

Liquid Holdup Correlation for Inclined Two-Phase Stratified Flow in Pipes

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Abstract:

In this study, set of explicit equations to predict the liquid holdup in inclined stratified, two-phase, gas-liquid flow has been developed. This study is similar to Abdul Majeed's study for predict the liquid holdup in horizontal pipes. The experimental tests conducted from several sources published in the literature and other unpublished selected from the Iraq-Oil wells. The comparison achieved using the FPR function versus many of previous studies. The comparison reveals that the present correlation is semi-identification with the original correlation by Taitel-Dukler model and it displays that the suggested correlation has the best performance than the others.

الخلاصة:

في هذه الدراسة ، تم استحداث مجموعة معادلات لحساب نسبة احتجاز السائل للجريان المنفصل المائل ، ثنائي الطور ، نوع غاز - سائل. هذه الدراسة مشابهة لدراسة عبد المجيد للجريان الثنائي الطور للأنايب الأفقية تم تجميع البيانات العملية اللازمة للمقارنة من مصادر مختلفة وأخرى غير منشورة مختارة لأبار نفطية عراقية. تمت المقارنة باستخدام دالة (أف بي آر) مع عدة طرق سابقة. المقارنة دلت على شبه التطابق بين الموديل الحالي والموديل الأصلي لـ (تايتل- دكلر) ، كما بينت المقارنة الأداء الأفضل للموديل المقترح بالمقارنة مع الموديلات الأخرى.

Introduction:

There are many studies had been achieved to predict the liquid holdup in pipes. Some of them, studied the stratified flow in horizontal pipes while the other studied the flow in inclined pipes. Moreover, most of them were empirical studies and developed model is already valid for their experimental data only. Taitel-Dukler (1976) studied the stratified flow and they derived a set of equations to predict the liquid holdup. Their study was depending on the implicit solution of two-phase momentum equation. Usually, this solution adopts the iteration method to locate the independent term ($\bar{h}_L$). The final step of their study was drawing a compact plot of the independent term ($\bar{h}_L$) versus Lockhart-Martinelli parameter (X).

The plot consists of multi-curve; each one specialized to the inclination term (Y). Their procedure still used to this time. Barnea (1987) used this procedure to develop their unified model to predict the flow pattern in two-phase, air-water flow for the whole inclination angles. Xiao et al. (1990) also used Taitel-Dukler procedure to develop their mechanistic model to predict the flow patterns, liquid holdup and pressure drop in horizontal pipelines. Abdul Majeed (1996) suggests new procedure to simplify and to modify the model of Taitel-Dukler by converting it to two explicit equations to predict the liquid holdup. These equations are expressed the dependent term ($\bar{h}_L$) and the Lockhart-Martinelli parameter as independent term and for horizontal flow only. There are may investigators adopted the model of Taitel-Dukler as Gokcal et al (2006), studied the effects of the viscosity on the flow pattern, liquid holdup and pressure drop depending of the model of Taitel-Dukler. Lastly, Andritsos et al (2008) studied the stratified two-phase flow by using the model of Taitel-Dukler.

In the present study, the procedure of Abdul Majeed will extended to cover the upwardly inclined flows and set of equations will suggested to predict the liquid holdup in inclined pipes.

Experimental Data:

There are no experimental tests developed in the present study, the used experimental data conducted by some studies published in the literature and other conducted from the Iraqi-Oil wells, these data has been examined by using the flow pattern map of Mukherjee-Brill (1985) [2] to prove the existing of the stratified flow. These sources of data displayed in table (1) and table (2).

Table (1): The Data Sources

R	References	No. of Data	Inclination Angles	Diameter (m)	System Fluids
1	Abdul Majeed	20	0°	0.0508	Air-Kerosene
2	Mukherjee and Brill	12	0°	0.0508	Air-Kerosene
3	Iraq-oil wells	27	0°	0.1524 0.2032	Natural gas-crude oil
4	Smith	199	2°	0.0508	Air-Oil

Table (2) displays the ranges of the tests undertaking in the present work.

Table (2): Flow Conditions Ranges

R	The Property	Minimum	Maximum
1	Superficial gas velocity m/sec	0.315	24.75
2	Superficial liquid velocity m/sec	0.001	0.095
3	Average Pressure KPa	307.5	929
4	Average Temperature °C	21	47.2
5	Liquid Holdup dimensionless	0.00151	0.6061

The properties of each phase could be predicted by the facilities systems and as follows [1 and 10]:

$$\rho_K = 832.34 - 0.8333 T_{av}$$

$$\rho_A = P_{av} / [287 \times (273 + T_{av})]$$

$$\rho_o = 823.2 \text{ kg/m}^3$$

$$\mu_K = 0.001 \text{EXP} (0.0664 - 0.0207 T_{av})$$

$$\mu_A = 10^{-5} \left(1.7044 + 0.00613 T_{av} - 0.0000314 T_{av}^2 \right)$$

$$\mu_o = 6.61 \text{ mPa.s}$$

The subscripts; K: kerosene A: Air o: Oil

Naji and Al-Kayiem (2001) [2] displayed set of equations to estimate the flow properties of natural gas and live crude oil for Iraqi oil wells.

Taitel-Dukler (1976) Model:

Taitel and Dukler (1976) derived the two-phase, stratified, gas-liquid momentum equation as:

$$X^2 \left[\left(\frac{\tilde{V}_L}{\tilde{d}_L} \right)^{-n} \frac{\tilde{V}_L^2 \tilde{s}_L}{\tilde{A}_L} \right] - \tilde{V}_g^2 \left[\frac{\tilde{s}_g}{\tilde{A}_g} + \frac{f_i}{f_g} \left(\frac{\tilde{s}_i}{\tilde{A}_g} + \frac{\tilde{s}_i}{\tilde{A}_L} \right) \right] \left(\frac{\tilde{V}_g}{\tilde{d}_g} \right)^{-n} - 4Y = 0 \quad (1)$$

The superscript ($\approx$) over any variable in equation (1) represents it in dimensionless form. Lockhart-Martinelli parameter (X) and the inclination parameter (Y) are defined as:

$$X^2 = \left[\frac{V_{sg} \rho_g \mu_L}{V_{sL} \rho_L \mu_g} \right]^m \frac{\rho_L V_{sL}^2}{\rho_g V_{sg}^2}$$

$$Y = \frac{g d (\rho_L - \rho_g) \sin \theta}{2 C \rho_g V_{sg}^2} \left(\frac{\rho_g V_{sg} d}{\mu_g} \right)^m \quad \text{and } (\theta) \text{ is the inclination angle.}$$

For turbulent flow, it uses $m=0.2$ and $C=0.046$ while uses $m=1$ and $C=16$ for laminar flow. (f_i) and (f_g) are representing the interfacial and gas-wall friction factors respectively.

Now, all variables in equation (1) are possible to be estimated by using figure (1) depending on the dimensionless liquid leveling height ($\tilde{h}_L$).

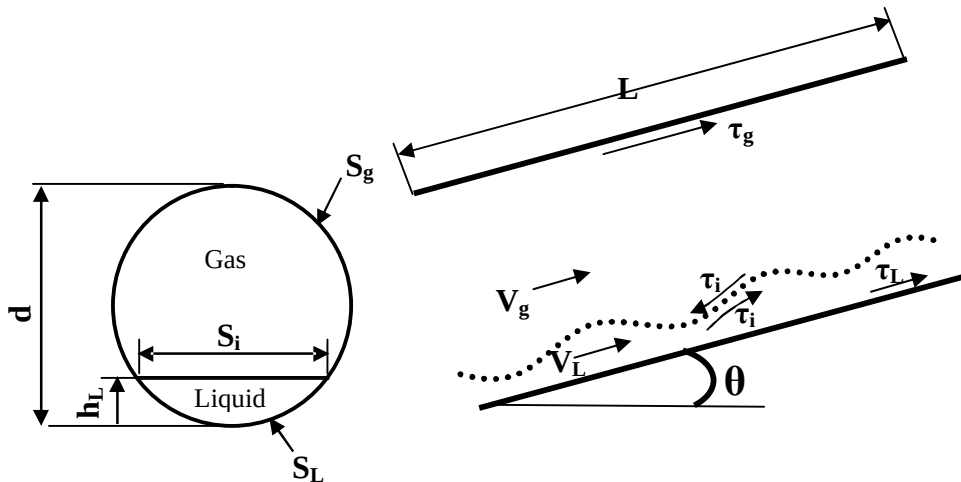


Figure (1): The Configuration of stratified flow

As in the following:

$$\begin{aligned} \tilde{h}_L &= \frac{\tilde{h}_L}{d} & \tilde{s}_g &= \cos^{-1} \left(2\tilde{h}_L - 1 \right) & \tilde{s}_L &= \pi - \tilde{s}_g & \tilde{s}_i &= \left(1 - (2\tilde{h}_L - 1) \right)^{0.5} \\ \tilde{A}_g &= 0.25 \left(\tilde{s}_g - (2\tilde{h}_L - 1) \tilde{s}_i \right) & \tilde{A}_L &= 0.25 \left(\pi - 4 \tilde{A}_g \right) \\ \tilde{V}_L &= 0.25 \pi / \tilde{A}_L & \tilde{V}_g &= 0.25 \pi / \tilde{A}_g \\ \tilde{d}_L &= 4 \tilde{A}_L / \tilde{s}_L & \tilde{d}_g &= 4 \tilde{A}_g / \left(\tilde{s}_i + \tilde{s}_g \right) \end{aligned}$$

Xiao et al (1990) developed equation to estimate the liquid holdup in stratified flow, this equation is function of $(\tilde{h}_L)$ also:

$$H_L = \frac{\psi - \sin \psi}{2\pi} \quad (2)$$

Where: ψ is the wet angle by the liquid and it is calculated by:

$$\psi = 2 \cos^{-1} \left(2\tilde{h}_L - 1 \right)$$

Taitel and Dukler (1976) proposed that the ratio f_i / f_g is unity, hence they could predict the liquid holdup by the following procedure:

1. Solve the equation (1) iteratively for $(\tilde{h}_L)$.
2. Estimate the liquid holdup by using the equation (2).

Abdul Majeed (1996) Model:

Abdul-Majeed studied the horizontal flow, in this state the value of (Y) will be zero and equation (1) is reduced to:

$$X^2 \left[\left(\frac{\tilde{V}_L}{\tilde{d}_L} \right)^{-n} \frac{\tilde{V}_L^2 \tilde{s}_L}{\tilde{A}_L} \right] - \tilde{V}_g^2 \left[\frac{\tilde{s}_g}{\tilde{A}_g} + \frac{f_i}{f_g} \left(\frac{\tilde{s}_i}{\tilde{A}_g} + \frac{\tilde{s}_i}{\tilde{A}_L} \right) \right] \left(\frac{\tilde{V}_g}{\tilde{d}_g} \right)^{-n} = 0 \quad (3)$$

Since $(\tilde{h}_L)$ is rounded between zero and unity, he suggested that the equation (3) could be solved for (X), and by using the equation (2) he correlated (H_L) as a function to (X). He suggested two equations: one for turbulent flow while the other was for laminar flow as in the following:

1. For Turbulent Flow:

$$H_L = \text{EXP} \left(-0.9304919 + 0.5285852 R - 9.219634 \hat{1} 10^{-2} R^2 + 9.02418 \hat{1} 10^{-4} R^4 \right)$$

2. For Laminar Flow:

$$H_L = \text{EXP} \left(-1.099924 + 0.6788495 R - 0.1232191 \hat{1} 10^{-2} R^2 - 1.778653 \hat{1} 10^{-3} R^3 + 1.626819 \hat{1} 10^{-3} R^4 \right)$$

Where; $R = \ln(X)$

Present Correlation:

Currently, equation (1) could be solved for (X) related to the change of (h_L) from zero to unity for each magnitude of (Y) and H_L could be calculate by using equation (2), a curve fitting achieved for H_L as function to the parameter (X), this process led to the following equation:

$$H_L = 10^Z \quad (4)$$

Where: $Z = \sum_{i=0}^N a_i \times b^i$

N: No of coefficients (a_i)

a_i : Coefficients get from table (3) or table (4) according to the flow type (turbulent or laminar).

$$b = \log_{10}(X)$$

The following procedure will be suggested:

1. Calculate (Y) according to inclination angle.
2. Calculate the value of Lockhart – Martinelli Parameter (X).
3. For turbulent flow: According to the value of (Y), use table (3) to locate the coefficients of (a).
4. For laminar flow: According to the value of (Y), use table (4) to locate the coefficients of (a).
5. The linear interpolation will be suggested for non-available values of (Y).
4. Use equation (4) to predict the liquid holdup (H_L).

Work Procedure:

In the present work, two steps have been achieved:

- Development of the model:

Abdul-Majeed (1996) present a procedure to predict the liquid holdup for horizontal flow using equations (2) and (3) and he suggested using his model instead of Taitel-Dukler model. This procedure adopted in the current work to develop the present model to predict the liquid holdup for inclined flow including the horizontal flow.

- Testing the model:

The present model has been tested relate to the model of Taitel-Dukler, the testing includes the following steps:

1. Assume various magnitudes for (X) and (Y).
2. Use the present model to predict the liquid holdup.
3. Use Taitel-Dukler to predict the liquid holdup.

Now, the values of (H_L) by the present model and those predicted by the Taitel-Dukler model plotted against (X) as shown in figure (2), and the values of the average error between the predictions of them displayed in table (5) for selected values of (Y), these values had been displayed in the results of Taitel-Dukler (1976) work. The other values of (Y) are currently undertaking to cover a wide range of the inclination effects.

Table (3): A-Coefficients for the Present Model in 'Turbulent Flow

Y	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉
0	-402952716	.5342773418	- .2190226335	- .003665895311	.01524176067	.0002726620402	- .5.679301919D-04			
0.1	-410618571	.5290948764	- .2007345728	.003060706626	.008563991565	.0007375468104				
1	-422654761	.5655009398	- .2203388977	- .01526195363	.01747649755	.001324044557	- .0008182478063			
2	-440258195	.5908214257	- .2201183131	- .02452062157	.0189995029	.002161640712	- .001004719826			
5	-484338399	.645849742	- .2152364112	- .04414761239	.02133399436	.003935374793	- .00135574638			
10	-540039907	.7024341924	- .2028189522	- .06352458645	.0221281223	.005697050412	- .001631832676			
20	-618178037	.7641596157	- .17781697	- .08347876137	.02057096291	.007562363384	- .00181586433			
50	-772308943	.8517718415	- .04362934611	- .157718447	- .02225430057	.034988426	- .000245608113	- .002804351499	.0003801492594	
100	-895039525	.8532438767	.03622611375	- .1253470648	- .05359661018	.02715521186	.004562294208	- .002425415032	.0001728444755	
200	-1.01819766	.8209672472	.08829378235	- .055415087	- .07429737906	.006806860082	.009432029712	- .0008140804272	- .0002216339255	
500	-1.16534668	.7093448449	7.89093549D-02	.1792111149	- .105022036	- .09476953711	.04371565358	.008761046678	- .00611522477	.0007175166
10 ³	-1.29745432	.7200298391	.145173055	.07170381643	- .1079464715	- .00966795885	.01712056185	- .002526911603		
2 × 10 ³	-1.40129302	.6878495922	.08818048089	.1258920551	- .08276439123	- .02571033007	.01718887234	- .002049888589		
5 × 10 ³	-1.52327343	.8089659988	- .2000280195	- .006885400142	.2448152141	- .07473784268	- .05657971795	.02930993128	- .003576244076	
10 ⁴	-1.6498943	.8605767834	- .1517919301	- .1595564449	.2609519074	- .00630811183	- .084166057	.03103733041	- .00326317084	

Table (4): A-Coefficients for the Present Model in Laminar Flow

Y	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉
0	-.477479304	.6871894111	-.2898260205	-.0185406336	.02667689445	.001537472272	-.001349638561			
1	-.507245658	.7287004396	-.2820719737	-.0412973654	.02920325293	.004587335863	-.002021806431			
2	-.533313761	.7613145313	-.2736439676	-.05887095501	.03056198285	.006948571769	-.002507382872			
5	-.597718829	.82822811001	-.2423706674	-.09472050852	.02715994251	.01257025522	-.002749197476	-.00016268199		
10	-.684828715	.9206036142	-.1551549157	-.2085864056	.000339822957	6.2571784671D-02	-.005900573774	-.006574584053	.001276189514	
20	-.795372693	.9663037701	-.05232338841	-.2272648072	-.0433755769	.07121231271	-.0002638747841	-.8.1017318D-03	.001314441462	
50	-.968070882	.9639407043	.08977078425	-.1651373361	-.1117703817	.05328177428	.01283610184	-.007348872806	.0007023433637	
100	-1.10458645	.9166140547	.1644085643	-.6.363493D-02	-.1506065657	.02177856656	.02316290236	-.005165817236		
200	-1.23947353	.8486474691	.2117887239	.04399059259	-.1870175339	-.0005333625618	.03386761398	-.006290185287		
500	-1.37038954	.809737521	-.1195773708	.24865696	.1402025538	-.2160316901	-.007254947825	.04773813658	-.01212226387	.00076195
10 ³	-1.49078152	.8768715709	-.2462230691	.07708509104	.360163696	-.1708556494	-.09462739611	.06481602166	-.00934251695	
2 × 10 ³	-1.62526312	.970802615	-.2665179779	-.2124233603	.5291248092	-.06107903778	-.1986568634	9.03931401D-02	-.01127600126	
5 × 10 ³	-1.82299246	.7066174084	.1293186563	.1285748831	-.1035842905	.01452521796				
10 ⁴	-1.92992796	.7325900146	.02543949153	.1393562835	-.6.76605-02	.005574313399				

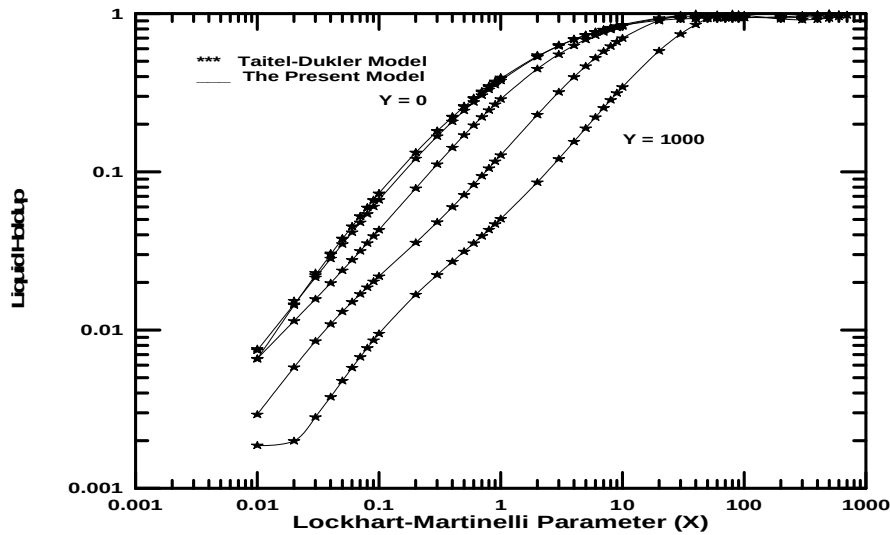


Figure (2): Comparison between the Present work and Taitel-Dukler Model for different Values of (X) and (Y)

Table (5): The error between the present work and the original model of Taitel-Dukler for different values of (Y)

Magnitudes of (Y)	Average Error
0	- 0.0002899
0.1	- 0.0005046
1	- 0.0005027
10	- 0.0003689
100	- 0.0001495
1000	- 0.0007539

The figure (2) and table (5) show the coincidence state of these models; therefore; it is suggested to use the present model instead of the model of Taitel-Dukler in the prediction in the horizontal and inclined flows.

The Activity of the Present Model:

The present model has been tested using several models, some of them used in the prediction in horizontal flow as Abdul Majeed (1996) and other used for inclined pipe as Gozhov et al. (1967), Mukherjee-Brill (1985) and Beggs-Brill (1986). The testing achieved using the experimental data shown in table (1) and table (2) by using the relative performance factor (F_{PR}) which is defined as:

$$F_{PR} = \frac{\left| \frac{E_1}{E_{1max}} - \frac{E_{1min}}{E_{1min}} \right|}{\left| \frac{E_1}{E_{1max}} - \frac{E_{1min}}{E_{1min}} \right|} + \frac{\frac{E_2}{E_{2max}} - \frac{E_{2min}}{E_{2min}}}{\frac{E_2}{E_{2max}} - \frac{E_{2min}}{E_{2min}}} + \frac{\frac{E_3}{E_{3max}} - \frac{E_{3min}}{E_{3min}}}{\frac{E_3}{E_{3max}} - \frac{E_{3min}}{E_{3min}}} +$$

$$\frac{\left| \frac{E_4}{E_{4\max}} - \frac{E_{4\min}}{E_{4\min}} \right|}{\left| \frac{E_4}{E_{4\max}} - \frac{E_{4\min}}{E_{4\min}} \right|} + \frac{\frac{E_5}{E_{5\max}} - \frac{E_{5\min}}{E_{5\min}}}{\frac{E_5}{E_{5\max}} - \frac{E_{5\min}}{E_{5\min}}} + \frac{\frac{E_6}{E_{6\max}} - \frac{E_{6\min}}{E_{6\min}}}{\frac{E_6}{E_{6\max}} - \frac{E_{6\min}}{E_{6\min}}} \quad (5)$$

It is observed that the relative performance factor depends on many parameters defined in the following equations:

$$1. \text{ Average Error: } E_1 = \frac{1}{n} \sum_{i=1}^n E_i \quad (6)$$

$$2. \text{ Absolute Average Error: } E_2 = \frac{1}{n} \sum_{i=1}^n |E_i| \quad (7)$$

$$3. \text{ Standard Deviation: } E_3 = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (E_i - E_1)^2} \quad (8)$$

$$4. \text{ Average Percent of Error: } E_4 = \frac{1}{n} \sum_{i=1}^n PE_i \quad (9)$$

$$5. \text{ Absolute Average Percent of Error: } E_5 = \frac{1}{n} \sum_{i=1}^n |PE_i| \quad (10)$$

$$6. \text{ Percent Standard Deviation: } E_6 = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (PE_i - E_1)^2} \quad (11)$$

Where the error (E) and the percent error (PE) are defined as:

$$E_i = H_{L\text{cal}} - H_{L\text{meas}} \quad \text{and} \quad PE_i = \frac{E_i}{H_{L\text{meas}}} \times 100 \%$$

The range of this factor is limited between zero and 6. The zero value indicates the best performance [Ansari et al. (1994), Abdul-Majeed (1997 and 2000) and Naji, A. Saib and Al-Kayiem, H. H. (2001)].

The Results and Discussion:

The statistical results of the present model and the other models has been displayed according to the inclination angle:

Horizontal Flow Data:

The results were displayed graphically in figure (3) through figure (6) and in tabular form as in table (6). The graphs show the best spread of the predicted holdup related to the experimental holdup than the other while the behavior of the Abdul-Majeed was the second best. It is observed that the present model has a superior results than the other models where it has ($F_{PR}=0$) as shown in table (6).

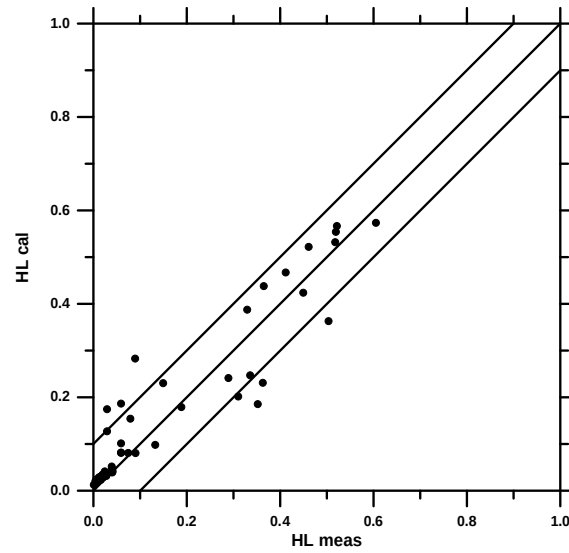


Figure (3): The calculated liquid holdup by the present model for horizontal flow data.

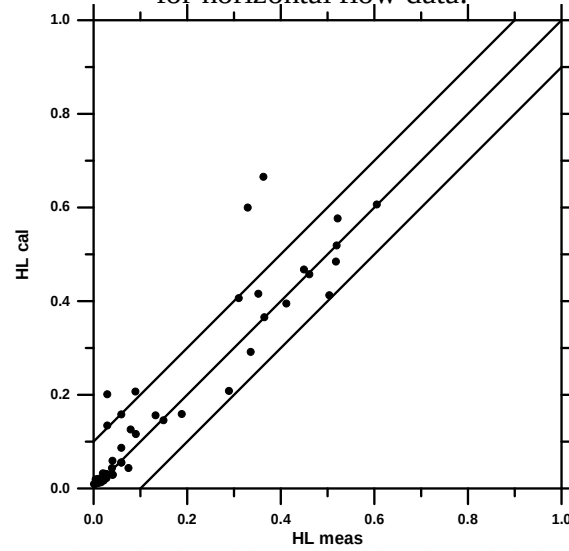


Figure (4): The calculated liquid holdup by Abdul-Majeed model for horizontal flow data.

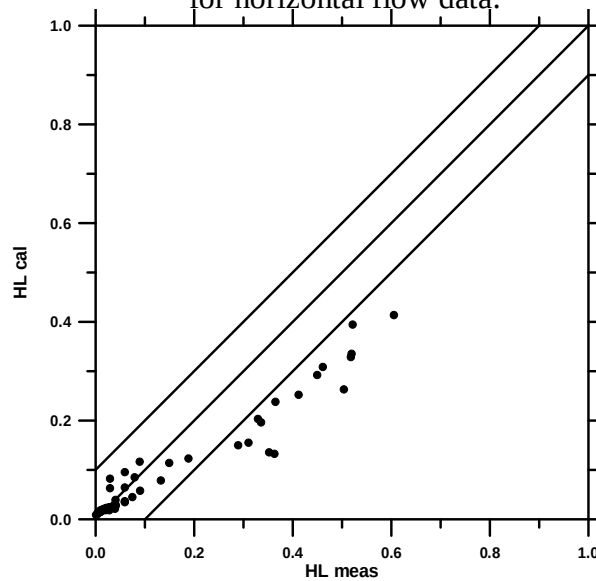


Figure (5): The calculated liquid holdup by Beggs-Brill model for horizontal flow data.

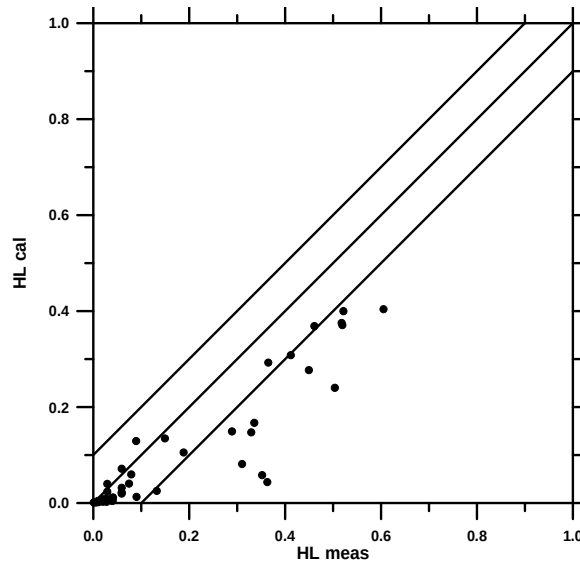


Figure (6): The calculated liquid holdup by Mukherjee-Brill model for horizontal flow data.

Table (6): The Statistical Results Using Horizontal Data

	The Methods	AE $\hat{1} 10^{-5}$	AAE $\hat{1} 10^{-5}$	SDE $\hat{1} 10^{-5}$	APE	AAPE	SDPE	F _{PR}
1	Present Model	$8 \hat{1} 10^{-4}$	$8 \hat{1} 10^{-4}$	0.012	$2 \hat{1} 10^{-5}$	$2 \hat{1} 10^{-5}$	$3 \hat{1} 10^{-4}$	0
2	Abdul-Majeed	-3.62	3.62	58.0	-0.093	0.093	1.496	1.818
3	Beggs-Brill	-5.08	-5.08	81.5	-0.131	0.131	2.10	2.553
4	Mukherjee-Bril	-11.9	11.9	191	-0.308	0.308	4.94	6

Inclined Flow Data:

The results of the whole models have been displayed graphically in figure (7) through figure (10) and the statistics shown in table (7). The results show that the present model is the best in the prediction of the liquid holdup than the others. It is observed also, the bad results of the models by Mukherjee -Brill (1985) and Beggs -Brill (1986) in spite of these methods are specified for the inclined flow. Finally, the model of Gozhov et al. (1967) has good results as shown in table (7)

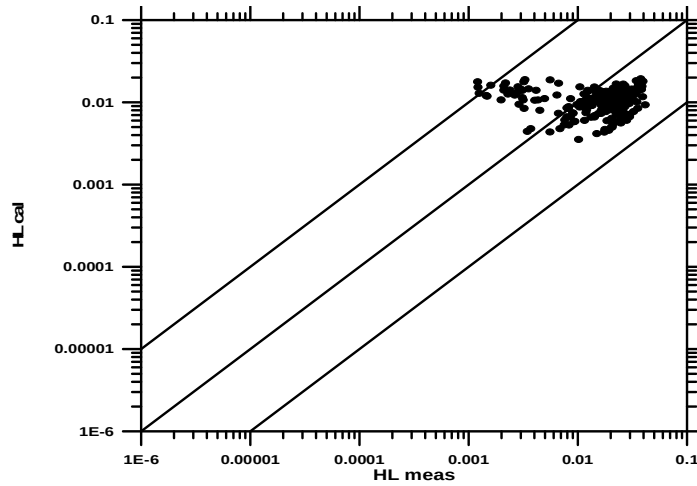


Figure (7): The calculated liquid holdup by the Present model for Inclined flow data.

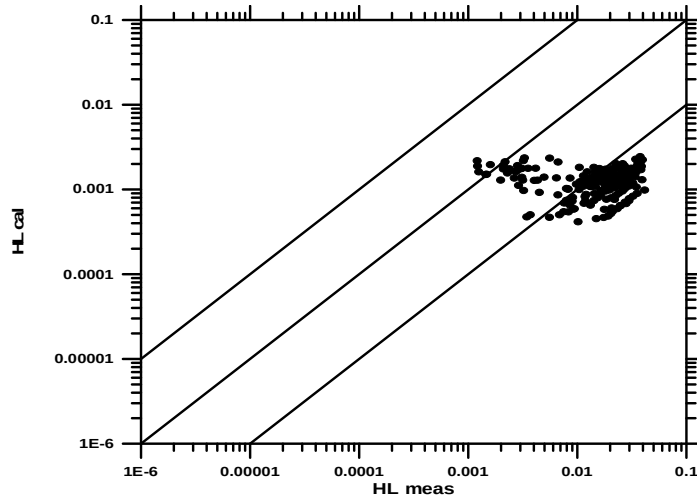


Figure (8): The calculated liquid holdup by Beggs-Brill model for Inclined flow data.

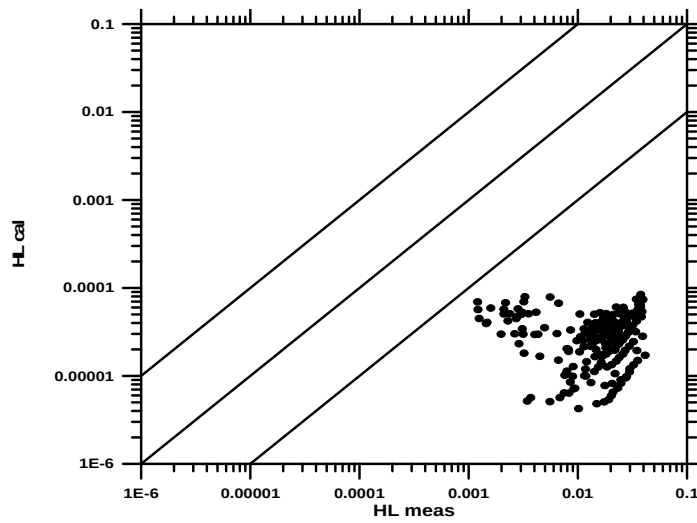


Figure (9): The calculated liquid holdup by Mukherjee-Brill model for Inclined flow data.

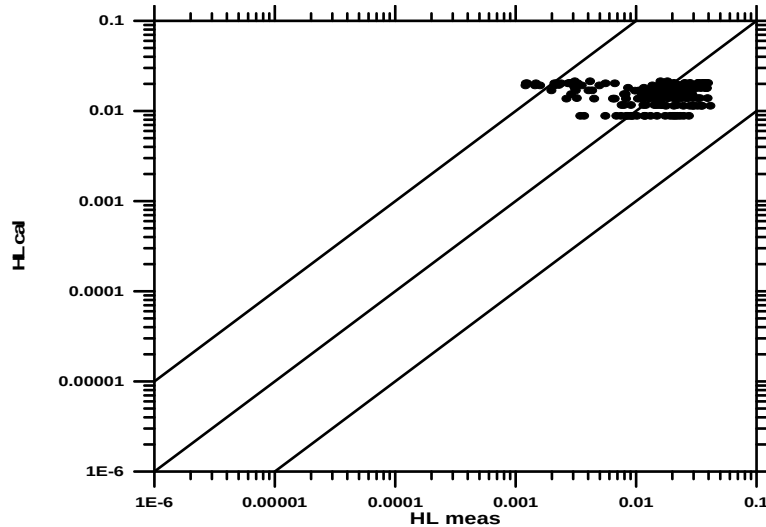


Figure (10): The calculated liquid holdup by Gozhov et al model for Inclined flow data.

Table (7): The Statistical Results Using Inclined Data Only

	The Methods	AE $\hat{1} 10^{-5}$	AAE $\hat{1} 10^{-5}$	SDE $\hat{1} 10^{-5}$	APE	AAPE	SDPE	F _{PR}
1	Present Model	-0.701	0.701	10.63	-0.091	0.091	1.386	0
3	Beggs-Brill	-3.02	3.02	45.9	-0.394	0.394	5.98	5.33
4	Mukherjee-Brill	-3.32	3.32	50.3	-0.432	0.432	6.56	6
5	Guzhov et al	1.686	1.686	25.6	0.22	0.22	3.33	2.26

Conclusions:

The used correlations of Mukherjee-Brill (1985), Beggs-Brill (1986) and Guzhov et al (1967) are developed by regardless the liquid leveling concept, therefore, they gave the bad results, it is found that this factor locates the encountered flow pattern [17]. This concept adopted by the mechanistic model of Taitel-Dukler. The procedure of estimation the liquid holdup in the last model need to an iterative technique. The present work converted this lengthy technique to simple equation including the inclination impact. Therefore, The model of Taitel-Dukler and the present correlation gave convergent results as shown in table (5), and gave best performance comparing with the other methods, as displayed in tables (6) and figure (3) to (6) for horizontal flow, and as shown in table (7), the figures (7) to (10) for inclined flow.

From the whole tables and figures, the following conclusions may be reveal:

1. The results of the present correlation and the model of Taitel-Dukler are very convergent, therefore, it is recommended to use the present method instead of the model of Taitel-Dukler.

2. The results of the present method is the best performance the other used models for both horizontal and inclined flow due to it adopts the effect of the inclination.
3. The magnitudes of the liquid holdup in horizontal flow is larger those in inclined flow because of the inclination effects.

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Nomenclature:

A	cross sectional area	(m ²)
D	pipe diameter	(m)
f	friction factor	(dimensionless)
F _{PR}	relative performance factor	(dimensionless)
H _L	Liquid Holdup	(dimensionless)
$\bar{h}_L$	Liquid level height	(dimensionless)
P _{av}	average pressure	(N/m ²)
QL	liquid discharge	(m ³ /s)
Re	Reynolds number	(dimensionless)
s	phase perimeter	(m)
V	velocity	(m/s)
X	Lockhart-Martinelli Parameter	(dimensionless)

Greek symbols:

ε	Relative roughness	(dimensionless)
τ	Shear stress	(N/m ²)
ρ	Density	(kg/m ³)
μ	Viscosity	(N s/m ²)
θ	Angle of inclination	(deg.)

Subscripts:

wg	wall-gas	wL	wall-liquid
g	gas	L	liquid
i	interfacial		
sg	superficial gas	sL	superficial liquid
t	translation		

Superscripts:

$\approx$	Dimensionless	*	critical value
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