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ON-MΘ-C-FUNCTIONS

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ABSTRACT

In this work we introduce anew concept namely MΘ -C- function. These are the functions $f:X\to Y$ such that the inverse of every compact subset in Y is Θ - closed in X. We proved several theorems about MΘ-C- functions and we study the relations of MΘ-C- function with other types of functions.

Introduction

let $f:X\to Y$ be a Θ- continuous function from topological space X in to a Θ-T2 space Y,

if $K\subset Y$ is compact , then K is Θ-closed.

But f is Θ-continuous, so f-1 (K) is Θ-closed.

This means that the inverse of each compact set in Y is Θ-closed in X.

This motivates the definition of MΘ-C- function.

Preliminaries

In this section we recall the Basic definitions needed this work.

Definition [3]

If $f:X\to Y$ is a function, then the Graph of f(G(f)) is the subset $\{(x,y): x\in X.\ \ y = f(x)\}$ of $X\times Y$.

Definition [2]

1- Apoint x is the Θ-closure of subset K of a space ($x\in Cl\Theta(K)$ if each $V\in\sum(x)$ satisfies $K\cap Cl(V)\neq\varphi$ (where $\sum(x)$ denoted the family of open subset which contain x).

2- A subset K is Θ-closed iff $Cl\Theta(K) = K$.

Definitions

1- A function $f:X\to Y$ has a closed graph (relative to $X\times Y$) iff G(f) is a closed subset of the product space $X\times Y$[4].

2- function $f:X\to Y$ has Θ-closed graphs iff for each $(x,y)\in X\times Y- G(f)$ there are sets $V\in\sum(x)$ in X and $W\in\sum(x)$ in Y with $(Cl(V)\ X\ Cl(W))\cap G(f)=\varphi$ [2].

Definition [7]

Let D be a non empty set and let ≥ be a binary relation on D we say that the relation ≥ directs D iff the following three conditions hold :

1- for every $a\in D$, $a\geq a$.

2- if $a\geq b$ and $b\geq c$ then $a\geq c$.

3- for each $a,b\in D\ \exists\, c\in D\ni\ c\geq a$ and $c\geq b$.

The pair (D, ≥) is called directed set.

2- A net in the space X is a function $f:D\to X$ where (D, ≥) is directed set.

Definitions [2]

1. A net $\{x_\alpha\}$is in a space X is converges to x in X ($x_\alpha\to x$) if $\{x_\alpha\}$is eventually in $V\in\sum(x)$.

2. A net $\{x_\alpha\}$is in space X is Θ-converges to x in X ($x_\alpha\to_\Theta x$) if $\{x_\alpha\}$ is eventually in Cl(V) for each $V\in\sum(x)$.

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Definitions

1.A topological space (X,T) is said to satisfy Θ-T_2-space if given a pair of distinct points $x,y \in X$ $\exists$ two Θ-open sets U,V s.t $x \in U$ and $y \in V$, $U \cap V = \varphi$.

2. A point x is said to be a Θ limit point of a set K if every open set U with $x \in U$, $(Cl(U)-\{x\}) \cap K \neq \varphi$.

3.Let (X,Y) be a topological space ,$K \subseteq X$ is called a Θ-neighboruhood of a point $x \in X$ if $\exists$ Θ-open set $U \in T$ with $x \in U$ s.t $U \subset K$.

Definition [5]

We will say that a function $f:X \to Y$ has a Θ-subclosed graph if for each $x \in X$ and net$\{x_\alpha\}$in X-{x} with $x_\alpha \to_\Theta x$.

And net $\{y_\alpha\}$ in Y with $y_\alpha = f(x_\alpha)$ for α and $x_\alpha \to_\Theta y$ in Y, we have y = f(x).

Definition [6]

A function $f:X \to Y$ is Θ-continuous at $x \in X$ if for each $W \in \sum f(x)$ in Y there is $V \in \sum(x)$ in X with $f(Cl(V) \subset Cl(W))$.

Remark

Every continuous function is Θ- continuous

M Θ-C-function

In this section we introduce a new concept, namely M Θ-C-function definee as follows:

Definition

Let $f:X \to Y$ be function we say that f is M Θ-C-function iff for each compact $K \subset Y$ then $f^{-1}(K)$ is Θ-closed in X.

If $f:X \to Y$ were also continuous then we that f is an M Θ-C-mapping .

We will need the following fact (see[2]).

Fact[2]

Let $f:X \to Y$ be function from a space X in to Y then f has Θ-closed graph iff f has a Θ-closed point images.

Theorem

If $f:X \to Y$ is a function with Θ-subclosed graph then f is an M Θ-C-function.

Proof

Let K be a compact subset in Y

We must prove that $f^{-1}(K)$ is Θ-closed in X

Let a be a Θ-limit point of $f^{-1}(K)$.

There are nets $\{x_\alpha\}$in X-{a} and $\{y_\alpha\}$in K with $x_\alpha \to_\Theta x$ and

$y_\alpha = f(x_\alpha)$ for each α .

some sub net $\{y_{\alpha n}\}$of $\{y_\alpha\}$ converges to some $y \in K$

this gives y = f(a) since f has Θ-subclosed graph so $a \in f^{-1}(K)$ then $f^{-1}(K)$ is Θ-closed in X

Corollary

If $f:X \to Y$ is a function with Θ-closed graph then f is M Θ-C-function.

If $f:X \to Y$ is an M Θ-C-function, the G(f) is not necessarily Θ-closed.

The following theorem show that if $f:X \to Y$is an M Θ-C-function from a space X into a locally compact Θ-T_2-space Y, the G(f) will be Θ-closed.

Theorem

Let $f:X \to Y$ be an M Θ-C-function from space X,X into a locally compact Θ-T_2-space Y, the G(f) will be Θ-closed.

Proof

Suppose $(x,y) \in G(f)$, then

y = f(x) and so there exists disjoint open sets U_1, U_2 with $y \in U_1$ and $f(x) \in U_2$,_further there is a compact neighborhood W of y such that $W \subset U1$ then $f^{-1}(W)$ is Θ-closed in X and $x \notin f^{-1}(W)$ thus there is an Θ-open set V such that $x \in V$ and $V \cap f\text{-}1(W) = \varphi$.

Hence V×W is a Θ- neighborhood of (x,y) which misses G (f) and so G(f) is Θ-closed.

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Theorem

If $f:X\to Y$ is a continuous function from a space X into a Θ-T_2-space Y then f is an M Θ-C-mapping.

Proof

Let K is compact subset in Y since Y is Θ-T_2-space then K is Θ-closed in Y {A compact subset of Θ-Hausdorff space is Θ-closed.[6]}

Then f is M Θ-C-mapping.

Example

Let $f:(R,T_u)\to(R,T_u)$ be a function defined as following

$f(x) = x^2$ then f is an M Θ-C-mapping.

Before we state our next theorem , we recall the definition of compact function.

3-8 Definition

Let $f:X\to Y$ be a function, we say that f is a compact function iff the inverse of each compact set K in Y is compact in X.

Theorem

If $f:X\to Y$ is a compact function from Θ-T_2-space X into space Y, then f is an M Θ-C-function.

Proof

Let K is a compact subset in Y since f is compact function. Then $f^{-1}(K)$ is compact in X.

Since X is Θ-T_2-space then $f^{-1}(K)$ is Θ-closed in X.

Then f is an M Θ-C-function.

Theorem

If $f:X\to Y$ is a M Θ-C-function and let $W\subset X$ .then $f|W : W\to Y$ is M Θ-C-function.

Proof

Let $g = f|W : W\to Y$ and let K is a compact subset in Y, since f is an a M Θ-C-function . then $f^{-1}(K)$ is Θ-closed in X.

Now $g^{-1}(K) = f^{-1}(K)\cap W$

g-1(K) is Θ-closed in W.

$g = f|W$ is an M Θ-C-function.

Theorem

If $f:X\to Y$ is an M Θ-C-function and let $W\subset X$ then $(f|W)_p :W\to f(W)$ is M Θ-C-function.

Proof

Let $h = (f|W)_p :W\to f(W)$ and let K is compact subset in f(W).

Hence K is compact in Y.

Since f is M Θ-C-function then $f^{-1}(K)$ is Θ-closed in X.

Now $h^{-1}(K) = f^{-1}(K) \cap W$

$h^{-1}(K)$ is Θ-closed in W

$h = (f|W)_p :W\to f(W)$ is an M Θ-C-function.

Theorem

Let $f:X\to Y$ be a continuous function from a Θ-T_2-space X into Y, and let $g : Y\to Z$ be a M Θ-C-function.

Proof

Let K be a compact subset in Z .

Since g is M Θ-C-function then g-1(K) is Θ-closed in Y.

Since f is continuous then $f^{-1}(g^{-1}(K))$ is Θ-closed in X i.e

$(g\circ f)^{-1}(K)$ is Θ-closed in X then $g\circ f$ is an M Θ-C-function.

Theorem

Let $f:X\to Y$ be a M Θ-C-function from a space X into a compact space Y and let $g : Y\to Z$ be a M Θ-C-function then $g\circ f$ is an M Θ-C-function.

Proof

Let K be a compact subset in Z since g is M Θ-C-function then $g^{-1}(K)$ is Θ-closed in Y.

Since Y is compact then $g^{-1}(K)$ is compact

Since f is M Θ-C-function

$f^{-1}(g^{-1}(K)) = (g\circ f)^{-1}(K)$ is Θ-closed in X

then $g\circ f$ is M Θ-C-function.

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الدوال MΘ-C

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الخلاصة

في هدا البحث استنتجنا مفهوماً جديداً يدعى بالدالة –MΘ-C وهذه الدوال $f:X \to Y$ بحيث ان كل مجموعة متراصة في Y المعكوس لها هو Θ – مغلقة في X. ثم برهنا عدة مبرهنات عن الدوال MΘ-C وقمنا بدراسة العلاقة بين الدالة MΘ-C وبقية انواع الدوال.