(2.1360)
Yb^{172}	Ground state Band $K^+=0^+$	0^+	0.0000	0.0000
		2^+	0.07819	0.0787
		4^+	0.26063	0.2602
		6^+	0.54723	0.5398
		8^+	0.93793	0.9115
		10^+	1.43261	1.3698
	Bata Band $K^+=2^+$	2^+	1.04117	1.0429
		3^+	0.98811	1.11785
		4^+	1.17211	1.2865
		5^+	1.46105	1.5375
		6^+	1.85458	1.8536
		7^+	2.35241	2.2125
Yb^{174}	Ground state Band $K^+=0^+$	0^+	0.0000	0.0000
		2^+	0.07603	0.07648
		4^+	0.25341	0.253123
		6^+	0.53206	0.526029
		8^+	0.91189	0.8895
		10^+	1.39277	1.3362
	Bata Band $K^+=2^+$	2^+	1.48652	1.48743
		3^+	1.37520	1.56101
		4^+	1.55453	1.71540
		5^+	1.83607	1.9091
		6^+	2.21952	---------
		7^+	2.70447	---------
Yb^{176}	Ground state Band $K^+=0^+$	0^+	0.0000	0.0000
		2^+	0.08211	0.08213
		4^+	0.27369	0.27169
		6^+	0.57470	0.5648
		8^+	0.98507	0.9541
		10^+	1.50473	1.4312
	Bata Band $K^+=2^+$	2^+	1.28363	(1.2609)
		3^+	1.24286	(1.336)

	4^+	1.43583	(1.4356)
	5^+	1.73889	---------
	6^+	2.15182	---------
	7^+	2.67433	**---------**

Table 3. the comparative between experimental and theoretical the values of square of rotational energy , the moment of inertia

Isotopes	**I^+_i--I^+_f**	**square of rotational energy in(Mev)^2**		**the moment of inertia in(Mev)$^{-1}$**	
		This work	Experimental [13]	This work	**Experimental[12]**
Yb^{170}	$2^+\rightarrow0^+$	0.002309	0.002367	72.08939	**71.20817**
	$4^+\rightarrow2^+$	0.020418	0.020423	50.465	**50.45954**
	$6^+\rightarrow4^+$	0.086921	0.08437	37.77019	**38.33688**
	$8^+\rightarrow6^+$	0.252473	0.235226	30.05109	**31.13325**
	$10^+\rightarrow8^+$	0.586046	0.521185	24.92212	**26.42743**
Yb^{172}	$2^+\rightarrow0^+$	0.002038	0.002054	76.73616	**76.43312**
	$4^+\rightarrow2^+$	0.018022	0.017974	53.716	**53.78761**
	$6^+\rightarrow4^+$	0.076721	0.074663	40.20247	**40.75282**
	$8^+\rightarrow6^+$	0.222861	0.210478	31.98533	**32.91278**
	$10^+\rightarrow8^+$	0.517357	0.472986	26.52501	**27.74128**
Yb^{174}	$2^+\rightarrow0^+$	0.001927	0.00195	78.91622	**78.45188**
	$4^+\rightarrow2^+$	0.017037	0.016998	55.24644	**55.30908**
	$6^+\rightarrow4^+$	0.072527	0.070892	41.34872	**41.82279**
	$8^+\rightarrow6^+$	0.210658	0.20044	32.8987	**33.72681**
	$10^+\rightarrow8^+$	0.488982	0.450067	27.28376	**28.43886**
Yb^{176}	$2^+\rightarrow0^+$	0.002247	0.002248	73.07271	**73.05491**
	$4^+\rightarrow2^+$	0.019873	0.019584	51.15276	**51.52932**
	$6^+\rightarrow4^+$	0.084617	0.081727	38.28084	**38.95184**
	$8^+\rightarrow6^+$	0.245825	0.230611	30.45469	**31.44324**
	$10^+\rightarrow8^+$	**0.570757**	**0.516339**	**25.2537**	**26.55115**

Results and Discussion:

The whole Hamiltonian is then diagonal zed in the selected model space . The interaction parameters are determined by using IBM-1 program to the energy spectra of the $Yb^{170-176}$ even-even isotopes which its contain of (70) proton and (100-106) neutron. The (20) proton fall out closed shell of (50). There are some pointed refer to this isotopes belong to the rotational limit SU(3) .

The number of protons and neutrons in this isotopes fall at near half closed shell (50-82) ,(82-126) respectively [17,18].The ratio of energy levels

Also the Hamiltonian parameters which it show in table (1) refers to this isotope belong to the rotational limit SU(3).[4] .

refers to this isotopes belong to the rotational limit SU(3).[12,16].

$$\frac{E0_2^+}{E2_1^+}, \frac{E4_1^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}$$

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