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DESIGN OF NONLINEAR ROBUST PROPORTIONAL CONTROLLER FOR ACTIVE BRAKING SYSTEM

Dr. Mohsin N. Hamzah*, Dr. Shibly Ahmed Al-Samarraie,**
and YasirKhudhair Abbas**

* University of Technology/ Machinery and Equipments Eng. Dept..

** University of Technology/ Control and Systems Eng. Dept.

ABSTRACT

This work presents nonlinear proportional feedback controller design for the active braking system. The controller objective is to maximize the deceleration braking force by forcing the slip ratio to attain the maximum value, where a magic formula is utilized to model the friction force to slip ratio for various road conditions. The validity of the proposed controller is proved, first, based on Liapunov function approach taking into consideration the uncertainty in system model. Then, the simulation results using MATLAB/Simulink showed the effectiveness of the proposed nonlinear controller in reducing the vehicle velocity to the desired velocity (5 km/hr) in a minimal period of time in presence of model uncertainty and for various road conditions.

Keywords: Braking System Model, Active Control Design, Magic Formula, & Robust Control.

تصميم مسيطر تناسبي لاخطي متين لمنظومة فرملة فعالة

الخلاصــة

يقدم هذا العمل تصميم لمسيطر تناسبي لاخطي ذو تغذية استرجاعية لمنظومة فرملة فعالة. هدف المسيطر هو تعظيم قوة الفرملة التباطؤية بقسر نسبة الانزلاق للوصول الى اعلى قيمة لها، حيث تم استخدام ما يسمى "بالمعادلة السحرية" لنمذجة قوة الاحتكاك لنسب الانزلاق لمختلف الظروف للطرق. تم اثبات صلاحية المسيطر المقترح، اولا، وذلك بالاعتماد على طريقة دالة ليابونوف آخذين بعين الاعتبار مبدأالريبةفي نموذج المنظومة. بعدها، تم عمل محكاة باستخدام (MATLAB/Simulink)،حيث اظهرت النتائج الخاصة بالمحاكاة فعالية المسيطر اللاخطي المقترح في تخفيض سرعة المركبة الى السرعة المرغوبة (5كم/ساعة) في فترة زمنية قياسية وبوجود مبدأ الريبة في النموذج ولمختلف انواع الطرق او ظروفها.

NOMENCLATURE

λ	Longitudinal wheel slip	
$\dot{\omega}_f$, $\dot{\omega}_r$	Angular decelerations of the front and rear wheels	
ω_f, ω_r	Angular speed of the front and rear wheels	
F_x	Braking force	N
F_y	Lateral force	N
F_z	Vertical force	N
F_{xf}	Front longitudinal tyre–road contact force	N
F_{xr}	Rear longitudinal tyre–road contact force	N
J_f	Polar moment of inertia of the front wheels	kg.m^2
J_r	Polar moment of inertia of the rear wheels	kg.m^2
m	Mass of the vehicle	kg
r	Effective radius	m
v	Longitudinal speed of m	m/s
$\dot{v}$	Longitudinal deceleration of m	m/s^2
T_f	Front braking torque	N.m.
T_r	Rear braking torque	N.m.

1. INTRODUCTION

Active braking system is a very vital electro-mechanical system that greatly improves the stability of a vehicle in extreme circumstances, since it can maximize the longitudinal tyre–road friction while keeping large lateral forces that ensure vehicle drivability.

The design of active braking control systems is highly dependent on the braking system characteristics. As is well known, standard ABS systems for wheeled vehicles equipped with traditional hydraulic actuators mainly use rule-based control logics [1].In designing such kind of braking system, it is required that the system should react quickly enough to the wheels locking to lessen the stopping distance or to provide greater vehicle control in a panic situation and reduce skidding as possible.

Recently, the vehicle by which a safety device like anti-lock braking system is carried has spread. Wheels lock if the driver brakes suddenly. If the wheels lock, it will become impossible to control the direction of wheel. Therefore, even if it turns wheel to the right, vehicle runs in unstable manner and cannot be controlled. Then, the driver pumped the brake and need to take care not to lock it. Even if he presses the brake, control which wheel does not lock is performed automatically. This is the anti-lock braking system. This shows power on the road surface which got wet to rain, snowy, and the griddle road used at the time of road repairing [2].

Different control techniques where used to improve the braking system design or to minimize the stopping distance, the work of Schinkel and Hunt[3] started with a brief history of the design of anti-lock brakes (ABS). They explained the advantages of ABS. For their analysis and controller design they used a sliding mode approach with a non-linear longitudinal car model were derived. They showed that the dynamics can be separated conveniently into two different linear dynamics dependent on the tyre slip. The analysis of the dynamics shows the highest possible braking performance. It is further shown that a continuous feedback law will not achieve the maximum braking performance. They, also, suggest a sliding mode like controller design approach to achieve the maximum braking performance.

Wang *et. al.*[4] proposed a robust sliding mode-like fuzzy logic controller for an anti-lock brake system (ABS) with self-tuning of the dead-zone parameters. The main control strategy was to force the wheel slip ratio tracking the optimum value 0.2 as they assumed. The proposed controller for anti-lock braking systems provides a stable and reliable performance under the uncertainties in vehicle brake systems.

Driving safety, increasing stability and reduces fatal traffic accidents are main features of good design of anti-lock braking systems which tackled by many researchers [5, 6]. A more recent works were done by different researchers to satisfy these objectives. Kasahara*et al.* [2] used the sliding mode control for the braking control, the model used was quarter vehicle model, the aim was to design a high robustness for disturbance that fulfills matching conditions with a try to attain optimal braking control to switch wheel speed following to slip ratio following. A more recent works were done by Junwei and Jian [7], They deal with the strong nonlinearity in the design of ABS controller, a variable structure controller has been designed and index reaching law and integral switching surface with saturation function methods are used to reduce chattering. They considered in their analysis several situations such as braking in dry road, wet road and snow road.

The main objective of the present paper is to design a nonlinear robust proportional controller for the active braking system to maximize braking force to attain optimal value; the proposed controller is based on Liapunov function taking into consideration the uncertainty in system model. The analysis is based on a half-vehicle model taking into account the load transfer effect. Two separated brake torques for front and rear wheels are important considered parameters for vehicle dynamic.

2. FRICTION MODEL

The tyre allows contact between the rigid part of the wheel – the hub – and the road surface to take place on all surfaces and in every road condition. Three forces are identified between the tyre and the road, the vertical force F_z, traction and braking force F_x and lateral force F_y, as shown in Figure 1.

The forces F_x and F_y are depending on many factors according to road, tyre, and suspensions. Most often, they can be described as:

$$F_x = f(F_z, \alpha_t, \gamma, \lambda) \quad \textbf{(1)}$$

$$F_y = f(F_z, \alpha_t, \gamma, \lambda) \quad \textbf{(2)}$$

Where α_t is the tyre sideslip angle which represents the angle between the tyre longitudinal axis and the speed vector of the contact point, as shown in Figure 1, γ is the camber angle, λ is the longitudinal wheel slip, *i.e.*, the normalized relative velocity between the road and the tyre, which, in case of zero tyre sideslip angle may be expressed as:

$$\lambda = \frac{v - r\omega}{v} \quad \textbf{(3)}$$

Where v is the wheel ground contact point velocity and $r\omega$ is the linear speed of the tyre (with radius r and angular speed ω) at the contact point.

Physically$\lambda \in [0,1]$. In particular, $\lambda = 0$ corresponds to a pure rolling wheel and $\lambda = 1$ to a locked wheel.

In this work it is assumed that sideslip and camber angles are small, *i.e.*, $\alpha_t \cong 0$ and $\gamma \cong 0$and thus consider the longitudinal force only. Therefore, the longitudinal tyre–road force can be expressed as:

$$F_x = F_z \mu(\lambda) \quad \textbf{(4)}$$

The friction model employed in the current work is the Burckhardt model [8] or the so called the magic formula, as it is particularly suitable for analytical purposes while retaining a good degree of accuracy in the description of the friction coefficient$\mu(\lambda)$. Based on this model, the longitudinal coefficient has the following form:

$$\mu(\lambda;\vartheta) = \vartheta_1\left(1 - e^{-\lambda\vartheta_2}\right) - \lambda\vartheta_3 \tag{5}$$

Notice that the vector θ has three elements only. By changing the values of these three parameters, many different tyre–road friction conditions can be modeled. Table 1gives the parameters values of θ for four different roads.

3. VEIHCLE BRAKE MODEL

In the present work a half-vehicle brake model is used for better model and designs the active braking system by considering the load transfer effect. The yaw, and rolling rotations are assumed to be small and neglected in the current analysis, and only the bounce and pitching motions are considered. Consider now the vehicle shown in Figure 2, the equations of motions for this vehicle may be written as;

$$\begin{aligned} rF_{xf} - T_f &= J_f\dot{\omega}_f \\ rF_{xr} - T_r &= J_r\dot{\omega}_r \\ m\dot{v} &= -(F_{xf} + F_{xr}) \end{aligned} \tag{6}$$

Where $\dot{\omega}_f$ and $\dot{\omega}_r$are the angular decelerations of the front and rear wheels due to braking, respectively. v is the longitudinal speed of the vehicle center of mass.T_f and $T_r$are the front and rear braking torques, respectively.F_{xf} and$F_{xr}$are the front and rear longitudinal tyre–road contact forces, respectively,$J_f$and$J_r$are the polar moment of inertia of the front and rear wheels, respectively. r is the effective radius of the front and rear wheels. Note that, for simplicity, it is assumed that the front and rear wheels effective radii are equal.

Similarly,

$$\begin{aligned} F_{zf} + F_{zr} &= mg \\ F_{zr}d_r - F_{zf}d_f &= m\dot{v}h \end{aligned} \tag{7}$$

which yields

$$\begin{aligned} F_{zf} &= \frac{m}{(d_f + d_r)}(gd_r - \dot{v}h) \\ F_{zr} &= \frac{m}{(d_f + d_r)}(gd_f + \dot{v}h) \end{aligned} \tag{8}$$

Differentiating equation (3) and simplifying yields,

$$\dot{\lambda} = \left(\frac{r}{v^2}\right)(\omega\dot{v} - \dot{\omega}v) \tag{9}$$

Which can be expressed for the front and rear wheels as

$$\begin{aligned} \dot{\lambda}_f &= \left(\frac{r}{v^2}\right)(\omega_f\dot{v} - \dot{\omega}_f v) \\ \dot{\lambda}_r &= \left(\frac{r}{v^2}\right)(\omega_r\dot{v} - \dot{\omega}_r v) \end{aligned} \tag{10}$$

Equation (3) linked the state variables v and ω_f or ω_r with λ_f or λ_r. Therefore, using the third equation of equations (6) with equations (4) and (8) yields

$$\dot{v} = -\frac{g[d_r\mu(\lambda_f)+d_f\mu(\lambda_r)]}{[d_f+d_r]-h[\mu(\lambda_f)-\mu(\lambda_r)]} \quad \textbf{(11)}$$

Using the first two equations of equations (6) together with equations (8), (3), and (10).This leads to

$$\dot{\lambda}_f = \left(\frac{r}{vJ_f}\right)(\mathcal{H}_f\dot{v} - \mathcal{R}_f + T_f)$$
$$\dot{\lambda}_r = \left(\frac{r}{vJ_r}\right)(\mathcal{H}_r\dot{v} - \mathcal{R}_r + T_r) \quad \textbf{(12)}$$

or

(13)

Where,

$$\mathcal{H}_f = \frac{J_f}{r}(1-\lambda_f) + \frac{mhr}{[d_f+d_r]}\mu(\lambda_f)\ , \mathcal{H}_r = \frac{J_r}{r}(1-\lambda_r) + \frac{mhr}{[d_f+d_r]}\mu(\lambda_r)\ ,$$

$$\mathcal{R}_f = \frac{mgrd_r}{[d_f+d_r]} \quad , \quad \mathcal{R}_r = \frac{mgrd_f}{[d_f+d_r]},$$

$$F_f = \left(\frac{r}{vJ_f}\right)(\mathcal{H}_f\dot{v} - \mathcal{R}_f) \ , \quad F_r = \left(\frac{r}{vJ_r}\right)(\mathcal{H}_r\dot{v} - \mathcal{R}_r)\ ,$$

$$G_f = \left(\frac{r}{vJ_f}\right) \quad \text{and} \quad G_r = \left(\frac{r}{vJ_r}\right)$$

4. BRAKING CONTROL DESIGN

In this section a nonlinear proportional controller (NP) is proposed for the front and rear braking torques. The nonlinear proportional term will use a saturation function in its formula. This will make the controller nonlinear and add robustness for the proposed controller. Namely:

$$T_f = \quad -T_{fo} - kp_f * Sat_{\varepsilon_{1f}}(e_f) \quad \textbf{(14-a)}$$

and

$$T_r = \quad -T_{ro} - kp_r * Sat_{\varepsilon_{1r}}(e_r) \quad \textbf{(14-b)}$$

Where$kp_f$and kp_r are the gains of the proportional controller terms for the front / rear braking torques respectively, alsoe_f and e_r are the error function for the front and rear slip ratio. The function$Sat_\varepsilon(e)$,in the control formula, is the saturation function defined by:

$$Sat_{\varepsilon_f}(e_f) = \begin{cases} 1 & |e_f| > \varepsilon_f \\ \frac{e_f}{\varepsilon_f} & |e_f| \le \varepsilon_f \end{cases} \quad \textbf{(15-a)}$$

$$Sat_{\varepsilon_r}(e_r) = \begin{cases} 1 & |e_r| > \varepsilon_r \\ \frac{e_r}{\varepsilon_r} & |e_r| \le \varepsilon_r \end{cases} \quad \textbf{(15-b)}$$

In addition, the braking torques$T_{fo}$and $T_{ro}$is the torques used to cancel the effect of the functions$F_f/G_f$and $F_r/G_r$with nominal system parameter values.

The control law parameters could be determined by considering the candidate Liapunov function:

$$V(e_f, e_r) = \frac{1}{2}e_f^2 + \frac{1}{2}e_r^2 \qquad (16)$$

The error functions e_f and e_r are defined by

$$\left.\begin{aligned} e_f &= \lambda_f - \lambda_{fd} \\ e_r &= \lambda_r - \lambda_{rd} \end{aligned}\right\} \qquad (17)$$

Where λ_{fd} and λ_{rd} are the reference slip ratios for the front and rear wheels respectively.

The derivative of the Liapunov function is:

$$\dot{V} = e_f\dot{e}_f + e_r\dot{e}_r = e_f\dot{\lambda}_f + e_r\dot{\lambda}_r$$

$$= e_f(F_f + G_fT_f) + e_r(F_r + G_rT_r)$$

$$= G_fe_f\left(\frac{F_f}{G_f} + T_f\right) + G_re_r\left(\frac{F_r}{G_r} + T_r\right) \qquad (18)$$

Let the functions $\frac{F_f}{G_f}$ and $\frac{F_r}{G_r}$ be written as a sum of certain (nominal) and uncertain terms as in the following:

$$\frac{F_f}{G_f} = \left(\frac{F_f}{G_f}\right)_o + \left(\frac{F_f}{G_f}\right)_\delta \qquad (19\text{-a})$$

$$\frac{F_r}{G_r} = \left(\frac{F_r}{G_r}\right)_o + \left(\frac{F_r}{G_r}\right)_\delta \qquad (19\text{-b})$$

where the subscript$o$ refer to the nominal term whileδ refer to the uncertain term.

By substituting equations (14) and (19) into equation (18) gives:

$$\dot{V} = G_fe_f\left\{\left(\frac{F_f}{G_f}\right)_o + \left(\frac{F_f}{G_f}\right)_\delta - T_{fo} - kp_f * Sat_{\varepsilon_f}(e_f)\right\}$$

$$+G_re_r\left\{\left(\frac{F_r}{G_r}\right)_o + \left(\frac{F_r}{G_r}\right)_\delta - T_{ro} - kp_r * Sat_{\varepsilon_r}(e_r)\right\}$$

Let $T_{fo} = \left(\frac{F_f}{G_f}\right)_o$ and $T_{ro} = \left(\frac{F_r}{G_r}\right)_o$, then $\dot{V}$ becomes:

$$\dot{V} = G_fe_f\left\{\left(\frac{F_f}{G_f}\right)_\delta - kp_f * Sat_{\varepsilon_f}(e_f)\right\} + G_re_r\left\{\left(\frac{F_r}{G_r}\right)_\delta - kp_r * Sat_{\varepsilon_r}(e_r)\right\}$$

Now consider the following:

$G_f, G_r > 0$

$e_f = |e_f| * sgn(e_f)$ and $e_r = |e_r| * sgn(e_r)$

$sgn(e_f) * Sat_{\varepsilon_f}(e_f) = Sat_{\varepsilon_f}(|e_f|) \geq 0$ and

$sgn(e_r) * Sat_{\varepsilon_r}(e_r) = Sat_{\varepsilon_r}(|e_r|) \geq 0$

consequently, we get

$$\dot{V} = -G_f|e_f|\left\{kp_f * Sat_{\varepsilon_f}(|e_f|) - sgn(e_f) * \left(\frac{F_f}{G_f}\right)_\delta\right\}$$
$$-G_r|e_r|\left\{kp_r * Sat_{\varepsilon_r}(|e_r|) - sgn(e_r) * \left(\frac{F_r}{G_r}\right)_\delta\right\}$$
$$\leq -G_f|e_f|\left\{kp_f * Sat_{\varepsilon_f}(|e_f|) - \left|\left(\frac{F_f}{G_f}\right)_\delta\right|\right\}$$
$$-G_r|e_r|\left\{kp_r * Sat_{\varepsilon_r}(|e_r|) - \left|\left(\frac{F_r}{G_r}\right)_\delta\right|\right\}$$

For $|e_f| \geq \varepsilon_f$ and $|e_r| \geq \varepsilon_r$ we have

$Sat_{\varepsilon_f}(|e_f|) = 1$ and $Sat_{\varepsilon_r}(|e_r|) = 1$

and define the following set

$$\Phi_{\varepsilon_f,\varepsilon_r} = \{(\lambda_f, \lambda_f): |e_f| < \varepsilon_f \text{ and } |e_r| < \varepsilon_r\} \tag{20}$$

Now for the following choice of the proportional gains:

$$kp_f > \left|\left(\frac{F_f}{G_f}\right)_\delta\right| \text{and} kp_r > \left|\left(\frac{F_r}{G_r}\right)_\delta\right| \tag{21}$$

we get:

$$\dot{V} < 0 \quad , \quad \text{for}(\lambda_f, \lambda_f) \notin \Phi \tag{22}$$

By considering inequality (22), we can state the following:

For the control law in equation (14) and with the proportional gains satisfying inequality (21), the controller will be able to regulate the error state (e_f, e_r) to a set given in (20).

For higher accuracy, ε_f and ε_r can be taken very small. Ultimately one may select the following:

$$\varepsilon_f \to 0 \quad \text{and} \varepsilon_r \to 0 \tag{23}$$

This may expose the system to chatter near the origin as in the case of sliding mode control system (the chattering problem [9]).

5-SIMULATION RESULTS

The simulations done in this section are based on the model in Eq. (6) and the nominal system parameters given in Table (2) below. The uncertainty term is taken related to the nominal term in the following form

$$\left|\left(\frac{F_f}{G_f}\right)_\delta\right| \leq \alpha_f \left|\left(\frac{F_f}{G_f}\right)_o\right| \text{and} \left|\left(\frac{F_r}{G_r}\right)_\delta\right| \leq \alpha_r \left|\left(\frac{F_r}{G_r}\right)_o\right|$$

with $0 < \alpha_f, \alpha_r < 1$. The existence of α_f and α_r is proved in reference [10]. In this work $\alpha_f \& \alpha_r$ is chosen to be 0.2.

The optimal reference slip ratio is calculated for various road conditions which based on thematic formula presented in Equation (5). Hence the optimal slip ratio λ is

$$\lambda_{opt.} = \frac{1}{\vartheta_2} ln\left(\frac{\vartheta_1 \vartheta_2}{\vartheta_3}\right) \text{ where } \frac{d}{d\lambda}\mu(\lambda) = 0 \qquad (24)$$

For various road conditions the optimal slip ratioλ, using Table **(**1) where the parameters of the magic formula (ϑ_1, ϑ_2 *and* ϑ_3), are found as follows:

Road Condition	$\lambda_{opt.}$
Dry Asphalt	**0.17**
Wet Asphalt	**0.13**
Cobblestone	**0.4**
Snow	**0.06**

In addition the simulation is performed for vehicle initial velocity equal 100 km/hr and the controller objective is to lower this velocity to reaches preferred value, 5 km/hr. The breaking torques as a result to applying the control formula, as presented in equations (14), are plotted for the front and the rear in figures (3) and (4), respectively, for different road conditions. The error function between the slip ratio and its optimal value with time are plotted in Figures (5) and (6) for the front and the rear wheels. These figures show the ability of the proposed control formula in Eq. (14) in regulating the error for various road conditions. Furthermore the slip ratio attains the optimal value for less than 0.12 second for the case of cobblestone road condition and less than 0.04 for other road conditions as demonstrated in figures (7) to (10).

The main actuating torques objective is to maximizing the longitudinal tyre road force at the front and rear wheels in order to get maximum deceleration force for braking process as can be deduced from Equation (6). The longitudinal tyre road forces are clarified for various road conditions are shown in Figures (11) to (14) where the peak value for each case is attained when the slip ratio reach its optimal one.

The vehicle velocity deceleration for the dry asphalt condition is depicted in Figure (15). It can be seen that within 2.3 second the vehicle velocity reduces from 100 km/hr to 5 km/hr as a result of applying the braking torques according to the proposed control law given in Equation (14).

6-CONCLUSIONS:

In this paper, a nonlinear proportional control is proposed for active braking system, where the uncertainty in system model is taking into consideration. The validity of the proposed controller is proved via the candidate Liapunov function, which, also enables, via the inequality (21), calculating the gains of the proportional terms in the control formula.

The simulation results, were done using MATLAB 10.0, are depicted in Figures (3) to (15) for different roads types/conditions. From which it is clear that the assumed control action responds actively to attain the desirable (optimal) slip ratio. This optimized the braking forces to decelerate the vehicle to the required velocity within a certain time (minimal) as the controller objective required.

Table 1: Friction model parameters [8]

Road Condition	ϑ_1	ϑ_2	ϑ_3
Dry Asphalt	**1.28**	**23.99**	**0.52**
Wet Asphalt	**0.86**	**33.82**	**0.35**
Cobblestone	**1.37**	**6.46**	**0.67**
Snow	**0.19**	**94.13**	**0.06**

Table 2: Nominal Vehicle Parameters

v km/h	I_r kg.m^2	I_f kg.m^2	r mm	d_r (m)	d_f (m)	h mm	M (kg)
100	1.7	1.2	310	1.24	1.1.21	585	915

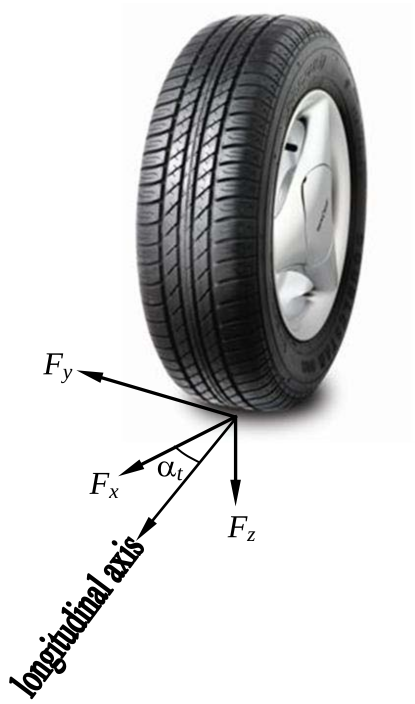

Figure 1: Tyre–road contact forces

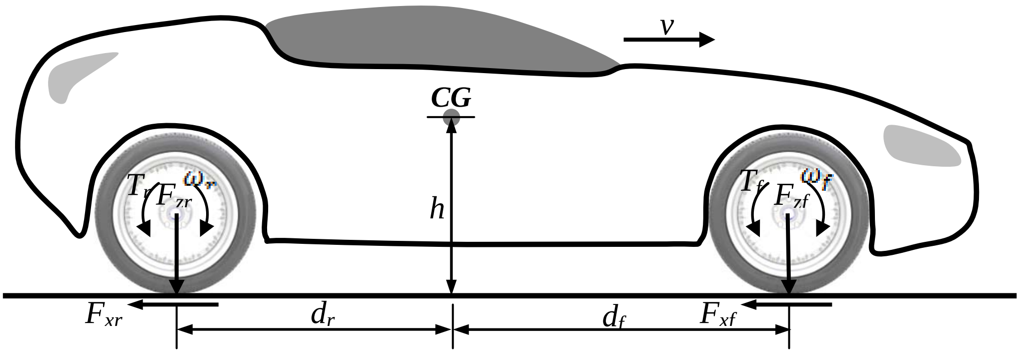

Figure 2: Vehicle free body diagram

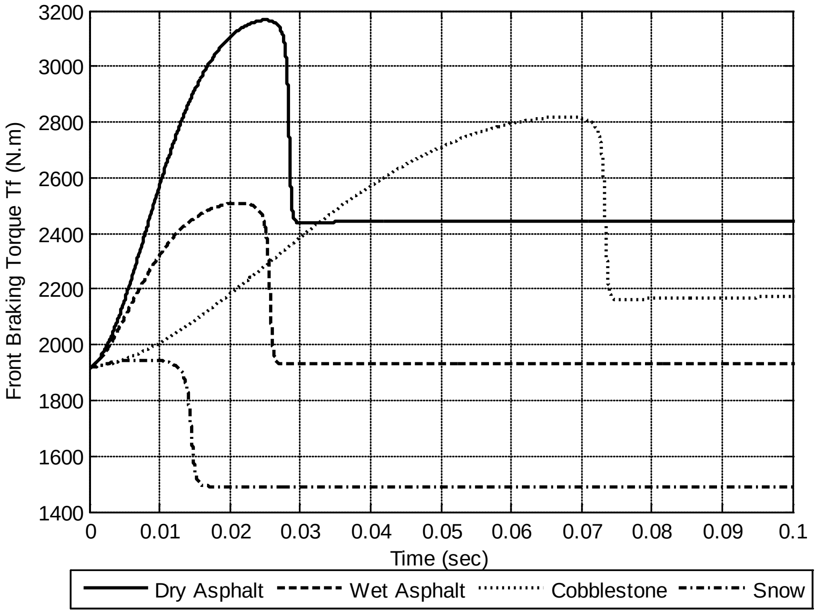

Figure 3: Braking torques of the front wheels for different road conditions

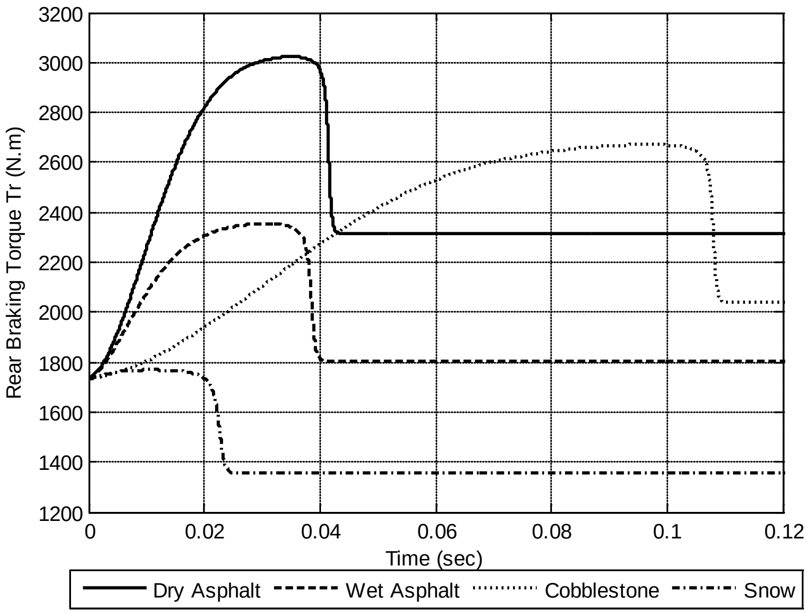

Figure 4: Braking torques of the front wheels for different road conditions

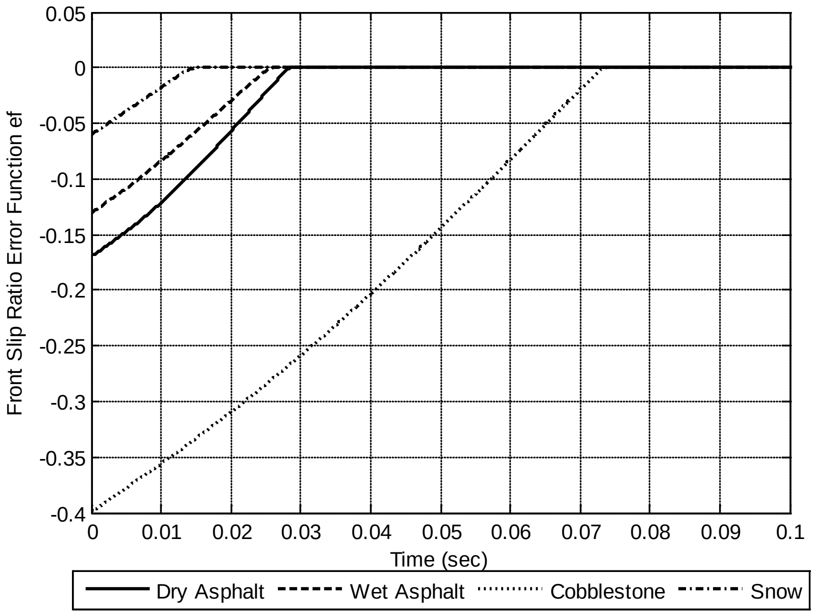

Figure 5: Plot of error function for front wheels for different road conditions

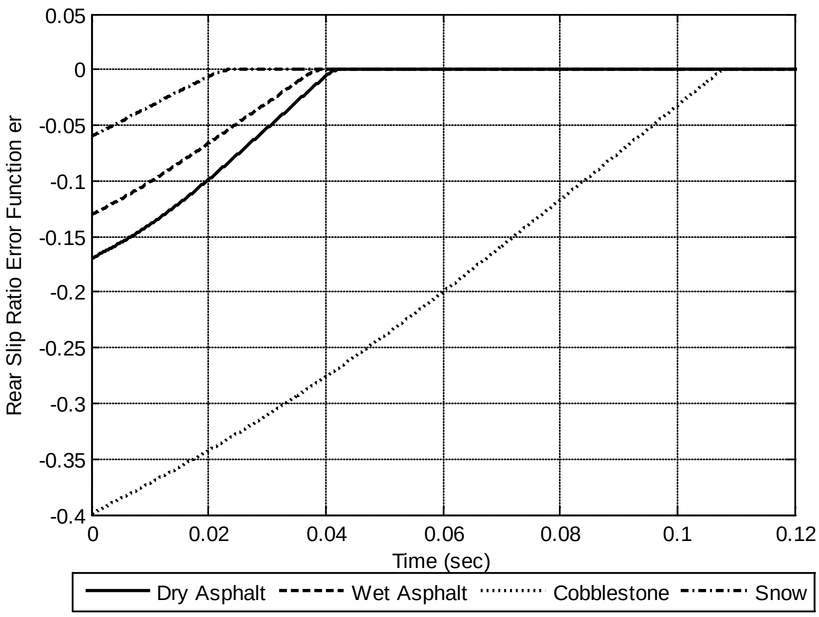

Figure 6: Plot of error function for front wheels for different road conditions

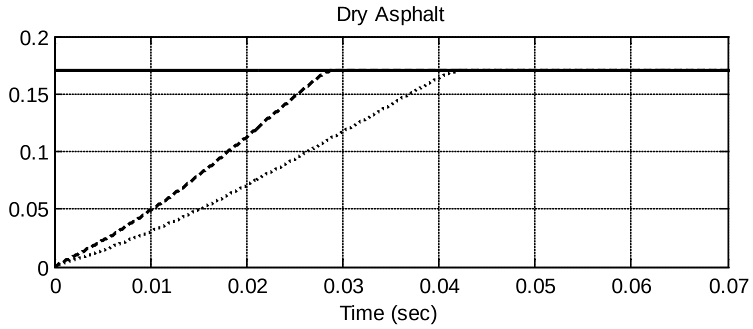

Figure 7: Plot of the slip ratio with time for dry Asphalt road

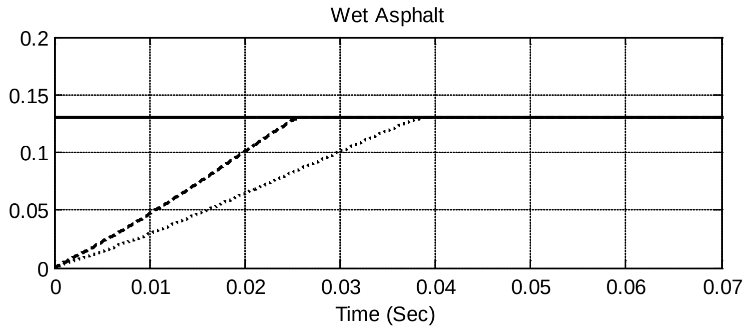

Figure 8: Plot of the slip ratio with time for wet Asphalt road

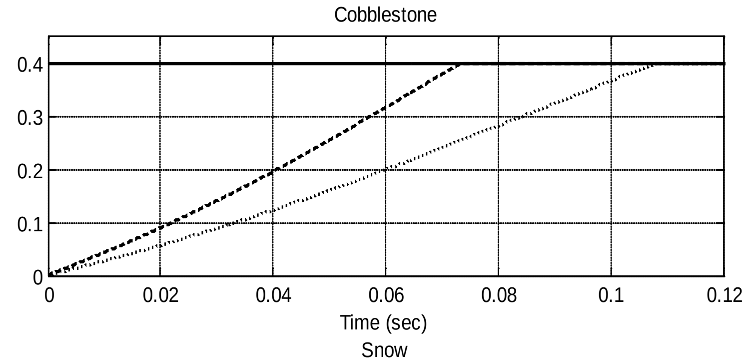

Figure 9: Plot of the slip ratio with time for Cobblestone road

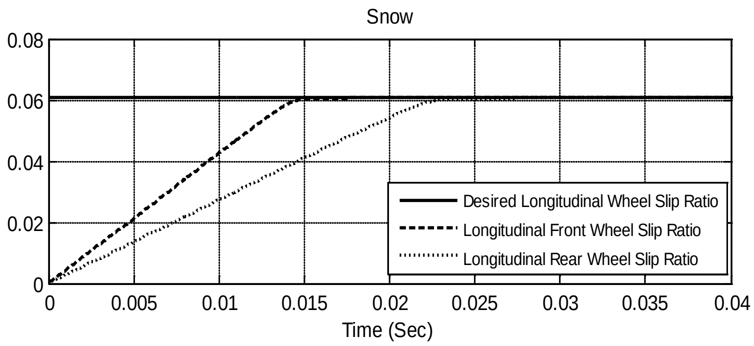

Figure 10: Plot of the slip ratio with time for Snowy road

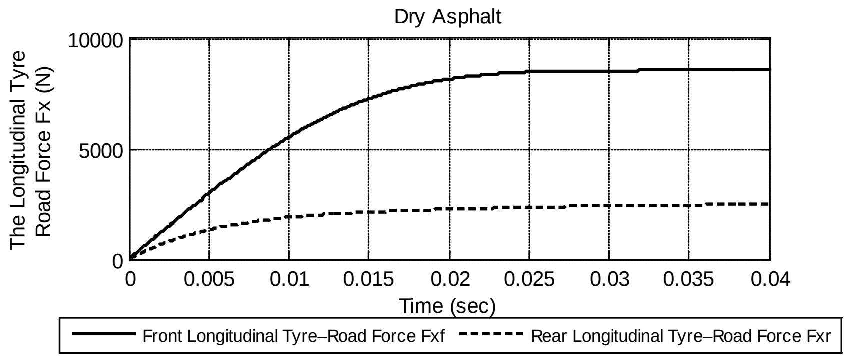

Figure11: The longitudinal tyre road forces for dry Asphalt road.

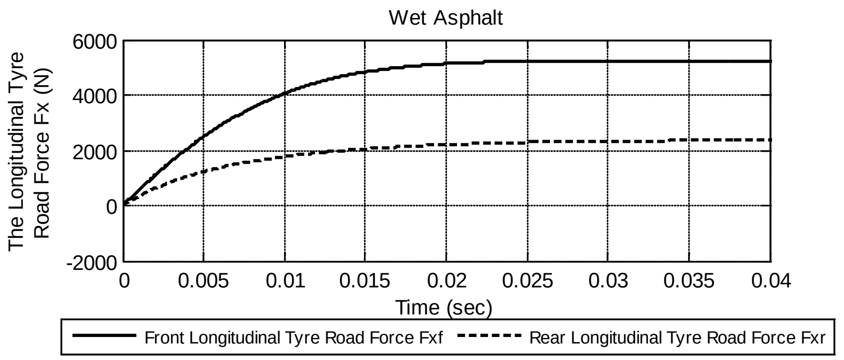

Figure12: The longitudinal tyre road forces for wet Asphalt road.

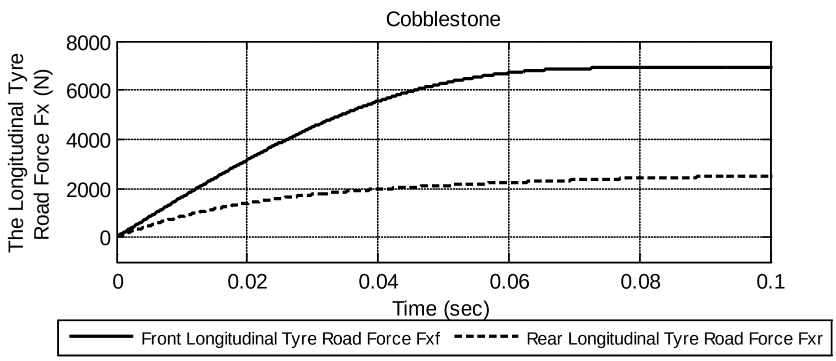

Figure13: The longitudinal tyre road forces for Cobblestone road.

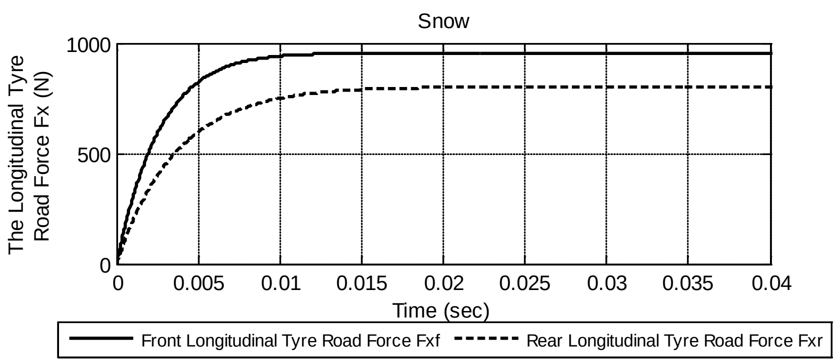

Figure14: The longitudinal tyre road forces for Snowy road.

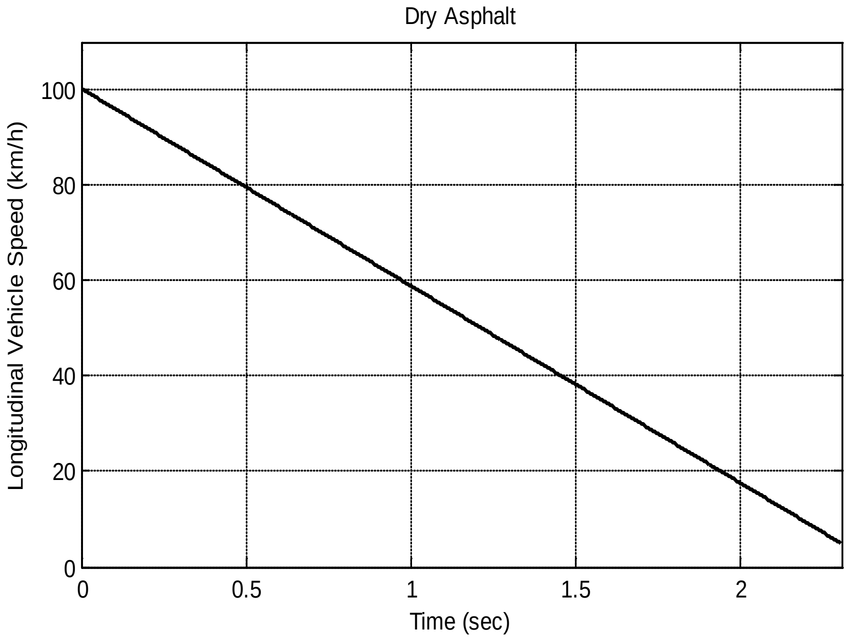

Figure15: The vehicle velocity deceleration for the dry Asphalt road

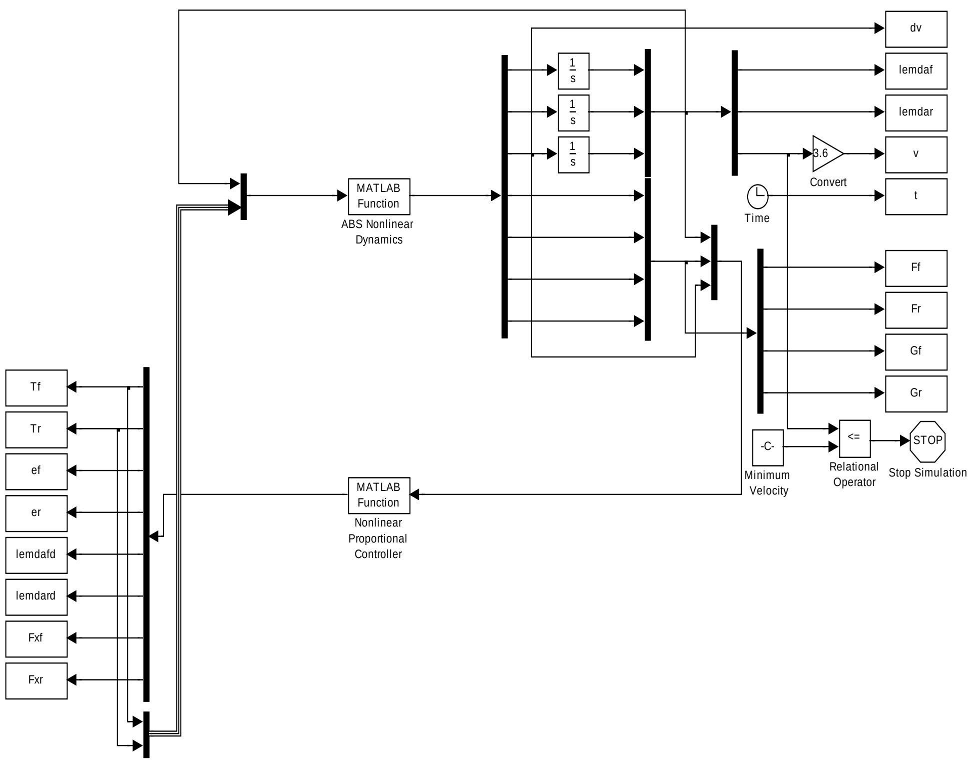

Figure16: MATLAB/ Simulink of ABS system using nonlinear robust proportional controller.

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