

Transient Behavior of Nanofluid Natural Convection in Equilateral Triangular Enclosure

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Abstract :

In this work, the unsteady state behavior of natural convection of nanofluid in triangular enclosure was studied. Numerical solution of laminar convection of Cu-Water nanofluid over a range of Rayleigh numbers (10^3 – 10^7) and volume fraction of nanoparticles (0-20%) were performed. The Non-dimensional form of the energy and the momentum equations are solved using the commercial package COMSOL (5). A uniform wall temperature (UWT) boundary condition was subjected to the enclosure. The base temperature was always higher than the inclined surfaces. The effects of Rayleigh number on the velocities, temperature and Nusselt number were examined. The effects of volume fraction of nanoparticles on thermal performance and the stability of the transient behavior of the heat transfer and the fluid flow were examined. Based on the numerical results, it was shown that the addition of Cu nanoparticles enhanced the heat transfer rate for all values of Rayleigh. Moreover, it was found that both the stability of the transient behavior of the natural convection and the steady state profiles of temperature and velocities were affected by the existence of the Cu nanoparticles.

Keywords : transient, natural convection, triangular enclosure, nanofluid, Cu, COMSOL.

الخلاصة :

يتطرق هذا البحث الى دراسة التصرف غير المستقر لانتقال الحرارة بالحمل الحر في مائع نانوي في حيز مغلق مثلث الشكل. وقد تم اجراء الحل العددي لجريان طبقي للمائع النانوي المتكون من (الماء-النحاس) لمدى رقم رالي (10^3 – 10^7) ونسبة خلط حجمية (0-20%) محسوبة على اساس الدقائق الصلبة النانوية. لقد تم استخدام برنامج (كومسول 5) في حل الصيغة عديمة الوحدات لمعادلات الطاقة والزخم التي تمثل المعادلات الحاكمة للمسألة مع تطبيق الشروط الحدودية التي تشير الى ثبات درجات الحرارة على جدران الحيز المغلق حيث تكون درجة حرارة جدار القاعدة اعلى من درجة حرارة الجدارين المائلين. لقد تم فحص تأثير قيم رقم رالي على كل من السرعة ودرجات الحرارة وقيم معدلات انتقال الحرارة متمثلة برقم نسلت. كذلك تم دراسة تأثير قيم التركيز للدقائق النانوية (النسبة الحجمية) على الاداء الحراري واستقرارية السلوك الزمني لكل من انتقال الحرارة وجريان المائع. لقد اشارت النتائج الى ان اضافة الدقائق النانوية للنحاس يحسن دائما معدلات انتقال الحرارة لكل قيم رقم رالي. كما ان وجود هذه الدقائق سيغير السلوك الحراري وقيم درجات الحرارة والسرعة في الحالتين المستقرة وغير المستقرة لظاهرة انتقال الحرارة للحمل الحر في هذه الاشكال المغلقة.

الكلمات المفتاحية: غير مستقر، حمل طبيعي، حيز مغلق مثلث، مائع نانوي، نحاس، كومسول

I. Introduction

Due to its wide applications, the natural convection is one of the most important phenomena in thermal engineering systems. The natural convection heat transfer is a main design parameter in many applications [Yang,1987;Raithby *et. al.*,1998;Jaluria, 2003].

Different technique can be used to improve and enhance the heat transfer. Using the nanofluids is one of the promising methods. The conventional heat transfer fluids have low thermal conductivity that can be improved by using a dispersion of nanonparticles in the base fluid. The nanofluids have been studied extensively in many fields such as cooling of electronics [Turgut *et. al.*,2014; Vasu *et. al.*, 2009], solar water heating [Mahian *et. al.*,2013],nuclear reactor cooling [Nematolahi *et. al.*,2015; Buongiorno *et. al.*, 2009; Buongiorno *et. al.*, 2008], chillers [Anandakumar, 2015; Liu *et.*

al., 2011], engine cooling/vehicle thermal management [Cui *et. al.*, 2012; Raja *et. al.*, 2013; Azimi *et. al.*, 2013], cooling and heating in buildings [Kulkarni *et. al.*, 2009], biomedical applications [Khosroshahi *et. al.*, 2011; Buteică *et. al.*, 2010; Tombácz *et. al.*, 2008; Zakaria *et. al.*, 2015], fuel cell [Sharma *et. al.*, 2008; Zakaria *et. al.*, 2015; Vanaki *et. al.*, 2016]. Even though the natural convection heat transfer research has received less attention than the forced convection. Therefore, it is still required to study the effect of the nanoparticles on the heat transfer characteristics in natural convection applications. Many workers study the natural convection of nanofluids in enclosures. Different shapes of enclosure were investigated numerically as well as experimentally, [Wang *et. al.*, 2006; Aboualim *et. al.*, 2012; Abu-Nada *et. al.*, 2010; Windi, 2014; Ho *et. al.*, 2010; Itoh *et. al.*, 1970].

Recently, natural convection was studied in an isosceles triangular enclosure with a heat source located at its bottom wall and filled with an Ethylene Glycol–Copper nanofluid [Ghasemi *et. al.*, 2010]. A numerical analysis of unsteady state natural convection heat transfer in an isosceles triangular cavity filled with Al_2O_3 -water nanofluid was carried out with non-uniform hot temperature on the bottom wall of the cavity. [Rahman *et. al.*, 2014].

In this work, an attempt was made to investigate the natural convection in a triangular enclosure of copper water nanofluid subjected to constant different temperature at the lower and the upper halves. The effect of addition of nanoparticles on the stability of thermal behavior of the enclosure will be examined.

I. PROBLEM DESCRIPTION

Fig. (1) shows the geometry of the triangular enclosure filled with nanofluids. The inclined wall of the enclosure is maintained at a uniform low temperature (T_c) while the bottom wall is maintained at a uniform high temperature (T_h). The enclosure is filled with a water-Cu nanofluid with different volume fraction of solid particles. It is assumed that the flow is laminar and the fluid is Newtonian and incompressible. Also it is assumed that the base fluid and the nanoparticles are in thermal equilibrium. The thermophysical properties of the nanoparticles and base fluid are given in Table 1. Constant thermophysical properties are considered for the nanofluids except for the density variation in the buoyancy forces determined by using the Boussinesq approximation.

Table (1) Thermo physical properties of Copper and Water.

Property	Cu	Water
C_p (J/kg K)	385	4179
ρ (kg/m ³)	8933	997
k (W/m K)	400	0.61
β (1/K)	1.67×10^{-5}	2.1×10^{-4}

The thermal conductivity and the viscosity of the nanofluid are given by the following models:

$$\frac{k_n}{k_f} = \frac{k_p + 2k_f - 2(k_f - k_p)\phi}{k_p + 2k_f + (k_f - k_p)\phi} \quad (1)$$

$$\mu_n = \frac{\mu_f}{(1-\phi)^{2.5}} \quad (2)$$

The governing equations for a Boussinesq incompressible laminar flow under unsteady state conditions in a two-dimensional geometry take the following form:

Continuity equations:

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

Momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot [-\rho_n^{-1} p \mathbf{I} + \nu_n (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \nu_n (\nabla \cdot \mathbf{u}) \mathbf{I}] + \mathbf{F} \quad (4)$$

$$\text{Where } \mathbf{F} = \begin{Bmatrix} 0 \\ g \beta_n (T - T_\infty) \end{Bmatrix}$$

Energy equation:

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot (\alpha_n \nabla T) \quad (5)$$

The following dimensionless variables are used:

$$\begin{aligned} X &= \frac{x}{L}; \quad Y = \frac{y}{L}; \quad \tau = \frac{t \alpha_f}{L^2} \\ U &= \frac{uL}{\alpha_f}; \quad V = \frac{vL}{\alpha_f}; \quad P = \frac{pL^2}{\rho_f \alpha_f^2}; \quad \theta = \frac{T - T_c}{T_h - T_c} \\ k &= \frac{k_n}{k_f}; \quad \alpha = \frac{\alpha_n}{\alpha_f}; \quad \mu = \frac{\mu_n}{\mu_f}; \end{aligned}$$

The governing equations are re-written in above dimensionless form as follows:

$$\nabla \cdot \mathbf{U} = 0 \quad (6)$$

$$\frac{\partial \mathbf{U}}{\partial \tau} + (\mathbf{U} \cdot \nabla) \mathbf{U} = (\chi(\rho))^{-1} \nabla \cdot [-P \mathbf{I} + \mu Pr (\nabla \mathbf{U} + (\nabla \mathbf{U})^T) - \frac{2}{3} \mu Pr (\nabla \cdot \mathbf{U}) \mathbf{I}] + \mathbf{F}' \quad (7)$$

$$\frac{\partial \theta}{\partial \tau} + \mathbf{U} \cdot \nabla \theta = (\chi(\rho C_p))^{-1} (k \nabla^2 \theta) \quad (8)$$

$$\text{where } \mathbf{F}' = \begin{Bmatrix} 0 \\ Ra Pr \chi(\beta) \theta \end{Bmatrix}$$

and $\chi(\lambda) = 1 - \varphi + \varphi \frac{\lambda_p}{\lambda_f}$ is the property function; e.g;

$$\chi(\beta) = 1 - \varphi + \varphi \frac{\beta_p}{\beta_f}$$

The following dimensionless initial boundary conditions are imposed:

Initial conditions

$$\text{at } \tau = 0: \theta = 0; \mathbf{U} = 0; V = 0 \quad (9)$$

Boundary conditions

$$\theta = 0; \mathbf{U} = 0; V = 0 \quad @ \Gamma_1 \quad (10_a)$$

$$\theta = 1; \mathbf{U} = 0; V = 0 \quad @ \Gamma_2 \quad (10_b)$$

The local Nusselt number along the hot wall is defined as

$$Nu = k \frac{-L}{(T_h - T_c)} \frac{\partial T}{\partial x} \Big|_{\Gamma_2} = -k \frac{\partial \theta}{\partial X} \Big|_{\Gamma_2} \quad (11)$$

and the average Nusselt number is expressed as

$$Nu_{av} = \int_0^1 Nu \, dX \quad (12)$$

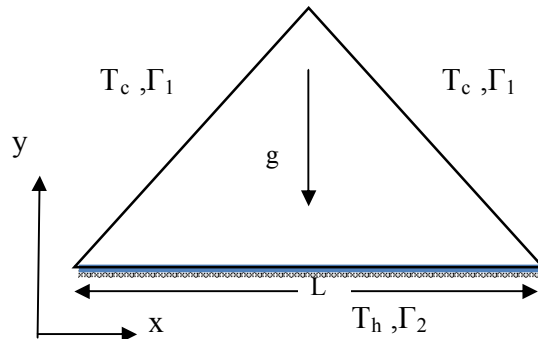


Fig. (1): Physical model.

II. NUMERICAL SOLUTION

With the aid of COMSOL Script, a code was written to solve the set of the dimensionless governing equations (6-8) and the associated initial and boundary conditions. The program COMSOL uses the finite element method to solve multiphysics problems. In order to have an independent grid size numerical solutions, The numerical solution was carried out by applying an optimum number of triangular element in order to ensure a grid independent solution. Fig.(2) shows the effect of grid size represented as a (Number of Elements) on the value of the Nusselt number. It is clearly shown that is suitable to have convenient solutions of this problem with a grid of about (10^3) elements.

III. RESULTS AND DISCUSSION

Based on the numerical simulations, the transient behavior of the heat and fluid flow of nanofluid is investigated in the equilateral triangular enclosure. A uniform heating with constant temperature through the horizontal and uniform cooling through the inclined surfaces is prescribed. The effect of Rayleigh number and nanoparticle volume fraction are discovered.

A typical example of this problem for Rayleigh number of 1×10^6 for clear water with $Pr=6.2$ is illustrated in Fig. (3). The isotherms and the stream lines are shown at different time steps. It is shown that at the first time steps there were three small plumes of temperature isotherms appeared and then collapsed to a large stable plume.

Fig. (4) shows the steady state solution of problem for different values of for Rayleigh number equal and less than 1×10^6 . At $Ra=10^3$ the conduction is dominated as a mode in heat transfer. As Ra exceeds a value of 10^3 the heat transfer controlled by natural convection. The temperature isotherms and velocity streamlines confirm such behavior. It is observed that this solution is symmetric with respect to the geometric midline of the enclosure.

The solution image is characterized by single plume in temp isotherm and two cells rotating in opposite directions for velocity streamlines.

As the Rayleigh number exceeds a value of 10^6 , Fig. (5), the thermal behavior is observed to be unstable with unsustainable periodic variation in temperature-velocities profiles.

The numerical analysis was extended to investigate the heat transfer coefficient on the surfaces of the enclosure. Figure (6) shows the average Nusselt numbers on the base of the enclosure with time for different values of Ra . The calculated results exhibit similar features as previously discussed.

The effect of addition of nanoparticles to the base fluid of water on the average Nusselt numbers can be explained with the aid of Fig. (7). It can be shown that the existence of nanoparticles will not only enhance the heat transfer but also stabilize the thermal behavior.

As (χ) increases, the system becomes stable and the Nu increase also. At $Ra=4 \times 10^6$, the average value of the average Nu is about (24) for pure water whereas the enhancement due to addition of copper nanoparticles with (20%) volume fraction reaches to 1.5 times than they are in the pure water at the same Ra .

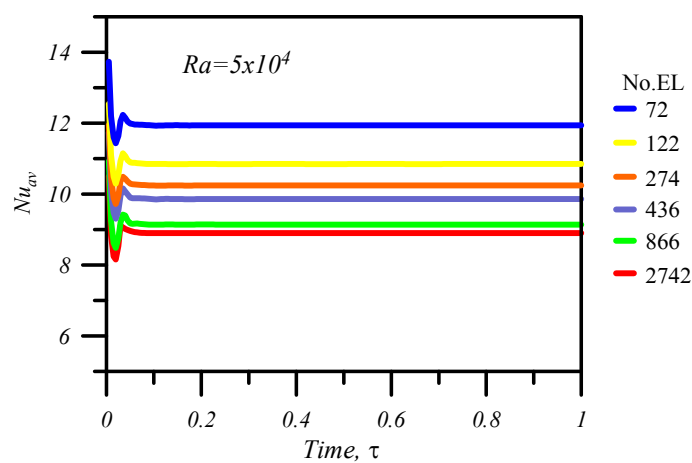


Fig.(2): Transient Average Nusselt number at different values of Number of Elements

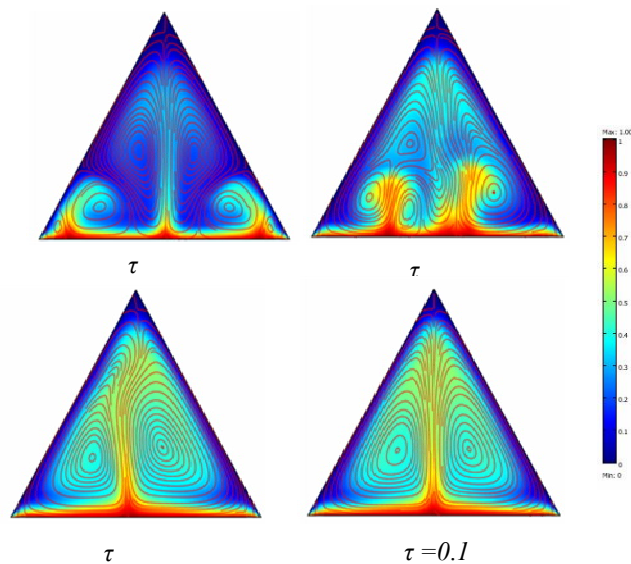


Fig.(3): Snapshot of dynamic behavior of clear water for $Ra=1 \times 10^6$:

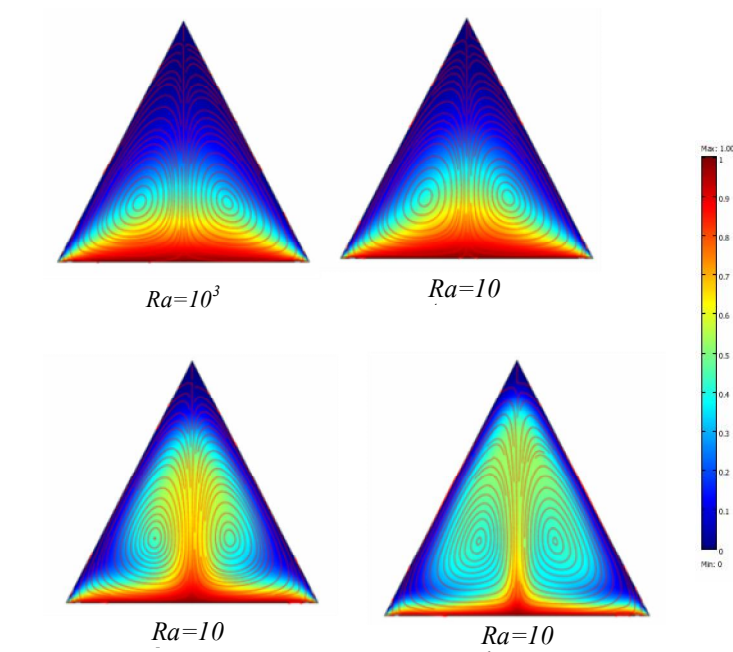


Fig.(4): Steady state profile of clear water at different values of Ra .

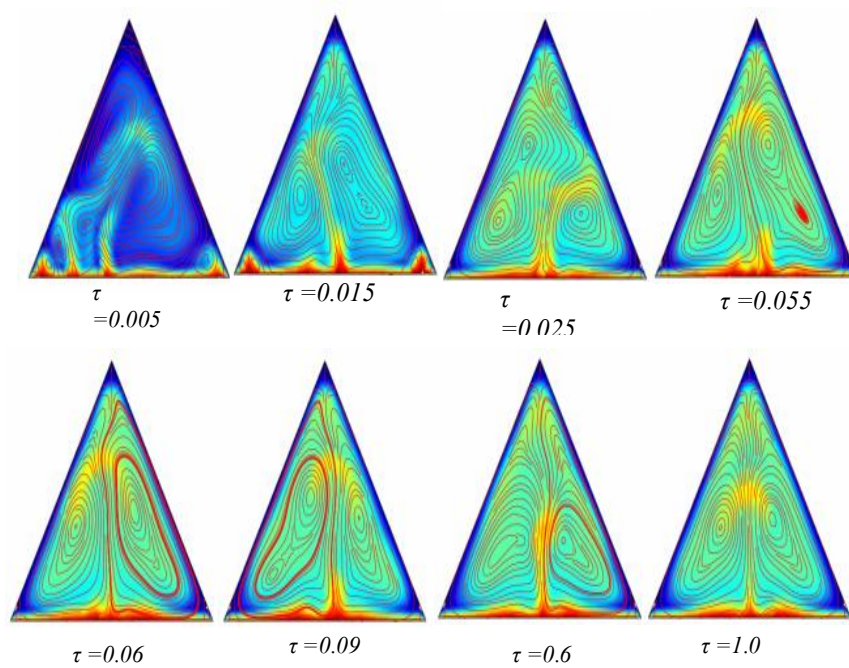


Fig.(5): Snapshot of dynamic behavior of clear water for $Ra=7 \times 10^6$:

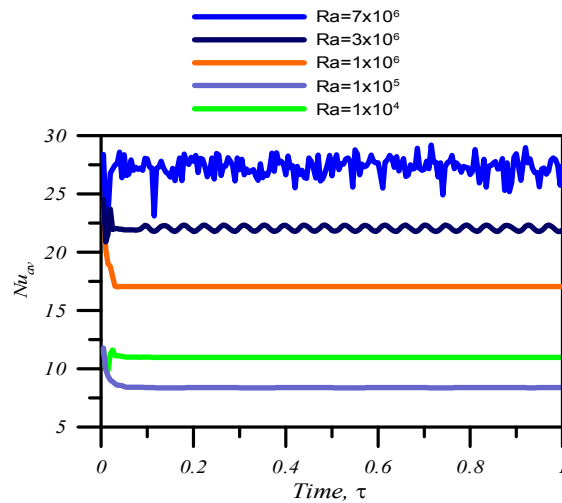


Fig.(6): Variation of average Nusselt Number with time for different values of Rayleigh number

IV. CONCLUSIONS

The numerical analysis of the transient behavior of natural convection heat transfer of copper-water nanofluid filled an equilateral triangular cavity has been done in this work. The convection fluid flow through the enclosure was caused by the potential difference in temperature between the upper (cold) and the lower (hot) sides of the cavity. The commercial package COMSOL Multiphasics (5.0) was used to formulate and solve the model equations of the problem. The computations in this work include the effects of some factors such as the volume fraction (χ) of solid particles of nanoparticles (Cu) and Rayleigh number (Ra), based on base fluid (water), on the temperature, velocities and the Nusselt number (Nu). It is observed that the existence of nanoparticles make the transient behavior of the natural convection (velocities and temperature) for nanofluid more stable than the behavior for the pure fluid. Moreover it is found that for all values of solid volume fraction of nanoparticles, increasing Rayleigh number results in enhancing the heat transfer and the Nusselt number due to strengthening of the buoyancy forces.

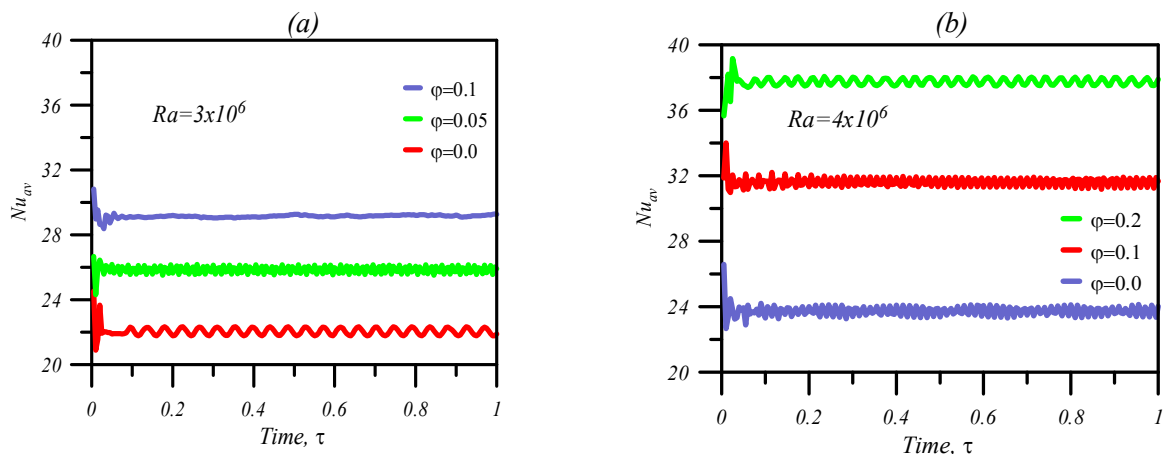


Fig.(7): Variation of average Nusselt Number with time for different values of volume fraction (ϕ) at Rayleigh number (a) $Ra=3 \times 10^6$ (b) $Ra=4 \times 10^6$

NOMENCLATURE

Cp	Specific heat (J /kg K)	T	Temperature (K)
g	Gravity acceleration (m/s ²)	t	Time (s)
Gr	Grashof number ($g \beta (T_h - T_c) L^3 / \nu^2$)	u	Velocity in x direction (m/s)
k	Thermal conductivity (W /m K)	U	Dimensionless x component of velocity
L	Side length of the enclosure (m)	v	Velocity in y direction (m/s)
Nu	Nusselt number	V	Dimensionless y-component of velocity
p	Pressure (Pa)	x	Distance in x direction (m)
P	Dimensionless pressure	X	Dimensionless x coordinate
Pr	Prandtl number ($\mu C_p / k$)	y	Distance in y direction (m)
Ra	Rayleigh number (Gr Pr)	Y	Dimensionless y coordinate
<i>Greek symbols</i>		<i>Subscripts</i>	
α	Thermal diffusivity (m ² /s)	c	cold
β	Thermal expansion coefficient (1/K)	f	fluid
Γ	Boundary of cavity	h	hot
θ	Nondimensional temperature	n	nanofluid
μ	Dynamic viscosity (kg/ms)	p	particle
ρ	Density (kg/m ³)	∞	environment
τ	Nondimensional time		
χ	Nanoparticle fraction		
ν	Kinematic viscosity (m ² /s)		

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