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COMPARING DIFFERENT ESTIMATORS OF PARAMETERS AND RELIABILITY OF ONE DISTRIBUTION OF FAILURE

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ABSTRACT

This research deals with estimating the parameters and reliability function of the well known distribution, which represents the distribution of time to failure, called Weibull distribution, denoted by WEI (λ , θ) (λ is called the shape parameter and θ is the scale parameter). The method of estimation are Maximum likelihood to obtain ($\hat{\theta}_{MLE}$), Minimax estimator (d_1^*), and the proposed Bayes estimator (d_2^*). The comparison between these estimators was done through simulation experiment for three sample size (n = 25, 50, 100), and it is also applied on real life data which represent the time to failure of (30) independent machines from State Company for Cotton Industries. From the results we fond that the best estimator for scale parameter (θ) is the Minimax estimator (d_1^*) compared with likelihood ($\hat{\theta}_{MLE}$) and Bayes estimator (d_2^*), which works on minimizing the maximum expected risk function.

KEYWORDS: Mixed Weibull Distribution, Scale parameter, Shape parameter, Reliability function, Maximum likelihood, Minimax estimator, Bayes estimator.

الخلاصــة : -

يهتم هذا البحث بتقدير معلمات ومعولية التوزيع الاحتمالي لوقت الاشتغال لحين الفشل (توزيع ويبل ذي المعلمين θ , λ)، حيث تشير (λ) الى معلمة الشكل، في حين تشير (θ) الى معلمة القياس، وهذا التوزيع يمثل اوقات الاشتغال لحين الفشل مقاسة بالاسابيع لـ (30) ماكنة من نفس النوع في الشركة العامة للصناعات القطنية – بغداد، حيث اجري اختبار على هذه الاوقات ووجد انه يتبع توزيع ويبل ذي المعلمين، لذلك تم التركيز على تقدير المعلمات (θ , λ) ومن ثم تقدير معولية هذا التوزيع بثلاثة طرق هي طريقة الامكان الاعظم (Maximum likelihood) وطريقة المقدر (Minimax) ومقدر بيز (Bayes). تم في البحث ايضا مقارنة المقدرات الثلاث بواسطة المحاكاة لثلاث حجوم عينات (n = 25, 50, 100)، واظهرت النتائج ان المقدر (Minimax) هو الافضل مقارنة بالمقدرين الاخرين لانه يحقق اصغر متوسط مربعات خطأ ممكن.

ABBREVIATIONS:
f(t) (p.d.f): The probability density function.
F_T(t) (c.d.f): The cumulative distribution function.
R(t): Reliability function.
μ = E(t): Mean of distribution.
λ : Shape parameter.
θ : Scale parameter.
$\hat{\theta}_{MLE}$: Maximum likelihood.
d_1^* : Minimax estimator.
d_2^* : Bayes estimator.

1. INTRODUCTION:

Technological developments have witnessed the industrial sector in the last century, to the rapid development of industrial process and the emergence of electronic devices and systems in various areas of complex life such as medicine, the field of communications, space research, and military operations etc. The imposition of this development approach many researchers to configure constant and private non – hybrid (such as Weibull distribution, Gamma distribution, exponential distribution, and normal distribution), constantly technological advances received distributions mixed wide interest to prominence in many fields, including the field of reliability, the times are associated with failure assessment with reliability function mixture from two distributions or more, this mixture may due to the same distribution or two different distributions or more [Alkutbi 2005, Alfawzan 2000, Veber 2004].
Weibull distribution is one of the continues probability distribution that deals with the distribution of time to failure of machines and equipments, it is denoted by WEI (t, λ, θ), where (λ , θ) are parameters, (λ) represents the shape of distribution, (θ) represents the scale parameter, and (t) is values of random variable (T) [Webb 2002, Dan 2009, Gong 2005, Bather 2000].
The probability density function (p.d.f) of it is defined in eq. (1):

$$f(t;\ \theta,\beta) = \frac{\lambda}{\theta}\ t^{\lambda-1}\exp\left[-\frac{t^{\lambda}}{\theta}\right] \qquad t,\lambda,\theta > 0 \qquad (1)$$

2. METHODS OF ESTIMATION:

2.1. Maximum Likelihood Method:

Let; $T_1, T_2, \dots, T_n$ be independent identically distribution from $(p.d.f)$ in eq. (1), then the likelihood function is:

$$L(t_1, t_2, \dots, t_n; \lambda, \theta) = \prod_{i=1}^{n} f(t_i; \lambda, \theta)$$

$$= \frac{\lambda^n}{\theta^n}\prod_{i=1}^{n} t_i^{\ \lambda-1}\exp\left[\sum_{i=1}^{n}\frac{t_i^{\ \lambda}}{\theta}\right] \qquad (2)$$

Taking logarithm for eq. (2);

$$\log L = n \log \lambda - n \log \theta + (\lambda - 1) \sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} \frac{{t_i}^{\lambda}}{\theta}$$

$$\frac{\partial \log L}{\partial \theta} = -\frac{n}{\theta} + \sum_{i=1}^{n} \frac{{t_i}^{\lambda}}{\theta^2} = 0$$

when ($\lambda\ is\ known$), then:

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{{t_i}^{\lambda}}{n} \tag{3}$$

while ($\lambda\ is\ known$) it can be estimated from:

$$\frac{\partial \log L}{\partial \lambda} = -\frac{n}{\lambda} + \sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} \frac{{t_i}^{\lambda} \log t_i}{\theta} \tag{4}$$

Eq. (4) is nonlinear equation can be solved by Newton Raphson method, therefore the maximum likelihood estimator of reliability function is:

$$\hat{R}(t) = exp\left[-\frac{t^{\lambda}}{\hat{\theta}_{MLE}}\right] \tag{5}$$

$$\hat{R}(t) = exp\left[-\frac{t^{\hat{\lambda}_{MLE}}}{\hat{\theta}_{MLE}}\right] \tag{6}$$

2.2. The Minimax Esimator:

This estimator of parameter (θ) is one which minimizes the maximum expected loss (L) [Makhdoom 2011, Roy 2002], where:

$$L = \left(\frac{\theta - d_1}{\theta}\right)^2 \tag{7}$$

This estimator (d_1^*) is obtained considering (θ) as random variable have prior distribution:

$$\pi(\theta) \propto \frac{1}{\Gamma(\alpha)\beta^{\alpha}}\ \theta^{-(\alpha+1)}\ e^{-\frac{1}{\beta\theta}} \qquad \theta, \alpha, \beta > 0 \tag{8}$$

First we must find the posterior distribution of (θ) given ($t_1, t_2, \dots, t_n$).

$$\pi(\theta|\underline{t}) = \frac{\pi(\theta) \prod_{i=1}^{n} f(t_i; \lambda, \theta)}{\int_0^{\infty} \pi(\theta) \prod_{i=1}^{n} f(t_i; \lambda, \theta)} \tag{9}$$

Applying eq. (9), we get:

$$\pi(\theta|\underline{t}) = \frac{\left(\sum_{i=1}^{n} t_i^{\lambda} + \frac{1}{\beta}\right)^{n+\alpha}}{\Gamma(n+\alpha)} \left(\frac{1}{\theta}\right)^{\alpha+n+1} exp\left[-\frac{1}{\theta}\left(\sum_{i=1}^{n} t_i^{\lambda} + \frac{1}{\beta}\right)\right] \tag{10}$$

i.e. $\pi(\theta|\underline{t}) \sim Gamma(n+\alpha), \left(\sum_{i=1}^{n} t_i^{\lambda} + \frac{1}{\beta}\right)$

According to square error loss function:

$$L = \left(\frac{\theta - d_1}{\theta}\right)^2$$

The risk $R(d,\theta)$ is:

$$R(d,\theta) = E(loss) = \int_0^\infty \left(\frac{\theta - d_1}{\theta}\right)^2 \pi(\theta|\underline{t})d\theta$$

$$= \pi(\theta|t) - 2d_1 E\left(\frac{1}{\theta}\middle| t\right) + d_1^2 E\left(\frac{1}{\theta^2}\middle| t\right)$$

$$\frac{\partial R(d,\theta)}{\partial d} = -2E\left(\frac{1}{\theta}\middle| t\right) + 2d_1 E\left(\frac{1}{\theta^2}\middle| t\right)$$

$$d_1^* = \frac{E\left(\frac{1}{\theta}\middle|t\right)}{E\left(\frac{1}{\theta^2}\middle|t\right)}$$

$$E\left(\frac{1}{\theta}\middle|t\right) = \int_0^\infty \frac{1}{\theta}\,\pi(\theta|t)d\theta = \frac{n+\alpha}{\left(\sum_{i=1}^n t_i^\lambda + \frac{1}{\beta}\right)}$$

$$E\left(\frac{1}{\theta^2}\middle|t\right) = \int_0^\infty \frac{1}{\theta^2}\,\pi(\theta|t)d\theta = \frac{(n+\alpha)(n+\alpha+1)}{\left(\sum_{i=1}^n t_i^\lambda + \frac{1}{\beta}\right)^2}$$

Therefore the minimax estimator:

$$d_1^* = \left(\frac{\sum_{i=1}^n t_i^\lambda + \frac{1}{\beta}}{n+\alpha+1}\right) \qquad (11)$$

$$d_1^* = k_1\left(T + \frac{1}{\beta}\right)$$

where;

$$k_1 = \frac{1}{n+\alpha+1} \quad , \quad T = \sum_{i=1}^n t_i^\lambda$$

also the risk obtained from this decision (d_1^*) is denoted by $R(\theta)$,

$$R(\theta) = E\left(\frac{d_1^* - \theta}{\theta}\right)^2 \qquad (12)$$

$$= \frac{1}{\theta^2}\left[k_1^2 E\left(T + \frac{1}{\beta}\right)^2 - 2k_1\theta E\left(T + \frac{1}{\beta}\right) + \theta^2\right] \qquad (13)$$

$$\because\ t_i \sim Weibull\ (\lambda,\theta)$$

$$\therefore T = \sum_{i=1}^n t_i \sim Gamma\ (n,\theta)$$

$$h(t) = \frac{1}{\theta^n \Gamma(n)}\, t^{n-1}\, e^{-\frac{t}{\theta}} \qquad \theta > 0, t \geq 0$$

then;

$$E\left(T+\frac{1}{\beta}\right)=n\theta+\frac{1}{\beta}$$

$$E\left(T+\frac{1}{\beta}\right)^2=E(T^2)+\frac{2}{\beta}E(T)+\frac{1}{\beta^2}$$

$$=n(n+1)\theta^2+\frac{2}{\beta}n\theta+\frac{1}{\beta^2}$$

So eq. (13) summarized to:

$$R(\theta)=k_1^2\frac{E\left(T+\frac{1}{\beta}\right)^2}{\theta^2}-\frac{2k_1E\left(T+\frac{1}{\beta}\right)}{\theta}+1$$

$$=n(n+1)k_1^2+\frac{2k_1(nk_1-1)}{\beta\theta}+\left(\frac{k_1}{\beta\theta}\right)^2-2nk_1+1$$

but when $\beta\to\infty$, then the risk function estimator is:

$$R(\theta)=n(n+1)k_1^2-2nk_1+1$$

which is independent of (θ).

2.3. Bayes Estimator:

This method of estimation of parameters, consider one or more parameters, are random variables and have random distribution, obtained from experience and past data. Here we consider the scale parameter (θ) as random variable having;

$$g_2(\theta)=k\,\frac{e^{-a/\theta}}{\theta^c}$$

$$0<\theta<\infty,\quad a,c>1,\quad \theta>0$$

Then;

$$g(\theta|t)\propto L(t;\lambda,\theta)g(\theta)=k\,\frac{\lambda^n}{\theta^n}\prod_{i=1}^{n}t_i^{\lambda-1}\ \exp\left[-\frac{\sum_{i=1}^{n}t_i^{\lambda}}{\theta}\right]g_2(\theta)$$

and the marginal distribution of (t) is:

$$f(t)=\int_0^{\infty}L(t_i;\lambda,\theta)\,g(\theta)d\theta$$

$$f(t)=k\int_0^{\infty}\lambda^n\prod_{i=1}^{n}t_i^{\lambda-1}\exp\left[-\frac{(a+\sum_{i=1}^{n}t_i^{\lambda})}{\theta}\right]\left(\frac{1}{\theta}\right)^{n+c}d\theta$$

$$=k\lambda^n\prod_{i=1}^{n}t_i^{\lambda-1}\int_0^{\infty}\left(\frac{1}{\theta}\right)^{n+c}\exp\left[-\frac{(a+\sum_{i=1}^{n}t_i^{\lambda})}{\theta}\right]d\theta$$

Let $\quad y=\frac{1}{\theta}\quad\Rightarrow\quad\theta=\frac{1}{y}\qquad d\theta=-\frac{1}{y^2}dy\qquad T=(a+\sum_{i=1}^{n}t_i^{\lambda})$

Marginal distribution of (t) is:

$$f(t) = k\lambda^n \prod_{i=1}^{n} t_i^{\lambda-1} \int_0^\infty y^{n+c} \exp[-y(t)] \frac{1}{y^2} d\theta$$

using formula:

$$\int_0^\infty y^{n-1} e^{-y} dy = \Gamma(n)$$

then;

$$f(t) = \frac{k\lambda^n \prod_{i=1}^{n} t_i^{\lambda-1}}{T^{n+c-1}} \Gamma(n+c-1)$$

Therefore:

$$g_2(\theta|t) = \frac{k\lambda^n \prod_{i=1}^{n} t_i^{\lambda-1} e^{-\frac{1}{\theta}(T)} \left(\frac{1}{\theta}\right)^{n+c}}{\frac{k\lambda^n \prod_{i=1}^{n} t_i^{\lambda-1}}{T^{n+c-1}} \Gamma(n+c-1)}$$

$$g_2(\theta|t) = \frac{T^{n+c-1}}{\Gamma(n+c-1)} \left(\frac{1}{\theta}\right)^{n+c} e^{-\frac{1}{\theta}(T)}$$

which is Gamma failure model with $(n+c-1, T)$.

From this we find the Bayes estimator under this proposed prior and squared error loss function,

$$Loss = E(d_2 - \theta)^2$$

and the posterior mean:

$$d_2^* = E_2(\theta|t) = \frac{T}{n+c-1} = \frac{(a+\sum_{i=1}^{n} t_i^{\lambda})}{n+c-1}$$

$L = (d_2 - \theta)^2$ is the posterior mean;

$$E(\theta|t) = \frac{T}{n} = \frac{(\sum_{i=1}^{n} t_i^{\lambda})}{n}$$

which is equal the maximum likelihood estimator.

3. PRACTICAL APPLICATION AND SIMULATION:

The data in table (1) represents the time to failure of (30) independent machines from The State Company for Cotton Industries. From these data we have:

n = 30 t_{min} = 0.9062 t_{max} = 34.071

mean = 11.19   standard deviation = 7.85

The data in table (1) were grouped in frequency distribution in five classes, table (2) shows the results of (χ^2) test for goodness of fit to see if the data follows Weibull two parameters distribution;

H_0 : data assumed Weibull

H_1 : data not assumed Weibull

The test equation is:

$$\chi^2 = \sum_{i=1}^{k} \frac{(E_i - O_i)^2}{E_i}$$

Where;
O_i : the observed frequency
E_i : the expected frequency

Range = 33.17 class interval = 6.634

By comparing the calculated value with the tabulated value of χ^2 test with ($\alpha = 0.05$) significant level, and ($k = 5 - 2 - 1 = 2$) degree of freedom, we found that $\chi^2_{0.95,2} = 5.99$ while $\chi^2_{cal.} = 1.55555$, that shows the calculated value is less than tabulated value, so we accept H_0 means, the distribution of working time to failure for the machines is Weibull two parameters distribution.

Also we estimate the parameter (θ) using ($\hat{\theta}_{MLE}, d_1^*, d_2^*$) and reliability function using simulation for ($n = 25, 50, 100$) and different values of $\alpha, \beta, and\ c$ when $\lambda = 1,\ a = 1$, the tables (3, 4, 5, and 6) shows the details.

From table (4), the estimator (d_1^*) have smallest mean square error than $\hat{\theta}_{MLE}$ and d_2^*, and from table (5), when $n = 100$ $MSE(d_1^*)$ is the smallest one.

From table (6), we can find the percentage preference for each reliability function estimator by dividing the total frequency for the largest one to the total number of the three sample size. Table (7) shows that minimax estimator of reliability function is the best.

4. CONCLUSIONS:

1. From table (3) we find that the minimax estimator (d_1^*) have smallest mean square error as compared with $\hat{\theta}_{MLE}$ and d_2 when ($n = 25$). Also when ($n = 50$), from table (4) the (d_1^*) is the best one and from table (5), we see that MSE(d_1^*) is the best. This indicate that the estimator (d_1^*)(minimax) estimator is the best, because it works on minimize the maximum loss.
2. The distribution of time to failure of (30 work time until failure) is shown to be Weibull, as indicated by χ^2 test shown in table (2).
3. The expected mean time to failure of machines from The State Company for Cotton Industries is about (11 weeks), so the Company must perform the maintenance works before that time.
4. The estimated reliability function from three estimators become decreases when time to failure increases, as the results in table (6) indicates that reliability in t_{min} = 0.9 week is greater than for reliability at t_{max}= 15 weeks and more.

Table (1) Time to failure for 30 machines measured by weeks.

0.9062	1.2868	3.152	34.071	2.4195	3.485
3.9702	3.9921	5.6094	3.4850	4.8306	11.4391
8.3432	9.0258	12.2847	7.8240	9.7943	22.5383
13.499	13.5532	15.6932	13.165	14.5857	14.5857
18.1497	18.6352	23.8066	17.4998	19.9496	9.2672

Table (2) Results of χ^2 test for goodness of fit

$Classes$	$f_i(O_i)$	E_i	$(E_i - O_i)^2/E_i$
0.9062 – 7.5402	10	9	0.11111
7.5402 – 14.1742	10	8	0.5000
14.1742 – 20.8082	7	9	0.44440
20.8082 – 27.4422	2	2	0.0000
27.4422 – 34.0762	1	2	0.50000
Total	30	30	1.55555

Table (3) Estimator of θ by three methods with mean square error when $\lambda = 1, n = 25, a = 1$, and different values of $\alpha, \beta, and\ c$.

α	β	c	$\hat{\theta}_{MLE}$	$MSE(\theta)$	d_1^*	$MSE(d_1^*)$	d_2	$MSE(d_2)$
0.5	0.5	-1	1.00577	0.0016178	1.02431	0.0014623	1.10793	0.0021491
		1	0.99593	0.0016389	1.01503	0.0014671	1.05483	0.0016049
		-2	0.99756	0.0016136	1.01657	0.0014469	1.12271	0.0023537
		2	1.00551	0.0015975	1.02407	0.0014439	1.04395	0.0015537
	1	-1	1.00560	0.0017058	0.98642	0.0015245	1.06694	0.0019541
		1	0.99655	0.0014724	0.97788	0.0013295	1.01623	0.0014253
		-2	0.99896	0.0014232	0.98015	0.0012824	1.08248	0.0018171
		2	1.00389	0.0017501	0.98480	0.0015663	1.00393	0.0016188
	1.5	-1	0.99962	0.0016602	0.96819	0.0015181	1.04723	0.0018179
		1	0.99879	0.0018150	0.96741	0.0016578	1.00535	0.0017456
		-2	1.00266	0.0015481	0.97107	0.0014111	1.07245	0.0018902
		2	0.98880	0.0014993	0.95799	0.0014005	0.97659	0.0014040
1	0.5	-1	0.99511	0.0016702	0.99547	0.0014319	1.07511	0.0018949
		1	0.99132	0.0016488	0.99196	0.0014135	1.03011	0.0015578
		-2	1.00029	0.0015325	1.00027	0.0013139	1.10257	0.0020172
		2	0.99417	0.0015996	0.99461	0.0013714	1.01355	0.0014302
	1	-1	0.99818	0.0018003	0.96128	0.0016034	1.03818	0.0018585
		1	0.99509	0.0015422	0.95842	0.0013905	0.99528	0.0014258
		-2	0.99368	0.0015174	0.95711	0.0013732	1.05499	0.0017000
		2	0.99368	0.0014533	0.95711	0.0013182	0.97543	0.0013168
	1.5	-1	1.00002	0.0015305	0.95064	0.0014096	1.02669	0.0015590
		1	1.01424	0.0018033	0.96380	0.0015915	1.00087	0.0016598
		-2	0.99897	0.001P5482	0.94967	0.0014287	1.04679	0.0017003
		2	0.99791	0.0016164	0.94869	0.0014910	0.96676	0.0014832
1.5	0.5	-1	1.00637	0.0015203	0.98761	0.0012612	1.06507	0.0016291
		1	0.99431	0.0017322	0.97664	0.0014523	1.01350	0.0015478
		-2	0.99841	0.0016794	0.98037	0.0014033	1.07862	0.0019272
		2	1.00595	0.0014791	0.98722	0.0012278	1.00568	0.0012686
	1	-1	0.99818	0.0015528	0.94380	0.0014096	1.01782	0.0015051
		1	0.98615	0.0014639	0.93286	0.0013838	0.96807	0.0013368
		-2	1.00125	0.0015560	0.94659	0.0014000	1.04146	0.0016253
		2	0.99800	0.0018107	0.94364	0.0016234	0.96128	0.0016128
	1.5	-1	0.99989	0.0016449	0.93323	0.0015377	1.00643	0.0015827
		1	0.99233	0.0018113	0.92636	0.0017119	0.96131	0.0016698
		-2	0.98748	0.0015653	0.92195	0.0015321	1.01435	0.0015679
		2	0.99754	0.0015188	0.93110	0.0014449	0.9480	0.0014085

Table (4) Estimator of θ by three methods with mean square error when $\lambda = 1, n = 50, a = 1$, and different values of $\alpha, \beta, and\ c$.

α	β	c	$\hat{\theta}_{MLE}$	$MSE(\theta)$	d_1^*	$MSE(d_1^*)$	d_2	$MSE(d_2)$
0.5	0.5	-1	0.99737	0.0003536	1.00716	0.0003342	1.04785	0.0004064
		1	1.00159	0.0003774	1.01125	0.0003582	1.03127	0.0003895
		-2	1.01028	0.0003864	1.01969	0.0003700	1.07177	0.0005032
		2	1.00625	0.0004194	1.01578	0.0003995	1.02579	0.0004157
	1	-1	1.00925	0.0004245	0.99927	0.0003995	1.03964	0.0004628
		1	0.98612	0.0004044	0.97681	0.0003883	0.99616	0.0003930
		-2	1.00710	0.0003776	0.99718	0.0003551	1.04811	0.0004384
		2	0.99230	0.0003604	0.98282	0.0003445	0.99250	0.0003465
	1.5	-1	1.00442	0.0004727	0.98811	0.0004480	1.02804	0.0004976
		1	0.99789	0.0003677	0.98177	0.0003532	1.00121	0.0003604
		-2	1.00348	0.0004075	0.98720	0.0003872	1.03762	0.0004524
		2	0.99709	0.0003601	0.98099	0.0003465	0.99066	0.0003477
1	0.5	-1	0.99348	0.0004301	0.99373	0.0003976	1.03348	0.0004517
		1	1.00215	0.0003894	1.00207	0.0003600	1.02171	0.0003836
		-2	1.00211	0.0004065	1.00203	0.0003758	1.05269	0.0004702
		2	0.99637	0.0004117	0.99651	0.0003806	1.00623	0.0003886
	1	-1	1.00137	0.0004265	0.98209	0.0004007	1.02137	0.0004356
		1	0.99857	0.0003816	0.97940	0.0003612	0.99860	0.0003667
		-2	0.99091	0.0003957	0.97203	0.0003800	1.02118	0.0004111
		2	1.01685	0.0004068	0.99697	0.0003710	1.00670	0.0003790
	1.5	-1	1.01314	0.0003922	0.98700	0.0003628	1.02648	0.0004028
		1	1.00452	0.0003697	0.97871	0.0003505	0.99790	0.0003550
		-2	1.00432	0.0004127	0.97852	0.0003904	1.02799	0.0004364
		2	1.00786	0.0004315	0.98192	0.0004043	0.99150	0.0004070
1.5	0.5	-1	1.00567	0.0003987	0.99588	0.0003614	1.03532	0.0004152
		1	1.00052	0.0004277	0.99097	0.0003896	1.01022	0.0004052
		-2	0.98985	0.0003816	0.98081	0.0003516	1.02990	0.0003974
		2	1.00318	0.0004448	0.99351	0.0004041	1.00311	0.0004113
	1	-1	0.99539	0.0003674	0.96703	0.0003546	1.00533	0.0003603
		1	0.99468	0.0004039	0.96636	0.0003884	0.98513	0.0003846
		-2	1.00194	0.0004187	0.97327	0.0003940	1.02199	0.0004283
		2	0.99583	0.0003869	0.96745	0.0003718	0.97680	0.0003682
	1.5	-1	1.00689	0.0003862	0.97164	0.0003655	1.01012	0.0003797
		1	1.00668	0.0004132	0.97144	0.0003903	0.99031	0.0003905
		-2	1.00072	0.0004053	0.96576	0.0003911	1.01410	0.0004093
		2	1.00512	0.0004332	0.96996	0.0004105	0.97933	0.0004086

Table (5) Estimator of θ by three methods with mean square error when $\lambda = 1, n = 100, a = 1$, and different values of $\alpha, \beta, and\ c$.

α	β	c	$\hat{\theta}_{MLE}$	$MSE(\theta)$	d_1^*	$MSE(d_1^*)$	d_2	$MSE(d_2)$
0.5	0.5	-1	0.99960	0.0000965	1.00453	0.0000938	1.02472	0.0001035
		1	1.00656	0.0001024	1.01139	0.0001003	1.02145	0.0001055
		-2	1.00350	0.0000962	1.00838	0.0000939	1.03385	0.0001094
		2	1.00875	0.0000961	1.01355	0.0000944	1.01858	0.0000970
	1	-1	0.99470	0.0000945	0.98985	0.0000924	1.00975	0.0000961
		1	0.99554	0.0000931	0.99069	0.0000911	1.00054	0.0000920
		-2	1.00075	0.0000969	0.99581	0.0000942	1.02097	0.0001032
		2	0.99751	0.0001036	0.99262	0.0001010	0.99755	0.0001015
	1.5	-1	1.00023	0.0000972	0.99202	0.0000949	1.01196	0.0000996
		1	0.99984	0.0000996	0.99163	0.0000974	1.00150	0.0000987
		-2	0.99684	0.0000892	0.98868	0.0000878	1.01366	0.0000928
		2	0.99692	0.0000871	0.98875	0.0000857	0.99366	0.0000857
1	0.5	-1	0.99722	0.0000976	0.99727	0.0000938	1.01722	0.0001005
		1	1.00690	0.0000927	1.00677	0.0000891	1.01674	0.0000932
		-2	1.00522	0.0001036	1.00512	0.0000996	1.03038	0.0001136
		2	1.00444	0.0000987	1.00435	0.0000949	1.00931	0.0000965
	1	-1	1.00235	0.0001096	0.99250	0.0001059	1.01235	0.0001111
		1	0.99869	0.0000976	0.98891	0.0000950	0.99870	0.0000956
		-2	0.99765	0.0001061	0.98789	0.0001034	1.01273	0.0001088
		2	1.00748	0.0001004	0.99753	0.0000961	1.00246	0.0000970
	1.5	-1	0.99505	0.0001024	0.98208	0.0001014	1.00172	0.0001022
		1	1.00009	0.0000966	0.98702	0.0000945	0.99679	0.0000948
		-2	1.00868	0.0000982	0.99544	0.0000939	1.02046	0.0001026
		2	0.99368	0.0001036	0.98073	0.0001029	0.98557	0.0001023
1.5	0.5	-1	1.00828	0.0000973	1.00319	0.0000920	1.02316	0.0001010
		1	1.00019	0.0000991	0.99530	0.0000945	1.00511	0.0000964
		-2	1.00379	0.0000914	0.99881	0.0000869	1.02380	0.0000969
		2	0.99853	0.0001014	0.99369	0.0000969	0.99857	0.0000975
	1	-1	0.99576	0.0000928	0.98123	0.0000917	1.00076	0.0000917
		1	1.00676	0.0000911	0.99196	0.0000869	1.00174	0.0000880
		-2	0.99924	0.0000988	0.98462	0.0000964	1.00925	0.0000996
		2	0.98960	0.0000857	0.97522	0.0000867	0.98001	0.0000853
	1.5	-1	1.00495	0.0001090	0.98695	0.0001052	1.00659	0.0001081
		1	1.00607	0.0001041	0.98803	0.0001002	0.99777	0.0001008
		-2	1.00179	0.0000963	0.98386	0.0000943	1.00846	0.0000970
		2	1.00412	0.0001078	0.98614	0.0001044	0.99098	0.0001043

Table (6) Values of reliability function estimator for estimating reliability function by all methods.

c	β	α	Reliability								
			$n=25$			$n=50$			$n=100$		
			R_1	R_2	R_3	R_1	R_2	R_3	R_1	R_2	R_3
1		1	0.82451	0.826174	0.825144	0.82951	0.830193	0.829804	0.829675	0.830168	0.829817
	1	1	0.651095	0.653085	0.653526	0.655822	0.656337	0.656997	0.658995	0.659559	0.65957
		2	0.058328	0.060467	0.067144	0.052692	0.053355	0.057135	0.05319	0.053168	0.055442
		1	0.908281	0.908815	0.908456	0.910324	0.910639	0.910403	0.911124	0.911236	0.911161
	1	1	0.806926	0.8083	0.807688	0.809254	0.809517	0.809621	0.812178	0.812655	0.812354
		2	0.225584	0.227907	0.235223	0.226859	0.229068	0.231762	0.227528	0.229739	0.22999'9
		1	0.938796	0.939191	0.938873	0.9392	0.939509	0.939236	0.939839	0.939888	0.939856
	2	1	0.866179	0.866936	0.866548	0.869084	0.869552	0.869258	0.869072	0.86923	0.869161
		2	0.367096	0.369324	0.374338	0.372079	0.375025	0.375685	0.372253	0.373349	0.374064
2		1	0.828093	0.82956	0.828698	0.829539	0.829519	0.829833	0.830226	0.830645	0.830368
	1	1	0.654347	0.655819	0.656731	0.656356	0.657886	0.657527	0.659055	0.659392	0.65963
		2	0.055266	0.057023	0.063964	0.053773	0.054303	0.058208	0.053	0.053526	0.055248
		1	0.908275	0.908926	0.90845	0.909894	0.910174	0.909974	0.911115	0.911197	0.911153
	1	1	0.806605	0.807572	0.807372	0.8093	0.809711	0.809667	0.811929	0.812575	0.812107
		2	0.227238	0.231087	0.236858	0.226473	0.22859	0.231373	0.2281	0.229627	0.23057
		1	0.93796	0.937984	0.938039	0.940289	0.94048	0.940323	0.940329	0.940492	0.940346
	2	1	0.865114	0.86601	0.865489	0.868839	0.869391	0.869013	0.870462	0.870411	0.870549
		2	0.367415	0.3714	0.374649	0.370952	0.372642	0.37457	0.371637	0.371946	0.37345
3		1	0.828908	0.830232	0.829507	0.829268	0.829633	0.829562	0.830414	0.830757	0.830555
	1	1	0.646506	0.649355	0.648997	0.651835	0.652989	0.653035	0.659648	0.660033	0.66022
		2	0.057818	0.059641	0.066587	0.054365	0.055533	0.058833	0.053235	0.053308	0.055488
		1	0.909236	0.909964	0.909408	0.909239	0.909589	0.909322	0.911355	0.911582	0.911393
	1	1	0.80447	0.805409	0.805252	0.808975	0.809166	0.809343	0.810913	0.810923	0.811092
		2	0.224315	0.226673	0.233967	0.225778	0.226815	0.230682	0.227087	0.228204	0.22956
		1	0.937844	0.938282	0.937924	0.939105	0.939355	0.939141	0.939521	0.939726	0.939538
	2	1	0.865756	0.866842	0.866127	0.86793	0.868221	0.868106	0.868986	0.868841	0.869074
		2	0.362254	0.36494	0.369588	0.373813	0.375784	0.377403	0.375086	0.376023	0.376882

R_1: Maximum Likelihood Estimator of Reliability Function.

R_2: Minimax Estimator of Reliability Function.

R_3: Bayes Estimator of Reliability Function.

Table (7) Percentage preference for reliability function estimator.

	R_1	R_2	R_3
Number of frequency	0	41	40
Percentage value	0 %	23.43 %	22.86 %

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