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NEW VACCUM FORMATION IN THE UNIVERSE

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ABSTRACT

Effects of gravitation on the decay of metastable state in the early universe are investigated. It is found that the decay is not possible under certain onditions, which involve the parameters of a field theory and the sign of the carvature of a cosmological. The corresponding constraints are the most essential in the case of supersymmetric models. A relation is found between the amount of in flation and the temperatur after reheating.

Introduction

Great attention has been paid recently to studies of Vaccum phase transition in the early universe. Due to the large unificant mass scale of fundamental interactions, $M_x \approx 10^4 - 10^{18} GeV$, significant role in the process of phase transition could be played by gravitation .

A full account of general relativity effect in phase transition phenomena seems to be a rather complicatid problem and as a matter of fact this question is not yet. investigated properly up to now despite its importance . In particular, gravitational effects are usually neglected in studies of the nucleation of a new phase bubble during the decay of a metastable state in the universe. Account of these effects effects was carrid out only in the case of the decay of a pure vacuum state in ref [1]. At some conditions gravitation makes the probability of vaccum decay smaller and it can even stabilize the false vacum, preventing vaccum decay altogether [1].

It was found in ref. [2] that the dcay of a metastable vaccum with account of general relativity effects could proceed via the simultaneous transition of the whole universe into the state with maximum of the effective potential rather than by means of the nucleation of isolated bubbles.

In the general case of a first order phase transition the spacetime isdescribed by different metrics inside and outside the new phase bubble. The same is true concering the universe before and after the phase transition. The phase separation surface should obey the Einstein equatins so one has to match the metrics on his hypersurface. In ref.[3,4] we have investigated the motion of the new phase bubble walls as well as old phase remants and domain walls, using the formilism developed by Israel [5] for thin shell in general relativity. In the present work we shall not be interested in the bubble motion but consider the possibility of junction of the metrics on the hypersurface which separates the new and old phase. In such a way we shall of course not obtain the decay probability of a metastable state, however, we can find the range of the parameters of a theory at which it should vanish. One has to Consiter the junction on

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both time–like (TL) and space (SL) Surface of phase separation.

The junction on the TL hypersurface gives us in particalar the real motion of bubble walls [3,4]. The junction on the SL hyperserface has no been considered up to now. We would like to note some physical situation to which we may suppose the SL junction could be applied:

A phase transition which proceeds through the nucleation isolated bubbles, collisions, and subsequent thermalization, but which finishes very rapidly but which finishes very rapidly (almost instantly).

A phase transition which proceeds by means of the Hawking–Moss (mechanism) [2].

A materialsization of a new phase inside on (large) bubble.

A phase transition in the fromework of the new inflationary scenario [6].

In the case (i) and (ii) the matching is carried out across the universe, while in the cases (iii) and (vi) we are restricted by the dimensions of the bubble. In the cases (ii) and (iii) we match two phases the transition between them being the quantum tunneling process. There for the applicability of the SL junction to these cases, is nuclear. Though this junction may prove to be rather unphysical we shall nevertheless formally investingate it too. In case (vi) we pay no attention to the quantum transition process and consider the whole region of space-time where the slow classical rolling of the field fluction occurs as the region still occupied by the old phase. To the region still occupied by the old phase. To the transition layer we attribute the region where the system rolls down rapidly to the mininum of the potential and then oscillates, the kinetic energy of the field being transformed into particle energy. The duration of both latre is of the order 1/Mx, which is much smaller than even the dimensions of the visible part of the universe. Therefore it is quite natural to use here the thin wall formalism.

We shall consider only sphericall symmetic surfaces of phase separation . The line element on such a hypersurface may be written as

$$\begin{aligned} dL^2 &= dT^2 - \rho^2(T)d\Omega^2(TL) \\ dl^2 &= -dq^2 - \rho^2(q)d\Omega^2(SL) \end{aligned} \quad \ldots\ldots(1)$$

Where $d\Omega^2$ is the solid angle element. The form of this surface is determined by the condition derived from the Einstein equation for a singular spherically symmetric shell [7] :

$$[K_2^2] = \varepsilon 4\pi S / M_{p1}^2 \ldots\ldots\ldots\ldots(2a)$$

$$[K_0^0] + [K_2^2] = \varepsilon 8\pi S_2^2 / M_{p1}^2 \ldots\ldots\ldots(2b)$$

$[K_i^j]$ being the discontinuity of the $[K_i^j]$ the component of the shell's outer curvature tensor: $S = S_0^0$, $\varepsilon \cong +1$ for a TL hypersurface and

$S \cong S_1^1$, $\varepsilon \cong -1$ for a SL hypersurface, respectively , where

$$S_i^j = \lim_{S\to 0} \int_{-\delta}^{\delta} dn T_i^j \quad ....(3)$$

Is the surface density of the energy-momentum tensor on the shell and n is the coordinate measured in the direction of the outer normal to the hypersurface. We shall denot all the quantities befor the phase transition by the index "b"and those after the transition by the index "a" , correspondingly the indices " in" and "out " will be used for the metrics inside and out side the bubble.

We first consider the TL junetion , caleuleding the outer curvature tensor for the TL shall in a spherically symmetrie metric of general form

$$ds^2 = e^{\upsilon} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2$$

$$\upsilon \equiv \upsilon(t,r) , \; \lambda \equiv \lambda(t,r) \quad ....(4)$$

We obtain $K_2^2 = -\sigma(\rho_T^2 + e^{-\lambda})^{\frac{1}{2}} \rho$ ....(5a)

$$K_0^0 = -\sigma[\rho_{\tau\tau} + \rho_\tau^2(\lambda' + \nu')/2 + \nu e^{-\lambda}/2] \times (\rho_\tau^2 + e^{-\lambda})^{\frac{1}{2}} - \sigma\lambda^0 \rho_\tau e^{(\lambda-\upsilon)/2} ..(5b)$$

where

$\rho_\tau \equiv d\rho / d\tau$,

$\lambda' \equiv \partial\lambda / \partial r$,

$\lambda^\bullet \equiv \partial\lambda / \partial t$,

$\sigma = \mp 1$

The case σ = +1 corresponds to increasing radii r in the outward direction while σ = -1 indicates the opposite case. Thus, for given inner and outer product metrics σ determines the global geometry. Substituting (5) into (5) one can detain the equation of motion of the shell determining $\rho(\tau)$ find σ . Here we are not interested in the explicit form of the function $\rho(\tau)$ but shall investigate the general junction condition. We find that at certain conditions the junction is impossible . such a situation may arise if the sign of σ_{out} (or σ_{in}) following from eqs.(2) does not agree with the given geometry of an outer (inner) region. It following immediately from eq.(2a) that $\sigma_{out} > 0$ if

$$\exp(-\lambda_{in}) - \exp(-\lambda_{out}) > 16\pi^2 (S_0^0)^2 \rho^2 / M_{Pl}^4 ....(6)$$

Correspondingly $\sigma_{out} < 0$ at the opposite sign of the inequality (6). One may easily write also similar in equalities. far σ_{in} .

Consider the case of a homogenous isotropic space described by the Robertson–Walker–Friedman metric:

$$ds^2 = dt^2 - R^2(t)[dr^2 /(1-kr^2) + r^2 d\Omega^2] ....(7)$$

This metric can be easily represented in the form (4). The Einstein equations, we obtain :

$$\exp(-\lambda) = 1 - 8\pi\varepsilon p^2 / 3M_{p1}^2 ....(8)$$

conve nience let us introduce the following notation :

$$\zeta \equiv M_{p1}^2 [\epsilon_{out}(t_{out}) - \varepsilon_{in}(t_{in})] / 6\pi [S_\circ^\circ(\tau)]^2 ....(9)$$

The correspondence between ζ and $\sigma's$ which follows from conditions (6) is shown in table 1, where we represented also the schematic views of the corresponding special sections

Discussion

We see that the case $k_{out}=+1$, ζ > -1 can be reduced to the case $k_{out}=+1$, ζ > 1 by exchanging the outer and inner regions, i.e. , these two cases are physically equivalent. The same is true for the cases ζ < 0 when -1 < ζ < +1 , so it is sufficient to consider the case ζ ≥ 0 . Note that in the case of spherically symmetric domains created in the course of spontaneous breaking of discrete symmetry (e.g for CP domains [8]. one has just ζ = 0. As one can see from table 1 the junction is not always possible. In the closed universe (k=+1) the junction is possible at any value of ζ, one cannot obtain any constrains on the parameters of a theory. In the open universe (k= -1) the junction is possible only if ζ > 1. It is clear that if such a metric describes all space time then the decay of a met stable state occur only if ζ > 1. However there may take place more complicated situation too, which occur if the coordinates (7) with k=-1 do not describe the whole space time but apart only. In these coordinates the junction is again impossible of ζ > 1 , however , this does not decay. It may be simply that the new phase bubble is nucleated in such a way that it goes out of the scope of the coordinate set (7) let as explain this by considering as an example the junction of two metrics with pure vacuum equations of state, i.e. , two sitter metrics and different ε's but $\varepsilon_{out>0}$. In the pure de Sitter world with ε > 0 one can always choose the coordinate in such a way that the section t = constant will be open or spatially flat. However, one needs two such coordinate systems to caver the whole manifold. At ζ > 1 the junction in terms of these coordinates is impossible while it may be easily shown that the junction of two de sitter metrics with $\varepsilon > 0$ is possible at any ζ . It is sufficient to note that such a metrics can be written in the form (7) with k=+1 and this coordinates set covers the whole space time. Thus we see that the junction is always possible in coordinates with k =+1 but it is impossible in coordinates with k=-1 (k=0) of ζ < 1. This property has also the following meaning, while it is meaningless to ask whether the "pure" de sitter world is open or closed because there is no preferable frame of reference, in the universe containing at least one bubble, such a preferable frame of reference does appear. Namely at ζ < 1 one can introduce only such coordinate t in the whole space time, for which the section t = constant is closed i.e., the universe i.e., the universe which contains a vacuum bubble is closed

Let us put now $\varepsilon_{out}=0$ i.e., the universe without a bubble is a Minkowski world which is described by one coordinate set of the type (7) with k=0. We see the gravitation stabilizes such a vacuum with respect into the state with negative energy density if $\zeta < 1$. This particular result was obtained in ref. [1] using completely different approach .It was found [1] that the probability of the decay of a pure vacuum state equal zero at $\zeta < 1$ if $\varepsilon_{out} = 0$ in agreement with our constraints following from the junction conditions .

Thus, the conclusion about the stability of a system with $\zeta < 1$ may be done always if the coordinates (7) with k = -1 describe the whole space–time. Therefore; such a conclusion may be drown, for example, with respect to the universe which is filled with radiation only [9]

In order to obtain from table (1) the constraints on the decay of a metastable state of the universe in tern of the parameters of a model of quantum field theory, one has yet to find relation between the time t and τ in eq.(9) . fortunately, in the cases of practical interest this procedure is necessary because of the large difference between ε and S^2 / M_{P1}^2 at $\zeta \leq 1$. Let us consider for example the super symmetric SU(5) model with a low scale of super symmetric breaking , $M << R_{GUT}^{-1}$ where R_{GUT}^{-1} is the radius of confinement during the grand unified phase. In such a theory a strong occurs at $\tau \approx R_{GuT}^{-1}$ and this when the phase transition occurs the energy density then being equal to $\varepsilon = \frac{1}{30} T^2 N R_{Gut}^{-4}$ where N is the total numbers of effectively mass less of freedom [N= 200 in the minimal SU(5) model]. In the energy density of the wall, of least at early stages of the expansion of the bubble the vacuum contribution dominates. thus we obtain that the non–closed universe in such a case, the transition is only possible if

$$S_{\circ}^{\circ} \leq (\pi N / 180)^{1/2} M_{P1} R_{GUT}^{-2} \sim 10^{37} (Gev)^3 \quad ....(10)$$

The constraints on the parameters of the decay vacuum following from table 1 may be useful in studies of any stages of the evolution of the universe and not only of the very early ones. In ref.[10] the constraint $\zeta > 1$ derived in ref.[1] for the particular case $\varepsilon_{out} = 0$ was used to show the stability of the present A=0 universe against decay into states with negative energy which arise in the framework of super gravity models. It should be noted, however, that the universe state is note in fact pure vacuum. Therefore; using the results of the present paper we may conclude that the decay probability of such a state could not equal zero in a closed universe. In the open universe the states considered in ref . [10] are undoubtedly stable for they satisfy the condition $\zeta < 1$. Of particular interest is the vacuum dominated case. Namely let the universe be radiation–dominated at high temperatures and be described by the metric (7) with k=-1. Further, let at a certain temperature, being there the epoch of domination of a met stable vacuum, and the parameters of a field theory to be such that ζ < 1. Will there occur a phase transition in this case or not? We believed that the answer to this question depends on the glebal structure at the epoch which preceds the vaccum, dominating stage [2] .

Let us now proceed to consider the junction on the SL hypersurface. Straightforward calculations of K_I^J for the spherically symmetric SL hypersurface Σ show t] hat in this case $\sigma_a b_\sigma = \sin g(dt_a / dt_b) \mid_\Sigma$ (remained that in the care of TL ; per surface we had

$$\sigma_{in}\sigma_{out} = sign(dr_{out} / dr_{in}) \mid_\Sigma .$$

Therefore, if we do not allow the time direction to reverse after the phase transition, we must require the signs of σ_a and σ_b to be the same. The tensor K_i^j for the SL supersurface may be obtained from $K_i^j(\tau)_{TL}$ given by eq.(5) by the continuation

$$K_l^m(q)_{SL} = iK_l^m(iq)_{TL}$$

Here we consider the isotropic homogenous case with the metric (eq.7), therefore $\delta_i^j = s\delta_i^i$, S = constant and it may by shown that only one of eqs . (2a) and (2b) is independent .it is convient to take eq .(2a):

$$\sigma_b[\rho_q^2 - 1 + \frac{8}{3}\pi(\varepsilon_b / M_{PL}^2)\rho^2]^{\frac{1}{2}}$$
$$= \sigma_a[\rho_q^2 - 1 + \frac{8}{3}\pi(\varepsilon_a / M_{P1}^2)\rho^2]^{\frac{1}{2}}$$
$$= -(4\pi / M_{P1}^2)\rho S_1^1 \quad \ldots\ldots\ldots\ldots\ldots(11)$$

Let us introduce the quality ξ defined as

$$\xi = [(\varepsilon_b - \varepsilon_a)/6\pi(S_1^1)^2]M_{p1}^2 \quad \ldots\ldots\ldots(12)$$

The time direction does not reverse at crossing over a SL hypersurface if while it reverses if -1< ξ <1

First we consider the junction between two phases connected by thetunneling process. In a pure vaccum care we found no constraints on a metastable vaccum decay from the SL junction because here we may not warry a bout the direction. We cannot calculated S in the case directly but we may suppose that it is equal to S_0^0 due to O(4) invariance of the Euclidean vacuum solution [11]. Considering the non-vacuum case we require the time not to change its direction at crossing a SL. Then the decay of a metastable state is possible only if $\xi > 1$.

Let us consider now the cases (i) and (vi) .eq. (11) in this case has to be understood as a condition on the parameters aftar the phase transition. Because of homogenous and isotropy the hypersurface of the phase separation is just the surface of constant t, where t is the cosmohogical time (eq.7). In proper coordinates the equation of such a hypersurface is $\rho_q^2 = 1 - k\rho^2 / R^2$ and (eq.11) takes now the form:

$$(\frac{8}{3}\pi\varepsilon_b / M_{P1}^2 - kT^2 / N^2)^{\frac{1}{2}}$$

$$-(\frac{8}{3}\pi\varepsilon_b T^4 / M_{P1}^2 - kT^2 / N^2)^{\frac{1}{2}} = -4\pi S / M_{P1}^2 \quad ....(13)$$

Where cT4 the radiation energy density, N=TR, N3 = s3/4c, s being the coordinate entropy density. For the inflationary scenario to work, it is necessary to have N $>$ 1028 [12]. Thus eq.(13) relates the amount of inflation with temperature after reheating. In general the value of k_{in} eq.(13) is not fixed. However, when the complete metric in O(4) invariant the metric inside the new phase bubble 13 of the open type (k=-1) [1,13]. In the new inflationary scenarios the fluetation passes the stage of vaccum expansion. One has therefore to expect in this case the visible part of the universe is described by (eq .7) with k=-1. The contributions to s come from the rapid rolling down to the minimum of potential and from subsequent particle creation processes.The first contribution can be easily calculated and equals to $S \approx -(2\lambda)^{\frac{1}{2}} t_0^3 / 3$, where t0 is the equibum value of the scalar field, λ being the quadratic coupling constant entering the potential as $-\lambda t^4$. The second contribution requires special consideration .

We have thus studies of the matching of two homogenous isotropic metrics on a spherically symmetric singular hypersurface of the phase separation and have found the conditions when junction is impossible, that means that the phase transition between these two states is forbidclen. We used the thin-wall approximation and found that the constraints on the parameters of field model arise at $\xi \approx 1$, i.e., just in the case of the validity of the thin-wall approximation [1]. We have seen that there is a non-trivial relation between the field theory model parameters and the signature of the universe. In particular: (I) Theories which predict $\xi < 1$ supplemented by the condition that the phase transition did occur, can be Compatible only with the conception of the closed universe. (2) In the open universe metastable states with $\xi < 1$ become safe for an observe living in such a universe. We would like to note that while the value $\xi < 1$ at MX << Mp1 in ordinary GUTs can appear only as a result of fine-tuning, such a value is quite natural [10] for super-symmetric models.

Of particular interest is the junction on the SL hyper surface. In the cases (2) and (3) mentioned at the beginning we arrived at the conclusions that the decay of a non-Vacuum state is possible only of $\xi > 1$; at the decay of a pure vacuum state no constraints arise but at the crossing of the SL hypersurface of a phase separation at $\xi < 1$ the time direction should change. In this connection, direct calculaion of the decay probability of a non-vaccum state with accounting of gravity effects and a comparison with the results of the junction of metrics on an SL hypersurface would be extrely interesting. In the cases (i) and (VI) we found the equation, which relates the amount of inflation with the temperature after reheating.

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Table: 1: the constraints on the decay of a metastable state of the universe in tern of the parameters of a model of quantum field theory

	σ_{IN}	σ_{OUT}	SPATIAL SECTION (SCHEMATIC) $K_{OUT}=+1$	$K_{OUT}=-1$	$K_{OUT}=0$
$\xi > 1$	+1	+1			
$-1 < \xi < 1$	+1	-1		JUNCTION	
$\xi < -1$	-1	-1		IMPOSSIBLE	

تشكيل الفراغ الجديد في الكون

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الخلاصة:
تأثيرات الجاذبية على تهدم المستويات الشبة مستقرة في الكون تمت دراستها، حيث وجد أن التهدم غير ممكن تحت الشروط الثابتة والتي تتضمن بارامترات النظرية الكمية وإشارة نصف قطر التكور للنموذج الكوزمولوجي التغير المقابل هو نماذج التماثل العالي ووجدت العلاقات بين الانتفاخ (التضخم) ودرجة الحرارة عند أعادة التسخين.