

EXPERIMENTAL AND THEORETICAL STUDY OF NATURAL CONVECTION HEAT TRANSFER OF HEATED SQUARE CYLINDER PLACED INSIDE A COOLED CIRCULAR ENCLOSURE FILLED WITH AIR

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Abstract:

In this paper, an experimental and theoretical study of natural convection heat transfer in a closed ends annulus region formed by a heated square cylinder placed concentrically in a larger isothermally cooled circular cylinder. Experiments are carried out for Rayleigh number based on the equivalent annulus gap length ranges from 10^3 up to 10^6 . Different aspect ratios, 0.2 up to 0.5, are considered. The numerical simulation for the problem is carried out by using commercial CFD code (FLUENT 6.3). The average Nusselt number from the experimental results is compared with that obtained from the CFD code. The fluid flow and heat transfer characteristics for different operating and geometric conditions are illustrated through stream lines and isotherms contours that obtained from the CFD code. The results showed that as the aspect ratio increases the heat transfer rate and the average Nusselt number increase. Also the result showed that as Rayleigh number increases the average Nusselt number increases.

Keywords: natural convection, heat transfer, horizontal annulus, circular enclosures, concentric cylinders.

دراسة عملية ونظرية لانتقال الحرارة بالحمل الحر من اسطوانة مسخنة مربعة موضوعة داخل حيز دائري مبرد مملوء بالهواء

الخلاصة:

في هذا البحث تم اجراء دراسة عملية ونظرية لانتقال الحرارة بالحمل الحر في حيز متكون من اسطوانة مسخنة ذات مقطع مربع موضوعة بمركز اسطوانة اخرى اكبر مبردة وذات مقطع دائري الشكل. شمل الجانب العملي مدى واسع من رقم رالي الذي تراوح بين 10^3 الى 10^6 والمحسوب على اساس طول الحيز بين الاسطوانتين. ايضا شملت الدراسة نسب مختلفة من متغيرات الشكل الهندسي والمتمثلة بالنسبة بين طول ضلع الاسطوانة المربعة الى قطر الاسطوانة الدائرية والتي تراوحت بين 0.2 الى 0.5. اما الجانب النظري فقد تم باستخدام برنامج الحجم المحددة (FLUENT 6.3). قيم متوسط رقم ناسلت المحسوب عمليا تم مقارنته مع قيم متوسط رقم ناسلت المحسوب نظريا. تم استعراض نتائج خصائص الجريان وانتقال الحرارة المحصلة من البرنامج بشكل خطوط الانسياب وخطوط ثبوت درجة الحرارة. اظهرت النتائج ان زيادة النسبة بين طول ضلع الاسطوانة الداخلية

المربعة الى قطر الاسطوانة الخارجية الدائرية تؤدي الى زيادة معدل انتقال الحرارة ومتوسط رقم ناسلت. كذلك بينت النتائج ان زيادة رقم راييلي تؤدي الى زيادة متوسط رقم ناسلت.

INTRODUCTION:

Natural convection within an annulus bounded by two concentric horizontal cylinders, with a temperature difference across the annulus, has received considerable attention in past experimental and analytical investigations due to its various engineering applications such as solar collectors design, thermal storage systems, nuclear reactors, cooling of electronic components, aircraft fuselage insulation, underground electrical transmission lines, etc. **Grigull and Hauf 1966** used thermal interferometer to visualize the temperature and flow fields in air-filled horizontal circular annuli. Their visual observation indicated that the flow patterns took the kidney shape and as the Grashof number increased the centers of the closed flow circulation moved up. **Kuehn and Goldstein 1976** carried out experimental and numerical investigations for natural convection flow in horizontal annuli for Rayleigh number in the range 2.11×10^4 to 9.76×10^5 . According to their observations, the flow and heat transfer results can be divided into several regimes. Below Rayleigh number of 100, the center of rotation was near 90° . A transition region existed for Rayleigh number between 10^2 and 3×10^4 . The flow remained of the same pattern but became strong enough to influence the temperature field. They observed steady laminar boundary layer regime exists between Rayleigh numbers of 3×10^4 and 10^5 . For air, the flow started to oscillate near Rayleigh numbers of 10^5 . Extensive survey on natural convection between two horizontal concentric cylinders was given by **Kuehn and Goldstein 1978**. They performed both experimental and theoretical-numerical studies for air and water at Rayleigh numbers (based on gap width, L) from 2.1×10^4 to 9.8×10^5 at a diameter ratio of 2.6. **Castrejon and Spalding 1988** conducted an experimental and theoretical study of transient free-convection flow between horizontal concentric cylinders in there experiment. They considered constant heat flux in the inner cylinder rather than constant temperature. The radius ratio used in their experiment was approximately 11.875. Their study considered only transient convection, meaning that they observed the behavior of the fluid only until the onset of turbulence. **Cesiniet et al. 1999** performed the numerical and experimental analysis of natural convection from a horizontal cylinder enclosed in a rectangular cavity. The influence of the cavity aspect ratio and the Rayleigh number on the distribution of temperature and Nusselt number was investigated. As a result, the average heat transfer coefficients increased with increasing Rayleigh number. **Asan 2000** numerically studied two-dimensional natural convection in an annulus between two isothermal concentric square ducts and obtained solutions up to a Rayleigh number of 10^6 . The results showed that dimension ratio and Rayleigh number had a deep influence on the temperature and flow fields. **El-Sherbiny and Moussa 2004** numerically investigated natural convection in air between two infinite horizontal concentric cylinders at different constant temperatures. The study covered a wide range of the Rayleigh number, Ra from 10^2 to 10^6 , and the Radius Ratio was changed between 1.25 and 10. A good agreement was shown between their correlation and previous data and correlations. **Teertstra et al. 2005** developed an analytical modeling technique to predict the total heat transfer rate in the 2D annular region formed between isothermal convex inner and concave outer boundaries having similar or different shapes. Agreement between the model and existing numerical and experimental data from the literature was quite good. **Kumar and Dalal 2006** studied

the natural convection around a heated square cylinder placed inside an enclosure with Rayleigh number ranged from 10^3 up to 10^6 . Effects of the enclosure geometry had been assessing using three different aspect ratios placing the square cylinder at different heights from the bottom. They found that the flow pattern and thermal stratifications were modified if the aspect ratio was varied. Overall heat transfer also changed as a function of the aspect ratio. **Kim et al. 2008** investigated numerically the characteristics of two dimensional natural convection problems in a cooled square enclosure with an inner heated circular cylinder. The immersed boundary method was implemented in a second-order accurate finite volume method to simulate the flow and heat transfer over an inner circular cylinder in the Cartesian coordinates. A detailed analysis for the distribution of streamlines, isothermals and Nusselt numbers were carried out to investigate the effect of the locations of the heated inner cylinder on the fluid flow and heat transfer in the cooled square enclosure for different Rayleigh number in the rang $10^3 \leq Ra \leq 10^6$. **Saeed and Ali 2009** studied heat transfer of a square eccentric body buried in porous media. The numerical results of heat transfer were presented for modified Rayleigh number in the range 50 to 400. They found that the increasing in Rayleigh number resulting in increasing in Nusselt number and increasing of body size causes increase of heat transfer rate. **Sidik and Abdul Rahman 2009** investigated the fluid flow behavior and heat transfer from a heated Squarecylinder inside an enclosure in the range of $10^3 \leq Ra \leq 10^6$. The computational results demonstrated that the flow, number, size and formation of vortices and also heat transfer mechanism are critically dependence on Ra and the position of heated inner square cylinder in enclosure. **Habeeb 2010** studied numerically the effect of a hot square cylinder placed on a cooled elliptical enclosure of a laminar natural convection. His study included different ratios of the geometry ($a/b = 1.5, 2 \text{ and } 3$), ($l/b = 0.25, 0.5$) and Rayleigh number from 10^3 to 10^6 . The results showed that, the increase of the major axis of the enclosure (a/b ratio) led to increase the average Nusselt number and decrease the flow strength for all Rayleigh numbers. In the present work, several experimental runs are carried out to show the effect of Rayleigh number and the aspect ratio on the average Nusselt number. Also, the results are extended through the numerical simulation of the present problem to investigate the effect of Rayleigh number and the aspect ratio between the internal and external cylinders.

Experimental Apparatus:

The experimental apparatus contains mainly on two concentric cylinders, a heater, a voltmeter, an ammeter, voltage regulator, digital thermometer, two vertical threading bars, 18 thermocouples, and a selector switch. The outer cooled circular cylinder is made from an aluminum plate with 20cm outer diameter, 0.2cm thickness, and 30cm length. It is insulated by a thermal insulator (glass wool). The inner heated square cylinder has different side length (4cm, 6cm, 8cm, and 10cm) and each pair was oriented horizontally as shown in **Fig. 1**. The two-vertical threading bars are used to fix the inner square cylinder in the middle of the outer circular cylinder. The inner square cylinder was heated by a nickel-chrome heater which provides a constant heat flux on the outer surface while the inner surface of the outer circular cylinder was kept at constant surface temperature by using circulating water system by which the water is pumped through a coil wrapped around the surface of the outer circular cylinder. Both ends of the annulus were closed to prevent heat leakage. Eight K-type

thermocouples were distributed circumferentially and embedded in the surface of the inner square cylinder at mid span distance to measure the surface temperature. Four thermocouples are distributed in the wall of the outer cylinder surface. Four thermocouples are used to measure the temperatures distribution through the annulus between the two cylinders by insertion fixed thermocouple probes from eight plugs located at mid-span distance. Another three thermocouples are used to measure the temperature of inlet and outlet cooling water and to measure the ambient air temperature. In order to confirm a uniform temperature distribution along the axial direction, three thermocouples were used at the ends and mid span of the tested cylinder. The readings of the thermocouples are taken by means of a digital thermometer with an accuracy of 0.1°C up to 200°C . An ammeter (0.001 Ampere resolution) and voltmeter (0.1 Volt resolution) are used to measure the power consumed by the heater.

Experimental Work:

In this study four pairs of cylinders were tested. Each pair was tested alone for the same orientation (horizontal orientation). The main parameter to be studied is the aspect ratio and Rayleigh number. **Table 1** shows the main dimensions of the four tested pairs. Experimental tests were carried out by maintaining constant heat flux on the inner cylinder and isothermally cooled outer one. Both ends of the annulus were closed. The study state condition was achieved after about three hours depending on the value of the heat flux. All thermocouples were used with leads and calibrated using the melting point of ice made from distilled water as reference point and the boiling points of several pure chemical substances. The calibration results are given in **Table 2**. Four various values of Rayleigh numbers based on the equivalent annulus gap length were utilized in the experiments which ranged from 10^3 to 10^6 . The average heat transfer coefficient, h_{avg} , Nusselt number, Nu_{avg} , and Rayleigh number are calculated as follows (Eldesouki, 2010):

$$h_{avg} = \frac{q''}{(T_{avg,i} - T_c)} \quad (1)$$

$$Nu_{avg} = \frac{h_{avg} * l}{k_{ref}} \quad (2)$$

$$Ra = \frac{g \beta q'' L_c^4}{k_{ref} \nu^2} Pr \quad (3)$$

Where; q'' , T_{avg} , T_c , l , L_c , ν , and k_{ref} are the net heat flux, the average temperature, cold wall temperature (outer cylinder), square cylinder side length, characteristic length ($\frac{D-1}{2}$), kinematic viscosity, and the air thermal conductivity, respectively. All properties of air are calculated at the film temperature.

Problem Specification:

The present study can be represented by an annulus space ranging from a square cylinder placed at the center of a circular cylinder filled with air. The inner square cylinder is heated under a constant heat flux q_H , while the external circular cylinder wall is cooled isothermally at temperature T_o . The physical model of the present problem is illustrated in **Fig. 2**. In this study it is assumed that the radiation effects are negligible and the fluid properties are constant except for the density in the buoyancy term, which follows the Boussinesq approximation. As the square cylinder and the circular enclosure are long enough, so the flow is considered steady, laminar, two dimensional and has a $Pr=0.71$. The present problem is governed by steady, two dimensional equations of continuity, momentum and energy. These equations can be written in the form (FLUENT user manual, 1998):

1- Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

2- Momentum Equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \quad (5)$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = \rho g - \frac{\partial P}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] \quad (6)$$

3- Energy Equation:

$$\rho C_p u \frac{\partial T}{\partial x} + \rho C_p v \frac{\partial T}{\partial y} = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \quad (7)$$

Fluent numerical code, version 6.3 which is employed for all numerical simulations solve the above differential equations (continuity, momentum, and energy equation respectively) and these equations can be written in the following form (FLUENT user manual, 1998):

1- Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_m \quad (8)$$

2- Momentum Equation:

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\bar{\tau}) + \rho \vec{g} + \vec{F}$$

(9)

4- Energy Equation:

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_i}[u_i(\rho E + p)] = \frac{\partial}{\partial x_j} \left(k_{\text{eff}} \frac{\partial T}{\partial x_j} + u_i(\tau_{ij})_{\text{eff}} \right) + S_h \quad (10)$$

Where E is the total energy and K_{eff} is the effective thermal conductivity $(\tau_{ij})_{\text{eff}}$ the deviatoric stress tensor.

Gambit 2.2.30 is used for the development of the computational grid. The computational domain resulted from the subtraction of the square cylinder section from circular section as shown in **Fig. 3**. Steady, laminar, model is employed to solve natural convection heat transfer. Because air flow is incompressible, continuity is satisfied using a semi-implicit method for pressure linked equations, which is referred to as the SIMPLE procedure. To reduce numerical errors, second order upwind discretization schemes were used in the calculations. The computational iteration is solved implicitly. The convergence of the computational solution is determined on scaled residuals for the continuity, energy equations and for many of the predicted variables. The settings for the scaled residuals for solution convergence are set to 10^{-4} for nearly all computed residuals. The only exception is the residual for the energy equation which is set to 10^{-6} **fluent user manual, 2012**. The solution is considered to be converged if all of the scaled residuals reach its default values.

Boundary Conditions:

The boundary conditions for the current problem are:

- 1- No slip condition along the inner and outer surfaces.
- 2- Constant heat flux at the surface of the inner cylinder, q'' .
- 3- Isothermal condition at the surface of the outer cylinder, $T=T_0$.

Several runs have been done for the case of $Ra=10^4$ and aspect ratio=0.2. **Table 3** shows the average Nusselt number for four tested grids. It can be seen that the results of the last two grids have little difference between them. So the grid with the total elements of 15000 is used for the subsequent calculations of the current study to decrease the time required for the solution convergence.

RESULT AND DISCUSSION:

Effect of Rayleigh number on the average Nusselt number:

A validation was done for the natural convection heat transfer between low temperature outer circular cylinder and a heated inner square cylinder. The validation data of the isotherm contours with the published results of **chang et al., 1983** for

Rayleigh number 10^4 and 10^5 and the aspect ratio 0.2 as shown in **Fig. 4**. A very good agreement has been found for the two cases. **Fig. 5** represents a comparison between the present experimental and numerical average Nusselt number and Rayleigh number (based on the equivalent gap length) for different aspect ratios. The results showed good agreement between the experimental and numerical results. It can be seen that as Rayleigh number increases the average Nusselt number increases.

Effect of the aspect ratio on the average Nusselt number:

Fig. 6. shows the effect of the aspect ratio on the average Nusselt number. It can be seen that as the aspect ratio increases the average Nusselt number increases for every tested Rayleigh number and the aspect ratio (0.5) gives the largest average Nusselt number while the aspect ratio (0.2) gives the lowest average Nusselt number.

Flow patrons and isotherm:

The effect of the Rayleigh number on the flow field characteristics and thermal field is illustrated in **Fig. 7**. It is observed that at low $Ra=10^3$, fluid near the cylinder wall is almost stagnant due to high viscous effect. The flow pattern is characterized by a primary flow, which consists of two big symmetrical and counter rotating vortices dragging upwards fresh air from the bulk of fluid placed below the square cylinder. The circulation of the flow is very weak and the flow velocity is too small to affect the temperature distribution. As a result, the main heat transfer mechanism at this Rayleigh number is by conduction. And this can be seen from the isotherm contours which they are almost equally spaced. As Rayleigh number increases to $Ra=10^4$, the circulation become stronger. And the center of the vortex moves upward in the square enclosure. At this Rayleigh number the buoyant force starts dominate the flow. And the isotherm contours distorts as a result of the effect convection. At $Ra=10^5$ the effect of convection become stronger and it starts to appear clearly in heat transfer and the thermal boundary layer on the surface of the square cylinder gets thinner. Isotherm contours are concentrated near the cylinder wall showing high temperature gradient as well as high heat transfer. The core of the circulating eddies is located only in the upper half of the enclosure. At $Ra=10^6$ the stream line indicate that the flow become stronger and isotherm contours shows the domination of the convection. The same behavior can be seen for all other aspect ratio. **Fig. 8** shows the stream lines and isotherms for the aspect ratio=0.3. It can be seen that as the aspect ratio increases the flow behaves as the previous case, but its strength becomes stronger especially at $Ra=10^5$ and $Ra=10^6$. The isotherm contours deformation becomes stronger. **Fig. 9** shows that As the aspect ratio increases to 0.4 the flow behaves as in the previous cases for low Rayleigh numbers (10^3 and 10^4), but for higher Rayleigh numbers (10^5 and 10^6) the flow is stronger and a vortex moves to the upper side of the square cylinder and the thermal mixing becomes stronger. **Fig. 10** shows that at $Ra=10^3$ three vortices appear, one in the middle region and another two vortices appears in the lower and the upper regions of the enclosure. As Rayleigh increases to 10^4 and 10^5 the lower vortex finishes and the flow strength increases. Isothermal contours are uniform and regular for low Rayleigh number but as Ra increases these lines redistributed irregularly from hot square cylinder to the circular enclosure due to the higher in convection contribution.

Conclusions:

The present study investigates experimentally and theoretically the flow and heat transfer characteristics of natural convection problem on square cylinder placed in a cooled circular enclosure. The following conclusion can be drawn:

- The average Nusselt number increases with the increase in Rayleigh number (based on the annulus gap length).
- As the aspect ratio increases the average Nusselt number increases. The aspect ratio=0.5 gives the highest value of the average Nusselt number.
- There is a very good agreement between the present numerical simulation and previous published simulation (Chang et al.).

Table 1 the main dimensions of the four tested pairs.

No. of pairs	l cm	D cm
1	4	20
2	6	20
3	8	20
4	10	20

Table 2 Experimental Accuracies.

Independent variables(v)	uncertainty interval (w)
Voltage of the heater	± 0.04 volt
Current of the heater	± 0.0003 Amp
Thermometer	$\pm (0.2 \% + 0.5 ^\circ\text{C})$

Table 3 average Nusselt number results for different grids sizes at Rayleigh= 10^4 and aspect ratio=0.2

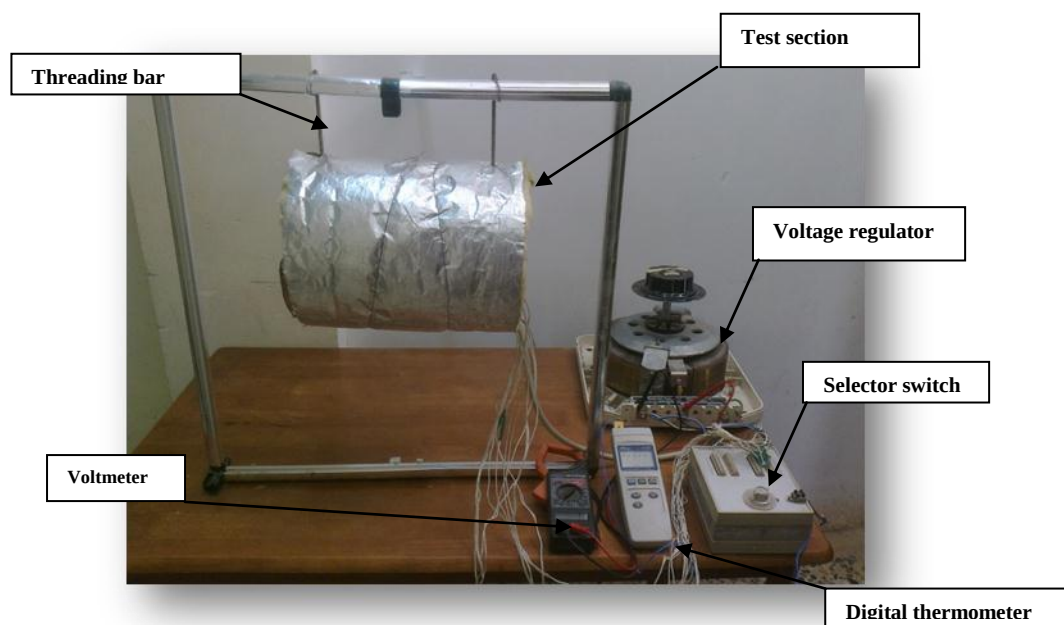
Grid total elements	Nu _{avg.}
8000	1.8
10000	2.01
15000	2.2
18000	2.21



(a)



(b)



(c)

Fig. 1 (a) the four tested cylinders, (b) fixing of the square cylinder inside the circular enclosure, (c) the experimental test-rig and the measuring devices

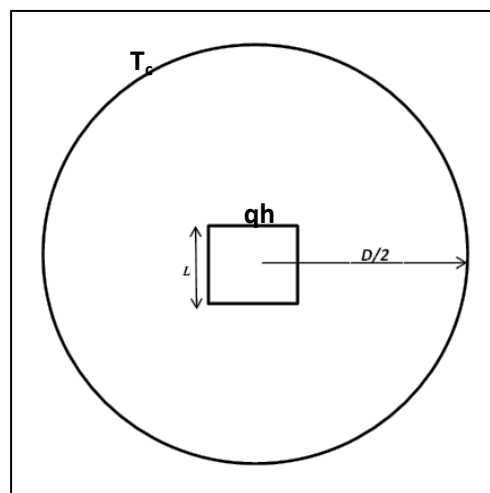


Fig. 2 A schematic view and geometry description of the physical domain

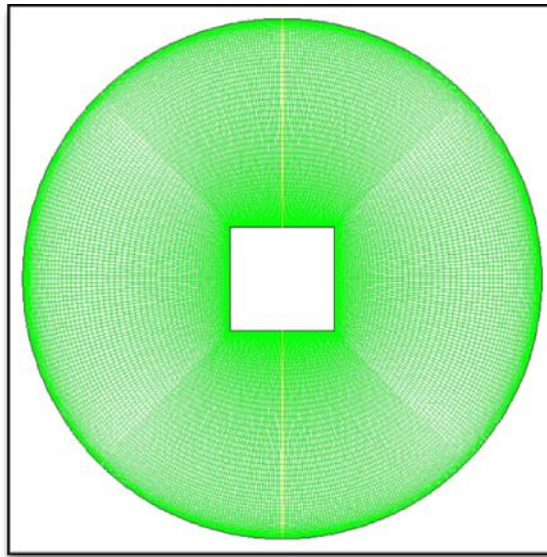


Fig. 3 the computational grid

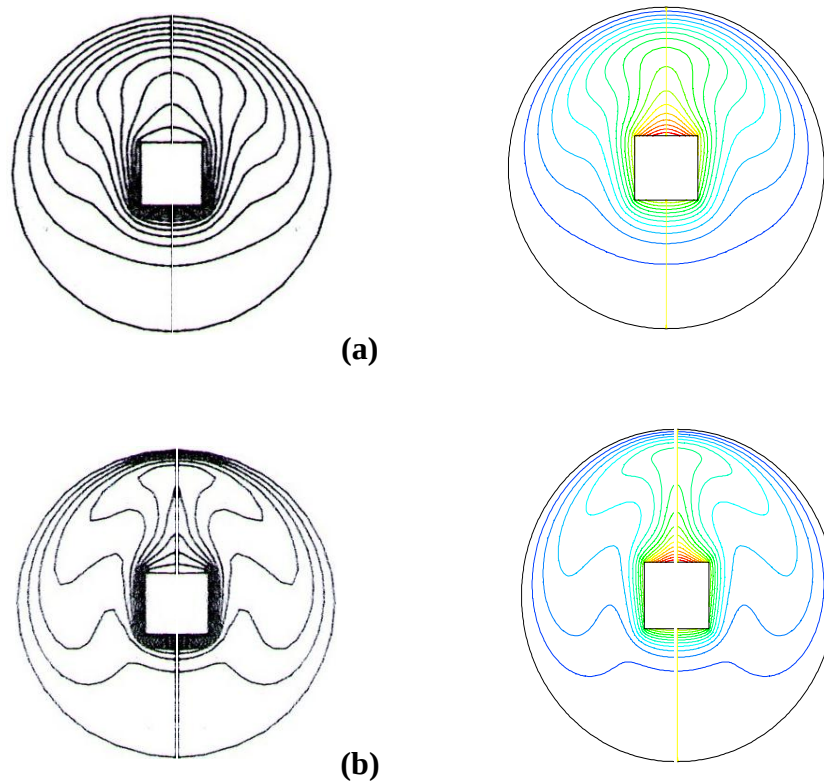


Fig. 4 A comparison between Chang et al. results (left) and present results (right): (a) at the aspect ratio=0.2 and $Ra=10^4$, (b) at the aspect ratio=0.2 and $Ra=10^5$

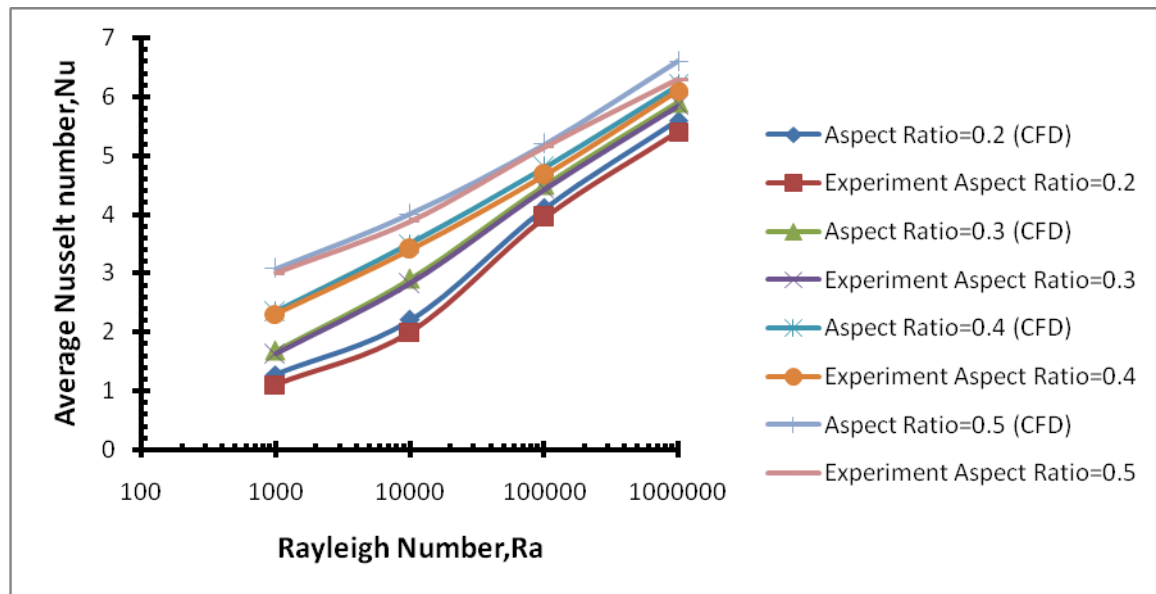


Fig. 5 A comparison between Experimental and numerical results of the average Nusselt number.

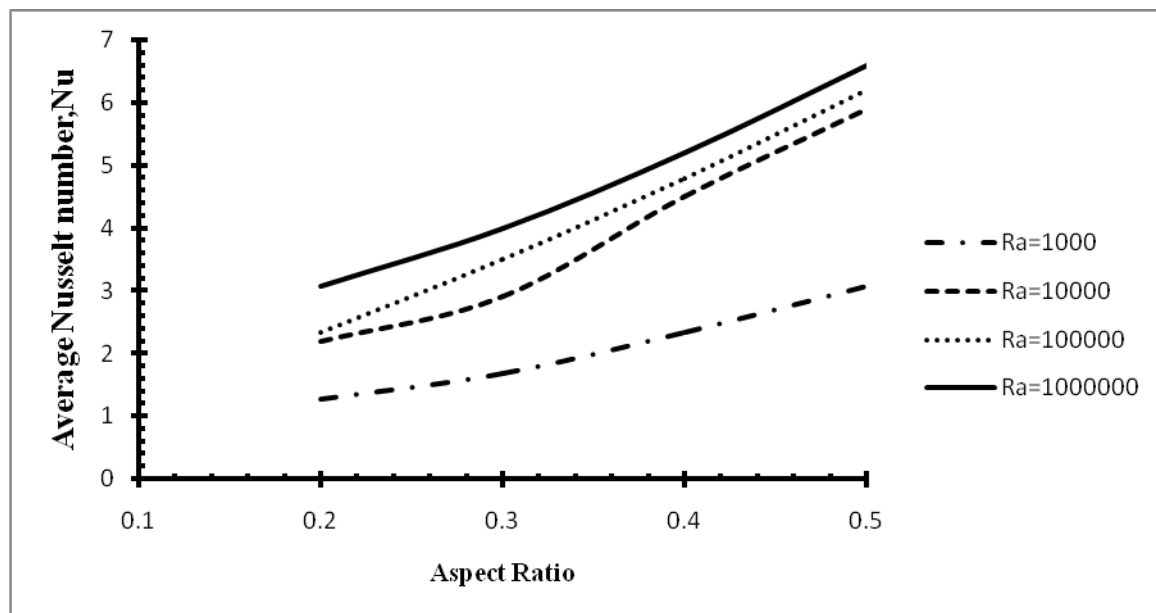


Fig. 6 The effect of the aspect ratio on the average Nusselt number.

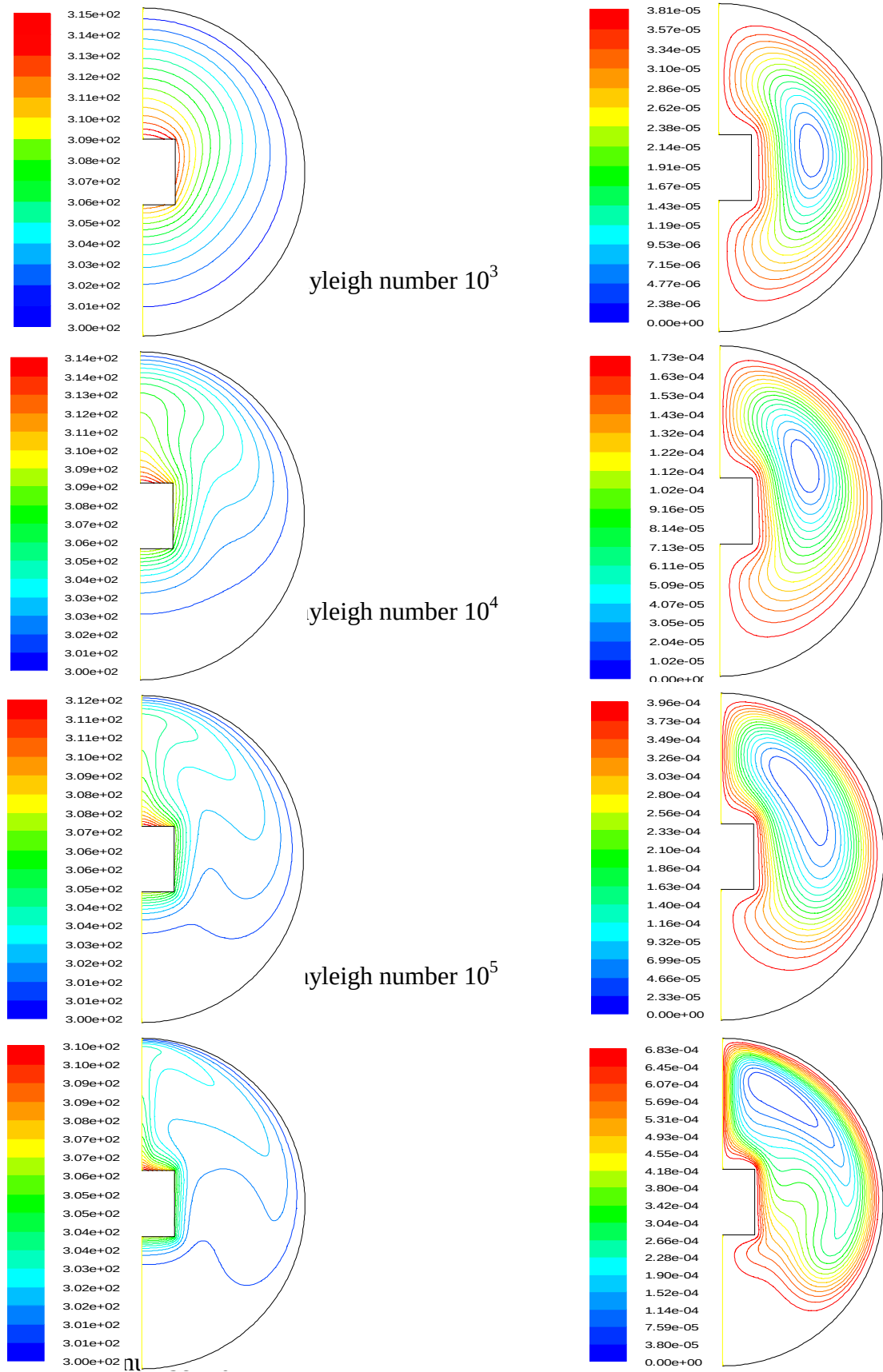


Fig. 7 temperature contours (left) and stream lines (right) at the aspect ratio 0.2

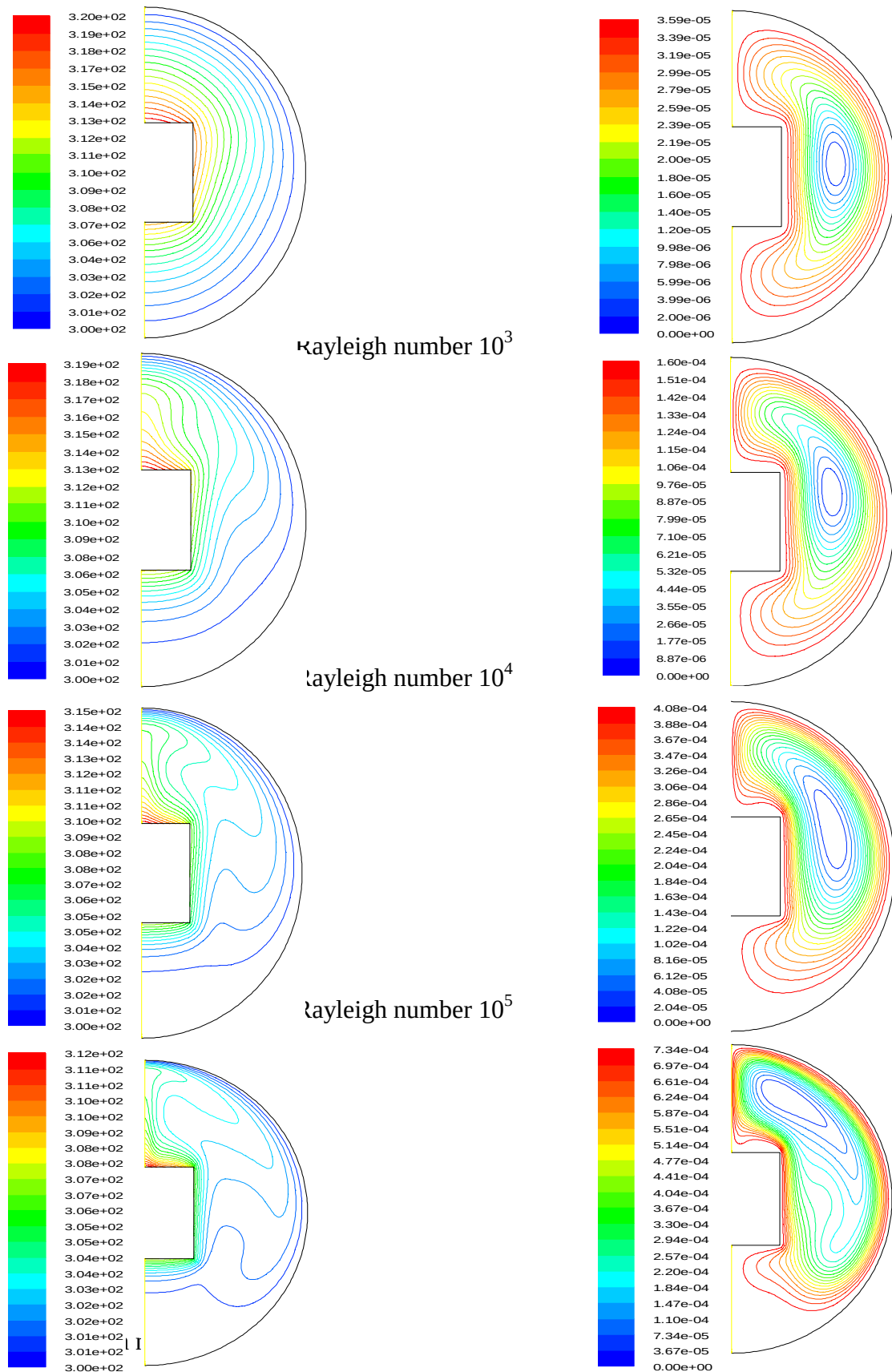


Fig. 8 temperature contours (left) and stream lines (right) at the aspect ratio 0.3

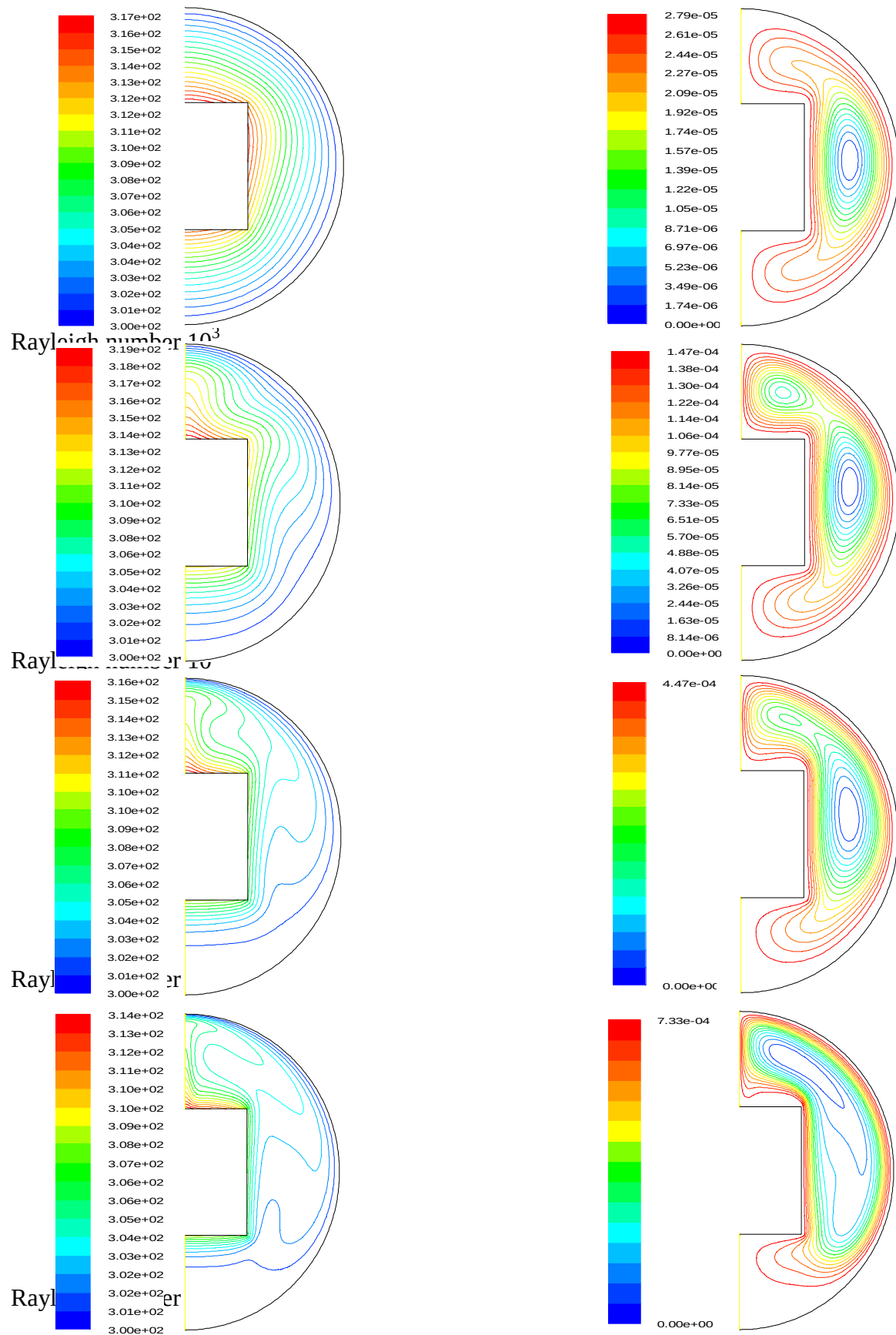


Fig. 9 temperature contours (left) and stream lines (right) at the aspect ratio 0.4

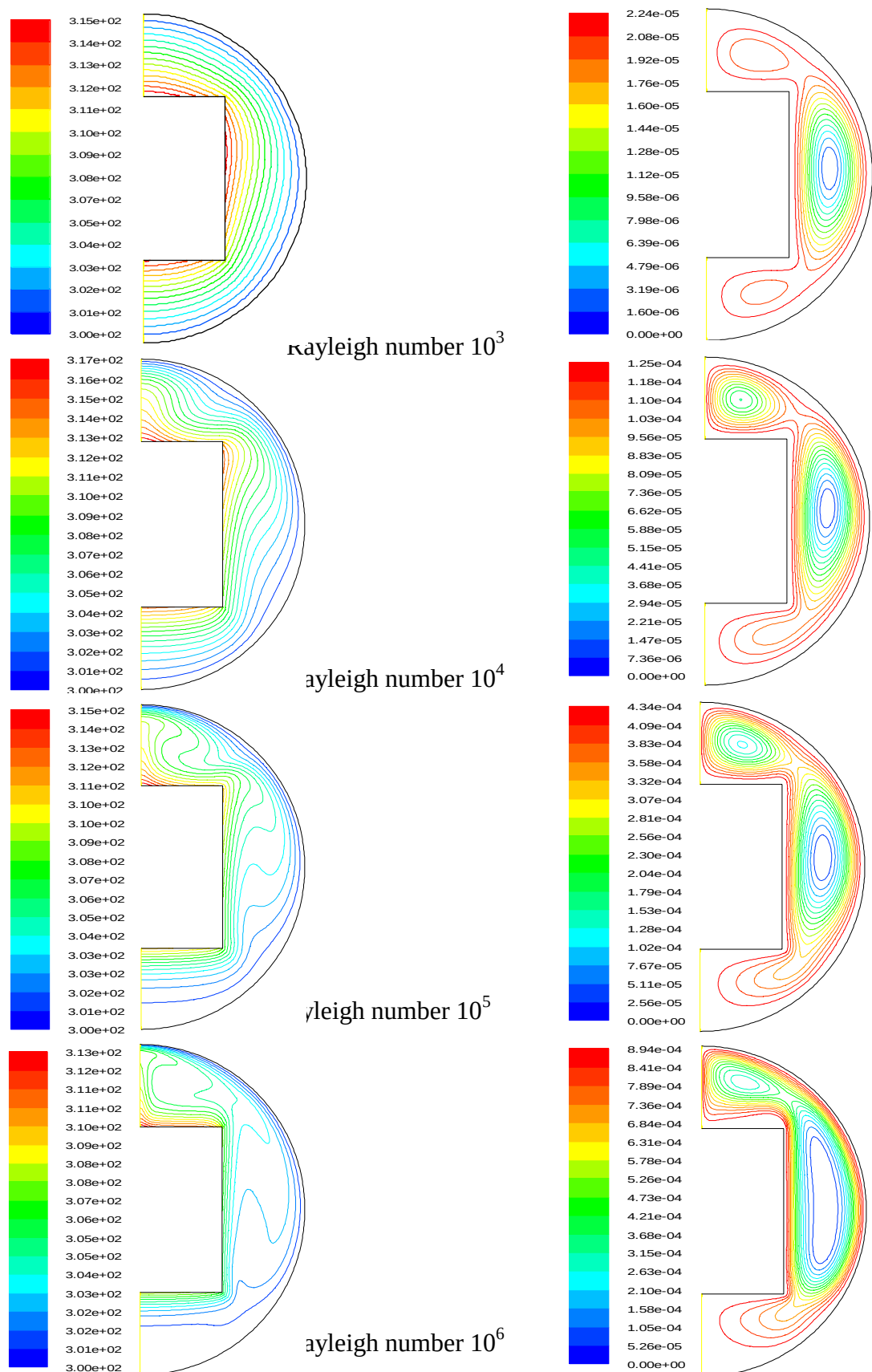


Fig. 10 temperature contours (left) and stream lines (right) at the aspect ratio 0.5

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NOMENCLATURE:

D	Diameter of the external circular cylinder.
CP	Specific heat at constant pressure, J/kg K.
h_x	Local heat transfer coefficient, W/m ² .k.
L	Equivalent annulus gap length, m
Nu _{ave}	average Nusselt number normal direction to the surface
Nu _i	local Nusselt number
p	pressure, Pa
q”	net heat flux, W/m ²
Ra _L	Rayleigh number based on the equivalent annular gap length
T _{ave}	average surface temperature, °C
T _c	cold wall temperature, °C
u	x-direction velocity component, m/s
v	y-direction velocity component, m/s

x,y Cartesian coordinate, m

Greek:

β coefficient of thermal expansion

ν kinematics viscosity, m^2/s

ρ density, kg/m^3