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Radar cross section of conducting bodies of revolution coated by dielectric layer

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ABSTRACT:

In this paper, the electromagnetic (EM) scattering from conducting bodies of revolution coated by dielectric layer was calculated by using moment of method with Galerkin's approach, and applying the equivalence principle to determine the electric and magnetic currents on the surface.

The mathematical analysis and the programs were checked to know the validity of the formulation by comparing our results with the published results. The computed results revealed the significant effect of the coated layer on the Radar cross section reduction with respect to the conducting body. Finally, this formulation was applied to study a proposed rocket which is more complex body than the previous shape, and calculate its radar cross section, and study of the effect of the dielectric constant and the thickness of the dielectric layer on the RCS.

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مساحة المقطع الراداري للأجسام الموصلة المطلية بطبقة عازلة والمتناظرة محورياً

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الـــخُـــلاصـــة:

تم حســاب الاســتطارة الكهرومغناطيســية من الأجســام الموصــلة المطلية بطبقة عازلة والمتناظرة محوريا باســتخدام طريقة العزوم مع تقريب كالركن. كما انه تم تطبيق مبدأ التكافؤ لاســتخراج كثافة التيارات الكهربائية والمغناطيســية على الســطح. تم التأكد من صــحة التحليل الرياضــي والبرمجيات لمعرفة مدى صــحة الصــيغة عن طريق مقارنة نتائجنا مع النتائج المنشــورة. كشــفت النتائج المحســوبة التأثير الملحوظ للطبقة المطلية على تقليل مســاحة المقطع الراداري فيما يتعلق بالجسم الموصل. أخيرا تم تطبيق هذه الصيغة لدراسة الصاروخ المقترح الأكثر تعقيدا من الأشكال السابقة وحساب مساحة المقطع الراداري ودراسة تأثير ثابت العزل وسمك الطبقة العازلة على مساحة المقطع الراداري.

الكلمات المفتاحية:

طريقة العزوم
مساحة المقطع الراداري
الاستطارة الكهرومغناطيسية
الأجسام المطلية
الأجسام المتناظرة محوريا

1. INTRODUCTION

The analytical solutions are important for researchers that are interesting in the electromagnetic scattering of the wave from conducting bodies for a limited number of uniformly shapes. In the numerical method the boundary conditions with the method of moment are used to develop a simple and efficient numerical technique to treat the electromagnetic problem. The accuracy of the solutions used in this method is appropriate for good conducting bodies of revolution (BOR) [1,2]. As the numerical technique to solving the scattering problem is developed, the accuracy of the results must be verified by comparing to this results with previous researches. In this technique the electric and magnetic currents surface ($\vec{J}$and$\vec{M}$) in addition to the RCS of the to the conducting body coated by dielectric substrate as a body of revolution are calculated by using of the Electric Field Integral Equation (EFIE) and Poggio-Miller-Chang-Harrington-Wu-Tsai.

2. THEORETICAL CONSIDERATION

Fig. (1) shows a general model of CBOR-Coated- dielectric layer. According to the equivalence principle, the body can be divided in two regions the first is the exterior region which is of volume V^e and filled with a homogenous material of permeability μ_e and permittivity ε_e. This region is formed by rotate the generating arc (S_{de}) around the axis of symmetry .The surface current densities on S_{de} are the electric current $\bar{J}_{de}$ and magnetic current $\vec{M}$ on S_{de} . **Fig (1b)** presents the homogenous interior region V^d that filled with a homogeneous material of permeability μ_d and permittivity ε_d are formed by rotating the generating arc $(S_{cd} + S_{de})$.

The surface current densities $\vec{J}_{de}$, $\vec{M}$are continuous , in the interior equivalent region , on the surface S_{de} but in opposite direction of the exterior equivalent region V^e . the vectors E^e and H^e are the electromagnetic fields in the interior region , while E^d and H^d are electromagnetic fields in the exterior region. This e and d are the space and dielectric region, respectively. Also $\hat{n}$ is the normal unit vector on the surfaces S_{de} and S_{cd}.

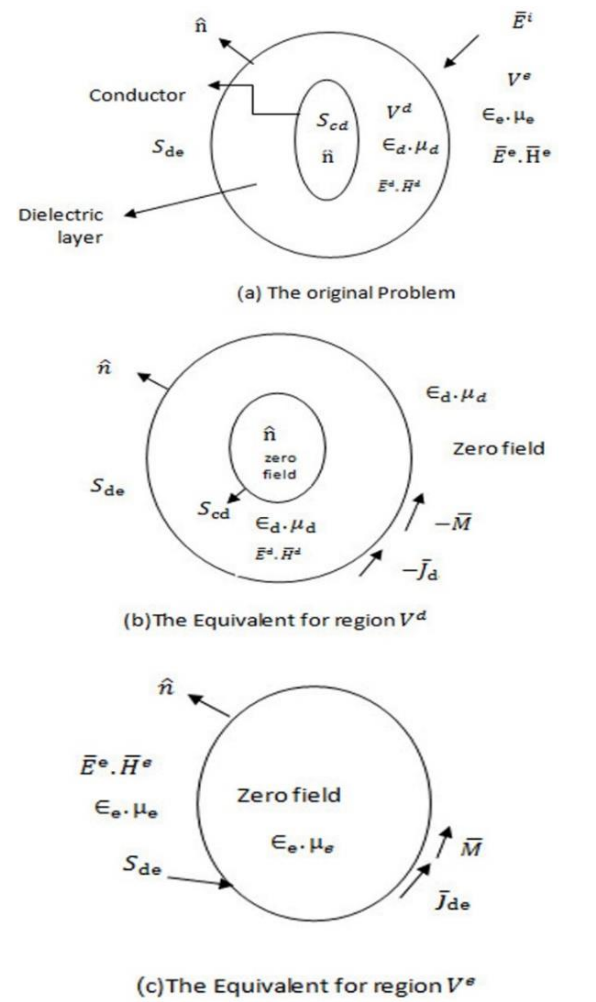


Figure 1: Equivalence principle for conducting bodies coated by dielectric layer.

The electric and magnetic currents surface can be calculated using the equivalence principle as [3]:

$$\vec{J}_{\mathrm{cd}} = \hat{\mathrm{n}} \times \vec{H} \qquad , on\ S_{cd} \qquad (1a)$$

$$\vec{J}_{\mathrm{de}} = \hat{\mathrm{n}} \times \vec{H}^{\mathrm{e}} \qquad , \mathrm{on}\ \mathrm{S}_{\mathrm{de}} \qquad (1b)$$

$$\vec{M} = -\hat{n} \times \vec{H}^{\mathrm{e}} \qquad , \mathrm{on}\ \mathrm{S}_{\mathrm{de}} \qquad (1c)$$

The surface equivalent electric and magnetic currents in equations (1) are unknown and can be calculated by applying the boundary conditions of the field vectors as shown in fig (1).

The boundary conditions equation for the problem can be described as follows [3]:

$$\hat{n} \times \vec{E}^{d} = 0 \qquad , on\ S_{cd} \qquad (2a)$$

$$\hat{n} \times \vec{E}^{d} = -\hat{\mathrm{n}} \times \vec{E}^{e}\ , \mathrm{on}\ S_{\mathrm{de}} \qquad (2b)$$

$$\hat{n} \times \vec{H}^{d} = \hat{n} \times \vec{H}^{e} \ , \mathrm{on}\ \mathrm{S}_{\mathrm{de}} \qquad (2c)$$

The body here is constructed from two types of material, conductor and dielectric, so there must be two types of boundary condition that satisfied equations (2). The conditions require that tangential electric field component is vanished on the S_{cd} surface, while the magnetic and electric fields is continuous on the S_{de} surface .

From Eqs. (2) a group of expressions can be obtained to give mathematical representation for the internal and external equivalent regions. This group is adequate to calculate the unknown surface currents densities $\bar{J}_{cd}, \vec{J}_{de}, \vec{M}$ as follow[4]:

$$\bar{E}^{\mathrm{d}}_{\mathrm{tan}}\left(\vec{J}_{\mathrm{cd}} + \vec{J}_{\mathrm{de}}, \bar{M}\right) = 0 \qquad (3a)$$

$$\vec{E}^{e}_{tan}\left(\vec{J}_{de}, \vec{M}\right) + \vec{E}^{d}_{tan}\left(\vec{J}_{cd} + \vec{J}_{de}, \vec{M}\right) = \vec{E}^{d}_{tan}\left(\vec{J}^{ie}\right) \qquad (3b)$$

$$\vec{H}^{e}_{tan}\left(\vec{J}_{de}, \vec{M}\right) + \vec{H}^{d}_{tan}\left(\vec{J}_{cd} + \vec{J}_{de}, \vec{\bar{M}}\right) = \vec{H}^{d}_{tan}\left(\vec{J}^{ie}\right) \qquad (3c)$$

The subscript (tan) denotes the tangential components of the EM fields on the surface $\vec{E}^{\mathrm{a}}(\vec{J}, \vec{M})\ \vec{H}^{\mathrm{a}}(\vec{J}, \vec{M})$ which represent the electric and magnetic fields produced due to the incident electric and magnetic currents ($\vec{J}, \vec{M}$) ,which in turn radiates in the media that characterized by (ε_a, μ_a) . The symbol (a) represents to the internal and external equivalent regions.

In this paper EFIE-PMCHWT were used to describe the scattering in the conducting bodies of revolution coated by dielectric substrate. The EFIE represents conductor part and PMCHWT represents dielectric part [5].

Equations' (3), represents the integral equations for the unknown electric and magnetic equivalence density current, can be described by magnetic and electric vector and scalar potential. The field vectors are [6].

$$\vec{E}^{a}\left(\vec{J}, \vec{M}\right) = -\mathrm{j}\omega\vec{A}^{\mathrm{a}}\left(\vec{J}\right) - \nabla\emptyset^{\mathrm{a}}\left(\vec{J}\right) - \frac{1}{\varepsilon_{\mathrm{a}}}\nabla \times \vec{F}^{\mathrm{a}}\left(\vec{M}\right) \qquad (4)$$

$$\vec{H}^{a}\left(\vec{J}, \vec{M}\right) = -j\omega\vec{F}^{a}\left(\vec{J}\right) - \nabla\Psi^{a}\left(\vec{M}\right) + \frac{1}{\mu_a}\nabla \times \vec{A}^{a}\left(\vec{J}\right) \qquad (5)$$

Where $\vec{F}^{a}(\vec{M})$ and $\vec{A}^{a}(\vec{J})$ are the electric and magnetic vectors potential and defined by:

$$\vec{A}^{a}\left(\vec{J}\right) = \mu_a \int_s \vec{J}(\vec{r}')G(\vec{r}, \vec{r}')ds' \qquad (6)$$

$$\vec{F}^{a}(\vec{M}) = \varepsilon_a \int_s \vec{M}(\vec{r}')G(\vec{r},\vec{r}')ds' \quad (7)$$

$\Psi^{a}(\bar{M})$ and $\emptyset^{a}(\bar{J})$ are the electric and magnetic scalar potential and defined by[7]:

$$\emptyset^{a}(\vec{J}) = \frac{1}{\varepsilon_a}\int_s \sigma(\vec{J})G(\vec{r},\vec{r}')ds' \qquad (8)$$

$$\Psi^{a}(\vec{M}) = \frac{1}{\mu_a}\int_s m(\vec{M})G(\vec{r},\vec{r}')ds' \quad (9)$$

While $\sigma(\vec{J})$ and $m(\vec{M})$ represented the electric and magnetic surface charge distribution defined by:

$$\sigma(\vec{J}) = \frac{j}{\omega}\vec{\nabla}_s.\vec{J}(\vec{r}) \qquad (10)$$

$$m(\vec{M}) = \frac{j}{\omega}\vec{\nabla}_s.\vec{M}(\vec{r}) \quad (11)$$

here ω is the angular frequency, and G is the scalar Green's function which can be expressed as[8]:

$$G(\vec{r},\vec{r}') = \frac{\exp(-jk_a|\vec{r}-\vec{r}'|}{4\pi|\vec{r}-\vec{r}'|} \qquad (12)$$

Where $K_a = \omega(\epsilon_a\mu_a)^{1/2}$ in the (a) region and called propagation constant.

Now, we can transform the integral equations to a group of linear equations and form a matrix by using the inner products of the weight function with the electric and magnetic field in eq. (3) as:

$$\begin{bmatrix} [Z^{1d}_{cd.cd}]_n & [Z^{2d}_{cd.de}]_n & \eta_d[Y^{3d}_{cd.de}] \\ [Z^{1d}_{de.cd}]_n & \left([Z^{2e}_{de.de}]_n + [Z^{2d}_{de.de}]_n\right) & \eta_e\left([Y^{3e}_{de.de}]_n + [Y^{3d}_{de.de}]_n\right) \\ [Y^{1d}_{de.cd}]_n & \left([Y^{2e}_{de.de}]_n + [Y^{2d}_{de.de}]_n\right) & \eta_e\left([Z^{3e}_{de.de}]_n + [Z^{3d}_{de.de}]_n\right) \end{bmatrix}$$

$$\begin{bmatrix} [I^{d}]_n \\ [I\ ]_n \\ [K\ ]_n \end{bmatrix} = \begin{bmatrix} [V^{d}_{cd}] \\ [V^{d}_{de}]_n \\ [I^{d}_{de}]_n \end{bmatrix} \quad (13)$$

3. THE IMPEDANCE ELEMENTS MATRIX $[\vec{Z}]$

The impedance element matrix is produced from the electric field which is a function of electric current density ($\bar{J}$) , and can be written as[9]:

$$\left(Z^{\alpha\beta}_{mn}\right)^{JE}_{ij} = \left\langle \vec{W}^{\alpha}_{mj}, \vec{E}^{a}(\vec{J}^{\beta}_{ni}, 0)\right\rangle = \left\langle \vec{W}^{\alpha}_{mj}, (-j\omega\vec{A}^{a}\left(\vec{J}^{\beta}_{ni}\right) - \nabla\emptyset^{a}\left(\vec{J}^{\beta}_{ni}\right))\right\rangle \quad (14)$$

or from the magnetic field produced by magnetic current density ($\vec{M}$), and can be written as:

$$\left(Z^{\alpha\beta}_{mn}\right)^{MH}_{ij} = \langle \vec{W}^{\alpha}_{mj}, \vec{E}^{a}(0, \vec{M}^{\beta}_{ni}\rangle = \langle \vec{W}^{\alpha}_{mj}, (-j\omega\vec{F}^{a}\left(\vec{M}^{\beta}_{ni}\right) - \nabla\psi^{a}\left(\vec{M}^{\beta}_{ni}\right))\rangle \quad (15)$$

the inner product between the electric field and the weighting function $\vec{W}^{\alpha}_{mj}$,yields to:

$$(Z^{tt}_n)_{ij} = -jk_a\eta_a \sum_{p=1}^{4}\sum_{q=1}^{4}\left[T_pT_q\left\{\sin\upsilon_p \sin\upsilon_q G_2 + \cos\upsilon_p \cos\upsilon_q G_1\right\} - \frac{G_1}{k_a^2}T'_pT'_q\right]$$

$$\left(Z^{t\emptyset}_n\right)_{ij} = -k_a\eta_a \sum_{p=1}^{4}\sum_{q=1}^{4}\left[T_pT_q\sin\upsilon_p G_3 + \frac{n}{k_a^2}\frac{T_q}{\rho_q}T'_pG_1\right]$$

$$\left(Z^{\emptyset t}_n\right)_{ij} = k_a\eta_a \sum_{p=1}^{4}\sum_{q=1}^{4}\left[T_pT_q\sin\upsilon_q G_3 + \frac{n}{k_a^2}T'_q\frac{T_p}{\rho_p}T'_pG_1\right]$$

$$\left(Z^{\emptyset\emptyset}_n\right)_{ij} = -jk_a\eta_a \sum_{p=1}^{4}\sum_{q=1}^{4}T_pT_q\left[G_2 - \left(\frac{n}{k_a^2}\right)^2\frac{G_1}{\rho_p\rho_q}\right] \quad (16)$$

Where

$$G_1 = \int_0^{\pi} \frac{e^{-jk_a R}}{R} \cos(n\emptyset)\, d\emptyset$$

$$G_2 = \int_0^{\pi} \frac{e^{-jk_a R}}{R} \cos(n\emptyset)\cos(\emptyset)\, d\emptyset \qquad (17)$$

$$G_3 = \int_0^{\pi} \frac{e^{-jk_a R}}{R} \sin(n\emptyset)\sin(\emptyset)\, d\emptyset$$

$$R = \sqrt{\left(\frac{\rho_p}{4}\right)^2 + 4\rho_p^2 sin^2(\frac{\emptyset}{2})} \quad (18)$$

4. THE ADMITTANCE SUB MATRIX ELEMENTS[$\boldsymbol{Y}$]

The admittance matrix element can be found from the magnetic field produced from electric current density, as follow [10]:

$$\left(Y_{mn}^{\alpha\beta}\right)_{ji}^{JH} = \langle \vec{W}_{mj}^{\alpha}, \vec{H}^{a}(\vec{J}_{ni}^{\beta}, 0)\rangle$$

$$= \langle \vec{W}_{mj}^{\alpha}, \left(\frac{1}{\mu_a}\nabla \times \vec{A}^{a}\left(\vec{J}_{ni}^{\beta}\right)\right)\rangle \quad (19)$$

or from electric field produced from magnetic current density, as follow:

$$\left(Y_{mn}^{\alpha\beta}\right)_{ji}^{ME} = \langle \vec{W}_{mj}^{\alpha}, \vec{H}^{a}(0, \vec{M}_{ni}^{\beta})\rangle$$

$$= \langle \vec{W}_{mj}^{\alpha}, \left(-\frac{1}{\varepsilon_a}\nabla \times \vec{F}^{a}\left(\vec{M}_{ni}^{\beta}\right)\right)\rangle \quad (20)$$

the inner product between the magnetic field and the weighting function$\vec{W}_{mj}^{\alpha}$,yields to the following:

$$(Y_n^{tt})_{ij} = j\sum_{p=1}^{4}\sum_{q=1}^{4} T_p T_q G_6 \Big[\rho_q cos\,\upsilon_{\ q} sin\,\upsilon_{\ p} + \left\{sin\,\upsilon_{\ q}(Z_p - Z_q) - \rho_p cos\,\upsilon_{\ p}\right\}\Big]$$

$$\left(Y_n^{t\emptyset}\right)_{ij} = \sum_{p=1}^{4}\sum_{q=1}^{4} T_p T_q \Big[\rho_q cos\,\upsilon_{\ p} G_4 + \left\{sin\,\upsilon_{\ p}(Z_p - Z_q) - \rho_p cos\,\upsilon_{\ p}\right\} G_5\Big]$$

$$\left(Y_n^{\emptyset t}\right)_{ij} = \sum_{p=1}^{4}\sum_{q=1}^{4} T_p T_q \Big[\rho_q cos\,\upsilon_{\ q} G_4 - \left\{sin\,\upsilon_{\ q}(Z_p - Z_q) + \rho_q cos\,\upsilon_{\ q}\right\} G_5\Big]$$

$$\left(Y_n^{\emptyset\emptyset}\right)_{ij} = j\sum_{p=1}^{4}\sum_{q=1}^{4} T_p T_q (Z_p - Z_q) G_6 \qquad (21)$$

Where

$$G_4 = \int_0^{\pi} \cos(n\emptyset)\frac{(1 + jk_a R)}{R^3} e^{-jk_a R} d\emptyset$$

$$G_5 = \int_0^{\pi} \cos(n\emptyset)\cos(\emptyset)\frac{(1 + jk_a R)}{R^3} e^{-jk_a R} d\emptyset \qquad (22)$$

$$G_6 = \int_0^{\pi} \sin(n\emptyset)\sin(\emptyset)\frac{(1 + jk_a R)}{R^3} e^{-jk_a R} d\emptyset$$

$\nu_P, \nu_q, \rho_p, \rho_q$ are the geometry model of body of revolution value, and t_p, t_q are the pulse function that can be calculated from[9]:

$$t_p = i + \frac{p-2\cdot 5}{2} \qquad (23)$$

$$t_q = j + \frac{q - 2\cdot 5}{2}$$

5. THE EXCITATION SUBMATRICES ELEMENTS [$\bar{\mathbf{R}}$]

The plane wave approximation depends on process and approximation used by the researchers J.R. Mautz&R.F. Harrington is used to calculate the excitation submatrices element [9]as:

$$\left(R_n^{t\theta}\right)_i = \sum_{q=1}^{4} C_q \left[\sin\left(\upsilon_{\ q}\right)\cos(\theta_r)\left(J_{(n+1)} - J_{(n-1)}\right.\right.$$
$$\left.\left. + 2j\cos\left(\upsilon_{\ q}\right)\sin(\theta_r)J_n\right)\right]$$

$$\left(R_n^{\phi\theta}\right)_i = \sum_{q=1}^{4} jC_q\cos(\theta_r)\left(J_{(n+1)} + J_{(n-1)}\right)$$

$$\left(R_n^{t\phi}\right)_i = \sum_{q=1}^{4} -jC_q\sin(\upsilon_{\ q})\left(J_{(n+1)} + J_{(n-1)}\right) \quad (24)$$

6. THE RADAR CROSS SECTION EVALUATION

Scattering problem in radar includes how to measure the amount of power, that is reflected back to radar, from the target (BOR). This amount can be express as a relation between the incident plane wave E^i and scattered wave E^s. In general the scattering wave is produced by the superposition of the two orthogonal components E_θ^s , E_ϕ^s while the incident wave is produced by the two orthogonal components E_θ^i , E_ϕ^i and the relation of the RCS can be given by [11]:

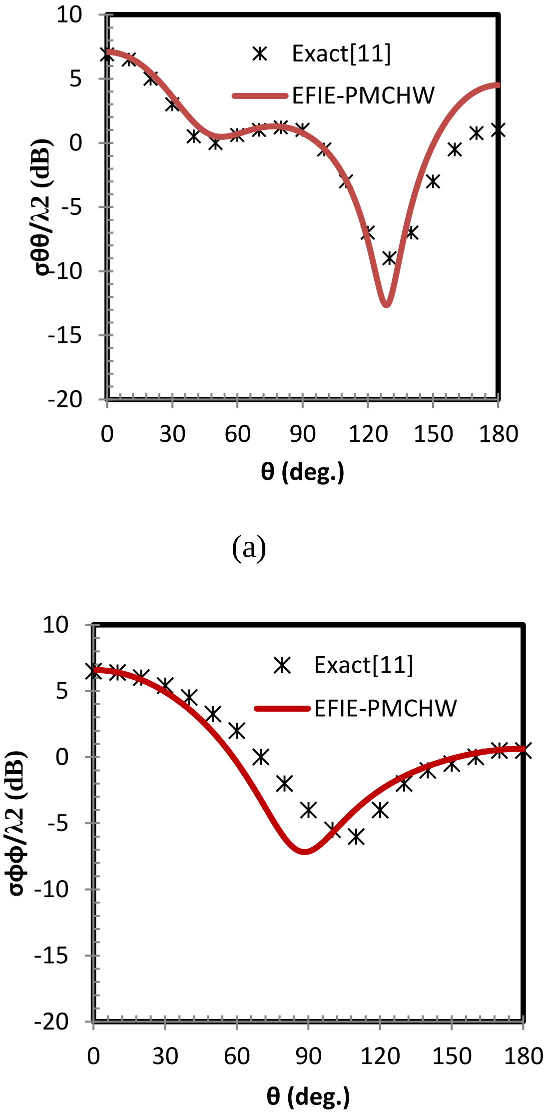
$$\sigma^{\theta\beta} = \frac{-j\omega\mu}{4\pi}\left|\sum_{-\infty}^{\infty} e^{-jn\phi}\sum_{i=1}^{N-1}\begin{bmatrix}[R_{ni}^{t\theta}]\\ [-R_{ni}^{\phi\theta}]\\ [R_{ni}^{t\phi}]\\ [-R_{ni}^{\phi\phi}]\end{bmatrix}\begin{bmatrix}[I_i^{t\beta}]\\ [I_i^{\phi\beta}]\\ [k_i^{t\beta}]\\ [k_i^{\phi\beta}]\end{bmatrix}\right|^2$$

$$\sigma^{\phi\beta} = \frac{-j\omega\mu}{4\pi}\left|\sum_{-\infty}^{\infty} e^{-jn\phi}\sum_{i=1}^{N-1}\begin{bmatrix}[R_{ni}^{t\phi}]\\ [-R_{ni}^{\phi\phi}]\\ [-R_{ni}^{t\theta}]\\ [R_{ni}^{\phi\theta}]\end{bmatrix}\begin{bmatrix}[I_i^{t\beta}]\\ [I_i^{\phi\beta}]\\ [k_i^{t\beta}]\\ [k_i^{\phi\beta}]\end{bmatrix}\right|^2$$

(25)

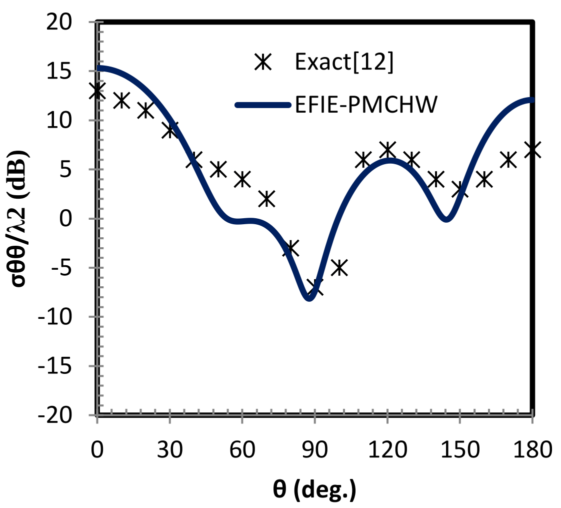
7. RESULTS AND DISCUSSION

To prove the validity of the mathematical analysis and the programs for CBOR coated by dielectric layer, conducting spheres of radii 0.311λ and 0.4777λ were selected and were compared the results with other researcher as in fig (2). The figure shows a good agreement between the two results.

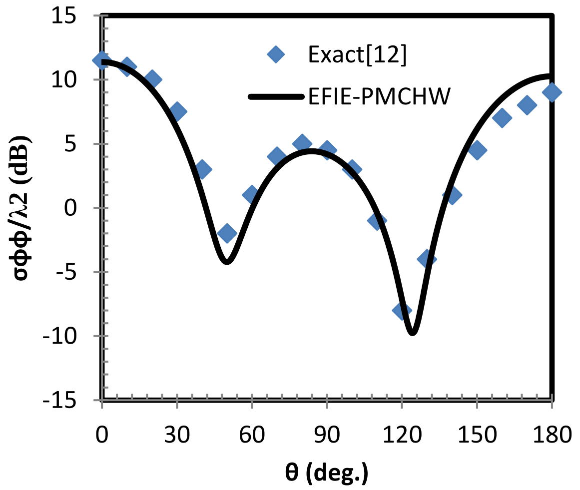


(b)

Figure 2: RCS for conducting sphere (a2=0.311λ) coated by dielectric layer(∈$_r$, a1=0.394λ)



(a)



(b)

Figure 3: RCS for conducting sphere (a2=0.319λ) coated by dielectric layer ($\epsilon_r = 4$, a1=0.4777λ)

8. PROPOSED MODEL

The proposed model chosen in this work is a rocket shown in fig(4), The effect of different values of dielectric constant (ϵ_r) and thickness (t) were studied:

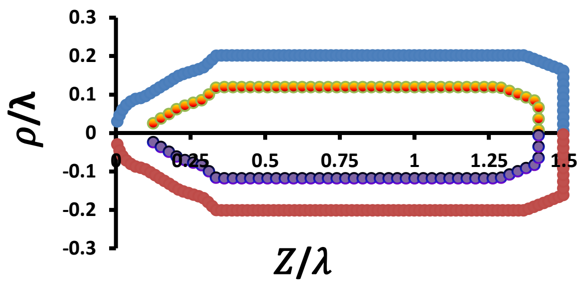


Figure 4: The proposed model.

Fig.(5) and Fig.(6) show the effect of dielectric constant (ϵ_r) and dielectric thickness (t) on radar cross section. These figures show that the value of RCS in E-plane and H-plane decreases as (ϵ_r) and t increase.

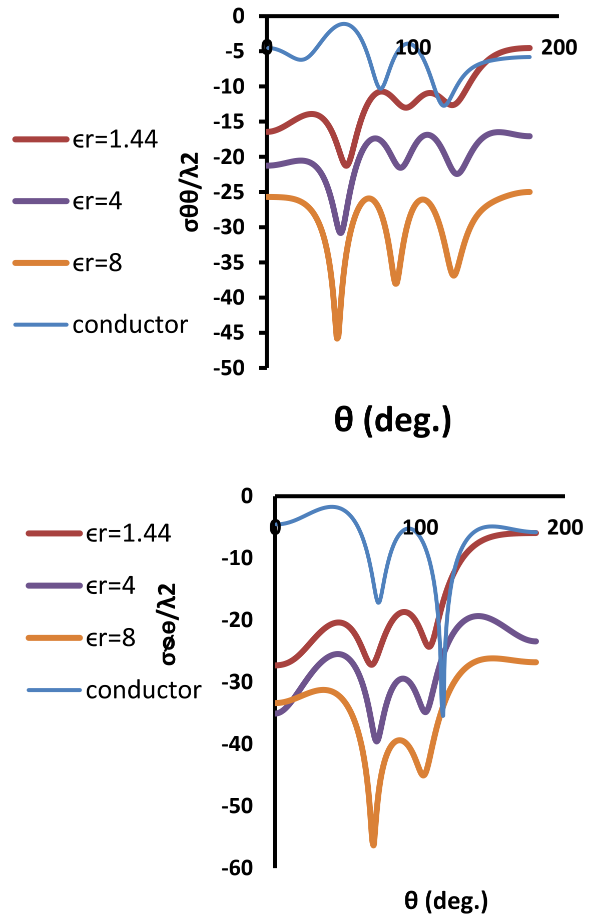


Figure 5: RCS as a function of dielectric constant ϵ_r.

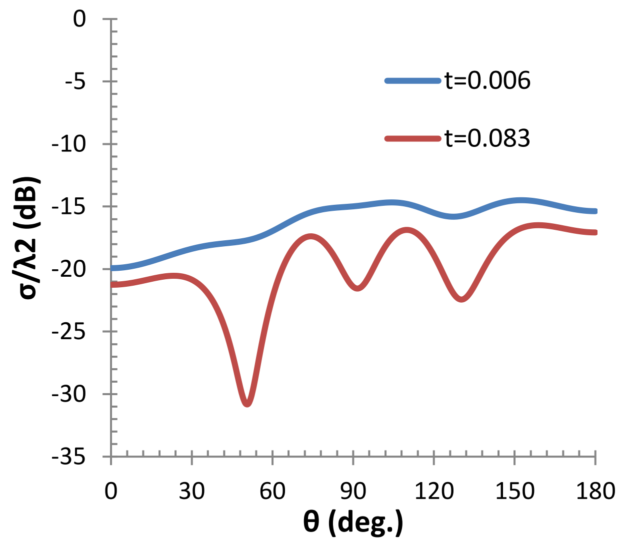


Figure 6: RCS of dielectric constant ϵ_r=4, as a function of layer thickness (t).

9. CONCLUSIONS:

In this research the method of moment was applied with Galerkin's approach to solve the problem of scattering from bodies of revolution, which is one of the most effective methods to processing these problems. The EFIE is used with PMCHWT formulation to processing coated bodies. The mathematical analysis and the program were checked to know the validity of the formulation by applied it to simple, regular and symmetrical bodies and comparing our results with the published results. From the study of the effect of dielectric constant and thickness of the substrate layer we observe that increase it reduces the RCS

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