Design of a Continuous Sliding Mode Controller for Path Tracking of an Articulated Vehicle

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Abstract– *This article presents design and evaluation of a sliding mode control scheme, being applied to the case of an articulated vehicle. The proposed Sliding Mode Controller (SMC) is based on a continuous sliding surface, being introduced for reducing the chattering phenomenon, while achieving a better tracking performance and a fast minimization of the corresponding tracking error. The derivation of the sliding mode controller relies on the fully nonlinear kinematic model of the articulated vehicle, while the overall stability of the control scheme is proven based on the Lyapunovs stability condition. The performance of the established control scheme is evaluated through circle path tracking scenario on a small-scale articulated vehicle.*

Index Terms*— Sliding mode control, path planning.*

I. INTRODUCTION

Recently, there has been a significant focus in designing automated vehicles for their utilization in the industry. Among the current vehicle types utilized in industry field, articulated ones are the most characteristic type that can be found most frequently, such as the Load Haul Dump (LHD) vehicles. In general, the articulated vehicles consist of two parts, a tractor and a trailer, linked with a rigid free joint. Each body has a single axle and the wheels are all non-steerable, while the steering action is performed on the joint, by changing the corresponding articulated angle, between the front and the rear part of the vehicle.

In the related literature there have been several research approaches for the problem of modeling articulated vehicles, either by considering point kinematic properties or based on the theory of multiple body dynamics [1–3]. Furthermore, from a control point of view, there have been proposed many traditional techniques for non–holonomic vehicles as the articulated vehicles. In [4] a linear control feedback has been applied, while in [5] a Lyapunov based approach has been presented. In [6] a control scheme based on LMIs has been evaluated and in [7] a pole placement technique has been applied. Moreover, in [8] a path-tracking controller based on error dynamics, and in [9] a Switching Model Predictive Control framework has been established. However, to the authors' best knowledge, the application of sliding mode control scheme and its merits, based on the non-linear error dynamics modeling of the articulated vehicle, has not appeared yet in the related literature.

In general, Sliding Mode Control (SMC) is a robust control scheme based on the concept of changing the structure of the controller, with respect to the changing state of the system in order to obtain a desired response [10]. The biggest advantage of the SMC is its insensitivity to variations in system parameters, external disturbances and modeling errors [11], a fact that can be achieved by forcing the state trajectory of the system to follow the desired sliding surface as fast as possible and in a minimum tracking error. Among the general advantages of utilizing the sliding mode control, it should be mentioned the ability of achieving a fast response, a good transient performance and an overall robustness regarding parameters' variations [12].

The aim of this article is to present a SMC scheme, being tuned and developed based on the nonlinear kinematic equations of an articulated vehicle model. Following this aim, the main

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contributions of the article are three. Firstly, a continuous sliding control surface for controlling the articulated vehicle, while reducing the chattering e□ects, and improving the reference tracking is being presented. Secondly, the overall stability of the proposed SMC is being proven based on Lyapunov's theory. Thirdly, the SMC's overall performance is being extensively evaluated by the utilization of a realistic small-scale articulated vehicle.

The rest of the article structured as it follows. In Section II, the non-linear full kinematic model of the articulated vehicle will be presented. In Section III, conventional and sliding mode control schemes, based on the error dynamic model of the articulated vehicle are presented, followed by multiple simulation results that prove the applicability and the overall performance of the proposed scheme in Section IV. Finally, concluding remarks are mentioned in Section V.

II. ARTICULATED VEHICLE MODEL

The articulated vehicle, considered in this article, can be described by two parts, linked with a rigid free joint, with lengths l_1 and l_2 respectively, while each part has a single axle and all wheels are non-steerable. The steering action is being performed on the middle joint, by changing the corresponding articulated angle γ, in the middle of the vehicle and $\dot{\gamma}$ is the rate of change for this articulated angle. The velocities v_1 and v_2 are considered to have the same changing with respect to the velocity of the rigid free joint of the vehicle, indicated by $\dot{\theta}_1, \dot{\theta}_2$, which are the angular velocities of the front and rear parts of the vehicle respectively. Overall, the articulated vehicle's geometry is depicted in Fig.1.

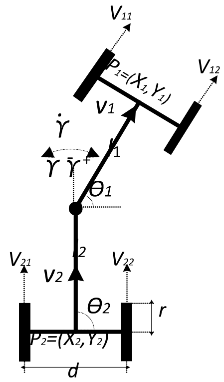

FIG.1 THE ARTICULATED VEHICLE MODEL

Based on the modeling approach presented in [13], the full kinematic model of the articulated vehicle, under the non-holonomic constraints, can be provided as it follows:

$$\dot{x}_1 = v_1 \cos\theta_1 \qquad \dot{y}_1 = v_1 \sin\theta_1 \tag{1}$$

$$\dot{\theta}_1 = \frac{v_1 \sin\gamma + l_2\dot{\gamma}}{l_1 \cos\gamma + l_2}$$

Where x_1 and y_1 are the position variables and $\dot{\theta}_1$, is the front orientation angle. In the presented approach, the longitudinal velocity v_1 and the steering angle $\dot{\gamma}$ are the control signals, while the kinematic equations in a state space formulation can be given as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta}_1 \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 \\ \sin\theta_1 & 0 \\ \frac{\sin\gamma}{l_1\cos\gamma + l_2} & \frac{l_2}{l_1\cos\gamma + l_2} \end{bmatrix} \begin{bmatrix} v_1 \\ \dot{\gamma} \end{bmatrix} \tag{2}$$

The velocities of the vehicle wheels are being controlled individually, but in a coordinated manner and in full symphony with the overall dimensions of the articulated vehicle and the articulation angle

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(γ and $\dot{\gamma}$), as depicted in the following geometrically derived equations [14]:

$$v_{11} = v_1 + \frac{v_1 \sin\gamma}{4l(\cos\gamma + 1)} - \frac{v_1 d\dot{\gamma}}{2(\cos\gamma + 1)}$$

$$v_{12} = v_1 - \frac{v_1 \sin\gamma}{4l(\cos\gamma + 1)} + \frac{v_1 d\dot{\gamma}}{2(\cos\gamma + 1)}$$

$$v_{21} = v_1 \cos\gamma + \frac{v_1 \sin^2\gamma}{2(\cos\gamma + 1)} + \frac{d \sin\gamma}{2l(\cos\gamma + 1)} + \frac{\dot{\gamma}(2l\, sin\,\gamma - d\, cos\,\gamma)}{2(cos\,\gamma + 1)}$$

$$v_{22} = v_1 \cos\gamma + \frac{v_1 \sin^2\gamma}{2(\cos\gamma + 1)} - \frac{d \sin\gamma}{2l(\cos\gamma + 1)} + \frac{\dot{\gamma}(2l \sin\gamma - d \cos\gamma)}{2(\cos\gamma + 1)}$$

Furthermore, it assumed that the reference path coordinates are being denoted by $\boldsymbol{x_r} = [x_r\ y_r\ \theta_r\ ]^T$ and that the vehicle is moving with a constant velocity of v_r. In the following derivation, the three error coordinates, expressing the deviation of the vehicle from the desired position and heading, are denoted as $\boldsymbol{x_e} = [x_e\ y_e\ \theta_e\ ]^T$, while in a state space formulation this error dynamics modeling, with respect to the reference path, can be formulated as it follows:

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 - x_r \\ y_1 - y_r \\ \theta_1 - \theta_r \end{bmatrix} \tag{4}$$

In the sequel it is being assumed that the vehicle is driven without lateral slip angles in all wheels and that the lengths of the front and rear parts are of the same length l. Moreover, it is assumed that the vehicle is moving on a flat surface and the curvature of the reference path is denoted by $c_r = \frac{1}{R}$, where R is the radius of this path. Di□erentiation of the error dynamic equations in (4) will lead into:

$$\dot{e}_x = -sin\theta_1\dot{\theta}_1(x_1 - x_r) + cos\theta_1(\dot{x}_1 - \dot{x}_r) + cos\theta_1\dot{\theta}_1(y_1 - y_r) + sin\theta_1(\dot{y}_1 - \dot{y}_r)$$

$$\dot{e}_y = cos\theta_1\dot{\theta}_1(x_1 - x_r) + sin\theta_1(\dot{x}_1 - \dot{x}_r) - sin\dot{\theta}_1(y_1 - y_r) + cos\theta_1(\dot{y}_1 - \dot{y}_r) \tag{5}$$

$$\dot{e}_\theta = \frac{v_1 \sin\gamma}{l(\cos\gamma + 1)} - \frac{v_1 \sin\gamma \cos e_\theta}{(l - e_y \sin\gamma)} + \frac{\dot{\gamma}}{\cos\gamma + 1}$$

By further assuming a small value of the distance between the vehicle's actual location and the reference path, the corresponding curvature can be calculated as $c_r = \frac{\sin\gamma}{l}$. By substituting (2) into (4) and by utilizing basic trigonometrical principles, the final nonlinear error dynamics of the articulated vehicle can be presented by:

$$\dot{e}_x = \dot{\theta}_1 e_y + v_1 cose_\theta - v_r$$

$$\dot{e}_y = -\dot{\theta}_1 e_x - v_1 sine_\theta \tag{6}$$

$$\dot{e}_\theta = \frac{v_1 \sin\gamma}{l(\cos\gamma + 1)} - \frac{v_1 \sin\gamma \cos e_\theta}{(l - e_y \sin\gamma)} + \frac{\dot{\gamma}}{\cos\gamma + 1}$$

Based on the equations in (6), the controller design, will be formulated in Section IV, under the following assumptions: a) The velocity v of the vehicle is forward and bounded, b) The heading and the displacement error is bounded $|e_\theta| < \pi/2$, c) The articulated steering angle is limited, and d) the rotation of the vehicle's wheels is bounded and assumed to achieve a maximum velocity of $|v_1| < v_{max}$

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III. SLIDING MODE CONTROL

A. Conventional Sliding Mode Control

To realize the concept of a general SMC, two parts need to be specified, the equivalent control u_{eq}, and the sliding surface part $s(x,t)$. In general, the selected control scheme is able to achieve a fast convergence and also presents robustness against external disturbances and uncertainties, such as localization errors and noise corrupted measurements [15, 16]. In the sequel, a single input nonlinear system, having the following general form will be considered [17]:

$$x^n = f(x) - b(x)u \tag{7}$$

Where $x = [x \;\; \dot{x} \;\; \dots . x^{(n-1)}]^T$ the state vector and u ∈ R is the control input. The function f(x) is a continuous nonlinear function and b(x) is not exactly known but the extents of the imprecision are bounded. For this system, the control problem can be defined as the problem of tracking the specific time varying reference states $\boldsymbol{x_r}$=$[x_r \; \dot{x}_r \;\; \dots . x_{(n-1)}\mathrm{r}\;]^T$. In the general case, the reference states x_r can vary randomly, however in the real application specific case of the articulated vehicle, x_r is being provided by a corresponding path planner component. In the described general case, the tracking error can be defined by:

$$x_e = x - x_r = [x_e \;\; \dot{x}_e \dots x_e^{(n-1)}]^T \tag{8}$$

Where x_e represents the tracking error vector. The SMC design procedure will start with the design of the proper control law for ensuring sliding condition and stability, or equivalently by selecting a suitable smooth discontinuous control function to eliminate chattering problems, while achieving robustness against disturbances. For tracking the reference path, all the errors should be driven to zero, or $x_e = 0$ and thus the following sliding surface s is being introduced as:

$$s(x,t) = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} x_e \tag{9}$$

The forward gain value λ is the slop of the switching line, which is a strictly positive constant, switched as a function of the state and time and denoted in the conventional sliding mode control by:

$$s(x,t) = \dot{x}_e + \lambda x_e \tag{10}$$

Based on the above formulation of the sliding surface s and for the commonly utilized value of λ = 1, the sliding surface can be graphically illustrated as depicted in Fig.2, while the surface in the phase plane is characterized by a straight line with a slop of −λ.

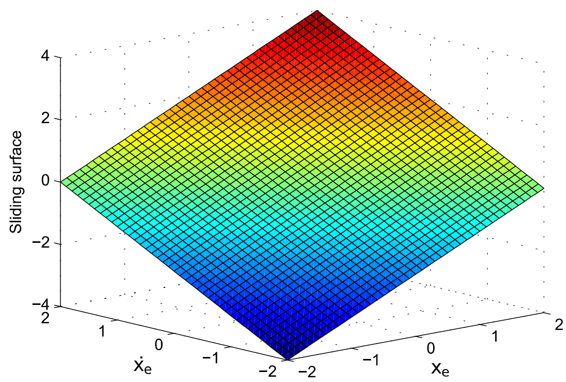

FIG. 2. SLIDING SURFACE OF THE CONVENTIONAL SLIDING MODE CONTROL.

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The convergence of the tracking error vector to zero can be achieved by choosing the sliding surface as s(x, t) = 0, which is often called as the switching line, defining the set of points in the phase plane where s = 0. A common selection for the structure of the sliding mode controller for such a system can be formulated as:

$$u = u_{eq} - k\, sgn(s(x,t)) \tag{11}$$

Where u_{eq} is the continuous control law, which derives the states to the sliding surface in finite time, and k is the forward gain value switched by the sgn function, while it is being chosen to satisfy the desired reaching condition. The conventional sliding surface term sgn(s(x,t)) is a discontinuous control function and also it is defined as a sliding control, which acts to keep the states in the sliding surface.

The utilization of a sgn(s(x,t)) function, often causes chattering in practice, especially when the sliding surface has been reached, a fact that degrades the overall performance of the control system and can even damage the mechanical parts and the actuators, especially when implemented in real life [18, 19]. For avoiding the chattering phenomenon, the saturation function, sat(s(x,t) φ), is commonly utilized instead of the sgn(s(x,t)) function, where φ defines the thickness of the boundary layer. Furthermore, various combinations of the (sgn and sat) functions cannot be further utilized for eliminating the chattering phenomenon. However, the control law has discontinuous control functions in sgn(s(x,t)) which leads to chattering.

B. Proposed Sliding Mode Control

In this article, one of the main contributions is the introduction and the evaluation of a SMC nonlinear sliding surface, without discontinuous control functions, which as it is going to be presented, is capable of preventing the chattering e□ects. The nonlinear sliding surface is being introduced by the fast excitations of the control input signal and is being defined as:

$$s(x,t) = \dot{x}_e + \lambda_1 x_e + \lambda_2 x_e|x_e| + \lambda_3 \dot{x}_e|x_e| \tag{12}$$

In Eq. 12, the sliding gains λ_i with i ∈ [1, 2,3] should be chosen appropriately for stabilizing the sliding surface vector s = 0. If s is asymptotically stable, then $\dot{x}_e$ and x_e converge to zero asymptotically. If s = 0, then $\dot{x}_e$ is being calculated by:

$$\dot{x}_e = \frac{-\lambda_1 x_e(1+\frac{\lambda_2}{\lambda_1}|x_e|}{1+\lambda_3|x_e|} \tag{13}$$

Based on Eq. 13 in order to guarantee an appropriate stabilization for the sliding surface, the parameter of λ_1 can be selected as positive constant, or $\lambda_1 > 0$, it can be derived that the selection of $\lambda_3 = \lambda_2,\ \lambda_1$ could satisfy the stability condition, since Eq. 13 is transformed into:

$$\dot{x}_e = -\lambda_1 x_e, with \quad \lambda_1 > 0 \tag{14}$$

In Fig. 3 the sliding surface of the proposed SMC is being displayed, where as it can be observed it is distributed in all the phase plane, while being able to regulate the states of the system to the origin point.

For verifying the performance of the proposed chattering part of the SMC, we can apply the new nonlinear function to a simple underwater vehicle model as in [17]. The underwater vehicle model is as follows,

$$u = m\ddot{x} + c\dot{x}|\dot{x}| \tag{15}$$

Where x defines position, u is the control input, m is the mass of the vehicle, and c is a drag coe□cient. The parameters m and c are not known exactly, but with a prior bounds assumed as below,

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$$1 \leq m \leq 5, \;\; 1.5 \leq c \leq 2.5 \tag{16}$$

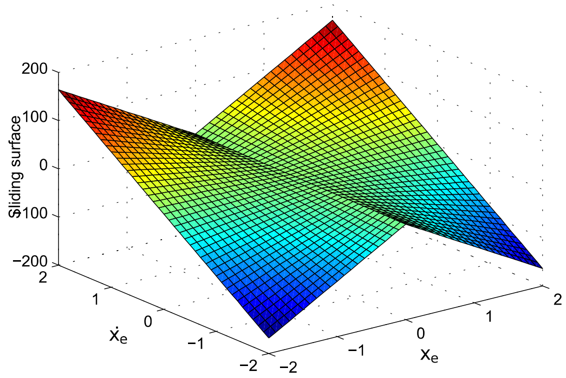

FIG. 3. SLIDING SURFACE OF THE PROPOSED SLIDING MODE CONTROL

In order to have the system track, a sliding surface s = 0 has been defined according to Eq. 11. Let's design the SMC as follow,

$$u = \hat{m}(\ddot{x}_r - \lambda \dot{x}_e) + \hat{c}\dot{x}|\dot{x}| - k\, sgn(s),$$
$$k = (F + \beta\eta) + \hat{m}(\beta - 1)|\ddot{x}_r - \dot{x}_e| \tag{17}$$

Where $F = 0.857\dot{x}|\dot{x}|$, $\beta = \sqrt{2}$, and the reference trajectory is x_r= sin(πt/2). In case of using the proposed SMC, let's design the control input of the SMC as follow,

$$u = \hat{m}(\ddot{x}_r - \lambda \dot{x}_e) + \hat{c}\dot{x}|\dot{x}| - k(s),$$
$$s = \dot{x}_e + \lambda_1 x_e + \lambda_2 x_e|x_e| + \lambda_3 \dot{x}_e|\dot{x}_e| \tag{18}$$

The trajectory, control input and tracking error of conventional and the proposed SMC are shown below in Figures 4, 5 and 6. In the simulation results, the performance of the proposed SMC has been compared with that of the conventional counterpart. The simulation parameters are; m = 2.5, c = 2, η = 3, and λ = 10. In addition, the parameters of the proposed SMC are; λ_1 = 175, λ_2 = 15 and λ_3 = 0.085. Figures 4, 5 and 6 show the output behavior of the actual and desired path with tracking error, and control inputs of underwater vehicle model. From these figures, it seems that the proposed SMC has good tracking performance and less chattering in control input.

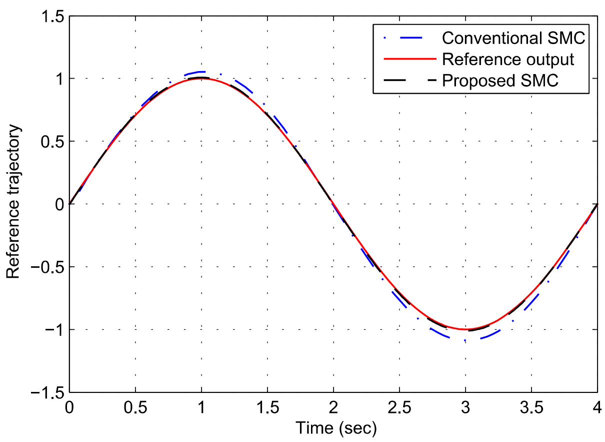

FIG. 4. TRAJECTORY TRACKING

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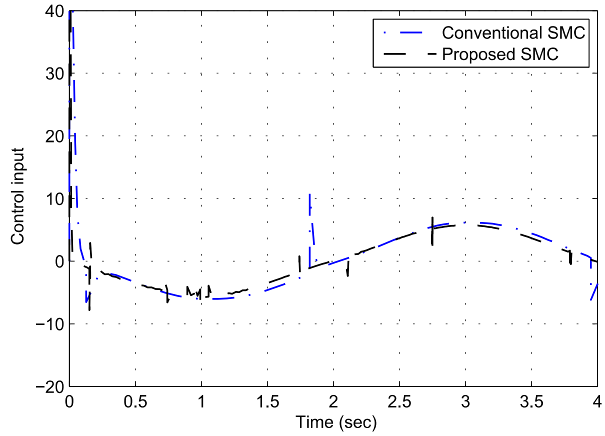

FIG. 5. CONTROL INPUT

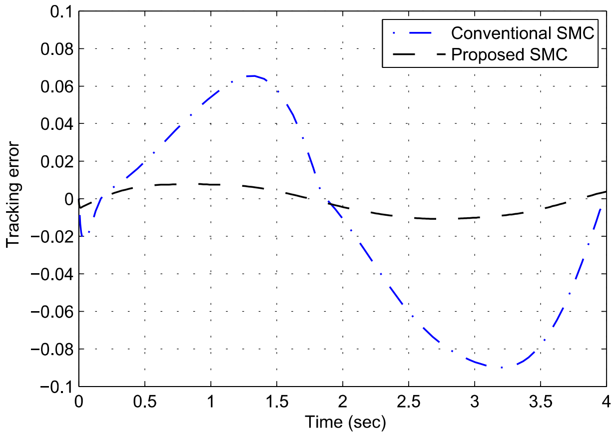

FIG. 6. TRACKING ERROR

IV. DESIGN OF THE SLIDING MODE CONTROL FOR THE ARTICULATED VEHICLE

In this Section, an articulated vehicle based sliding mode controller for tracking a reference path will be proposed. In the derivation of the proposed control scheme, the full nonlinear error kinematics model of the vehicle will be considered, in contrast to the existing approaches in the field that are oversimplifying the problem by utilizing the kinematics of a single point $\dot{x}$ = u.

Based on the kinematic equations of the articulated vehicle in Eq. 2, two control signals, the longitudinal velocity v_l and the rate of steering angle $\dot{\gamma}$ are applied to the system. Furthermore, the convergence of the tracking error vector to zero s(x, t) = 0 can be achieved by selecting the sliding surface as: $s = [s_1 \ \ s_2]^T$, with the proposed continuous sliding surfaces, according to Eq. 12, as it follows:

$$s_1 = \dot{e}_x + \lambda_1 e_x + \lambda_2 e_x|e_x| + \lambda_3 \dot{e}_x|e_x| \quad (19)$$
$$s_2 = \dot{e}_\theta + \lambda_4 e_\theta + \lambda_5 e_y|e_y| + \lambda_6 \dot{e}_y|e_y|$$

Where s_i is utilized to keep the states on the sliding line, while retaining a smooth sliding mode, without chattering phenomenon in reaching the origin point. In Eq. 19, if s_1 converges to zero, trivially ex also converges to zero, according to the stability condition as described in Eq. 13. For stabilizing the sliding surface or setting $s_2 = 0$, a similar procedure can be utilized, and thus it can be derived that is calculated by:

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$$\dot{e}_\theta = -\lambda_4 e_\theta - |e_y|(\lambda_5 e_y + \lambda_6 \dot{e}_y) \tag{20}$$

Based on Eq. 20 and by assuming that the parameters λ_5 and λ_6, are selected to satisfy the following criteria:

$$\lambda_5 = \propto_1 \; sgn(e_y) \qquad \lambda_6 = \propto_2 \; sgn(\dot{e}_y) \tag{21}$$

Where αi are positive key scaling factors, and $\lambda_4 > 0$. In this case, when $\dot{e}_y \le 0$, then $e_y \ge 0$ and if $\dot{e}_y \ge 0$ then $e_y \le 0$ and therefor, the equilibrium state of $(\dot{e}_y, e_y)$ is asymptotically stable. Thus, if these conditions for the sliding surface s_2 are satisfied, the convergence of the error state vector to zero is guaranteed. The sliding gains λ_i with i ∈ [1,2,3,4], and $\propto_i$ with i ∈ [1,2] should be chosen appropriately according to the conditions in Eq. 13 and Eq. 21, for stabilizing the sliding surface. In this case, the general structure of the proposed SMC input u_i without using sgn or sat functions for an articulated vehicle can be expressed as it follows:

$$u_1 = u_{eq1} - k_1 s_1 \qquad u_2 = u_{eq2} - k_2 s_2 \tag{22}$$

Where u_1 is the control longitudinal velocity, and u_2 is the rate of control steering angle. The parameters k_i are the gains of the control law, which are chosen large enough to satisfy the desired reaching condition. Moreover, Eq. 19 is utilized to satisfy the sliding condition of the sliding surface part and for the equivalent control part u_{eqi}, which is the solution of $\dot{s} = 0$, in order to bring the system states, from any initial position in the phase plane, to the sliding line. The control laws (u_{eq1}, u_{eq2}) that stabilize the sliding surfaces $(\dot{s}_1, \dot{s}_2)$ are calculated by solving the equations $\dot{s}_i = 0$, for the corresponding control input u_{eqi}. Therefore, from Eq. 19 after calculating the first time derivatives it can be obtained that:

$$\dot{s}_1 = \ddot{e}_x + \lambda_1 \dot{e}_x + \lambda_2 e_x \frac{e_x}{|e_x|} + \lambda_2 \dot{e}_x |e_x| + \lambda_3 \dot{e}_x \frac{e_x}{|e_x|} + \lambda_3 \ddot{e}_x |e_x|$$
$$\dot{s}_2 = \ddot{e}_\theta + \lambda_4 \dot{e}_\theta + \lambda_5 e_y \frac{e_y}{|e_y|} + \lambda_5 \dot{e}_y |e_y| + \lambda_6 \dot{e}_y \frac{e_y}{|e_y|} + \lambda_6 \ddot{e}_y |e_y| \tag{23}$$

For the articulated vehicle, the control law structure of the proposed SMC will allow the tracking error vector $\boldsymbol{x_e} = [x_e \; y_e \; \theta_e]^T$ to reach the sliding surface and thus achieving $\dot{s}_i = 0$. The best approximation of the continuous part u_{eq} of the control law, in the tracking case, can be constructed by utilizing the second derivatives of Eq. 6 and thus defining:

$$\ddot{e}_x = \dot{\theta}_1 \dot{e}_y + \ddot{\theta}_1 e_y - v_1 \dot{e}_\theta \sin e_\theta$$
$$\ddot{e}_y = -\dot{\theta}_1 \dot{e}_x - \ddot{\theta}_1 e_x - v_1 \dot{e}_\theta \cos e_\theta \tag{24}$$
$$\ddot{e}_\theta = \dot{\gamma} \frac{v_1}{l(1+\cos\gamma)} + \dot{\gamma}^2 \frac{\cos\gamma}{1+\cos\gamma} + \ddot{\gamma} \frac{1}{1+\cos\gamma} - \dot{\gamma} \frac{v_1 l \cos\gamma \cos e_\theta}{\left(l + e_y \sin\gamma\right)^2} + \frac{v_1 \dot{e}_\theta \sin\gamma \sin e_\theta}{l - e_y \sin\gamma}$$

From Eq. 23 and Eq. 24, the control laws u_1 and u_2, which stabilize the corresponding sliding surfaces are proposed as:

$$u_1 = \frac{-(\dot{\theta}_1 \dot{e}_y + \ddot{\theta}_1 \dot{e}_y)}{\dot{e}_\theta \sin e_\theta (1 + \lambda_3 |e_x|)} \left(\lambda_1 \dot{e}_x + \frac{\lambda_2 e_x^2}{|e_x|} + \lambda_2 \dot{e}_x |e_x| + \frac{\lambda_3 \dot{e}_x e_x}{|e_x|} \right) - k_1 s_1 \tag{25}$$
$$u_2 = \frac{-((l - e_y \sin\lambda)^2 + \cos\gamma \cos e_\theta + \cos^2\gamma \cos e_\theta)}{v_1 l \cos\gamma \cos e_\theta} (\lambda_4 \dot{e}_\theta + \lambda_5 e_y \frac{e_y}{|e_y|} + \lambda_5 \dot{e}_y |e_y| + \lambda_6 \dot{e}_y \frac{e_y}{|e_y|}$$
$$+ \left(-\dot{\theta}_1 \dot{e}_x - \ddot{\theta}_1 e_x - v_1 \dot{e}_\theta \cos e_\theta\right) \lambda_6 |e_y| - k_2 s_2$$

The gains of the control laws are computed in a way that make the sliding surface asymptotically

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stable. By selecting a Lyapunov stability condition, with the time derivative of the Lyapunov function candidate [20], the Lyapunov function is defined as:

$$V = \frac{1}{2} s^T s \tag{26}$$

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2$$

From Eq. 19 and Eq. 23, $\dot{V}$ can be derived as:

$$\dot{V} = (\dot{e}_x + \lambda_1 e_x + \lambda_2 e_x|e_x| + \lambda_3 \dot{e}_x|e_x|)\left(\ddot{e}_x + \lambda_1 \dot{e}_x + \lambda_2 \frac{e_x^2}{|e_x|} + \lambda_2 \dot{e}_x|e_x| + \lambda_3 \dot{e}_x \frac{e_x}{|e_x|} + \lambda_3 \ddot{e}_x|e_x|\right) +$$
$$(\dot{e}_\theta + \lambda_4 e_\theta + \lambda_5 e_y|e_y| + \lambda_6 \dot{e}_y|e_y|)(\ddot{e}_\theta + \lambda_4 \dot{e}_\theta + \lambda_5 \frac{e_y^2}{|e_y|} + \lambda_5 \dot{e}_y|e_y| + \lambda_6 \dot{e}_y \frac{e_y}{|e_y|} + \lambda_6 \ddot{e}_y|e_y| \tag{27}$$

Replacing $\dot{e}_x, \dot{e}_y, \dot{e}_\theta, \ddot{e}_x, \ddot{e}_y, \ddot{e}_\theta$ from Eq. 6, and Eq. 24 in Eq. 27, it can be concluded the negative definite function for the first derivative of the selected Lyapunov function, while this selection can satisfy the sliding condition, since the right side term of Eq. 27 is a strictly negative constant by selecting $\lambda_i \geq 0$ [21]. In Fig. 7 an overview of the proposed block diagram control scheme, for achieving a stable and smooth motion of the vehicle is being depicted.

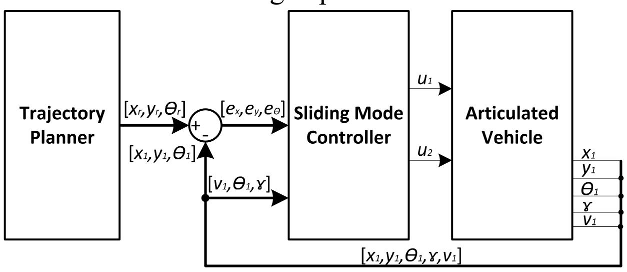

FIG. 7 BLOCK DIAGRAM OF THE PROPOSED SLIDING MODE CONTROL FOR THE ARTICULATED VEHICLE.

For verifying the performance of the proposed nonlinear hitting function, this function has been applied to the nonlinear kinematic articulated vehicle model. Figures (8, 9, 10 and 11) show the circle tracking performance, control input for sgn, and sat functions. Both of these functions cause the chattering phenomena as depicted in figures (9 and 11). The tuning parameters of λ_i for sgn and sat functions have been selected as follows: $\lambda_1 = 0.75$, $\lambda_2 = 1.25$ and $\lambda_3 = 0.5$. The simulation tests have been collected into three groups which refer to the vehicle trajectory tracking a circle path and the steering angle control signal for three cases. By following the condition of $\lambda_1 > 0$, can get that $\frac{\lambda_2}{\lambda_1} = \lambda_3$ could satisfy the stability condition. Figures (12 and 13), show the trajectory reference circle path tracking and control signal results of the proposed SMC, for the parameters of ($\lambda_1 = 10$, $\lambda_2 = \lambda_3 = 15$). Since the control signal is $\dot{\gamma}$, then the chattering will be neglected by using the integral function in the kinematic model to derive the steering angle γ. From these figures, it's obvious that the proposed hitting nonlinear function (12) has good performance as well as the conventional functions. The simulation results show that the hitting function has a chattering free feature.

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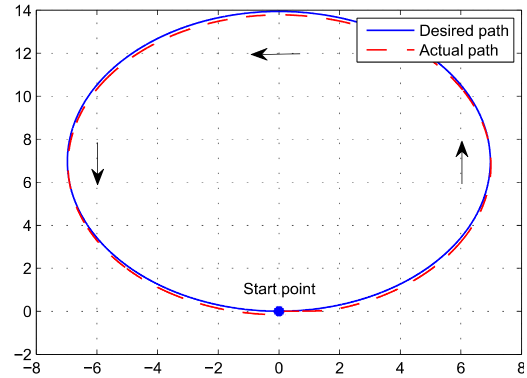

FIG. 8. CIRCLE TRAJECTORY TRACKING OF SGN FUNCTION.

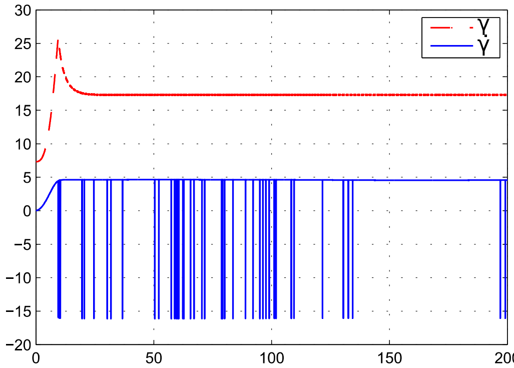

FIG. 9. CONTROL STEERING ANGLE OF SGN FUNCTION.

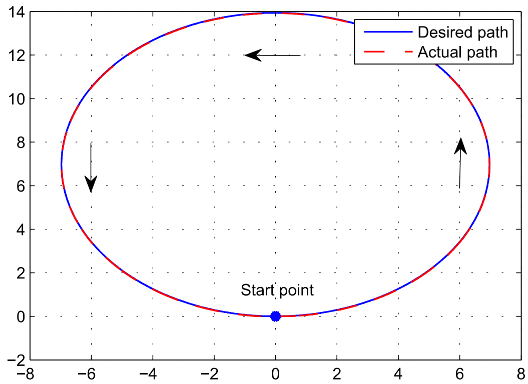

FIG. 10. CIRCLE TRAJECTORY TRACKING OF SAT FUNCTION.

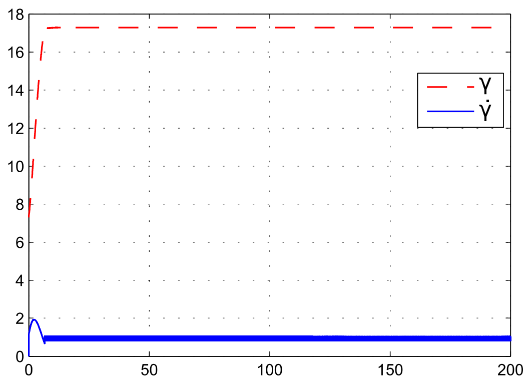

FIG. 11 CONTROL STEERING ANGLE OF SAT FUNCTION.

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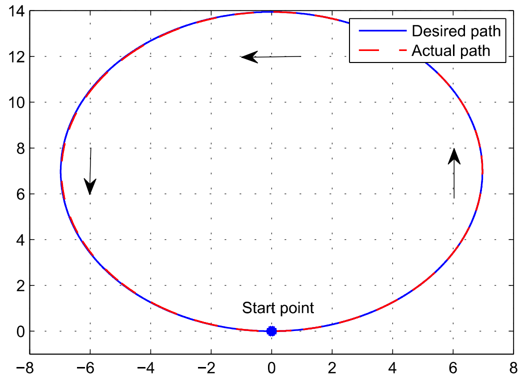

FIG. 12 CIRCLE TRAJECTORY TRACKING OF THE PROPOSED FUNCTION.

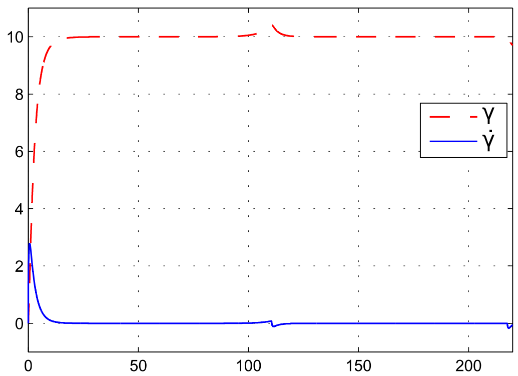

FIG. 13 CONTROL STEERING ANGLE OF THE PROPOSED FUNCTION.

V. CONCLUSIONS

In this article the design and evaluation of a sliding mode control scheme, being applied to the case of an articulated vehicle has been presented. The proposed SMC was based on a continuous sliding surface, which has been introduced for reducing the chattering phenomena, while achieving a better tracking performance and a fast minimization of the corresponding tracking error. The derivation of the sliding mode controller relied on the fully nonlinear kinematic model of the articulated vehicle, while the overall stability of the control scheme was proven based on the Lyapunov's stability condition. The performance of the established control scheme was evaluated through multiple scenarios on a small scale and fully realistic articulated vehicle.

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