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Parametric solutions for Camassa-Holm equation of fractional order

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Abstract

In this article, we consider a Camassa-Holm equation in $2+1$ dimensions in terms of Jumarie's fractional derivatives. By applying fractional complex wave transform to reduce the number of independent variables, a system of nonlinear ordinary differential equations is obtained. The extended tanh method is next used to construct some types of parametric solutions for the three fields. Kink and antikink solutions are found and plotted for some values of the parameters.

Introduction

Kraenkel el al. [1] derived a higher order Camassa-Holm equation in two spatial dimensions under asymptotic analysis in a perturbative manner; it describes the changing and developing of an unbounded free surface of fluid in the theory of shallow water waves. This is stated by:

$$m_t - um_x - 2mu_x = 0, \tag{1}$$
$$-mu_y + u_x - u_{xx} - 2p_x = 0, \tag{2}$$
$$-m^2p_y + mp_x + mp_{xx} - pm_x - m_xp_x = 0, \tag{3}$$

where $u(x,y,t)$ stands for fluid velocity and $m(x,y,t)$ stands for fluid momentum. When y- dependence is absent, the Eq. 2 and Eq. 3, after integrating, give $p = \frac{1}{2}(u - u_x + \varepsilon)$ and $m = p + p_x$, and as a result Eq. 1 becomes $1+1$ dimensional Camassa-Holm equation [2], that is

$$u_t - 2\varepsilon u_x - u_{xxt} - 3uu_x + 2u_xu_{xx} + uu_{xxx} = 0. \tag{4}$$

Lie symmetry analysis for the system of Camassa-Holm Eqs. 1,2 and 3 was investigated in [3]. The spectral problem and some type of particular solutions were examined by Zenchuk in [4]. Gordoa and his collaborators [5] showed that the Camassa-Holm equation in two spatial dimensions admitted a weak Painlevé property. Lie point symmetries lying on lax pair by using differential form method was sought in [6]

Here, we consider Camassa-Holm equation with space-time Jumarie's fractional derivative. This is given by

$$m_{t^\delta} - u m_{x^\mu} - 2m u_{x^\mu} = 0, \tag{5}$$

$$-m u_{y^\nu} + u_{x^\mu} - u_{x^{2\mu}} - 2p_{x^\mu} = 0, \tag{6}$$

$$-m^2 p_{y^\nu} + m p_{x^\mu} + m p_{x^{2\mu}} - p m_{x^\mu} - m_{x^\mu} p_{x^\mu} = 0, \tag{7}$$

where m, u and p are fields of two spatial dimensions x and y plus time t, and $0 < \delta, \mu, \nu \leq 1$. The subscripts mean the fractional partial derivatives regarding the independent variables. A modified version of Riemann-Liouville fractional derivative was brought in by Jumarie [7], this is expressed by

$$V_{t^\delta} = \frac{\partial^\delta V}{\partial t^\delta} = \begin{cases} \frac{1}{\Gamma(-\delta)} \int_0^t (t-\zeta)^{-\delta-1} \times (V(x,y,\zeta) - V(x,y,0)) d\zeta, \ \delta < 0, \\ \frac{1}{\Gamma(1-\delta)} \frac{d}{dt} \int_0^t (t-\zeta)^{-\delta} \times \left(V(x,y,\zeta) - V(x,y,0)\right) d\zeta, \quad 0 < \delta < 1, \\ (V^{\delta-n}(t))^{(n)}, \ n \leq \delta < n+1, \ n \geq 1, \end{cases} \tag{8}$$

where Γ is Euler's gamma function given by $\Gamma(\upsilon) = \int_0^\infty \tau^{\upsilon-1} e^{-\tau} d\tau, \upsilon \in \mathbb{C}$, with the following formula of Jumarie's fractional derivative [8]

$$D_z^\delta z^\lambda = \Gamma(1+\lambda)\Gamma^{-1}(1+\lambda-\delta) z^{\lambda-\delta}, \lambda > 0. \tag{9}$$

The theory of calculus and equations involving integrals and derivatives of arbitrary non-integer order have been a source of interest to many researchers in variety fields of sciences, such as, mechanical engineering, macroeconomic models, Energy harvesting in dynamical systems, neural networks, fluid mechanics, fractional kinetics of charge carriers in super capacitors, polymer, biosciences, theory of viscoelasticity, thermodynamics, circuit design, control theory, robotics, social sciences, chaotic dynamics, electromagnetic waves, nanotechnology, particle physics, disordered media, optics, etc. [9-14], we also would like to refer the reader to some books in this area [15-23] and references therein. The rest of the paper is lined up as follows, in section two, we overview the concept of fractional complex transform and the extended tanh method which they will be used to reduce the nonlinear system of Camassa-Holm equations of fractional order to nonlinear ordinary differential equations and then solved it. Section three is devoted to gain parametric solutions for fractional Camassa-Holm equation. The last section is conclusion.

Preliminaries

We review, in the current section, the principle of fractional complex transform and the idea of the extended tanh approach for solving differential equations. The fractional complex transformation was introduced in [24]. It is basically employed to convert fractional differential equations of arbitrary non-integer order to ordinary differential equations as they may be appeared with no difficult to solve. The fractional complex transform and its extension [25].

$$\zeta = \frac{\tilde{\alpha}}{\Gamma(1+\mu)} x^\mu + \frac{\tilde{\beta}}{\Gamma(1+\nu)} y^\nu - \frac{\tilde{\gamma}}{\Gamma(1+\delta)} t^\delta, \tag{10}$$

where $\tilde{\alpha}, \tilde{\beta}$ and $\tilde{\gamma}$ are arbitrary constants, and $\Gamma(.)$ is gamma function, they have been a useful tool to get exact solutions of differential equations, see for instance [26-30].

The idea of tanh technique was originally proposed by Malfliet [31], and later the method was modified and generalized by many researchers see for instance [32]. In the next section, we shall use the extended version of tanh method or so called tanh-coth method [33]. In this method, one looks for solution in the following form

$$\Lambda = \sum_{\kappa=0}^{N} a_\kappa X^\kappa + \sum_{\kappa=0}^{N} b_\kappa X^{-\kappa} \tag{11}$$

where a_κ and b_κ, $\kappa = 0,1,2,\cdots,N$ are constants to be obtained, and $X = \tanh(\mu\zeta)$, where μ is a wave number. The number of terms, positive number N, in the ansatz above, Eq. 11, can be deduced by balancing the highest order derivative terms and the nonlinear terms in the given equation. Substituting determined Eq. 11 into given equation and next collecting the coefficients of the $X^\kappa, \kappa = 0, \mp1, \mp2, \mp3, \cdots, \mp N$ and then putting each coefficient to zero to end up with nonlinear algebraic equations can be cracked to get solutions of equations under consideration.

Main Results

We focus, in this section, on solving the nonlinear system of Camassa-Holm equation of fractional order lying on fractional complex wave transform and the extended tanh method. We start off by assuming that the solution of nonlinear Eqs. 5,6 and 7 in the form

$$u(x,y,t) = \Phi(\zeta), m(x,y,t) = \Theta(\zeta), p(x,y,t) = \Psi(\zeta), \tag{12}$$

where ζ is the fractional complex wave transform (10). Using the chain rule

$$D_z^\lambda w = \rho \frac{dW}{d\zeta} D_z^\lambda \zeta,\ w(x,y,t) = W(\zeta), \tag{13}$$

where ρ is a fractional indices [34], and the formula (9), the system of Eqs. 5,6 and 7 becomes

$$-\gamma\Theta_\zeta - \alpha\Phi\Theta_\zeta - 2\alpha\Theta\Phi_\zeta = 0,$$
$$-\beta\Theta\Phi_\zeta + \alpha\Phi_\zeta - \alpha^2\Phi_{\zeta\zeta} - 2\alpha\Psi_\zeta = 0,$$
$$-\beta\Theta^2\Psi_\zeta + \alpha\Theta\Psi_\zeta + \alpha^2\Theta\Psi_{\zeta\zeta} - \alpha\Psi\Theta_\zeta - \alpha^2\Theta_\zeta\Psi_\zeta = 0,$$

where $\alpha = \rho_1\tilde{\alpha}, \beta = \rho_2\tilde{\beta}$ and $\gamma = \rho_3\tilde{\gamma}$. Rewriting the equations

$$-(\gamma\Theta^{\frac{1}{2}} + \alpha\Phi\Theta^{\frac{1}{2}})_\zeta = 0, \tag{14}$$
$$\beta\Theta\Phi_\zeta + (\alpha^2\Phi_\zeta - \alpha\Phi + 2\alpha\Psi)_\zeta = 0, \tag{15}$$
$$(-\beta\Psi + \frac{\alpha\Psi+\alpha^2\Psi_\zeta}{\Theta})_\zeta = 0. \tag{16}$$

Integrating Eq. 14 to get

$$\Theta = \frac{A^2}{(\alpha\Phi+\gamma)^2}, \tag{17}$$

where A is a constant of integration. Plugging Eq. 17 into Eq. 15 and Eq. 16 and simplifying yields

$$\Phi_\zeta = \frac{\beta A^2}{\alpha^3(\alpha\Phi+\gamma)} + \frac{1}{\alpha}\Phi - \frac{2}{\alpha}\Psi + \frac{1}{\alpha^2}C, \tag{18}$$

$$\Psi_\zeta = \frac{\beta A^2}{\alpha^2(\alpha\Phi+\gamma^2)}(B+\beta\Psi) - \frac{1}{\alpha}\Psi, \tag{19}$$

where B and C are constants, and $\alpha \neq 0$. From Eq. 18 we have

$$\Psi = -\frac{\alpha}{2}\Phi_\zeta + \frac{\beta A^2}{2\alpha^2(\alpha\Phi+\gamma)} + \frac{1}{2}\Phi + \frac{1}{2\alpha}C, \tag{20}$$

differentiating Eq. 20 with respect to ζ to get

$$\Psi_\zeta = -\frac{\alpha}{2}\Phi_{\zeta\zeta} - \frac{\beta A^2}{2\alpha(\alpha\Phi+\gamma)^2}\Phi_\zeta + \frac{1}{2}\Phi_\zeta. \tag{21}$$

Inserting the last equation and Eq. 20 into Eq. 19 yields

$$\Phi_{\zeta\zeta} + \frac{A^2(2B\alpha+\beta C-\beta\gamma)}{\alpha^4(\alpha\Phi+\gamma)^2} + \frac{A^4\beta^2}{\alpha^5(\alpha\Phi+\gamma)^3} - \frac{1}{\alpha^2}\Phi - \frac{1}{\alpha^3}C = 0,$$

multiplying by Φ_ζ and integrating to obtain

$$\Phi_\zeta^2 = [\Phi^4 + 2(\frac{\gamma}{\alpha}+\frac{C}{\alpha})\Phi^3 + (\frac{\gamma^2}{\alpha^2}+4\frac{\gamma C}{\alpha^2}+D\alpha^2)\Phi^2 + 2(2A^2\frac{B}{\alpha^3}+\beta\frac{C}{\alpha^4}A^2-\beta\frac{\gamma}{\alpha^4}A^2 + \frac{\gamma^2}{\alpha^3}C+\gamma\alpha D)\Phi + (4A^2B\frac{\gamma}{\alpha^4}+2A^2\beta C\frac{\gamma}{\alpha^5}-2A^2\beta\frac{\gamma^2}{\alpha^5}+A^4\frac{B^2}{\alpha^6}\gamma^2 D]/(\alpha\Phi+\gamma)^2, \tag{22}$$

where D is a constant of integration.

Now, let $\Phi(\zeta) = \Lambda(\sigma)$ where $d\sigma = \frac{d\zeta}{\alpha\Phi+\gamma}$ and

$$\zeta = \int (\alpha\Lambda(\sigma)+\gamma)d\sigma, \tag{23}$$

then Eq. 22 becomes

$$(\frac{d\Lambda}{d\sigma})^2 = [\Lambda^4 + 2(\frac{\gamma}{\alpha}+\frac{C}{\alpha})\Lambda^3 + (\frac{\gamma^2}{\alpha^2}+4\frac{\gamma C}{\alpha^2}+D\alpha^2)\Lambda^2 + 2(2A^2\frac{B}{\alpha^3}+\beta\frac{C}{\alpha^4}A^2-\beta\frac{\gamma}{\alpha^4}A^2 + \frac{\gamma^2}{\alpha^3}C+\gamma\alpha D)\Lambda + (4A^2B\frac{\gamma}{\alpha^4}+2A^2\beta C\frac{\gamma}{\alpha^5}-2A^2\beta\frac{\gamma^2}{\alpha^5}+A^4\frac{B^2}{\alpha^6}\gamma^2 D]. \tag{24}$$

Setting some physical boundary conditions $\Theta, \Theta_\zeta \to 0$ and $\Psi, \Psi_\zeta \to 0$ as $|\zeta| \to \infty$ in Eq. 18 and Eq. 19 to get the relation

$$\frac{\beta A^2}{\gamma} + C = 0 \quad \text{and} \quad \frac{BA^2}{\gamma^2} = 0,\ \gamma \neq 0.$$

From this relation, it is easy to see that $A \neq 0$, since if $A = 0$ then that makes $\Theta = 0$ and Eq. 17 becomes ill-defined so must be $B = 0$. By taking $A = 1, B = 0, C = -\frac{1}{2}, D = \frac{9}{4}, \alpha = \beta = 1$ and $\gamma = 2$ in Eq. 24 that gives

$$(\frac{d\Lambda}{d\sigma})^2 = \Lambda^4 + 3\Lambda^3 + \frac{9}{4}\Lambda^2. \tag{25}$$

Upon balancing $(\frac{d\Lambda}{d\sigma})^2$ that has the exponent $(N+1)^2$ with Λ^2 that has the exponent $4N$, that is $(N+1)^2 = 4N$ which gives $N = 1$, and from Eq. 11, the extend tanh method asserts the finite expansion

$$\Lambda = a_0 + a_1X + \frac{b_1}{X} \tag{26}$$

where a_0, a_1 and b_1 are constant to be gained, and $X = \tanh(\mu\zeta)$, and μ is a wave number to be found. Plugging Eq. 26 into Eq. 25 and collecting the coefficients of like powers of $X^i, i = 0, \mp 1, \mp 2, \mp 3, \mp 4$ to get the following nonlinear algebraic equations

$$\mu^2 a_1^2 - a_1^4 = 0, -4a_0 b_1^3 - 3b_1^3 = 0, \mu^2 b_1^2 - b_1^4 = 0, -4a_0 a_1^3 - 3a_1^3 = 0,$$
$$-4a_0^3 a_1 - 12a_0 a_1^2 b_1 - 9a_0^2 b_1 - 9a_1^2 b_1 - \tfrac{9}{2} a_0 a_1 = 0,$$
$$-2\mu^2 a_1^2 - 2\mu^2 a_1 b_1 - 6a_0^2 a_1^2 - 4a_1^3 b_1 - 9a_0 a_1^2 - \tfrac{9}{4} a_1^2 = 0,$$
$$-4a_0^3 b_1 - 12a_0 a_1 b_1^2 - 9a_0^2 b_1 - 9a_1 b_1^2 - \tfrac{9}{2} a_0 b_1 = 0,$$
$$-2\mu^2 a_1 b_1 - 2\mu^2 b_1^2 - 6a_0^2 b_1^2 - 4a_1 b_1^3 - 9a_0 b_1^2 - \tfrac{9}{4} b_1^2 = 0,$$
$$\mu^2 a_1^2 + 4\mu^2 a_1 b_1 + \mu^2 b_1^2 - a_0^4 - 12a_0^2 a_1 b_1 - 6a_1^2 b_1^2 - 3a_0^3 - 18a_0 a_1 b_1 - \tfrac{9}{4} a_0^2 - \tfrac{9}{2} a_1 b_1 = 0.$$

Solving the algebraic equations to get the following set of solutions

The first set of solution is $\mu = \mp\frac{3}{4}, \quad a_0 = -\frac{3}{4}, \quad a_1 = 0, \quad b_1 = \frac{3}{4}$.

The second set of solution is $\mu = \mp\frac{3}{4}, \quad a_0 = -\frac{3}{4}, \quad a_1 = 0, \quad b_1 = -\frac{3}{4}$.

The third set of solution is $\mu = \mp\frac{3}{4}, \quad a_0 = -\frac{3}{4}, \quad a_1 = \frac{3}{4}, \quad b_1 = 0$.

The fourth set of solution is $\mu = \mp\frac{3}{4}, \quad a_0 = -\frac{3}{4}, \quad a_1 = -\frac{3}{4}, \quad b_1 = 0$.

The fifth set of solution is $\mu = \mp\frac{3}{8}, \quad a_0 = -\frac{3}{4}, \quad a_1 = \frac{3}{8}, \quad b_1 = \frac{3}{8}$.

The sixth set of solution is $\mu = \mp\frac{3}{8}, \quad a_0 = -\frac{3}{4}, \quad a_1 = -\frac{3}{8}, \quad b_1 = -\frac{3}{8}$.

The corresponding family of solutions are

$$\Lambda = -\tfrac{3}{4} - \tfrac{3}{4}\tanh(\tfrac{3}{4}\sigma), \quad \Lambda = -\tfrac{3}{4} + \tfrac{3}{4}\tanh(\tfrac{3}{4}\sigma), \tag{27}$$
$$\Lambda = -\tfrac{3}{4} + \tfrac{3}{4}\coth(\tfrac{3}{4}\sigma), \quad \Lambda = -\tfrac{3}{4} - \tfrac{3}{8}\tanh(\tfrac{3}{4}\sigma) - \tfrac{3}{8}\coth(\tfrac{3}{4}\sigma), \tag{28}$$
$$\Lambda = -\tfrac{3}{4} + \tfrac{3}{8}\tanh(\tfrac{3}{4}\sigma) + \tfrac{3}{8}\coth(\tfrac{3}{4}\sigma), \quad \Lambda = -\tfrac{3}{4} - \tfrac{3}{4}\coth(\tfrac{3}{4}\sigma). \tag{29}$$

The corresponding ζ can be calculated from Eq. 23 as listed below

$$\zeta = \tfrac{5}{4}\sigma + \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) - 1] + \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) + 1],$$
$$\zeta = \tfrac{5}{4}\sigma - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) - 1] - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) + 1],$$
$$\zeta = \tfrac{5}{4}\sigma - \ln[\tanh(\tfrac{3}{4}\sigma) - 1] + \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) + 1] + \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) - 1],$$
$$\zeta = \tfrac{5}{4}\sigma + \ln[\tanh(\tfrac{3}{4}\sigma) - 1] - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) + 1] - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{4}\sigma) - 1],$$
$$\zeta = \tfrac{5}{4}\sigma - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{8}\sigma) - 1] - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{8}\sigma) + 1] - \tfrac{1}{2}\ln[\coth(\tfrac{3}{8}\sigma) - 1]$$
$$- \tfrac{1}{2}\ln[\coth(\tfrac{3}{8}\sigma) + 1],$$
$$\zeta = \tfrac{5}{4}\sigma + \tfrac{1}{2}\ln[\tanh(\tfrac{3}{8}\sigma) - 1] - \tfrac{1}{2}\ln[\tanh(\tfrac{3}{8}\sigma) + 1] + \tfrac{1}{2}\ln[\coth(\tfrac{3}{8}\sigma) - 1]$$
$$+ \tfrac{1}{2}\ln[\coth(\tfrac{3}{8}\sigma) + 1].$$

The solutions for the two fields Θ and Ψ can be concluded from Eq. 17 and Eq. 20 respectively. In the figurs, 1, 2, 3 and 4, we draw the parametric solutions (27); they represent antikink and kink type solutions and the parametric solutions (28); they represent wave type solutions respectively.

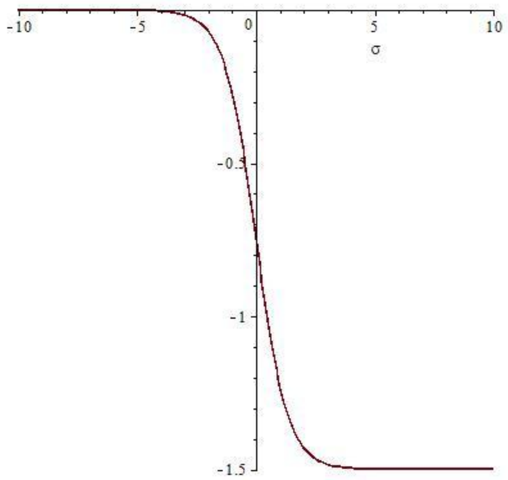

Fig. 1: Antikink type solution

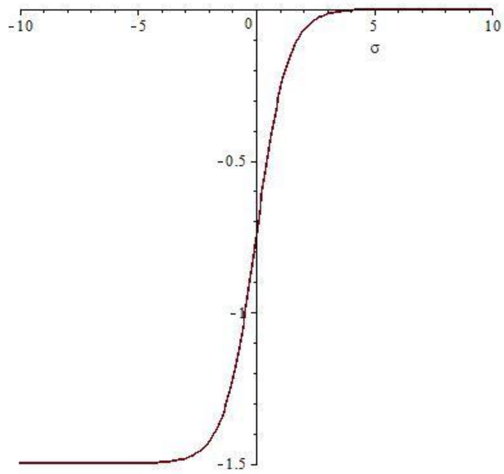

Fig. 2: Kink type solution

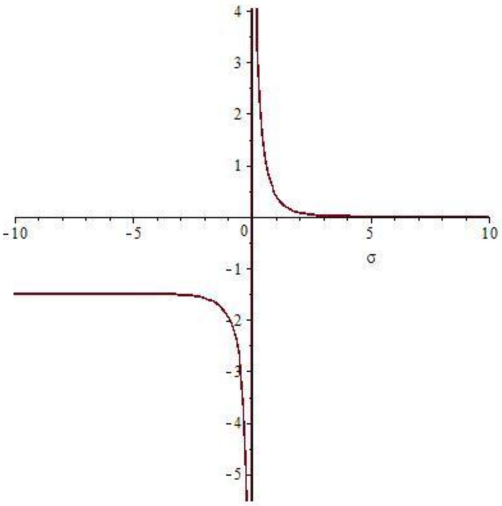

Fig. 3: Wave type solution

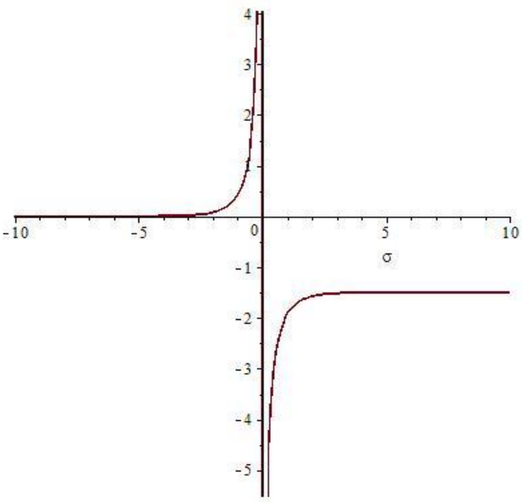

Fig. 4: Wave type solution

Conclusion

We have dealt with Camassa-Holm equation in terms of Jumarie's fractional derivative. The fractional complex transform to the independent variables is used to decrease the number of independent variables to only one, as a result, the nonlinear system of Camassa-Holm equation of fractional order is converted to a nonlinear system of differential equations. The last system is solved by adopting the extended tanh method. Type of parametric solutions are established, kink, antikink and wave solutions are extracted and plotted for some values of the parameters. For other values of the parameters, one may have different types of solutions and that will be a task we intend to look at in the near future.

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الحلول المترية لمعادلة كاماسا - هولم ذات الرتبة الكسرية

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معادلة كاماسا – هولم في 2+1 من الابعاد، مشتقات جوماري الكسرية، تحويل الموجة الكسرية المعقدة، طريقة دالة الظل الزائدية الموسعة، الحلول المترية.

الخلاصة:

في هذا البحث، تناولنا معادلة كاماسا – هولم في 2+1 من الابعاد بدلالة مشتقات جورماري الكسرية. استخدمنا تحويل الموجة المعقدة الكسرية للمعادلة التفاضلية الجزئية الكسرية لتقليل عدد المتغيرات المستقلة وبالنتيجة تم الحصول على نظام من المعادلات التفاضلية الاعتيادية غير الخطية. طريقة دالة الظل الزائدية الموسعة استخدمت للحصول على بعض أنواع الحلول المترية للحقول الثلاثة. بعض أنواع الحلول الموجية رمست لبعض قيم المعلمات.