



GRILLAGE ANALYSIS OF STRUCTURES CONSIST OF STEEL I-BEAMS IN TWO DIRECTIONS WITH TRANSVERSE SHEAR EFFECT.

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ABSTRACT:

The present study is concerned with the behavior of structures made up of I-steel beams in two directions. The grillage or grid framework method as simplified method of analysis is used in this study to study deflection and stresses caused by moments from applied normal loads. A method is suggested to derive the required section rigidities (the flexural and torsional rigidities) of the grillage members. Warping effect was included in the calculation of the torsional rigidities of the grillage members and it was found when warping is prevented the results became approach to the experimental results . Also effect on deflections by transverse shearing forces was found to give more acceptable results in comparison with experimental results.

استخدام طريقة المشبكات في تحليل المنشآت المتكونة من عتبات حديدية باتجاهين ودراسة تأثير القص العرضي للعتبات .

الخلاصة:

تهتم الدراسة الحالية بسلوك المنشآت المتكونة من عتبات حديدية باتجاهين. في هذا البحث تم استخدام طريقة المشبكات لدراسة الهطول والاجهادات الناتجة من الأحمال المسلطة. وتم اقتراح طريقة لحساب جساءة المقاطع الانحنائية و الالتوائية للعتبات. وقد تم إدخال تأثير الاعوجاج في حساب جساءة المقطع الالتوائية للعتبات وقد وجد انه عندما يمنع الاعوجاج فإن النتائج تقترب أكثر من النتائج العملية. كذلك تم التوصل إلى نتائج أكثر قبولاً للهطول مقارنة مع النتائج العملية عند إدخال تأثير قوى القص العرضية للعتبات.

1. Introduction

The material steel, is relatively modern human creation and it has been vastly improved both in materials and in methods and types of applications. Steel structures of note at present include the bridges, high-rise steel framed- buildings and towers. This is not to say that steel offers the builder an answer to all structural problems. The other major common building materials (concrete, masonry, and wood) all have their place and in many situations will offer economies that will dictate their use. But for building applications in which the ratio of strength to weight (or the strength per unit weight) must be kept high, steel offers feasible options^[8].

It was found (Cheung et al. 1986) ^[1] that the presence of transverse beams (diaphragms) do reduce the stresses in the longitudinal girders by improving the load distribution over the bridge assumed bolted connection between the transverse diaphragms and the main longitudinal girders. Test on continuous steel bridges by (Kennedy and Grace 1983) have shown that, when transverse steel diaphragms of I-section are welded to the longitudinal girders by means of moment connection, a rigid grid work is formed.

In this paper the structural responses of single-span structures are examined with respect to the use of welded diaphragms. The theoretical analyses were verified and substantiated by results from tests on simple-span model made up by (Kennedy and Grace) ^[6].

2. Theoretical Analysis

Steel structures which consists of I-steel beams connected together in two directions were analyzed by the grillage (or grid-frame work) method using a computer program to study the behavior of steel structures and used to study the deflection and stresses caused by the applied loads. The longitudinal and transverse steel beams are assumed rigidly connected (welded connections), because many engineers thought that welds had reduced fatigue strength, compared with riveted and bolted connections^[5]. The grillage mesh is assumed to be coincident with the center-lines of the main steel beams.

An equivalent grillage of interconnected beams can be constructed to give an adequate behavior of the distribution of forces and deflections within the steel structure, Although the method is necessarily approximate, it has the advantage of almost complete generality. At the joints of the grillage any normal form of restraint to movement may be applied so that any support condition

may be represented. This measure of usefulness, combined with economy in computing, input preparation and interpretation of out put, makes the grillage analogy a popular and widely used method in design offices.

Finally, the grillage analogy involves the representation of effectively a three-dimensional steel structure by a two-dimensional assemblage of discrete one- dimensional interconnected beams in bending and torsion and the proposed method in this work is applied to a practical problem and the results are checked with available solutions by other methods and with the available experimental work. The proposed method is found to give acceptable solution regarding the analysis and design of steel structures.

3. Stiffness Matrix for a Grillage Member

To construct the assembled stiffness matrix of a structure, the stiffness matrix of each individual structural member must be formulated before. The sign conventions used herein are shown in Fig. (1) for a grillage member (1-2). The grillage member has a length of L and a flexural rigidity (EI) and torsional rigidity (GJ). The moments and rotations are assumed positive in clockwise direction from the local coordinates (x',y') viewpoint (or right- hand rule). The forces and the displacements are positive downwards.

The stiffness matrix for a typical member is (6x6). The effect of transverse shear deformation on the deformation of the member can be included by the use of shearing rigidity (GA).

The stiffness matrix $[K']$ which relates the action vector $\{F'\}$ to the nodal displacement vector $\{\delta'\}$ in local coordinates is:

$$\{F'\} = [K'] \{\delta'\} \quad (1)$$

Transformation of the member stiffness matrix $[K']$ from local coordinates to stiffness matrix $[K]$ in global coordinates can be achieved by using the transformation matrix $[T]$ ^[4], where for one node, (node1),

$$\begin{bmatrix} M_{x1} \\ M_{y1} \\ V_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_{x1} \\ M_{y1} \\ V_1 \end{bmatrix} \quad (2a)$$

Thus, for two nodes of grillage beam,

$$[T] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2b)$$

where α is the angle of inclination from the global to the local axis of a grillage member as shown in Fig.2.

The relation between the member stiffness matrix $[K']$ in local coordinates and the member stiffness matrix $[K]$ in global coordinates is:

$$[K] = [T]^T [K'] [T] \quad (2c)$$

The assumed stiffness matrix for the whole grillage is obtained by the assemblage process where the final structure is generated by assembling structures consisting of one member at a time. After imposing the boundary conditions, the solution gives the unknown node rotations and transverse displacements. By back substitutions, the bending and twisting moments and transverse shearing forces in the grillage beams will be obtained ^[2].

So, for the beam element shown in Fig. (1), the stiffness matrix in the local-coordinates, is shown in the following Eq.(3);including the transverse shear effects:

$$\begin{Bmatrix} MX'_1 \\ My'_1 \\ V_1 \\ Mx'_2 \\ My'_2 \\ V_2 \end{Bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & -a_1 & 0 & 0 \\ & a_3 & a_2 & 0 & a_4 & -a_2 \\ & & a_5 & 0 & a_2 & -a_5 \\ & & & a_1 & 0 & 0 \\ & & & & a_3 & -a_2 \\ & & & & & a_5 \end{bmatrix} \begin{Bmatrix} \theta x'_1 \\ \theta y'_2 \\ w_1 \\ \theta x'_2 \\ \theta y'_2 \\ w_2 \end{Bmatrix} \quad (3)$$

Symm

Where g is a factor for transverse shear deflection,

$$g = \frac{12EI_y}{GA_v L^2}$$

$$a_1 = \frac{GJ}{L} \quad a_2 = \frac{6EI_y}{L^2} \frac{1}{1+g} \quad a_3 = \frac{4EI_y}{L} \frac{4+g}{4(1+g)}$$

$$a_4 = \frac{2EI_y}{L} \frac{2-g}{2(1+g)} \quad a_5 = \frac{12EI_y}{L^2} \frac{1}{1+g}$$

Using the transformation matrix [T], the stiffness matrix [K] for the beam in the global coordinates is obtained. Therefore, the governing matrix equation in the global coordinates for the beam including the transverse shear effect is:

$$\{F\} = [K] \{\delta\} \quad (4a)$$

$$\begin{Bmatrix} M_{x_1} \\ M_{y_1} \\ V_1 \\ M_{x_2} \\ M_{y_2} \\ V_2 \end{Bmatrix} = \begin{bmatrix} K_1 & K_2 & K_4 & K_7 & K_8 & K_{10} \\ & K_3 & K_5 & K_8 & K_9 & K_{11} \\ & & K_6 & K_4 & K_5 & K_{12} \\ & & & K_1 & K_2 & K_{10} \\ & & & & K_3 & K_{11} \\ & & & & & K_6 \end{bmatrix} \begin{Bmatrix} \theta_{x_1} \\ \theta_{y_1} \\ w_1 \\ \theta_{x_2} \\ \theta_{y_2} \\ w_2 \end{Bmatrix} \quad (4b)$$

Symm

Where

$$K_1 = \frac{GJ}{L} C_x^2 + \frac{4EI_y}{L} \frac{4+g}{4(1+g)} C_y^2 \quad K_2 = \left[\frac{GJ}{L} - \frac{4EI_y}{L} - \frac{4+g}{4(1+g)} \right] C_x C_y$$

$$K_3 = \frac{GJ}{L} C_y^2 + \frac{4EI_y}{L} \frac{4+g}{4(1+g)} C_x^2 \quad K_4 = -K_{10} = -\frac{6EI_y}{L^2} \frac{1}{(1+g)} C_y$$

$$K_5 = -K_{11} = -\frac{6EI_y}{L^2} \frac{1}{(1+g)} C_x \quad K_6 = -K_{12} = -\frac{12EI_y}{L^2} \frac{1}{(1+g)}$$

$$K_7 = -\frac{GJ}{L}C_x^2 + \frac{2EI_y}{L} \frac{2-g}{2(1+g)} C_y^2 \quad K_8 = \left[-\frac{GJ}{L} - \frac{2EI_y}{L} - \frac{2+g}{2(1+g)} \right] C_x C_y$$

$$K_9 = -\frac{GJ}{L}C_y^2 + \frac{2EI_y}{L} \frac{2-g}{2(1+g)} C_x^2$$

and $C_x = \cos \alpha$; $C_y = \sin \alpha$

4. Evaluation of Elastic Section Rigidities of Grillage Members

The elastic rigidities of the grillage members should be derived from the section properties of the actual composite steel structure so that an adequate behavior of the steel section under the applied loadings can be obtained from the equivalent grillage. The elastic section rigidities required for the sections of the equivalent steel grillage members are as follows:

1-Bending (or flexural) rigidity (EI). 2-Torsional rigidity (GJ). 3-Shearing rigidity (GA_v).

Herein, suggestions are presented for these quantities and adopted in this work.

4.1 Bending (or Flexural) rigidity:

Flexural rigidities of the equivalent grillage members play an important role in the calculation of deflections and in the distribution of moments. In analyzing the steel beam structure by the grillage analogy, the flexural rigidity of the equivalent grillage members is calculated as follows^[9]:

$$EI = E \left[\left(\frac{b_f(h_w + 2t_f)^3}{12} \right) - 2 \left(\frac{\left(\frac{b_f - t_w}{2} \right) h_w^3}{12} \right) \right] \quad (5)$$

4.2 Torsional Rigidity of I-steel Section

The torsional stiffness of I-steel section may be estimated as follow^[3]:

A) **Free to warp:** it is generally accepted that the torsional stiffness of a linearly elastic beam free to warp is given by^{[9] [10]}:-

$$GJ_s = G_s \left[\frac{1}{3} (2b_f t_f^3 + h_w \cdot t_w^3) \right] \quad (6)$$

B) Warping prevented (or restrained): if the beam ends are fixed against warping, then the relationship between the torque and the total angle of twist is given by ^[7]:-

$$\theta_m = \frac{T}{C_w \cdot E_s \cdot \beta_1^3} \left[\beta_1 \cdot L - \frac{2 \tanh(\beta_1 \cdot L)}{2} \right] ,$$

Where

$$\beta_1 = \left[\frac{J_s \cdot G_s}{C_w \cdot E_s} \right]^{\frac{1}{2}}$$

$$C_w = \frac{(h_w + t_f)^2 \cdot t_f \cdot b_f^3}{24}$$

then

$$GJ'_s = \left[\frac{T \cdot L}{\theta_m} \right] \quad (7)$$

where: T=applied torque

L=length of the beam

θ_m =total angle of rotation when the ends of beam are fixed (Rad).

4.3 Shearing Rigidity

The vertical (or transverse) shearing force across a steel section causes the flanges and webs to bend independently out of plane (as a result of shearing deformation). It is known that the transverse shearing deformation is usually small compared with deformation due to bending. But in some cases, such as in

short deep members subjected to high shearing forces, it is necessary to consider the transverse shearing deformation in order to obtain a more accurate description of the behavior of the beam. A shearing rigidity (GA_V) is assigned to the stiffness matrix of a grillage member to take into account the effect of transverse shearing forces on the deformation of that member.

In the grillage analogy, the ability of the steel structure to resist distortion can be approximately achieved by providing the grillage members an equivalent shear area (A_V). The independent bending moments, which are developed in the webs and in the flanges are caused by the shearing forces generated in these components. The transverse shearing rigidity for a steel member in the present work will be computed by two methods as follows:

$$GA_v = G_s \cdot t_w \cdot h_v \quad (8)$$

Where:

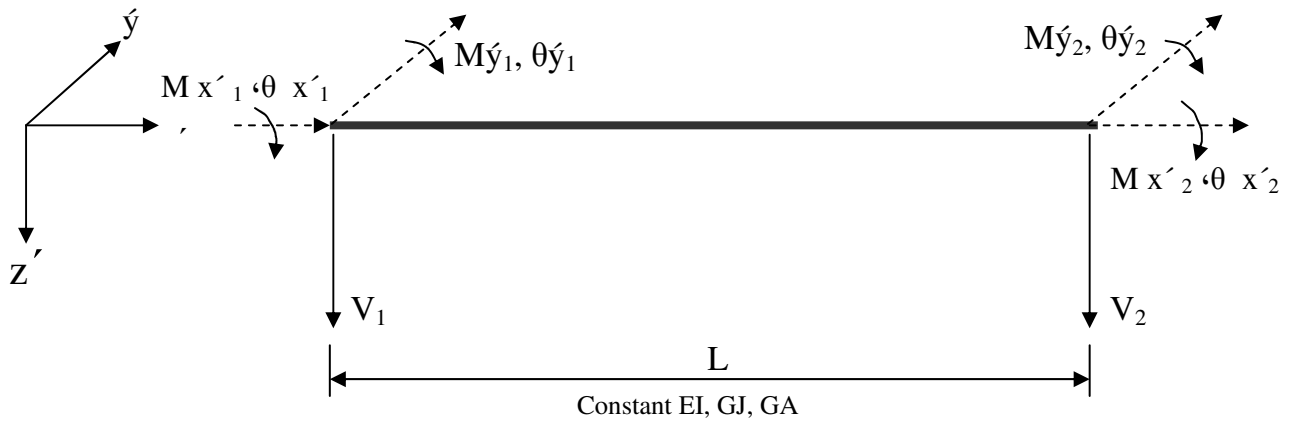


Fig. 1. Grillage beam in local coordinates.

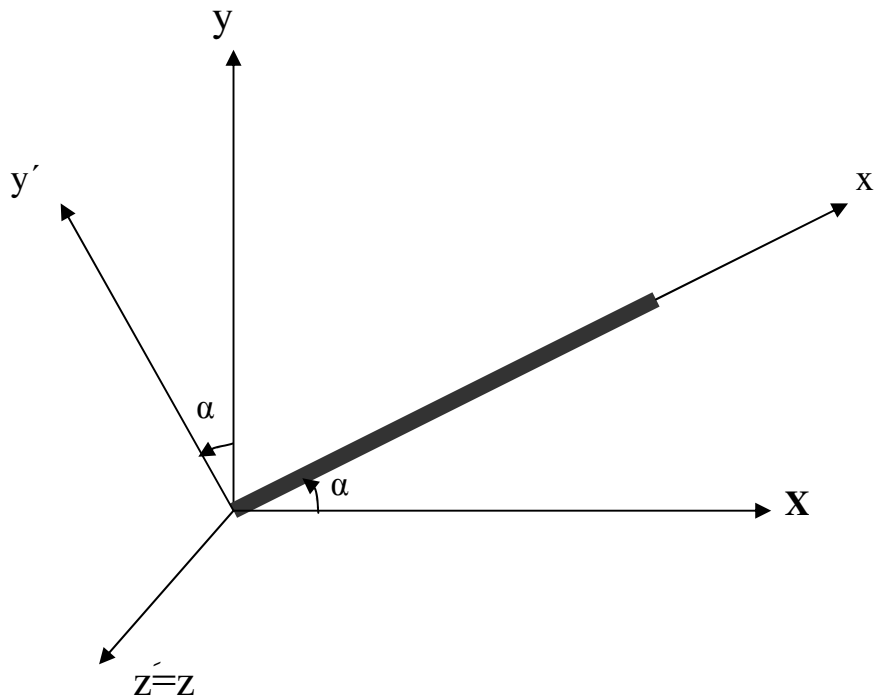


Fig. 2. Relation between global and local coordinates for a grillage beam.

5. Applications

5-1 First Model:

A steel structure is selected from the available reference to assess the accuracy of the grillage method. The theoretical results of Kennedy model^[6] were derived by the finite element method using the orthotropic plate element; also an experimental study was made for this model. The steel model considered here is simply supported at two opposite edges and being free at the longitudinal edges. This type of construction is used in bridge decks, the connection between I-steel beams is welded-diaphragm steel grid. The structure dimensions are shown in Fig.(3), and material properties are as follows:

Longitudinal and transverse steel beams

Depth of steel beam $h_2 = 152.2$ mm

Flange width of steel beam $b_f = 152.2$ mm

Thickness of flange of steel beam $t_f = 6.6$ mm

Thickness of web of steel beam $t_w = 5.84$ mm

Cross sectional area of steel beam $A = 2858$ mm²

Moment of inertia of steel beam $I = 12112334.49$ mm⁴

Modulus of elasticity of steel beam $E = 200000$ MPa

Poisson's ratio of steel beam $\nu = 0.3$

Shear modulus of elasticity of steel beam $G_s = 76923$ N/ mm² (calculated from $G = E/(2(1+\nu))$).

Evaluating the elastic rigidities for each grillage member as given in section (3)

For longitudinal and transverse steel beams:

$$EI = 2.39 \times 10^{12} \text{ N.mm}^2$$

-When the members are free to warp

$$GJ_s = 2.96 \times 10^9 \text{ N.mm}^2$$

-But if the warping is prevented, then:

For longitudinal beams: 1- edge beam $GJ'_s = 8.06 \times 10^{10} \text{ N.mm}^2$

2-Interior beam $GJ'_s = 9.77 \times 10^{10} \text{ N.mm}^2$

For transverse beams $GJ'_s = 1.76 \times 10^{11} \text{ N.mm}^2$

The shearing rigidity is constant for all grid members and it can be calculated as shown:

$$GA_s = 0.0 \text{ (without steel shear area)}$$

$$GA_v = 68.37 \times 10^6 \text{ N (for steel shear area)}$$

The loading condition is considered center point load of 89 kN is applied over the bridge (point no. 13, Fig. (3)).

In Fig.(4), the vertical deflections at the mid- span cross- section (section A-A) are plotted for the loading condition. The comparisons of the maximum deflections in the steel structure as calculated by the suggested method for the loading condition are listed in table (1). In the grillage analysis the maximum deflections are calculated for:

Case (I): with transverse shear effect and the member is free to warp.

Case (II): without steel shear area and the member is free to warp.

Case (III): with steel shear area and warping is prevented.

Case (IV): without steel shear area and warping is prevented.

Tab. 1. Comparisons of maximum deflections (steel model) (percentage differences with respect to experimental results)

Method of analysis		Max. Deflection (mm)	Percentage Difference (%)
Grillage analogy	Case (I)	5.16	1.57
	Case (II)	4.79	5.71
	Case (III)	5.11	0.59
	Case (IV)	4.73	6.88
Orthotropic plate method		5.09	0.20
Experimental result ^[6]		5.08	0.0

From the above comparison, it is clear that when the effect of transverse shear area (A_v) is calculated the deflections obtained by the grillage analogy are rather in acceptable agreement with the experimental and finite element results (applied to the equivalent orthotropic plate). Also this effect is shown in Figure (4). Comparisons between the results are also given in Table (2) also percentage differences with respect to experimental results are listed in table (3).

Tab. 2. Vertical deflections (in mm) at mid- span of steel model under loading condition.

Node no.	Experimental	Ortho.	Grill. case I	Grill. case II	Grill. case III	Grill. Case IV
23	2.79	3.30	3.02	2.91	3.07	2.97
18	3.81	4.06	4.28	4.11	4.26	4.08
13	5.08	5.09	5.16	4.79	5.11	4.73
8	3.81	4.06	4.28	4.11	4.26	4.08
3	2.79	3.05	3.02	2.91	3.07	2.97

Tab. 3. percentage differences with respect to experimental results.

Node no.	Perce. Diff. Of Ortho. (%)	Perce. diff. of Grill. case I (%)	Perce. diff. of Grill. case II (%)	Perce. diff. Of Grill. case III (%)	Perce. diff. Of Grill. Case IV (%)
23	18.27	8.24	4.30	10.03	6.45
18	6.56	12.34	7.87	11.81	7.08
13	0.20	1.57	5.71	0.59	6.88
8	6.56	12.34	7.87	11.81	7.08
3	18.27	8.24	4.30	10.03	3.2

Comparisons between the variations of center deflection with an applied central load shown in table (4) and Fig (5).

Tab. 4. Comparisons between the variations of center deflection with an applied central load.

Method of analysis		Max.Deflection (mm)			
		Load of center (kN)			
		22.25	44.5	66.75	89
Grillage analogy	Case (I)	1.29	2.58	3.87	5.16
	Case (II)	1.19	2.39	3.59	4.79
	Case (III)	1.28	2.55	3.83	5.106
	Case (IV)	1.18	2.37	3.55	4.73
Experimental result ^[6]		1.27	2.54	3.81	5.08

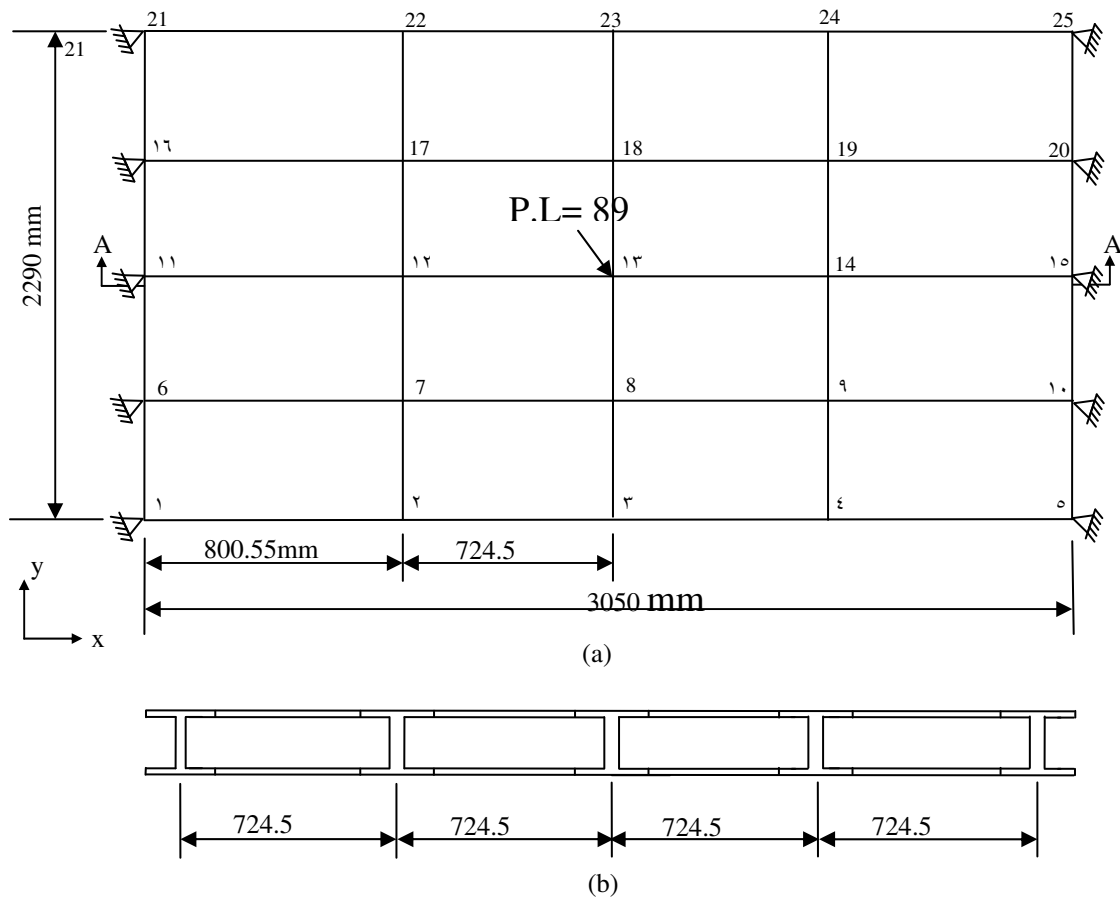


Fig. 3. Details of steel model.(a) Plan view, (b) Section (A-A).

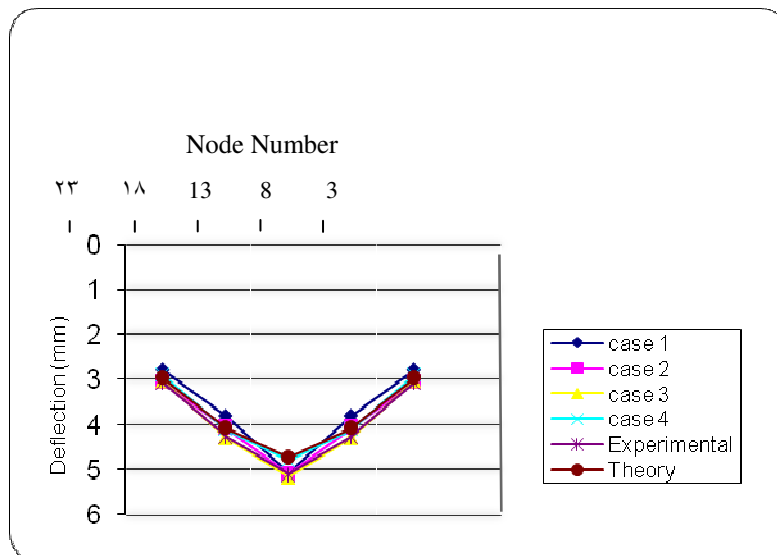


Fig. 4. Vertical deflections at mid span section of steel model.

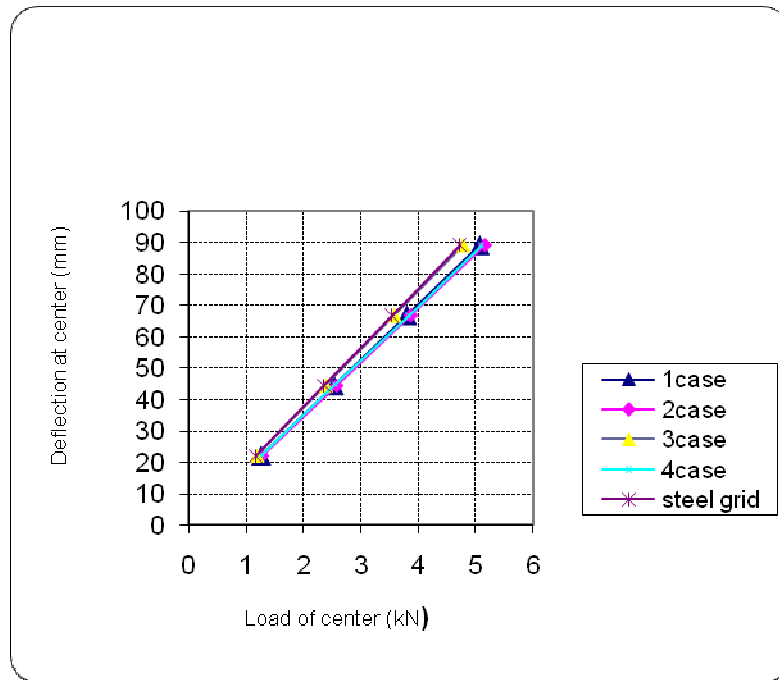


Fig. 5. Deflection curve at center of Kennedy's steel model.

5-2 Second Model:

This model is solved to explain the program working method the steel structure model consist of UB457×152×52 in x-direction and UB305×127×37 in y-direction, all members are assumed to be prevented to warp, $E = 200000 \text{ Mpa}$, $G_s = 82700 \text{ N/mm}^2$, and the details of steel structure and I-steel sections are shown in Fig.6.

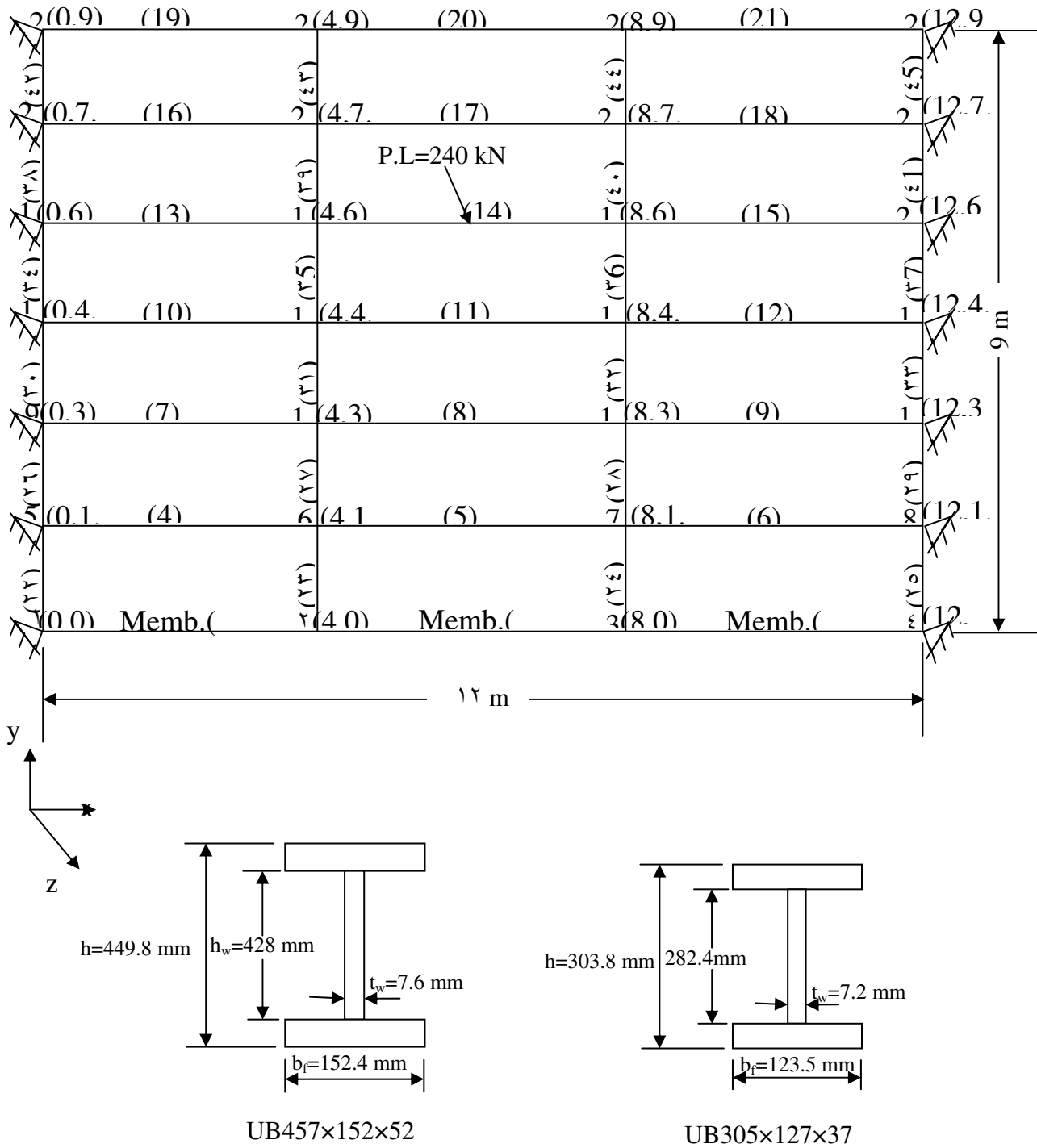


Fig. 6. Details of steel model

Here in the data which input to the program are:

No. of members=45, No. of nodes=28, the coordinates of each node, the incision of each member such like that the incision of member 7 is (9,10).

No. of supports=14 @ nodes(1,5,9,13,17,21,25,4,8,12,16,20,24,28).

Types of supports are simply supports.

The load is transmitted from memb.(14) to nodes (18) and (19) thus the no. of loaded joint=2 @ 18 and 19.

$F_z=120$, $M_x=0$, $M_y=0$

For members (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21):

$EI=4.1937052 \times 10^{13} \text{ N.mm}^2$

$GJ'_s=6.563234524 \times 10^{10} \text{ N.mm}^2$

$GA_v=2.82708296 \times 10^8 \text{ N}$

Formembers(22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45):

$EI=1.405986411 \times 10^{13} \text{ N.mm}^2$

$GJ'_s=9.042315225 \times 10^{10} \text{ N.mm}^2$

$GA_v=1.80894672 \times 10^8 \text{ N}$

OUTPUT

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NODE DISPLACEMENT (mm radians)

NODE	Z-TRANS	X-ROTAT	Y-ROTAT
1	0.000000	-0.000004	-0.001174
2	-3.974746	-0.007890	-0.000589
3	-3.974742	-0.007890	0.000589
4	0.000000	-0.000004	0.001174
5	0.000000	-0.000001	0.002365
6	7.970492	-0.008157	0.001182
7	7.970498	-0.008157	-0.001182
8	0.000000	-0.000001	-0.002365
9	0.000000	-0.000002	0.006086
10	20.512016	-0.008451	0.003043
11	20.512018	-0.008451	-0.003043
12	0.000000	-0.000002	-0.006086
13	0.000000	-0.000002	0.009681
14	32.636337	-0.006999	0.004842
15	32.636330	-0.006999	-0.004842
16	0.000000	-0.000002	-0.009681
17	0.000000	-0.000000	0.011716
18	39.507912	-0.000488	0.005861
19	39.507893	-0.000488	-0.005861
20	0.000000	-0.000000	-0.011716
21	0.000000	0.000001	0.010119
22	34.114380	0.005990	0.005061
23	34.114365	0.005990	-0.005061
24	0.000000	0.000001	-0.010119
25	0.000000	0.000004	0.006986

26	23.530020	0.007435	0.003491
27	23.530022	0.007435	-0.003491
28	0.000000	0.000004	-0.006986

MEMBER END FORCES (kn kn.m)

MEMB.	NODE	SHEAR-Z	TORSION	MOM.-Y
1	1	3.17	0.00	0.00
	2	-3.17	0.00	0.00
2	2	-0.00	0.00	0.00
	3	0.00	0.00	0.00
3	3	-3.17	0.00	0.00
	4	3.17	0.00	0.00
.				
.				
.				
44	23	18.23	0.00	0.00
	27	-18.23	0.00	0.00
45	24	-0.13	0.00	0.00
	28	0.13	0.00	0.00

SUPPORT REACTION (kn kn.m)

NODE	FORCE-Z	MOM.-X	MOM.-Y
1	3.31	0.00	0.00
4	3.31	0.00	0.00
5	-6.25	0.00	0.00
8	-6.25	0.00	0.00
9	-15.95	0.00	0.00
12	-15.95	0.00	0.00
13	-25.47	0.00	0.00
16	-25.47	0.00	0.00
17	-30.88	0.00	0.00
20	-30.88	0.00	0.00
21	-26.67	0.00	0.00
24	-26.67	0.00	0.00
25	-18.10	0.00	0.00
28	-18.10	0.00	0.00

6. Conclusions

The main concluding remarks that have been achieved in this study may be summarized as follow

1. The grillage method can be used to analyze steel structures by using members coinciding with the centerlines of the steel beams. The results of deflections and moments are acceptable for design purposes.
2. The grillage method is suitable at the design stage because of the simplicity and ease of preparing the input and interpretation of the output and the very short computing time.
3. When warping is prevented at the end of members the result came be more acceptable.

4. Effect on deflections by transverse shearing forces is found to give more acceptable results compared with experimental and orthotropic plate results.
5. This method could be applied on any type of materials like wood in the same structures just changing the properties of sections.

7-References:

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