

ON forecasting by Dynamic Regression models

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ABSTRACT

This research include the application of some statistical technique for studying the time series of the average monthly humidity as an output series with one of the variables which affect on it, which is the series of the average monthly relative rainfall as an input which is measured at the meteorological station of Duhok the techniques used are the modeling by an(ARIMA) model as well as the dynamic regression model. So that the perfect dynamic regression model selected was suitable for determining the future forecasting values.

Introduction

The series of the humidity and relative rainfall were examined and determined that they are stationary in the mean and the variance, also both the auto correlation and partial auto correlation function were studied for the humidity and relative rainfall series and determined that there is an observed correlation for these phenomena; therefore, a suitable model was determined for both series from order ARMA(1,1). Relative the dynamic regression models (it is that model which take the time into account),the modeling of the dynamic regression shows how is that output result from the input and that is depends upon:

- 1- the relation of the lag time with the input and output.
- 2- The time composition for the turbulence series.

Then the model which was identified by the statistical measures as well as the cross correlation function for the residual between the residual series (α_t) of the input series, it was found that these two series are independent and the model of the transformation function was suitable. As well as the examination of the auto correlation for the residuals series (a_t) by the statistical test shows that all values were insignificant and it is prove that the turbulence series is a white noise series. In

this paper we compare between series ($\alpha_t^{\wedge}$) and series ($a_t^{\wedge}$) and we conclusion that the correlation between two series ($\alpha_t^{\wedge}$) and series ($a_t^{\wedge}$) is significant.

DYNAMIC REGRESSION (DR)

Preliminary

A Dynamic Regression model is a regression model which allows lagged value of the explanatory variable(s) to be included, the relationship between the forecast variable and the explanatory variable is model using a transfer function. A DR model states how an output (Y_t) is linearly related to current and past value of one or more input ($X_{1,t}, X_{2,t}, X_{3,t}, \dots$), it is usually assumed that observations of the various series occur at equally spaced time intervals. While the output may be affected by the inputs, a crucial assumption is that the inputs are not affected by the outputs this means that we are limited to single equation models [6].

1.1 TRANSFER FUNCTION [1],[2],[6]

For simplicity we will discuss just one input. The ideas which developed here are easily extended to multiple inputs if Y_t depends on X_t in some way we may write this as

$$Y_t = f(X_t) \quad (1)$$

Where $f(.)$ is some mathematical function. The function $f(.)$ is called a transfer function. The effect of a

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change in X_t is transferred to Y_t in some way specified by the function $f(.)$. In general, however, there are other factors causing variation in Y_t besides changes in the specified input, we capture those other factors with an additive stochastic disturbance (N_t) that may be auto correlated N_t represents the effects of all excluded inputs on the variability of Y_t . The input – output relationship may also have an additive the constant term (C). This is a buffer term that captures the effect of excluded inputs on the overall level of Y_t , thus we are considering models of the form:

$$Y_t = C + f(X_t) + N_t \quad (2)$$

Where Y_t : is the output

X_t : is the input

C : is the constant term.

$f(X_t)$: is the transfer function

N_t : is the stochastic disturbance which may be auto correlated

N_t is assumed to be independent of X_t

input N_t output

$X_t \rightarrow [\text{transfer function}] \rightarrow Y_t$

1-2 IMPULSE RESPONSE FUNCTION [5],[6]

We can write a linearly distributed lag transfer function in back shift form by defining $v(B)$ as

$$v(B) = v_0 + v_1 B + v_2 B^2 + v_3 B^3 + \dots \quad (3)$$

where B is the backshift operator defined such that

$$B^k X_t = X_{t-k}$$

We can write the transfer function $f(X_t)$ as a liner combination of current and past X_t value:

$$Y_t = f(X_t) = v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + v_3 X_{t-3} + \dots \quad (4)$$

Using equation (5),(4) may be rewritten as

$$Y_t = v(B) X_t \dots \quad (5)$$

Equation (5) is a compact way of saying that there is a linearly distributed lag relationship between change in X_t and changes in Y_t . The individual v_k weights in $v(B)$, ($v_0, v_1, v_2, v_3, \dots$) are called the impulse response weights we can estimate that the V_{k-b} weights as follows

$$V_k = \frac{\sigma_{\beta}}{\sigma_{\alpha}} \rho_{\alpha\beta}(k) \quad (6)$$

Where $\rho_{\alpha\beta}(k)$ estimates the cross correlation between α, β

σ_{β} : standard deviation of β

σ_{α} : standard deviation of α

1-3 DEAD TIME[5],[6]

Y_t might not react immediately to a change in X_t , some initial v weights may be zero. The number of v weight equal to zero (starting with v_0) is called dead time denoted as b , starting with v_0 , there is one v weight equal to zero ($v_0 = 0$), so $b=1$. Alternatively if $v_0 = v_1 = v_2 = 0$ and $v_3 \neq 0$ then $b=3$.

1-4 THE RATIONAL DISTRIBUTED LAG FAMILY[5],[6]

The Koyck impulse response function is just one member of the family of rational polynomial distributed lag models. This family is a set of impulse response functions $v(B)$ given by

$$v(B) = \frac{w(B)B^b}{\delta(B)} \quad \text{where} \quad (7)$$

$$w(B) = w_0 + w_1 B + w_2 B^2 + \dots + w_h B^h \quad (8)$$

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \dots - \delta_r B^r \quad (9)$$

Where h : represents the order of (w)

r : represents the order of (δ)

Extending this frame work to m inputs, $i=1,2,\dots,m$, is straight forward. The result may be written compactly as

$$Y_t = \sum_{i=1}^m v_i(B) X_{i,t} = \sum_{i=1}^m \frac{w_i(B)B^{b_i}}{\delta_i(B)} X_{i,t} \quad (10)$$

1-5 BUILDING DYNAMIC REGRESSION MODELS(DR) [3],[5], [6].

A dynamic regression (DR) model with one input consists of a transfer function plus a disturbance. This may be written as:

$$Y_t = c + v(B) + N_t$$

Where

$$N_t = \frac{\theta(B^s)\theta(B)}{\phi(B^s)\phi(B)\Delta_s^D \Delta_d^d} a_t$$

and

a_t : is zero mean and normally distributed white noise

1-6 PREPARATION AND PREWHITENING OF THE INPUTS AND OUTPUTS SERIES

[1],[2],[5]

Rewriting this process, we may think of AR and MA operators as a filter that, when applied to X_t , produces an uncorrelated residual series

$$\alpha_t = \theta_x^{-1}(B)\phi_x(B)X_t$$

The series α_t (in practice $\alpha_t^\wedge$) is called the prewhitened X_t series now suppose we apply the same filter to Y_t : this will produce another residual series

$$\beta_t = \theta_y^{-1}(B)\phi_y(B)Y_t$$

1-7 IDENTIFICATION

a) ESTIMATION OF THE IMPULSE RESPONSE WEIGHTS [5]

Equation (7) shows that if we prewhiten the input, and apply the same filter to the output, then the v weights are proportion to the cross correlations of the residuals from these two filtering procedures. in practice we don't know the parameters on the right side of equation (7). Instead we substitute estimates of these parameters obtained from the data to arrive at the following estimated v weights.

$$v_k^\wedge = \frac{r_{\alpha\beta}(k)\sigma_\beta^\wedge}{\sigma_\alpha^\wedge}$$

b) IDENTIFICATION OF (r,s,b) FOR THE TRANSFER FUNCTION[6]

We obtain the identity

$$\begin{aligned} v_j &= 0; & j < b \\ v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \dots + \delta_r v_{j-r} + w_0; & j = b \\ v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \dots + \delta_r v_{j-r} - w_{j-b}; & j = b+1, b+2, \dots, b+s \\ v_j &= \delta_1 v_{j-1} + \delta_2 v_{j-2} + \dots + \delta_r v_{j-r}; & j > b+s \end{aligned}$$

c) DISTURBANCE SERIES [5].

We generate an estimate of the N_t series denoted by $N_t^\wedge$, the estimate disturbance series and it is computed as:

$$\begin{aligned} \hat{N}_t &= Y_t - v_1(B)X_t \\ &= Y_t - \frac{w_s(B)}{\delta_r(B)} B^b X_t \end{aligned}$$

This disturbance series (N_t) in a dynamic regression will often be autocorrelated.

$$\phi_N(B)N_t = \theta_N(B)a_t \text{ where}$$

a_t : is zero mean and normally distributed white noise

1-8 ESTIMATION [2],[3],[4]

At the identification stage we tentatively specify a rational form transfer function model of orders (b,r,s), and a disturbance series ARIMA model of orders (p,d,q) We identified the following DR model

$$Y_t = \frac{w(B)B^b}{\delta(B)} X_t + \frac{\theta_N(B)}{\phi_N(B)} a_t \quad (11)$$

At the second stage of our modeling strategy we estimate the parameters of the identified DR model using the available data. To estimate (11) we will use initial values to refer to coefficient values can often be found from identification stage information to estimate the coefficients in $w(B)$ and $\delta(B)$ the next step in estimation is to compute the SSR (sum of squared residuals)

$$SSR = \sum_{i=1}^n a_i^2$$

Is used to choose better model coefficients, by taking the minimum SSR

1-9 Diagnostic Check [1],[6]

We can Diagnostic Check time series model by examining

a) Residuals Cross Correlation function (RCCF) $r_k(\hat{a})$

where

$$r_k(\hat{a}) = \frac{\rho_{\alpha\alpha(k)}}{\hat{\sigma}_a \hat{\sigma}_a} \quad k=1,2,3,\dots \quad (12)$$

Where

$\rho_{\alpha\alpha(k)}$: the cross correlation between a, α

$\hat{\sigma}_a$: standard deviation

$\sigma_\alpha^\wedge$: standard deviation of α

$$\rho_{\alpha\alpha(k)} = \frac{\sum_{t=1}^{n-k} (\hat{a}_t - \bar{\hat{a}})(\hat{a}_{t+k} - \bar{\hat{a}})}{n} \quad k=1,2,3,\dots \quad (13)$$

Where $\hat{r}_k$ is only an estimate of parameter ρ_k , we may test the null hypothesis

$$H_0: \rho_k = 0$$

$$H_A: \rho_k \neq 0$$

If α_t and a_t are uncorrelated and normally distributed, and one of these two series is white noise, than $\hat{r}_k$ has the following approximate standard error

$$S(\hat{r}_k) = n^{-1/2} \quad (14)$$

When n is the smaller of the number of observation for $\hat{a}_t$ or $\hat{\alpha}_t$

Another useful statistic involves a test on all K residual CCF coefficients as a set. Consider the joint null hypothesis

$$H_0: \rho_1 = \rho_2 = \rho_3 = \dots = \rho_k = 0$$

$$H_A: \rho_1 \neq \rho_2 \neq \rho_3 \dots \neq \rho_k \neq 0$$

By using Ljung and Box(1978) below

$$s = n^2 \sum_{k=0}^K (n-k)^{-1} (r_k)^2 \quad (15)$$

When: n is the smaller of the number of observation for $\hat{a}_t$ or $\hat{\alpha}_t$

$s \sim \chi^2$ for degree of freedom $(K+1-m)$

m : is the number of parameters estimated in the transfer function part of the DR model. If the critical value less than the χ^2 for degree of freedom $(K+1-m)$, we accepted the H_0 . it means that the two series $\hat{a}_t$ and $\hat{\alpha}_t$ are independent.

b) Autocorrelation Check

we also check the adequacy of the ARIMA model for the disturbance series in the DR model by examine autocorrelation and partial autocorrelation of series $\hat{a}_t$ we test the null hypothesis

$$H_0: \rho_k = 0$$

$$H_A: \rho_k \neq 0 \quad k=1,2,3,\dots,K$$

Wrong transfer function model will also tend to produce significant residual autocorrelations, even if the disturbance ARIMA model is correct. We may also perform a joint test with the null

hypothesis

$$H_0: \rho_0(a) = \rho_1(a) = \rho_2(a) = \rho_3(a) = \dots = \rho_k(a) = 0$$

$$H_A: \rho_0(a) \neq \rho_1(a) \neq \rho_2(a) \neq \rho_3(a) \dots \neq \rho_k(a) \neq 0$$

The test statistic proposed by Ljung and Box(1978) is

$$Q^* = n(n+2) \sum_{k=1}^K (n-k)^{-1} r_k^2(\hat{a})$$

Under the null hypothesis Q^* is approximately χ^2 distributed with $K-m$ degrees of freedom, where m is the total number of parameters estimated in the disturbance ARIMA model. After calculate the Critical value we compare it with the tabulated value if Critical value is less than the tabulated value it means that the good model.

1-10 forecasting [1],[6]

We explain how forecasts of future value of Y_t are produced from the following DR model with $M=1$ input:

$$Y_t = \frac{w(B)B^b}{\delta(B)} X_t + \frac{\theta_N(B)}{\varphi_N(B)} a_t \quad (16)$$

Now equation(16) may be written

$$\delta^*(B) Y_t = w^*(B) X_{t-b} + \theta^*(B) a_t$$

where

$$\delta^*(B) = 1 - \delta_1^* B - \dots - \delta_{p+q+r}^* B^{p+q+r} = \delta(B) \nabla^d \varphi(B) \quad (17)$$

$$w^*(B) = w_0 + w_1^* B + \dots + w_{p+d+h}^* B^{p+d+h} = \nabla^d \varphi(B) w(B)$$

$$\theta^*(B) = 1 - \theta_1^* B - \dots - \theta_{q+r}^* B^{q+r} = \delta(B) \theta(B)$$

Suppose the current time period from which forecasts are to be made is period $t=n$ called the forecast origin. suppose also that we want to forecasts the future value Y_{n+I} , where $I \geq 1$ is called the forecast lead time. Using (17) write the value for Y_{n+I} as

$$Y_{n+I} = \delta_1^* Y_{n+I-1} + \dots + \delta_{p+q+r}^* Y_{n+I-p-d-r} + w_0^* X_{n+I-b} + \dots + w_{p+d+h}^* X_{n+I-b-p-d-h} - \theta_1^* a_{n+I-1} - \dots - \theta_{q+r}^* a_{n+I-q-r} + a_{n+I} \quad (18)$$

Forecasts are made using only information available through the forecast origin $t=n$. denote the information in the set of data available at time $n(Y_n, Y_{n-1}, \dots, X_n, X_{n-1}, \dots)$ as I_n . the forecast of Y_{n+I} given I_n . denoted a conditional expectation with square brackets. The from (18) the DR model forecast of Y_{n+I} is

$$\hat{Y}_n(I) = E(Y_{n+I} | I_n) = [Y_{n+I}] = \delta_1^* [Y_{n+I-1}] + \dots + \delta_{p+q+r}^* [Y_{n+I-p-d-r}] + w_0^* [X_{n+I-b}] + \dots + w_{p+d+h}^* [X_{n+I-b-p-d-h}] - \theta_1^* [a_{n+I-1}] - \dots - \theta_{q+r}^* [a_{n+I-q-r}] + [a_{n+I}] \quad (19)$$

The value of $[Y_{n+j}]$ and $[X_{n+j}]$ for $j \leq 0$ are the observed values of each series. Similar the value of $[a_{n+j}]$ for $j \leq 0$ are estimated by the residuals $(\hat{a}_{n+j})$ of the DR model. On the other hand, the value of $[Y_{n+j}]$ and $[X_{n+j}]$ for $j > 0$

Are their forecasts $\hat{Y}_n(j)$ and $\hat{X}_n(j)$, which are their respective condition expected values. And for $j > 0, [a_{n+j}] = 0$: At time n we have no estimate of a_{n+j} in the form of the DR model residual $\hat{a}_{n+j}$, so its expected value is zero.

If the DR model in rational form has a constant term we compute the forecast as:

$$F = C(1 - \sum_{i=1}^p \varphi_i)(1 - \sum_{i=1}^p \varphi_{is})(1 - \sum_{i=1}^{r_1} \delta_{i1}) \dots (1 - \sum_{i=1}^{r_m} \delta_{im}) \dots \quad (20)$$

(1.11) forecast error variance[1],[6]

To find the variance of the forecast errors, we first write the ARIMA model for X_t

$$X_t = [\theta_x(B)/\nabla^d \phi_x(B)] \quad (21)$$

Use (21) to substitute for X_t in (16) we obtain

$$Y_t = \eta(B)\alpha_t + \psi(B)\alpha_t \quad (22)$$

where

$$\eta(B) = \eta_0 + \eta_1 B + \eta_2 B^2 + \dots \\ = \omega(B)\theta_x(B)B^b / \delta(B)\nabla^d \phi_s(B)$$

and

$$\psi(B) = \psi_0 + \psi_1 B + \psi_2 B^2 + \dots = \theta(B)/\nabla^d \phi(B)$$

where $\psi_0 = 1$

Both $\eta(B)$ and $\psi(B)$ may be of infinitely high order. Using the definition of (B) , the η weights are found by equating coefficients of like powers of B on either side of this expression:

$$\delta(B)\nabla^d \phi_s(B)\eta(B) = \omega(B)\theta_x(B)B^b \quad (23)$$

Similarly, using the definition of (B) , the ψ weights are found by equating coefficients of like power of B on either side of this expression:

$$\nabla^d \phi(B\psi(B)) = \theta(B) \quad (24)$$

Now use (22) to write the future value Y_{n+I} , where $t=n$ is the forecast origin, as

$$Y_{n+I} = \eta_0 \alpha_{n+I} + \eta_1 \alpha_{n+I-1} + \eta_2 \alpha_{n+I-2} + \dots + \alpha_{n+I} + \psi_1 \alpha_{n+I-1} + \psi_2 \alpha_{n+I-2} + \dots \quad (25)$$

The forecast of Y_{n+I} is

$$\hat{Y}_n(I) = [Y_{n+I}] = \eta_I \alpha_n + \eta_{I+1} \alpha_{n-1} + \dots + \psi_I \alpha_n + \psi_{I+1} \alpha_{n-1} + \dots \quad (26)$$

That is, any α_t or α_t value for $t > n$ is neither known nor estimated from a model residual, given the data available through $t = n$. The expected value of these terms is zero; (26) shows the remaining nonzero terms. The lead I forecast error from origin n is

$$e_n(I) = Y_{n+I} - \hat{Y}_n(I)$$

thus, subtracting (26) from (25) gives the forecast error

$$e_n(I) = \eta_0 \alpha_{n+I} + \eta_1 \alpha_{n+I-1} + \dots + \eta_{I-1} \alpha_{n+I} + \alpha_{n+I} + \psi_I \alpha_{n+I-1} + \dots + \psi_{I-1} \alpha_{n+1} \quad (27)$$

The forecast error variance for lead I from origin n is the mathematical expectation

$$V(I) = E[e_n(I)]^2$$

2-1 Where the expected value of the forecast error is zero. Therefore squaring (27) and taking expected values gives **preparation and prewhitening of the inputs and outputs series**

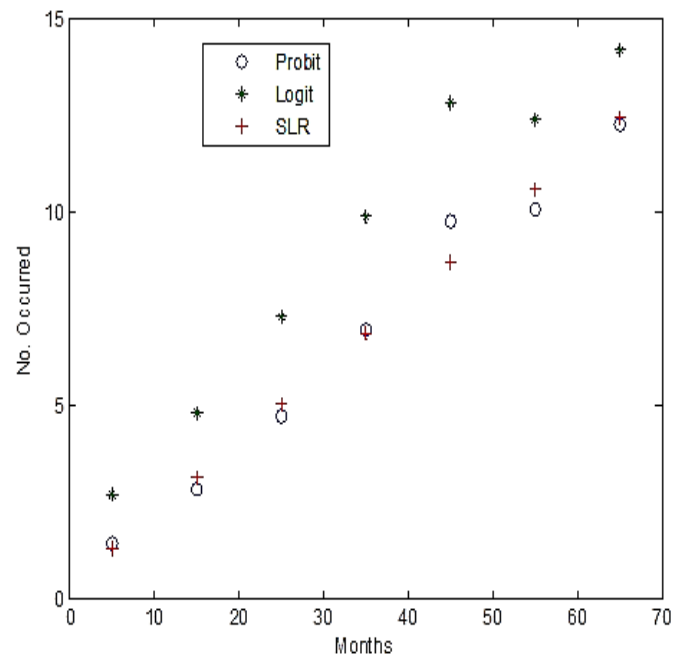
1) cross-correlation between output RH(Y_t) and input rainfall (X_t). we plot the time series of it by using software of Minitab (13.2) as in figures(1),(2) respectively we show that the series is stationary in mean and variance

$$V(I) = \sigma_a^2 \sum_{j=0}^{I-1} \eta_j^2 + \sigma_a^2 \sum_{j=0}^{I-1} \eta_j^2 \quad (28)$$

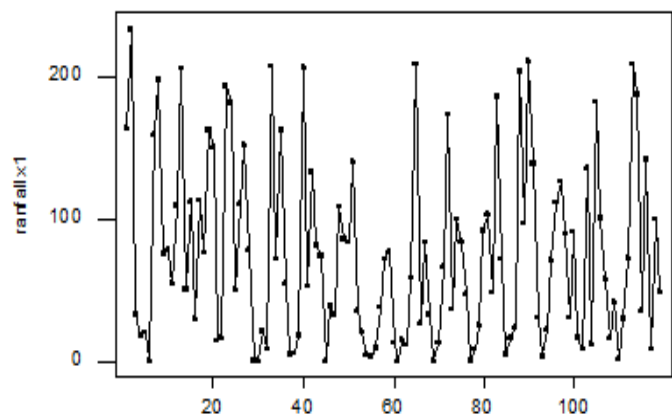
In finding this result we use the fact that α_t and α_t are mutually independent and not auto correlated.

2) APPLICATION

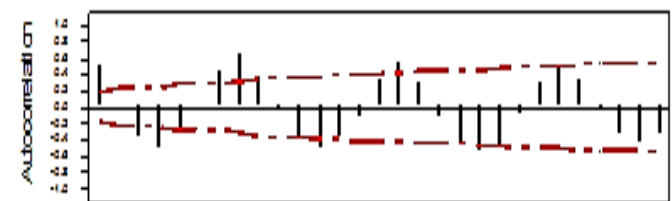
This section contains applying section two. The first method is testing of cross-correlation function between prewhitening of the input denoted by rainfall and of the output denoted by humidity (RH). We take the monthly average of the meteorological station of Duhok for the period (1992) to (2006), all data are shown in the tables(1) in the appendix (A). The second method is it used to test χ^2 between two series of input and output by using equation.



Figure(1): the time series plot of (RH)



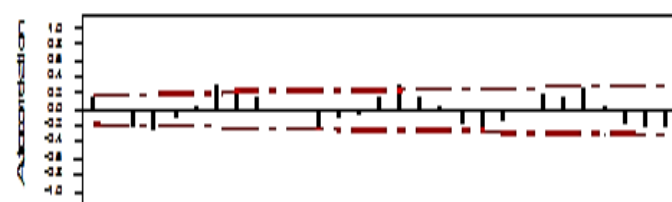
Figure(2):the time series plot of rainfall



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.53	5.77	31.15	10	-0.05	-0.28	191.51	19	-0.11	-1.31	352.19	28	-0.12	-1.57	352.67
2	0.05	0.17	31.50	11	-0.10	-0.16	212.95	20	-0.51	-3.32	405.15	29	-0.32	-1.15	353.79
3	-0.33	-2.90	45.26	12	-0.19	-0.57	215.15	21	-0.11	-1.75	432.99				
4	-0.51	-4.11	30.45	13	-0.37	-1.51	263.59	22	-0.07	-0.29	433.72				
5	-0.32	-2.33	35.72	14	-0.10	-0.16	261.51	23	0.32	1.26	483.77				
6	0.00	0.01	35.72	15	0.33	1.55	379.71	24	0.52	2.06	490.27				
7	0.11	3.02	115.40	16	0.55	2.60	321.90	25	0.31	1.25	507.66				
8	0.66	4.21	171.27	17	0.32	1.13	335.26	26	-0.01	-0.16	507.91				
9	0.36	2.01	191.49	18	-0.11	-0.15	337.97	27	-0.32	-1.21	521.31				

We plot (Autocorrelation function) ACF for the RH and rainfall series in figures (3),(4) we show that the seasonality period (8) months.

Autocorrelation Function for rainfall x1



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.17	1.20	3.33	10	-0.00	-0.01	41.21	19	-0.12	-1.39	79.27	28	-0.20	-1.38	122.42
2	-0.00	-0.08	3.34	11	-0.03	-0.27	41.88	20	-0.22	-1.64	83.39	29	-0.21	-1.38	129.15
3	-0.23	-0.40	9.82	12	-0.21	-1.77	47.38	21	-0.18	-1.07	88.82				
4	-0.27	-0.71	12.81	13	-0.09	-0.77	49.03	22	0.01	0.09	88.80				
5	-0.09	-0.27	19.88	14	-0.07	-0.88	49.87	23	0.21	1.81	92.20				
6	0.04	0.41	19.81	15	0.17	1.38	83.15	24	0.16	1.14	97.12				
7	0.32	3.03	33.04	16	0.32	2.81	85.05	25	0.29	2.08	110.31				
8	0.21	1.28	32.74	17	0.18	1.14	71.23	26	0.03	0.19	110.43				
9	0.18	1.21	41.20	18	0.03	0.24	71.37	27	-0.19	-1.28	118.20				

Figure(4): (Autocorrelation function) ACF for rainfall (4)

We take first difference for the data as shown in the figures(5),(6) and plot ACF again for the differenced time series about rainfall and (PACF) in a figures(7),(8).

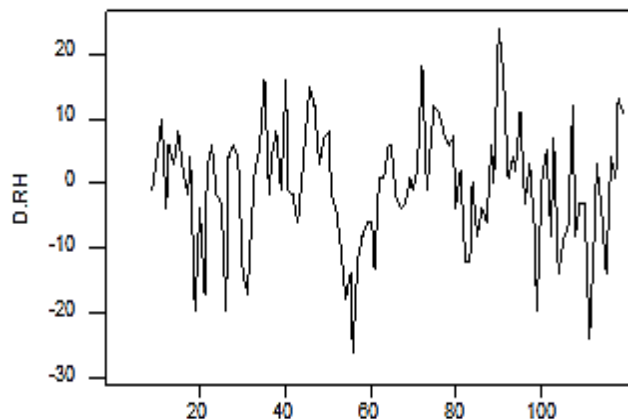
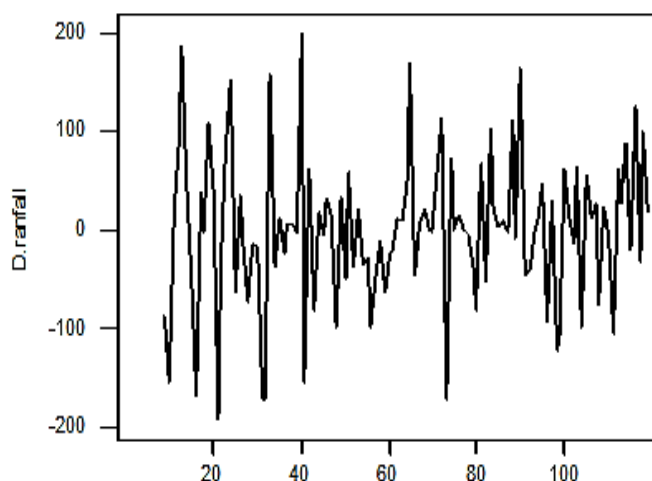
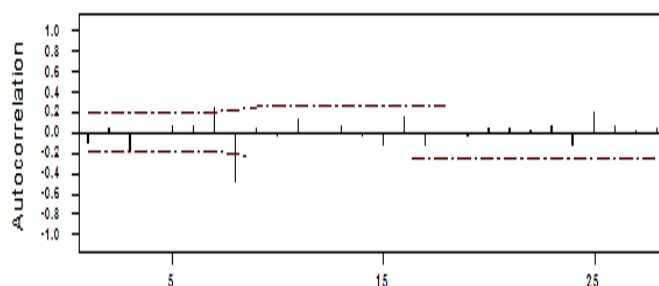


Figure (5): the plot for differenced timeseries of(rainfall)



Figure(6):the plot for differenced time series of(RH)



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	-0.12	-1.28	1.67	10	-0.06	-0.46	46.43	19	-0.04	-0.31	58.20	28	0.03	0.21	68.49
2	0.03	0.30	1.77	11	0.14	1.08	48.76	20	0.03	0.20	58.30				
3	-0.18	-1.88	5.58	12	-0.01	-0.05	48.77	21	0.03	0.26	58.46				
4	-0.02	-0.24	5.65	13	0.05	0.39	49.09	22	0.01	0.08	58.48				
5	0.05	0.49	5.94	14	-0.05	-0.37	49.37	23	0.08	0.57	59.29				
6	0.07	0.75	6.60	15	-0.14	-1.11	51.99	24	-0.14	-1.08	62.26				
7	0.25	2.50	14.12	16	0.15	1.19	55.10	25	0.20	1.48	67.96				
8	-0.51	-4.84	45.90	17	-0.15	-1.12	57.97	26	0.05	0.37	68.35				
9	0.03	0.25	46.01	18	0.00	0.02	57.97	27	0.01	0.06	68.36				

Figure (7): (Autocorrelation function) ACF for differenced time series of(rainfall)



20	19
-64.697	-1.822
40	39
-99.762	12.152
60	59
21.392	6.239
80	79
120.885	7.493
100	99
-73.079	25.509

by same method we can find the output series (RH) as below:

$$(1-\phi B)Y_t = (1-\theta B)\beta_t$$

$$Y_t - \phi Y_{t-1} = \beta_t - \theta \beta_{t-1}$$

$$\beta_t = Y_t - \phi Y_{t-1} + \theta \beta_{t-1}$$

$$\beta_1 = Y_1 \text{ where } Y_0 = 0$$

We can find the series $(\beta_t^\wedge)$ by using program (2) in the appendix (B)

Table (2): values $(\beta_t^\wedge)$ for output (RH)

t	1	2	3	4	5	6	7	8
$\beta_t^\wedge$	-1.0000	3.8813	10.3663	-2.4781	6.9165	4.5500	9.7732	4.5706
t	21	22	23	24	25	26	27	28
$\beta_t^\wedge$	1.9371	-15.411	-19.951	-14.717	-3.4988	0.2104	12.2148	-3.5612
t	41	42	43	44	45	46	47	48
$\beta_t^\wedge$	11.5237	12.9664	3.4899	0.7815	-6.1036	-	-	-
t	61	62	63	64	65	66	67	68
$\beta_t^\wedge$	-2.7684	-4.3263	-1.1596	15.3489	-1.2871	7.6189	12.6012	12.9740
T	81	82	83	84	85	86	87	88
$\beta_t^\wedge$	-0.8323	23.2391	19.1532	4.9863	7.7630	5.9149	14.8164	1.7947
T	101	102	103	104	105	106	107	108
$\beta_t^\wedge$	-4.9184	-5.1099	-	-	-2.4636	-8.6388	-	-1.9728
			26.2850	12.9377			18.7156	

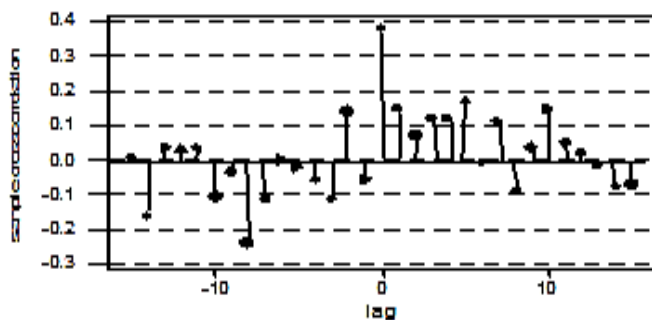
9	10	11	12	13	14	15	16	17	18	19	20
0.5875	6.1281	-17.579	-4.1614	-17.622	-0.5868	3.8725	-3.2328	-4.3644	-21.603	0.1602	2.9644
29	30	31	32	33	34	35	36	37	38	39	40
2.3354	6.9530	-1.0076	15.8744	0.7843	-0.4875	-4.8546	2.3349	11.5436	16.8026	15.4285	7.5587
49	50	51	52	53	54	55	56	57	58	59	60
-	-	-	-	-	-6.4322	-5.6759	0.0156	1.2413	-5.6382	-7.5635	-6.7325
69	70	71	72	73	74	75	76	77	78	79	80
11.1103	9.7931	11.1798	0.6521	5.7781	-8.3086	10.0498	0.3585	-7.6722	-4.6500	-7.0690	4.3105
89	90	91	92	93	94	95	96	97	98	99	100
7.0272	-2.9622	-	0.2410	5.2203	-7.2051	6.7771	-	10.0885	-9.0634	9.2828	-9.0597
109	110	111									
-3.9855	8.5610	8.4849									

- 1) correlation coefficient between $(\alpha_t^\wedge)$ and $(\beta_t^\wedge)$ by using equation (1) we can find the values of correlation coefficient in the table (3)

Table (3): the values of correlation coefficient between $(\alpha_t^\wedge)$ and $(\beta_t^\wedge)$

t	$r_{\alpha\beta}$	T	$r_{\alpha\beta}$	t	$r_{\alpha\beta}$	T	$r_{\alpha\beta}$
0	0.378	6	-0.004	12	0.025	18	-0.214
1	0.147	7	0.110	13	-0.011	19	-0.135
2	0.068	8	-0.090	14	-0.078	20	-0.062
3	0.122	9	0.038	15	-0.071	21	

4	0.118	10	0.148	16	-0.019	22	
5	0.171	11	0.048	17	-0.128	23	



Figure(9): correlation coefficient between $(\alpha_t^{\wedge})$ and $(\beta_t^{\wedge})$ determines the dead time for input(RH).It is clear from the figure (9) that the dead time is (b=0) close to zero which explains $\rho_{\alpha\beta}(0) \neq 0$ significant and all of values near to zero, we can say that there is feedback between output series (Y_t) and input (X_t). We find ACF between residual series and series $(\alpha_t^{\wedge})$.we can clear that below.

2) identification

2.1- estimation of the impulse response weights

We estimate of the impulse response weights between input (X_t) and output series (Y_t) in the table (4) below.

Table (4): the values of the impulse response of input variable (rainfall X_t)

t	V	T	v	t	V	t	v
0	0.0499	6	-0.0005	12	0.0033	18	-0.0282
1	0.0194	7	0.0145	13	-	19	-0.0178
2	0.0089	8	-	14	-	20	-
3	0.0161	9	0.0050	15	-	21	-
4	0.01558	10	0.0195	16	-	22	-
5	0.0225	11	0.00634	17	-	23	-

2)identification of (r,s,b) for the transfer function

It is clear from the figure (9) taking one change point will continue toward itself for a few periods(s=5). It transfer to the other side thus than (r=1) .the pattern can be written as:

$$Y_t = \frac{(w_0 - w_1 B - w_2 B^2 - w_3 B^3 - w_4 B^4 - w_5 B^5)}{(1 - \delta_1 B)} X_t + N_t \quad (29)$$

3) disturbance series

We find disturbance series by using the equation

$$N_t = Y_t - v_0 X_t - v_1 X_{t-1} - \dots - v_{20} X_{t-20} \quad (30)$$

By using equation (30) we can obtain the number of disturbance series which their values less than the input and output series values(t=21) so, we can apply them in program (3) in appendix(B) the values in the table (5).

Table(5): estimate values of disturbance series N_t

t	$N_t^{\wedge}$	T	$N_t^{\wedge}$	T	$N_t^{\wedge}$	t	$N_t^{\wedge}$	t	$N_t^{\wedge}$
1	8.3067	19	8.4527	37	1.8966	55	-	73	-
2	-	20	10.644	38	11.867	56	11.473	74	2.293
3	20.113	21	-	39	-	57	5.6367	5	-
4	0	22	-	40	6.6606	58	-	76	2.274
5	-	23	4.6793	41	14.288	59	-	77	-
6	13.863	24	13.980	42	-	60	-	78	14.25
7	0.8759	25	-	43	-3.686	61	-	79	-
8	24.677	26	-	44	-	62	8.8708	80	-
9	15.107	27	-	45	0.5373	63	28.049	81	-
10	1.3322	28	7.7617	46	1.8966	64	-	82	-
11	15.023	29	2.2580	47	11.867	65	-	83	-
12	7.1083	30	-0.346	48	-	66	8.5172	84	2.567
13	3.5126	31	1.6847	49	-	67	-4.315	85	-
14	-	32	-	50	7.4006	68	4.4084	86	-
15	11.730	33	7.6015	51	1.6043	69	6.57	87	-
16	-	34	2.5706	52	3.009	70	-	88	5.219
17	-	35	4.4363	53	4.5073	71	-	89	-
18	9.382	36	-	54	-	72	8.9425	90	-

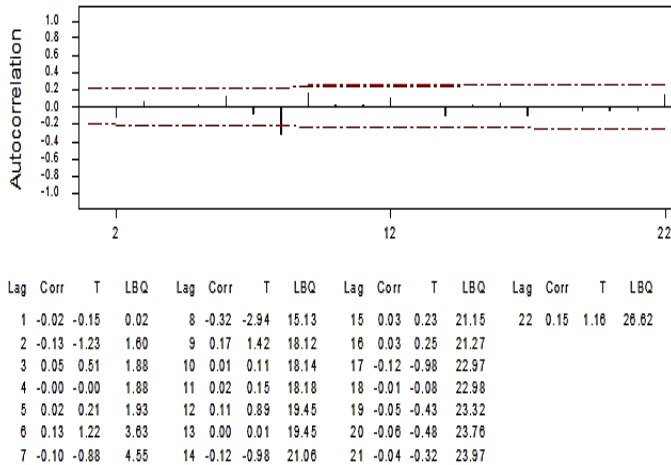


Figure (10): ACF and from the disturbance series ($\hat{N}_t$)

We plot ACF and PACF from the disturbance series ($\hat{N}_t$), as in the figures (10) and (11)

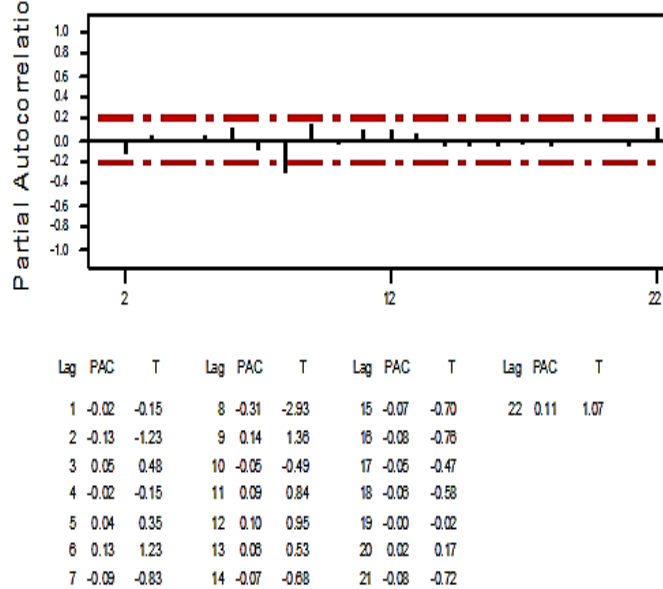


Figure (11): PACF and from the disturbance series ($\hat{N}_t$)

It is clear from the figure(11) that the disturbance series ($\hat{N}_t$) is equal residual series

$N_t = a_t$, thus the model of dynamic regression as shown in the equation below:

$$Y_t = \frac{(w_0 - w_1 B - w_2 B^2 - w_3 B^3 - w_4 B^4 - w_5 B^5)}{(1 - \delta_1 B)} X_t + a_t \quad (31)$$

We estimate the values of the model by using equation (31)

By using table(4) we find

$$v_0 = \delta_1 v_1 + w_0 \quad j = b$$

$$v_0 = w_0$$

$$v_1 = \delta_1 v_0 - w_1$$

$$v_2 = \delta_1 v_1 - w_2 \quad j = b+1, \dots, b+s$$

$$v_3 = \delta_1 v_2 - w_3$$

$$v_4 = \delta_1 v_3 - w_4$$

$$v_5 = \delta_1 v_4 - w_5$$

$$v_6 = \delta_1 v_5$$

$$\delta_1 = v_6 / v_5 = -0.0005 / 0.0225 = -0.222$$

$$w_0 = v_0 = 0.0499$$

$$w_1 = \delta_1 v_0 - v_1 = (-0.222)(0.0499) - (0.0194) = -0.0304778$$

$$w_2 = \delta_1 v_1 - v_2 = (-0.222)(0.0194) - (0.0089) = -0.0132066$$

$$w_3 = \delta_1 v_2 - v_3 = (-0.222)(0.0089) - (0.0161) = -0.01808$$

$$w_4 = \delta_1 v_3 - v_4 = (-0.222)(0.0161) - (0.01558) = -0.01915$$

$$w_5 = \delta_1 v_4 - v_5 = (-0.222)(0.01558) - (0.0225) = -0.025958$$

Search to Minimize Sum of Squared Residuals

The next step in estimation is to compute the SSR (Sum of Squared Residuals).

$$SSR = \sum_{t=1}^n a_t^2$$

$$Y_t = \frac{(0.0499 + 0.0305B + 0.0132066B^2 + 0.01808B^3 + 0.01915B^4 + 0.025958B^5)}{(1 + 0.222B)}$$

$$a_t^{\wedge} = Y_t + 0.222Y_{t-1} - 0.0499X_t - 0.0305X_{t-1} - 0.0132066X_{t-2} - 0.01808X_{t-3} - 0.01915X_{t-4} - 0.02595X_{t-5} - 0.222a_t \quad (32)$$

We find the values of series ($a_t^{\wedge}$) by using equation (32) and program (4) in appendix (B), The value as in the table(6).

Table (6): the values of series ($a_t^{\wedge}$)

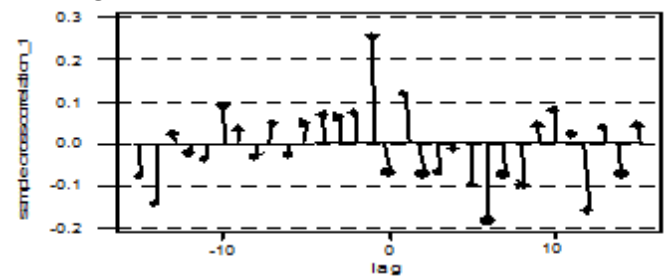
t	$a_t^{\wedge}$	t	$a_t^{\wedge}$	t	$a_t^{\wedge}$	t	$a_t^{\wedge}$	t	$a_t^{\wedge}$	t	$a_t^{\wedge}$
1	0.0000	20	-0.9160	39	-3.3955	58	1.1386	77	14.6687	96	-1.4370

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.7422	9.5889	4.9520	-3.4498	0.9387	-21.9821	-5.2100	-9.8956	4.5466	3.3031	-8.3265	-2.3224	-17.5623	2.1247	6.3294
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1.6774	5.2860	19.1778	1.2919	5.5449	5.3637	0.0144	9.9616	0.5953	-1.6047	-6.0637	2.3747	8.6837	16.8753	10.2357
40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
-8.4087	-17.9861	-11.8717	-23.2960	-8.0296	-6.5819	-2.2575	-2.6486	-9.9744	2.2018	1.4054	5.6577	0.3474	-3.4559	-4.2761
59	60	61	62	63	64	65	66	67	68	69	70	71	72	73
13.2296	1.9000	8.1710	9.5838	10.0785	6.9647	8.3027	6.7636	-0.7813	1.2307	-11.0673	-13.1957	-1.6501	-7.6427	-5.8736
78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
2.0145	1.8387	1.2348	7.7183	0.3255	4.0698	-4.2324	-14.4481	-0.0109	5.9083	-6.6499	7.5821	-11.2824	-10.1545	-8.6454
97	98	99	100	101	102	103	104	105	106					
-4.4783	-20.4011	-8.0027	2.3374	-6.7431	-14.3211	1.3423	-2.1231	9.6593	6.6924					

17	18	19
3.7500	-12.1944	-11.5427
36	37	38
6.8976	6.9511	8.7745
55	56	57
-5.5386	-1.1261	-3.1218
74	75	76
-6.2127	0.7505	-1.9606
93	94	95
11.4132	-6.8504	14.6687

4) Diagnostic Check

a) Residuals Cross Correlation function(RCCF) $r_k(\hat{a})$. We plot cross correlation between series $(\hat{\alpha}_t)$ and series $(\hat{a}_t)$ as in the figure (12), it is clear that there are significant values, this means that the correlation between two series $(\hat{\alpha}_t)$ and series $(\hat{a}_t)$ is significant.



Figure(12):cross correlation between series $(\hat{\alpha}_t)$ and series $(\hat{a}_t)$

By using Ljung and Box(1978) below
 $s = n^2 \sum_{k=0}^K (n-k)^{-1} (r_k)^2 = 8.1$

This value is less than the χ^2 critical value (11.07) for $K+1-m=10+1-6=5$ degrees of freedom at the 5% level. Therefore we do not reject the stated H_0 .

b) Autocorrelation Check

The test statistic proposed by Ljung and Box(1978) is

$$Q^* = n(n+2) \sum_{k=1}^K (n-k)^{-1} r_k^2(\hat{a}) = 11.9$$

this value is less than the χ^2 critical value (37.65) for $K-m=26-1=25$ degrees of freedom at the 5% level. Therefore we do not reject the stated H_0 .

4)Forecasting

We explain how forecasts of future value of Y_t are produced from the following DR model with equation below

$$y_t = \delta_1^* y_{t-1} + w_0 x_t - w_1 x_{t-1} - w_2 x_{t-2} - w_3 x_{t-3} - w_4 x_{t-4} - w_5 x_{t-5} + a_t$$

the forecast is

$$\hat{y}_n(I) = \delta_1^* [y_n] + w_0 [x_{n+1}] - w_1 [x_n] - w_2 [x_{n-1}] - w_3 [x_{n-2}] - w_4 [x_{n-3}] - w_5 [x_{n-4}] + a_{n+1}$$

$$\delta_1^* = 1 + \delta_1 = 1.0222 \text{ Where } t=n = 119$$

for $I = 1$, the forecast is

$$\begin{aligned} \hat{y}_{119}(1) = & 0.778[y_{119}] + 0.0499[x_{120}] + 0.0304778[x_{119}] + 0.013 \\ & 2066[x_{118}] + 0.01808[x_{117}] \\ & + 0.01915[x_{116}] + 0.025958[x_{115}] + a_{120} \end{aligned}$$

Therefore squaring (29) and taking expected values gives

$$\begin{aligned} V(I) &= \sigma_a^2 \sum_{j=0}^{I-1} \eta_j^2 + \sigma_a^2 \sum_{j=0}^{I-1} \eta_j^2 \\ \eta(B) &= \eta_0 + \eta_1 B + \eta_2 B^2 + \dots \\ &= \omega(B) \theta_x(B) B^b / \delta(B) \nabla^d \phi_s(B) \end{aligned}$$

$$\eta_0 = w_0 = 0.0499$$

$$\eta_1 = w_1 = -0.0304778$$

$$V(I) = \sigma_a^2 \sum_{j=0}^{I-1} \eta_j^2 + 0$$

$$\sigma_a^2 = 5433.89$$

$$V(I) = \sigma_a^2 \eta_0^2 = 13.53$$

$$SE = 3.6$$

$$V(2) = \sigma_a^2 (\eta_0^2 + \eta_1^2) = 18.577$$

$$SE = 4.31$$

The forecast for x_t is

$$x_{t+1} = \alpha_{t+1} - \theta \alpha_{t+1-1} + \phi x_{t+1-1}$$

$$x_{t+1} = \alpha_{t+1} - 0.9142 \alpha_{t+1-1} + 0.7955 x_{t+1-1}$$

And The forecast for Y_t is

$$Y_{t+1} = \beta_{t+1} - \theta \beta_{t+1-1} + \phi Y_{t+1-1}$$

$$Y_{t+1} = \beta_{t+1} - 0.9142 \beta_{t+1-1} + 0.7955 Y_{t+1-1}$$

Table (7): forecasting X_{t+1} and Y_{t+1} and Dynamic regression

time	time	Actual X_t	Forecasting X_t	Actual Y_t	Forecasting Y_t	Forecasting dynamic y_t
2006.	Jan.	72.9	101.463	66	60.6535	41.3421
	Feb.	209.3	100.494	60	63.6289	53.2040
	Mar.	188.6	92.162	47	61.5696	69.7538
	Apr.	35.9	94.081	56	55.4739	58.9298
	May.	142.6	49.055	40	54.4772	53.2323

2006 /2007	Oct.	8.2	9.542	44	37.8449	58.1119
	Nov.	100.4	4.061	55	38.1801	47.9720
	Des.	48.9	58.028		53.0353	48.0064
	Des.		90.406	55	62.9801	53.9366
	Jan.		106.214	61	65.3804	63.3803
	Feb.		91.721	65	63.0423	62.9003
	Mar.		88.656	56	56.8185	62.8561
	Apr.		44.103	57	54.7555	55.2476
	May.		11.214	43	38.2354	51.7219
	Oct.		2.226	40	37.6134	36.9045
	Nov.		53.536	59	52.1672	37.0295
	Des.		87.022	59	62.6513	49.9403

Conclusion

- 1) ACF for the RH and rainfall series , we show that the seasonality period (8) months, we suggest that the tentative model for the differenced series is ARMA(1,1)
- 2) After using cross- correlation between series ($\alpha_t^{\wedge}$) and series ($a_t^{\wedge}$), it is clear that there are significant values, which mean that the correlation between the two series ($\alpha_t^{\wedge}$) and series ($a_t^{\wedge}$) is significant, this is special case in dynamic regression.
- 3) to examine the value is less than the χ^2 critical value (37.65) for $K - m = 26 - 1 = 25$ degrees of freedom at the 5% level. Therefore we do not reject the stated H_0 .

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Appendix (A)

monthly average of the humidity and rainfall of the
meteorological station of Dohuk for the period (1992)
to (2006)

Year	Month	humidity	rainfall	year	month	RH	rainfall	year	month	RH	rainfall
1992	Jan.	11.1	111.1	1997	Jan.	53.0	53.0	2002	Jan.	111.1	111.1
	Feb.	234.7			Feb.	60	133.7		Feb.	54	48.0
	Mar.	32.8			Mar.	64	82.0		Mar.	52	106.8
	Apr.	18.2			Apr.	61	74.5		Apr.	59	72.1
	May.	19.8			May.	55	0.5		May.	33	4.3
	Oct.	0.0			Oct.	56	39.1		Oct.	40	16.1
	Nov.	159.9			Nov.	60	33.0		Nov.	50	23.7
	Des.	198.4			Des.	75	108.8		Des.	75	204.9
1993	Jan.	78.2		1998	Jan.	86.6	86.6	2003	Jan.	211.3	211.3
	Feb.	54.8			Feb.	68	83.5		Feb.	78	211.3
	Mar.	109.9			Mar.	62	140.2		Mar.	69	139.6
	Apr.	206.8			Apr.	57	36.0		Apr.	60	30.5
	May.	51.0			May.	45	20.9		May.	37	3.7
	Oct.	113.0			Oct.	38	4.0		Oct.	42	21.9
	Nov.	29.5			Nov.	46	3.0		Nov.	61	71.2
	Des.	29.5			Des.	49	9.2		Des.	72	112.0
1994	Jan.	76.4		1999	Jan.	71.8	71.8	2004	Jan.	89.5	89.5
	Feb.	163.6			Feb.	56	77.3		Feb.	71	89.5
	Mar.	150.8			Mar.	51	12.6		Mar.	49	30.3
	Apr.	13.7			Apr.	32	0.0		Apr.	60	91.1
	May.	16.0			May.	39	14.8		May.	42	16.9
	Oct.	194.1			Oct.	47	11.2		Oct.	34	8.3
	Nov.	181.9			Nov.	55	58.6		Nov.	68	136.2
	Des.	181.9			Des.	55	58.6		Des.	58	11.9
1995	Jan.	110.9		2000	Jan.	26.3	26.3	2005	Jan.	100.9	100.9
	Feb.	152.2			Feb.	58	83.6		Feb.	64	100.9
	Mar.	78.7			Mar.	52	83.6		Mar.	61	57.2
	Apr.	0.0			Apr.	48	33.3		Apr.	52	16.1
	May.	0.0			May.	33	0.0		May.	39	41.5
	Oct.	21.2			Oct.	38	12.8		Oct.	31	1.7
	Nov.	7.8			Nov.	49	66.8		Nov.	44	29.7
	Des.	7.8			Des.	73	174.1		Des.	50	72.9
1996	Jan.	71.7		2001	Jan.	100.5	100.5	2006	Jan.	108.6	108.6
	Feb.	163.1			Feb.	66	84.3		Feb.	60	108.6
	Mar.	55.1			Mar.	64	47.3		Mar.	47	35.9
	Apr.	4.9			Apr.	59	0.0		Apr.	56	142.6
	May.	5.5			May.	41	8.0		May.	40	8.2
	Oct.	17.7			Oct.	44	25.0		Oct.	44	100.4
	Nov.	207.5			Nov.	56	91.9		Nov.	55	48.9
	Des.	207.5			Des.	69	91.9		Des.		

Appendix (B)

The software Minitab(13.2) is used in the
following macro programs.

Program (1): the values of ($\alpha_t^{\wedge}$) variable input (rainfall)

```
gmacro
aa.macro
let c4(1)=-88
do k3=2:111
let c4(k3)=c2(k3)-0.7955*c2(k3-1)+
0.9142*c4(k3-1)
enddo
endmacro
```

Program (2): values ($\beta_t^{\wedge}$) for output
(RH)

```
gmacro
aa.macro
let c5(1)=-1
do k3=2:111
let c5(k3)=c1(k3)-0.7955*c1(k3-1)+
0.9142*c5(k3-1)
enddo
endmacro
```

Program (3): estimate values of disturbance series N_t by
using matlab program

```
For i=1:111
For k=1:21
Z(I,k)= v(k)*ul(i+21)-k);
end;
end
for i=1:111
s(i)=0;
for j=1:21
s(i)=s(i)-z(i,j);
end
end
for i=22:111
n(i)=y(i)+s(i-20)
end
```

program (4): the values of series ($a_t^{\wedge}$)

```
gmacro
aa.macro
let c3(5)=0
do k1=6:111
let c3(k1)=c1(k1)+0.222*c1(k1-1)-
0.0499*c2(k1)-0.0304778*c2(k1-1)-
0.0132066*c2(k1-2)-0.01808*c2(k1-3)-
0.01915*c2(k1-4)-0.025958*c2(k1-5)-
0.222*c3(k1-1)
enddo
let k4=sum(c3(k1)**2)
print k4
endmacro
```

حول التكهّن باستخدام الانحدار الحركي

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الخلاصة

يتضمن هذا البحث تطبيقات على بعض التقنيات الاحصائية لدراسة السلسلة الزمنية لمعدلات الرطوبة الشهرية كسلسلة مخرجات مع احد المتغيرات التي تؤثر عليها وهو سلسلة معدلات الامطار الشهرية التي تم قياسها في محطة دموك للأنواء الجوية والتقنيات التي استخدمت هي نموذج ARIMA والنمذجة بالانحدار الحركي . وبذلك يكون نموذج الانحدار الحركي التام المختار ملائماً لايجاد القيم التنبؤية للقيم المستقبلية.