Gabor Wavelet Transform in Image Compression

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Abstract-

In the present paper, an important mathematical transform which is called Gabor transform be used to develop a method for image compression. Gabor transform is a type of wavelet-based transform. It is embedded in the standard compression algorithm (JPEG2000) as a mother wavelet. Based on the obtained results we believe that Gabor wavelet transform can provide the optimal resolution in both the time and frequency domains, as well as it seems to be the optimal basis to extract local features such as discontinuities. These advantages of Gabor wavelet transform make it an efficient transform for compression. The proposed compression system is a new approach that achieves a high compression ratio with acceptable visual quality by exploiting the advantages of Gabor wavelets .The simulation results show that the proposed model gives a good compression images that could be competitive to JPEG2000 using Haar wavelet.

Keyword: Gabor wavelet ,JPEG2000, compression, milt-resolution, milt-orientation.

1.Introduction

Fourier transform is one of the most important *mathematical* transforms in applied mathematics and other scientific fields such as engineering and computer sciences. The aim of this transform is to transform the mathematical functions that dependent on time variable to functions dependent on frequency variable[2]. Fourier transform has been the most commonly used mathematical tool for analyzing frequency properties of a given signal, while after transformation, the information about time is lost and it's hard to tell where a certain frequency occurs. To solve this problem, we can use kinds of Fourier transform technique called short-time Fourier transform (*Gabor transform)* which is a *mathematical transformation* Firstly proposed by Dennis Gabor in 1946 [15].The Gabor transform makes it possible to analyze a signal f(x,y) in time and in frequency simultaneously, The idea is to make a Fourier Transform inside a window that will be translated along the signal. Gabor wavelets allow local frequency information to be extracted from an image. Gabor wavelets estimate the strength of certain frequency bands and orientations at each location in the image, giving a result in the spatial domain. the Gabor wavelets are recognized to be good feature detectors[4], that are directly related to the local image features by Eq.(2). This means that an optimized wavelet has e.g. ideally the exact position and orientation of a local image feature.

Gabor wavelets are a popular *mathematical tool* for *analyze* the *mathematical functions* and found an optimal solutions .Especially ,It used for *signal analysis*, and have found widespread use in computer vision. Its applications have included *object tracking* [11],*extract an* *feature* [4], *edges detection*[3][5] ,*faces classification* [9] , and *texture analysis* [3]. The use of Gabor wavelets is supported by its properties. Accordingly Gabor wavelets have also found applications in *mathematics engineering*, *computer science*, *biologically*, *radiologically* and *medicine* inspired early vision systems used it[4,10,5].

In computer science, Gabor wavelet transformation supplied to solve many application problems such as, a problems of image compression ,filter design and edge detection [4][5] . In this paper ,it used to perform the best resolution for the image compression in JPEG2000 algorithm. Gabor functions provide the optimal resolution in both the time and frequency domains, and the Gabor wavelet seems to be the optimal basis functions to extract local features such as discontinuity feature from an image by using its important properties [3]. The great properties makes Gabor wavelet transform suitable for image compression [4]. Based on these advantages of Gabor wavelet transform, we use it efficiently.

In this paper we have to modify the JPEG2000 algorithm by using Gabor wavelet as a mother wavelet .In practice, it is calculated through a multi resolution pyramidal representation [2]. JPEG2000 is a new standard for lossy graphics compression ,developed in 1995[2]. It was introduced that makes use the DWT because of their data reduction capacity. The JPEG2000 standard is devised with the aim of providing the best quality or performance and capabilities to market the evolution. Many researches worked with JPEG2000 algorithm, such that, in 2005, James S. Walker and Ying-Jui Chen compare the new still-image compression standard, JPEG2000, with two competing algorithms based on the Wavelet Difference Reduction (WDR) technique of encoding in this research [12] .Also, Anusorn Jitkam and Satra Wongthanavasu presents an image compression algorithm using modified Haar wavelet and vector quantization in 2007[14]. In 2009, Michael Dyer, Saeid Nooshabadi, David S. Taubman performed on optimizing the throughput of the block coding system within JPEG2000 in their research [13].

From the simulation results, they show that the proposed model given a good compression images that could be competitive to JPEG2000 with Haar wavelet and above researches results.

The paper is organized as follows; Section 2,deals with Gabor wavelet, Section 3 deals with JPEG2000 Algorithm, section 4 deals with the gives the overview of algorithm with results and last section5 ends the paper with conclusion

2. Gabor Wavelet (GW)

For analyze the signals with different frequency components, Gabor wavelet transform is developed. This transform uses Gabor wavelets such a basis functions to create an expansion of a non-stationary signal *f(x,y)* in terms of time-frequency atoms .This wavelet constructed by a window function $g_{\tau,\omega}(x,y)$ (which is a Gaussian function represented by Eq.(1) , combined to a complex exponential[10].

The design of Gaussian function is given in the form[3]:

$$g(x,y)=\frac{\left\|\vec{k_i}\right\|^2}{\sigma_x\sigma_y}e^{-\frac{\left\|\vec{k_i}\right\|^2}{2}\left(\frac{x^2}{\sigma_x^{\,2}}+\frac{y^2}{\sigma_y^{\,2}}\right)} \quad \ldots(1)$$

The Gabor wavelet in 2-D is expressed as[3][10]:

GW=

$$\psi_i(x,y)=\frac{\left\|\vec{k_i}\right\|^2}{\sigma_x\sigma_y}e^{-\frac{\left\|\vec{k_i}\right\|^2}{2}\left(\frac{x^2}{\sigma_x^{\,2}}+\frac{y^2}{\sigma_y^{\,2}}\right)}\left[e^{jk_v(x\cos\alpha_\mu+y\sin\alpha_\mu)}-e^{\frac{-\sigma_x\sigma_y}{2}}\right]$$

…(2)

Where $x = -(R/2),...,(R/2)$,
and $y = -(C/2),...,(C/2)$, such that, R and C=128.

Each $\psi_i(x, y)$ is a plane wave characterized by the vector k_i enveloped by a Gaussian function, where σ_x, σ_y is the standard deviation of this Gaussian along x and y-axis. The center frequency of i[th] window is given by the characteristic wave vector, where wave vector equation is

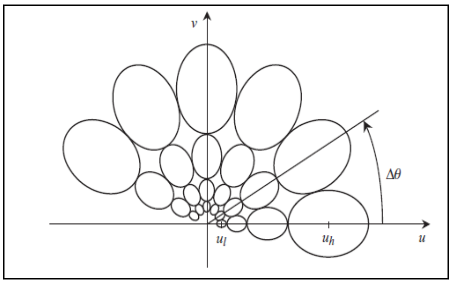
$$\vec{k_i} = \begin{pmatrix} k_{ix} \\ k_{iy} \end{pmatrix} = \begin{pmatrix} k_v \cos\alpha_u \\ k_v \sin\alpha_u \end{pmatrix} \quad \ldots(3)$$

Having a scale and orientation given by (k_v, α_u)
where

$$k_v = 2^{-\frac{v+2}{2}}\pi \quad \ldots(4)$$

$$\alpha_u = u\frac{\pi}{U} \quad \ldots(5)$$

V, U are a max number of scaling and rotation factor respectively, where u, v are the direction and scale of Gabor wavelets, respectively , $v = 0,1,...V-1$ and $u = 0,1,...U-1$; (x,y) is the coordinate of a pixel in image. An extra parameter α_u provides selectivity for the orientation of the function[10]. The $\psi(x, y)$ of Eq.(2) can be used as the mother wavelet to decompose a signal into various levels and orientations. In this section, we showed how mother wavelets could be used in the design of discrete wavelet transform . The same procedure can be applied to the mother Gabor wavelet [3], and we will be applied in the application of this paper. Figure(1) shows the spectrum of the Gabor wavelet at four levels and six orientations[3]. Each level reflects characteristics of the image at a various directions and spatial resolution. Hence, it can analyze textures of image and detect edges of its with describe them at these orientations and resolutions. For this maters the Gabor wavelet transform concedered a best optimizations for image transformations [3].



Fig(1): Gabor Wavelet Spectrum Graph; The Contours Indicate The Half-peak Magnitude of The Wavelet Responses In The Gabor Wavelet Dictionary. The Wavelet Parameters Used are u_h =0.4, u_l =0.05, V= 4 and U_θ =6.

2.1 Important Properties of Gabor Wavelet.

1. Multi-orientationproperty.
2. Multi-resolution property.
3. Biorthogonal.
4. Continuity.
5. Separable.

3. JPEG 2000 Algorithm

JPEG 2000 is a new standard of image compression ,it is devised with the aim of providing the best quality or performance and capabilities to market evolution that the current JPEG standard fails to supply for.

This compression system model consists of two parts, the compressor and the decompression. The compressor method depended on three fundamental stages are transformation, quantization and coding which they are illustrate in figure (2) . The first stage explain the using of DWT(discrete wavelet transform) to transform the data from time domain to mathematical space. Wavelet compression technique uses the wavelet filters,which are Gabor wavelets for image decomposition, image is divided into approximation and detail sub image. The decomposition operation of image f(x,y) is represented in the following equations as[3]

$$f(x,y)=\frac{1}{\sqrt{MN}}\sum_{m}\sum_{n}W_{\phi}(s_0,m,n)\phi_{s_0,m,n}(x,y)$$

$$+\frac{1}{\sqrt{MN}}\sum_{i=H,V,D}\sum_{s=s_0}^{\infty}\sum_{m}\sum_{n}W_{\psi}^{i}(s,m,n)\psi_{s,m,n}^{i}(x,y)$$
...(6)

$$W_{\psi}(s,m,n)=\frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)\psi^{i}{}_{s,m,n}(x,y)$$
...(7)

Where the scaling basis functions and the wavelet basis function from Gabor wavelet are explained in the following, respectively as

$$\phi_{s,m,n}(x,y)=2^{-s/2}\phi(2^{-s}x-m,2^{-s}y-n)$$
…(8)

$$\psi^{i}{}_{s,m,n}(x,y)=2^{-s/2}\psi^{i}(2^{-s}x-m,2^{-s}y-n)$$
…(9)

$for......s>s_0$, and $i=\{H,V,D\}$.Where i ={H,V,D}, As in the one – dimensional case, s_0 is an arbitrary starting scale and the $W_{\phi}(s_0,m,n)$ coefficients define an approximation of *f (x, y)* at scale s_0 . The $W_{\psi}^{i}(s,m,n)$ coefficients add horizontal, vertical, and diagonal details for scales $s\geq s_0$. We normally let s_0 =0 and select N = M = 2^s so that s = 0,1,2,.......,S -1 and m=n= 0,1,2,......,2^S-1 .This operation involves two filters, one is corresponding to the scaling filter or lowpass filter, and the other is corresponding to wavelet filter or high pass filter. Moreover, it is noticed that low-pass filter leads to the scaling function and the high-pass filter leads to wavelet function [6].

The output coefficients from the stage one be quantized by using threshold. it's the second stage, Each subband can have a different quantization step size [7][8]. Each wavelet coefficient in the subband is divided by the quantization step size and the result is truncated. The quantization step size may be determined iteratively in order to achieve a goal bitrate (i.e., the compression factor may be specified in advance by the user) or in order to achieve a predetermined level of image quality.

In the third stage we can be coded the quantization coefficients by means of Huffman coding, run length coding or any other method[2] .

The decomposer can be further broken down into the stages shown in Figure (3). Here the decoding process is divided into two stages. The first, the decoding stage, takes the compressed file and reverses the original coding by mapping the codes to the original quantized values ,which they are de-quantized . Next, these values (de-quantization values) are processed by a stage that performs an inverse transformation IDWT (inverse discrete Gabor wavelet transform) to reverse the original representing process .

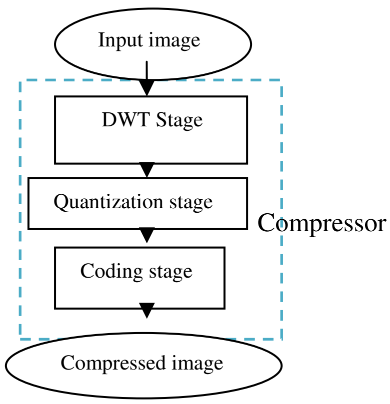


Fig (2):Compression Stages in JPEG2000

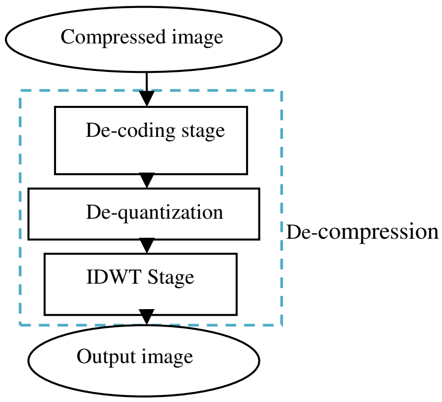


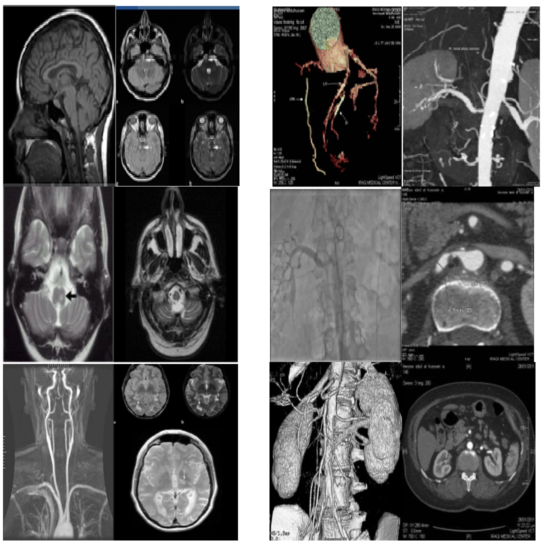
Fig (3):De-compression Stages in JPEG2000

4. Experimental Results

In this section a detailed experimental comparison of the above stated algorithms has been presented. We have used two gray image databases:

(1) MRI graphic Images database, an images obtained from Iraqi Medical Center /Iraq-karbala.

(2) CT Angiography Images database obtained in International Center for Cardio Thoracic and Vascular Diseases/India-Chennai. Figure(4) shown sample data base for MRI and CT Angiography Images which we used in this paper.



(1) (2)

Fig(4) : **Sample Data Base for MRI and CT angio Graphy Images**

This section provides the results of proposed compression system(PCS) for four test images (image 4, image 6) from Fig (4-1) and (image 4, image 5) from the Fig (4-2). The table (1) explain the results of this system which illustrate in Fig(5).This results explained by using some of mathematical process such as compression ratio, compression factor and peak signal to noise ratio , the types of process that are applied to still images will be explained and examined. The equations of them are[1][7]:

Compression ratio (CR)= $(1-(\frac{Compressed-size}{Orginal-size}))\times 100$ …(10)

Compression Factor (CF)= $\frac{size\_of\_input\_image}{size\_of\_output\_image}$ …(11)

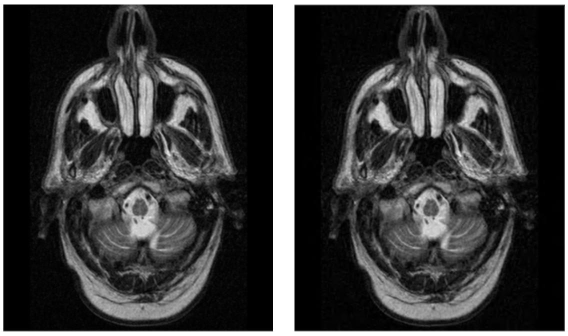
$$\text{PSNR} = 10\log_{10}\frac{(L-1)^2}{\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}[f(x,y)-f^*(x,y)]^2}$$ …(12)

where L is the number of gray levels(e.g., for 8 bits L=256).

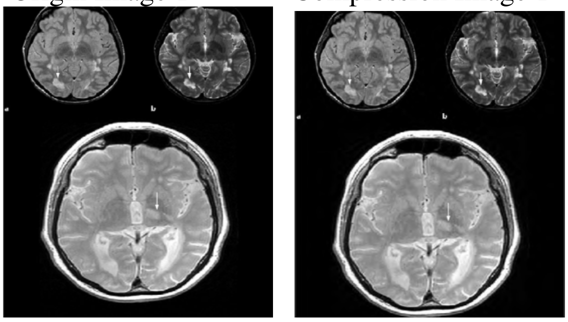
$f(x,y)$: the original image, $f^*(x,y)$: the decompressed image, x , y : row and column .

Table (1):The Result of Proposed System

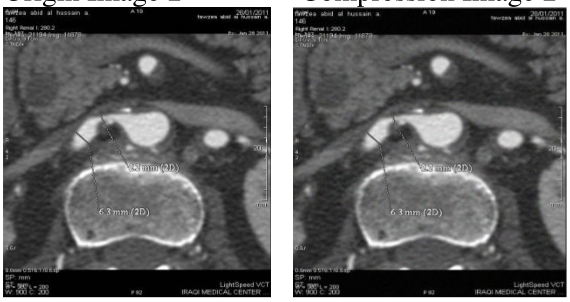
Image	O.I.size/ kb	C.I.size /kb	CR	CF	PSNR
Image 1	61.112	18.546	69.65%	3.29	33.45
Image 2	65.300	20.322	68.87%	3.21	34.12
Image 3	69.520	16.111	76.82%	4.31	32.01
Image 4	73.903	20.632	72.08%	3.58	31.04



Origin Image 1 Compression Image 1



Origin Image 2 Compression Image 2



Origin Image 3 Compression Image 3

Origin Image 4 Compression Image 4

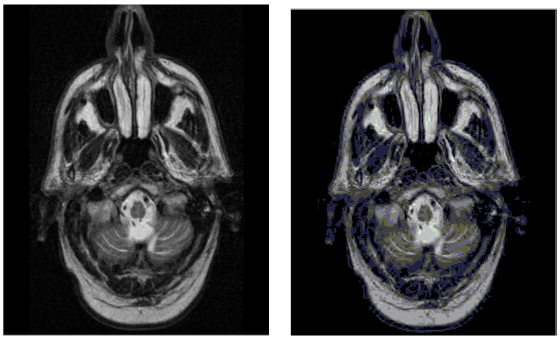
Fig(5):The Compression Images of The Proposed System

From these results, The compression system get high compression ratio, which greater than sixty ,and we can show that the compression images have high resolution by PSNR values, all PSNR values better thirty. Actively, Gabor wavelet assisted this system to get this resultant.

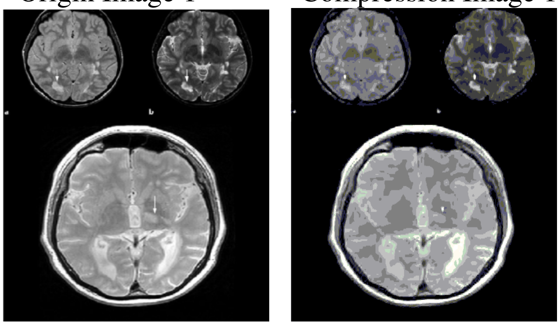
It's very nice to compare the proposed system performance with JPEG-2000 algorithm with Haar wavelet, the results of an images are summarized in Table(2), tested images shown in Figure(6).

Table (2): The Result of JPEG2000 Algorithm with Haar Wavelet.

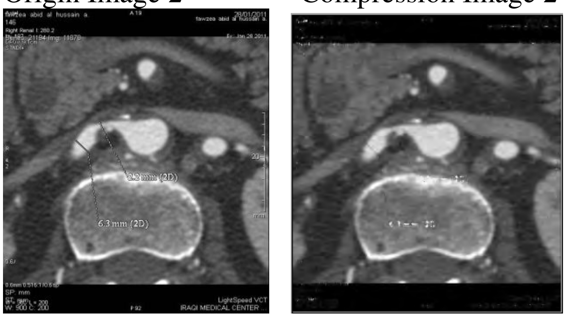
Image	CR	PSNR
Image 1	69.65	31.01
Image 2	68.72	30.03
Image 3	75.52	29.15
Image 4	72.07	28.35



Origin Image 1 Compression Image 1



Origin Image 2 Compression Image 2



Origin Image 3 Compression Image 3

Origin Image 4 Compression Image 4

Fig(6):The Compression Images of The comparison System

5. Conclusion

From the experimental results obtained from the proposed compression system (PCS) using different images ,we conclude:

1. The PCS is suitable to the different images complexity such as the images in figure (4).
2. The PCS give high compression ratios with higher qualities (resolution), all these ratios and qualities be suitable with respect to the *image applications* such as the *medicine treatment* and the *medicine diagnosis* .
3. Gabor wavelet gives the compression system more *activity* and *accuracy* than another wavelet such Haar wavelet ,because the properties of Gabor used to analyze an image and give more accuracy detection for *image features* especially for the *discontinuity (edges) regions* .
4. Gabor wavelet used to increase the PSNR values.

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