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Inference with new prior distribution in composite quantile regression

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Abstract: In this paper, we propose an efficient and easy method for achieving the coefficients estimated and variable selection in composite quantile regression via the employed new regularization method. It is the utilization of a new prior distribution that consists of a mixture of the normal distribution with the exponential distribution as an alternative form to the Laplace distribution. This composition will provide us with a good and efficient algorithm that is both fast and stable in estimating model parameters. The simulation scenario and real data were used in this study.
Keywords: Composite quantile regression, new hierarchical algorithm, new Gibbs sampler, new regularization approach

1. Introduction: It is not always convenient to model the relationship between the averages of a response variable (dependent variable) Y and a set of covariates X. Mean regression may not be appropriate in many application studies to describe the behaviour of the response variable (outcome variable) Y with covariates variable X. For instance, Abrevaya (2008) investigated the impact of maternal behaviour and demographic characteristics on the weight of a newborn, in the United States. This study concentrated on low-birth-weight newborns, which leads to numerous health issues. Standard quantile regression was used to analyse this data because the conditional mean was not a desirable strategy for a low-tail distribution. To deal with low tail distribution, Koenker and Bassett (1978) suggested quantile regression (Q Reg), an extension of conventional mean regression in conditional various quantiles of dependent variables. Complete details regarding the various quantiles of the stochastic relationships between the dependant and predictor variables can be obtained using the Q-Reg model. The Q-Reg model has drawn a lot of attention lately in both theoretical and practical research. Where the Q Reg model is implemented in several disciplines of knowledge, such as agricultural economics (Kostov and Davidova, 2013), body mass index (Yu et al., 2013), growth chart (Wei et al., 2006), ecological research (Cade and Noon, 2003), microarray study (Wang and He, 2007), and so on,. Q Reg model belongs to a robust regression models family (Koenker and Geling, (2001)). In cases where the random error is non-normal, it offers the highest statistical efficiency compared to classical regression models. Furthermore, the Q-Reg model is resilient to some economic issues. The selection of active explanatory variables is crucial while developing the Q-Reg model. The choice of significant subsets of explanatory variables has received a lot of attention in the literature recently. .. Numerous techniques have been developed in the field of variable selection (VS), including the elastic net method (Zou and Hastie, 2005), the SCAD method (Fan and Li, 2001), the lasso method (Tibshirani, 1996), and the adaptive lasso method (Zou, 2006). Additionally, V-S approaches are used in the Q-Reg model. For example, Koenker (2004) used the lasso penalty to the mixed-effect Q-Reg model or longitudinal data, and Li and Zhu (2008) proposed a solution path. The Bayesian technique is also used to estimate the coefficients of the lasso Q Reg model.By using a scale mixture of normal prior distributions on the regression parameters and exponential prior distributions on their variances, Park and Casella (2008) introduced the Bayesian Lasso in traditional linear regression models. By utilizing a scale mixture of normal (SMN) prior distributions on the regression parameters and exponential prior distributions on their variances, (Li et al., 2010) proposed the Bayesian Lasso Q Reg. Therefore, a straightforward and effective Markov chain Monte Carlo (MCMC) approach was presented by Li et al. (2010) for updating all model coefficients to conditional posterior distributions. A versatile method for determining the shrinkage parameter and regression coefficients is the Bayesian formulation. Lately, with regard to the traditional regression model. Numerous authors have demonstrated that, regardless of the residual distribution term, the relative value of composite Q Reg over ordinary least squares (OLS) is larger than 70%.A strong and effective penalized composite quasi-likelihood method for selecting very high-dimensional variables was presented by Bradic et al. (2011). For the purpose of estimating the nonparametric regression function, Kai et al. (2010) suggested a local composite estimator; Zhao and Xiao (2014) examined a weighted composite Q Reg and demonstrated its significant oracle qualities. A new Gibbs sampling method for composite quantile regression was proposed by Alhamzawi (2016), while Huang and Chen (2015) proposed a Bayesian method of composite Q-regression using the asymmetric Laplace distribution (ALD) for the error term and sampling the regression coefficients from its full conditional posterior distribution using a Gibbs sampler. We present a Bayesian with a new framework for composite Q Reg (Composite Q Reg) in the current paper. We mix composite Q Reg with new prior distribution via using the Bayesian approach, Real data and simulation examples are used to illustrate the Bayesian approach. Conversely, the findings indicate merging data from several quantiles.

2. Bayesian composite quantile regression

2.1 composite quantile regression

The quantile regression is concerned with estimating an infinite set of quantile regression lines to provide complete and comprehensive coverage. Via linear model has been Given.

$$y = \beta_0 + x^T\beta + \epsilon_i, \qquad i = 1, \dots\dots, n \qquad [1]$$

Where y is the dependent variable and β_0 is the intercept term $, x_i = (x_1, \dots\dots . x_k)^T$ are independent variables, $\beta = (\beta_1, \dots\dots \beta_k)^T$ are parameters victor of latent variable model and ϵ_i is random error term. To estimate the parameters of the model shown in equation (1) through the conditional quantile functions are given by,

$$Q_{y_{i|x_i}}(\theta|x_i) = \beta_0 + x_i^T\beta_\theta \qquad [2]$$

can be estimated the parameters of equation (2) by the minimization to the following problem,

$$\underset{\alpha_\theta,\beta_\theta}{min} \sum_{i=1}^{n} \rho_\theta\,(y_i - \beta_0 + x_i^T\beta_\theta) \qquad [3]$$

The coefficient estimation for equation (3) is very difficult, especially for much line quantile regression model (Reich et al., (2010)). To overcome this problem possible composite quantile regression has been used. Therefore, we can estimate the parameters of composite quantile regression via the minimization to the following equation,

$$(\hat{\alpha}1, \hat{\alpha}2, \dots, \hat{\alpha}\mathrm{H}, \hat{\beta}) = \underset{\alpha_{1\dots\dots\alpha_{2,\beta}}}{Min} \quad \sum_{h=1}^{H}\left\{\sum_{i=1}^{n} \rho_{\theta\,h}\,(y_i - \beta_0 + x_i^T\beta_\theta\right\}, \qquad [4]$$

The equation (4) is not differentiable at zero point. Koenker and D'Orey (1987) propose significant changes to the algorithm to implement the minimization of equation (4). According to the proposed **Fadel al-Husseini (2017)** the lasso composite quantile penalized regression solution is :

$$\hat{\beta} = argmin \sum_{q=1}^{Q}\left\{\sum_{i=1}^{n} \rho_{\tau_q}\;(y_i - (\beta_0 + x_i^T\beta_\theta)\right\} + \lambda \sum_{j=1}^{k}|\beta_j| \qquad [5]$$

Also, the equation (5) is not differentiable at zero. In this paper , to estimate the Composite quantile regression parameters, a Bayesian method is simple and an efficient Gibbs sampling is proposed for conditional posterior distributions inference.

2.2 **Prior Specification**

To continue with the Bayesian method., it assign a standard uniform prior distribution for β_{0_m}, $p(\beta_{0_m}) \propto 1$. by Laplace prior distribution for β taking the following form

$$f(\beta|\eta,\lambda) = \frac{\eta\lambda}{2}\, e^{-\eta\lambda|\beta|} \qquad [6]$$

We can reformulation equation (6) as a scale mixture of Uniform distribution mixing with standard exponential distribution as in the following proposition $\int'_{m>\lambda\eta|x|} \frac{\eta\lambda}{2}\, \lambda e^{-m} \frac{1}{\lambda}\, dm$, Mallick and Yi (2014) .

The equation 6 is equivalent The following equation $\int'_{m>\eta\lambda|\beta|} \frac{\eta\lambda}{2}\, e^{-m} \quad dm$ ((Remah ,2021)

where, the parameters of a, b, c and d are a set of fixed hyperparameters which take initial values. Following(Li et al. (2010)) and Hashem et al. (2015)), by assign a, b, c and d as very small values $(a = 0.1 , b = 0.1, c = 0.1 , d = 0.1)$ our Bayesian hierarchical approach is given by

$$y_i = \left(\beta_0 + x_i^T \beta_\theta + \delta_{1q}\, \tilde{v}_i + 2\, \delta_{2q}\, \eta^{-1/2} \sqrt{v_i}\, z_i\right)$$

$$p(\alpha_q) \propto 1$$

$$(\tilde{v}|\eta) \sim \eta^n exp\left(-\eta \sum_{i=1}^{n} \tilde{v}_i\right)$$

$$z \sim \left(\frac{1}{\sqrt{2\pi}}\right)^n \quad exp\left(-\frac{1}{2}\sum_{i=1}^{n} {z_i}^2\right)$$

$$\beta|\,\eta,\lambda \sim\ uniform(-\frac{1}{\eta\lambda}, \frac{1}{\eta\,\lambda})$$

$$m \sim\ standard\ exponential$$

$$\eta \sim\ \ \eta^{a-1}\, exp^{(-b\eta)}$$

$$\lambda \sim\ \lambda^{c-1} e^{-d\lambda}$$

$$w \sim Dirichlet\ (d_1, \dots, d_h)$$

(a,b,c and d) are hyper parameter

2.3 **Conditional Posterior Computation Inferences**

The current Bayesian above hierarchical model allows for improving mixing and generates an efficient and simple Gibbs sampling algorithm that action as follows:

- Fix the quantile levels $0 < \theta_1 < \theta_2 < \cdots < \theta_q < 1$,where $\theta_q = \frac{q}{Q+1}$ for $q = 1, \dots\dots, Q$.
- **Updating y_i**

The $[y_i, i = 1, \dots\dots, n]$, has a full conditional posterior distribution given by

$$\left[\prod_{i=1}^{n}\prod_{q=1}^{Q}\left(\frac{1}{\sqrt{2\pi\eta^{-1}\delta_{1q}^{2}\,\tilde{v}_i}}\right)^{c_{iq}}\right] exp\left[-\frac{1}{2}\sum_{i=1}^{n}\sum_{q=1}^{Q}\frac{\mathbf{c_{iq}}\left(y_i-\alpha_{\theta_q}-\mathbf{x}_i^{T}\boldsymbol{\beta}_{\theta}-\delta_{1q}\ \tilde{v}_i\right)^2}{\eta^{-1}\ \delta_{1q}^{2}\tilde{v}_i}\right]$$

- **Updating $\boldsymbol{\beta_j}$**

The full conditional posterior distribution of each β_j $for\ j=1,\dots,k, is\ N(\tilde{\beta}_j,\tilde{\sigma}_j^2)$, where $\tilde{\sigma}_j^2 = \frac{1}{\sum_{i=1}^{n}x_{ij}^2/h_{2i}}, and\ \tilde{\beta}_j = \frac{\sum_{1=1}^{n}x_{ij}h_{ij}}{\sum_{i=1}^{n}x_{ij}^2}$

- **Updating $\boldsymbol{\alpha_{q_\theta}}$**

The full conditional posterior distribution of $[\alpha_{q\theta}\ , q=1,\dots,Q], is\ N(\tilde{\alpha}_q,\tilde{\sigma}_q^2)$ where $\tilde{\alpha}_q$

$$=\frac{\dfrac{\sum_{i=1}^{n}c_n\eta\, y_{ik}/\delta_{2q}^2 v_i}{\sum_{i=1}^{n}c_{iq}/\delta_{2q}^2 v_i}}{\dfrac{1}{\sum_{i=1}^{n}c_{iq}/\delta_{2q}^2 v_i}}$$

and $\tilde{\sigma}_q^2 = \frac{1}{\sum_{i=1}^{n}\ c_{iq}\eta/\ Q_{2q}^2\eta v_i}$

- **Updating $\boldsymbol{c_i}$**

The full conditional posterior distribution of $\mathrm{C_i} = (\mathrm{C_{i1,}}\mathrm{C_{i2,}}\dots\dots,\mathrm{C_{iQ}})^{T}$ is a multinomial distribution ;

$$p(C_i|.) \propto \text{multinomial}(1,\hat{p}_1,\dots,\hat{p}_Q), \qquad (4.16)$$

$$\text{where } \hat{p}_q = \frac{\frac{w_q}{Q_{2q}}\left[exp\left(-\left(y_i-\alpha_{\theta_q}+\mathbf{x}_i^{T}\boldsymbol{\beta}_{\theta}-\delta_{1q}\,v_i\right)^2\middle|2\,\eta^{-1}\ \varphi_{1q}^2 v_i\right)\right]}{\sum_{q=1}^{Q}\frac{w_q}{Q_{2q}}\left[exp\left(-\left(y_i-\alpha_{\theta_q}+\mathbf{x}_i^{T}\boldsymbol{\beta}_{\theta}-\delta_{1q}\,v_i\right)^2\middle|2\,\eta^{-1}\ \varphi_{1q}^2 v_i\right)\right]}$$

- **Updating $\boldsymbol{w}$**

The full conditional posterior distribution of $\boldsymbol{w}$ is Dirichlete distribution $\boldsymbol{(n_1{+}d_1,\dots,n_q+d_q)}$.

- **Updating $\boldsymbol{\eta}$**

The full conditional posterior distribution of $\boldsymbol{\eta}$ **is** Inverse Gaussian as denoted $invG(\mu_i^T,\varphi^T)$,

$$where\ \mu_i^T = \sqrt{\frac{exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\sum_{q=1}^{n}\ \mathbf{c_{iq}}\ \left(y_i-\alpha_{\tau_q}+\mathbf{x}_i^{T}\boldsymbol{\beta}_{\tau}-\delta_{1q}\,v_i\right)^2\ \sum_{i=1}^{n}v_i+b\ \right\}}{\delta_{2q}^2 v_i}}$$

And $\varphi^T = \frac{3n}{2} + a + 1$

- **Updating λ**

The full conditional posterior distribution of λ **is** Gaussian as denoted $Gamma(k + c, d)$.

The sample distribution for every unknown parameter and the y and η of its complete conditional posterior distribution are obtained using the aforementioned Gibbs sampling mechanism., posterior conditional on all remaining unknowns. Every Gibbs sampling iteration cycles through all real parameters. of $(y, \alpha_{\theta q}, \beta, \eta_i, w_j, \lambda_{j)}),$. Our algorithm ran through 11,000 iterations, with the first 1,000 being eliminated as burnt.

3. Simulation study

To clearly demonstrate the superiority of our proposed method over other approaches in the context of composite quantile regression, a simulation approach is required. By creating an algorithm that is both efficient and simple to use. Two other methods (composite quantile regression for single-index models (Cqr) via employing the cqrReg package and Bayesian lasso composite quantile regression (BCqr) that will be compared to our proposed method (Bayesian composite quantile regression with new formulation of Laplace distribution (BcqrNLd) Five different sample size levels will be considered (n = 50, n = 100, n = 150, n = 200, n = 250), The explanatory variables are distributed normally and are correlated. $corr\,(X_i, X_j) = (0.5)^{i+j}$, $i = 1,2, \dots \dots n,$ $j = 1,2, \dots \dots p$

As for the model errors, they are generated according to three different types of errors:, normal distribution with mean 4 and variance 4 $\varepsilon \sim N(4,4)$, standard Laplace distribution with mean zero and variance 1 $\varepsilon \sim Lap(0,1)$), and chi-square distribution $\varepsilon \sim \chi^2_{(4)}$. We employed three quantile levels that are ($\tau = 0.25, \tau = 0.65$ and $\tau = 0.98$) . All methods involved in the experiment are evaluated using mean square error (MSE) that calculated $\frac{(x^T\hat{\beta} - x^T\beta^{true})^2}{r}$,In our current study, we will use two simulation studies.

3.1 Simulation Example One:

In this simulation study, we look at how our proposed method performs with very sparse models.. In particular, The true models are as follows:

$y_i = x_{1i} + +\varepsilon_i,$ i=1,2,….100

Seven explanatory variables from a multivariate normal distribution with mean zero and $cov(x_i, x_j) = 0.5^{|i-j|}$. As a result, the true coefficients of independent variables, including the intercept term, are as follows: are $\beta = (1,0,0,0,0,0,0)$

The results listed in Table 1 show that the mean square error (MSE) computed via our proposed method (BcqrNLd) is much smaller than the mean square error (MSE) computed by the other two methods (Cqr) and (BCqr). This mean the our proposed method have a good performance compared with other method .

Table 1. Mean Square Error for the simulation studies for One Example .

		$\varepsilon \sim N(4,4)$	$\varepsilon \sim Lap(0,1)$	$\varepsilon \sim \chi^2_{(4)}$
θ	Comparison Methods	RMSE	RMSE	RMSE
N=50	Cqr $_{q=4}$	0.985	1.075	**1.235**
	BCqr $_{q=4}$	0.854	0.963	**1.121**
	BcqrNLd $_{q=4}$	0.832	**0.811**	**0.718**
N=100	Cqr $_{q=4}$	1.121	0.928	1.023
	BCqr $_{q=4}$	1.082	**0.843**	**1.110**
	BcqrNLd $_{q=4}$	0.931	**0.734**	0.715
	Cqr $_{q=4}$	1.191	0.832	0.872

N=150	BCqr $_{q=4}$	1.117	0.743	0.609
	BcqrNLd $_{q=4}$	0.834	0.671	0.621
N=200	Cqr $_{q=4}$	**0.863**	**0.723**	0.819
	BCqr $_{q=4}$	0. 732	**0.634**	0.621
	BcqrNLd $_{q=4}$	0.627	**0.561**	0.623
N=250	Cqr $_{q=4}$	0.823	**0.671**	0.872
	BCqr $_{q=4}$	0.672	**0.618**	0.621
	BcqrNLd $_{q=4}$	0.528	**0.521**	0.518

3.2 Simulation Example two:

In this simulation study, we look at how our proposed method performs with very sparse models.. In particular, The true models are as follows:

$y_i = 0.85x_{1i} + 0.85x_{2i} + 0.85x_{3i} + 0.85x_{4i} + 0.85x_{5i} + 0.85x_{6i} + 0.85x_{7i} + \varepsilon_i,$ i=1,2,….100

Seven explanatory variables from a multivariate normal distribution with mean zero and $cov(x_i, x_j) = 0.5^{|i-j|}$. Thus the true coefficients of independent variables, including the intercept term, are $\beta = (0.85, 0.85, 0.85, 0.85, 0.85, 0.85, 0.85)$

The results that listed in table -2 - we see the mean square error (RMSE) is computed via our proposed method (BcqrNLd) is much smaller than root mean square error (RMSE) is computed by the other two methods (Cqr) and (Bcqr). This mean the our proposed method have a good performance compared with other method .

Table 2. Mean Square Error for the simulation studies for Two Example

		$\varepsilon \sim N(4,4)$	$\varepsilon \sim Lap(0,1)$	$\varepsilon \sim \chi^2_{(4)}$
θ	Comparison Methods	RMSE	RMSE	RMSE
N=50	Cqr $_{q=4}$	0.945	0.814	**0.962**
	BCqr $_{q=4}$	0.829	0.821	**0.971**
	BcqrNLd $_{q=4}$	0.784	**0.764**	**0.748**
N=100	Cqr $_{q=4}$	0.953	0.938	0.967
	BCqr $_{q=4}$	0.929	**0.905**	**0.910**
	BcqrNLd $_{q=4}$	0.734	**0.710**	0.820
N=150	Cqr $_{q=4}$	0.859	0.872	0.792
	BCqr $_{q=4}$	0.836	0.834	0.745
	BcqrNLd $_{q=4}$	0.671	0.573	0.702
N=200	Cqr $_{q=4}$	**0.834**	**0.673**	0.782
	BCqr $_{q=4}$	0. 722	**0.642**	0.644
	BcqrNLd $_{q=4}$	0.692	**0.671**	0.582
N=250	Cqr $_{q=4}$	0.686	**0.734**	0.834
	BCqr $_{q=4}$	0.561	**0.684**	0.723
	BcqrNLd $_{q=4}$	0.523	**0.508**	0.495

After receiving the simulation study results, We discovered that our proposed method works well, and the behavior of the estimated features can be seen in Figures (1) and (2).

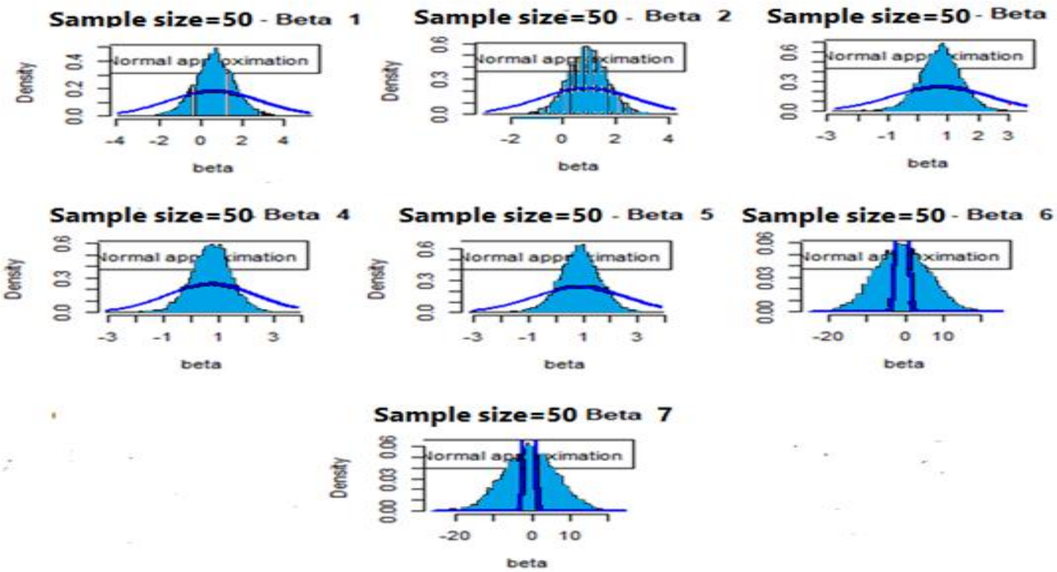

Figure (1) show the histograms BcqrNLd $_{q=4}$ of in density case and sample size is 50

The above figure, which depicts histograms of the estimated parameters in the density case at a sample size of 50, clearly shows that the estimated parameters follow a normal distribution and perfectly match the underlying distribution of the model's parameters. This figure shows that the estimated parameters perfectly match the true distribution of the model's parameters.

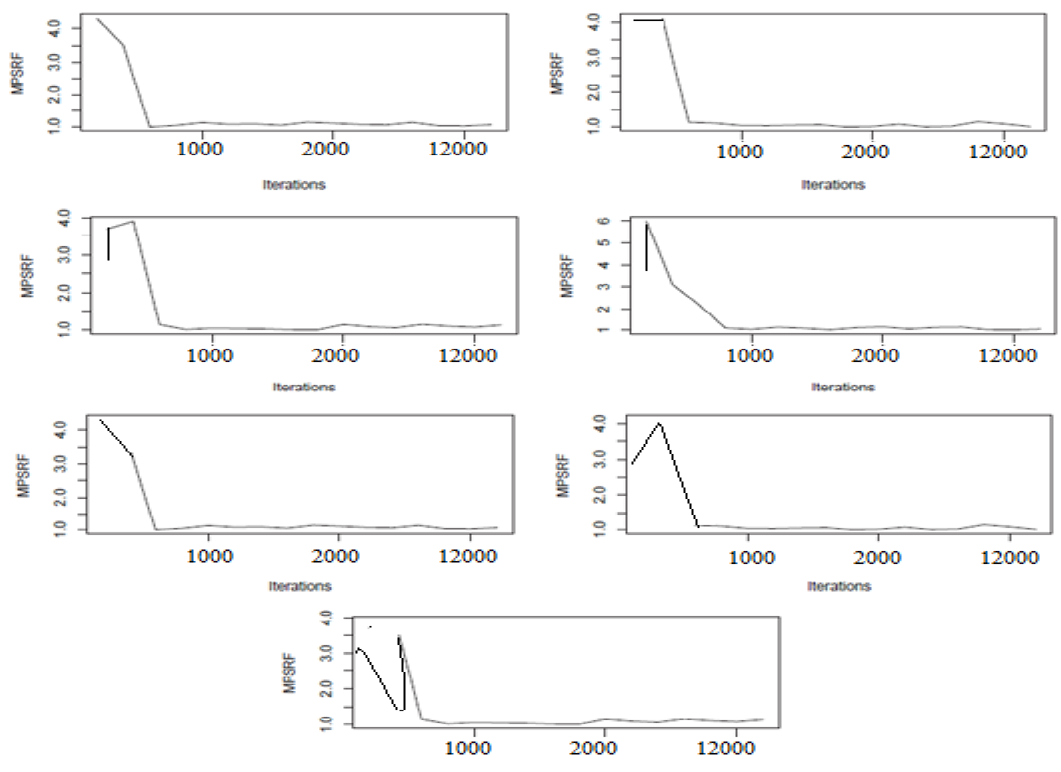

Figure (2) MPSRF for BcqrNLd $_{q=4}$ of in density case and sample size is 50

Figure (1) shows that the MPSRF becomes stable and close to 1 after about 3000 iterations. As a result, there is convergence of our proposed algorithm to the posterior was very fast, and the chain mixing was well done.

4. Real data

After demonstrating the efficiency of our proposed method through simulation in comparison to a set of methods in the same field, we must test its effectiveness with real-world data. Medical information was gathered from the hospital in Al-Hilla city. The dependent variable, y_i, which stands for "Baby weight at birth," and the eight independent variables that either directly or indirectly influence Baby weight at birth make up the data under investigation. These variables consist of the following:

1- x_1 : Maternal weight during pregnancy.
2- x_2:The mother's blood sugar levels during pregnancy.
3- x_3: Blood pressure level in the mother during pregnancy.
4- x_4: Hemoglobin levels in the mother's blood during pregnancy.
5- x_5 : Age of the mother during pregnancy.
6- x_6: The current child's birth order.
7- x_7: mother is a smoker during pregnancy.
8- x_8 :Number of visits to prenatal care clinics.

Following the collection of data, a regression model with one dependent variable and eight independent variables was constructed. The performance of our proposed method will be tested compared to the composite quantile regression for single-index models (Cqr)and Error (S.E). Bayesian lasso composite quantile regression (BCqr). The evaluation criteria used will be root Mean Squared Error (RMSE) and Standard

Table 1. RMSE and S.D for the real data.

Methods	RMSE	S D
Cqr $_{q=4}$	9.6231	5.2936
BCqr $_{q=4}$	5.7329	2.6232
BcqrNLd $_{q=4}$	1.431	0.2832

According to the results in Table 3, the values of the evaluation criteria, namely Mean Square Error (RMSE) and Standard Error (S.D), obtained using our proposed method (BcqrNLd), are significantly lower than the values obtained using the comparison methods (Cqr and BCqr). In terms of variable selection and parameter estimation, this result indicates that our proposed method outperforms the comparison methods.

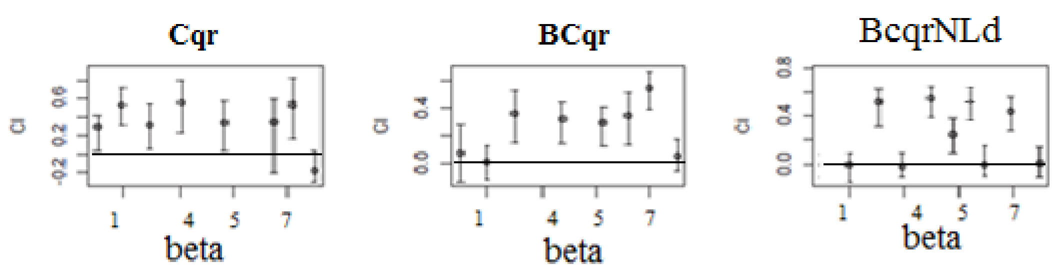

Figure -3- Show the confidence interval for parameters estimated at 95%

It can be seen the our proposed method (BcqrNLD) is a good performance to parameters estimation and variable selection compared with other method. In (BcqrNLD) ,we see the independent variables (x_1, x_3, x_6, x_8) are excluded from our model, because of it parameters estimation closed from zero exactly. In BCqr ,we see the independent variables (x_1, x_2, x_8) are excluded from our model, because of it coefficients estimation closed from zero exactly. In Cqr method,we see the independent variables (x_6, x_8) are excluded from our model, because of it coefficients estimation closed from zero exactly.

5. Conclusions and Recommendations

In this work, we created a straightforward and effective MCMC-based computing method for composite TQReg, which is based on combining uniform distribution and exponential distribution. Our suggested method is successful in estimating coefficients under various distributions, as demonstrated by simulation studies. We contend that to obtain an efficiency advantage, quantile information based on estimators at various quantiles must be combined, drawing from both simulation studies and real data analysis. There are plenty of easy methods to extend our suggested strategy. The suggested approach, for instance, can be used to right-censored data in the Bayesian composite quantile regression model.

References

1. Abrevaya, J., & Dahl, C. M. (2008). The effects of birth inputs on birthweight: evidence from quantile estimation on panel data. *Journal of Business & Economic Statistics*, *26*(4), 379-397.
2. Alhamzawi, R. (2016). Bayesian elastic net Tobit quantile regression.Communications in Statistics-Simulation and Computation, 45(7), 2409-2427.
Bilias, Y., S. Chen, and Z. Ying (2000). Simple resampling methods for censored regression quantiles. *Journal of Econometrics 68*, 303–338.
3. Cade, B. S. and Noon, B. R. (2003). "A gentle introduction to quantile regression for ecologists." *Frontiers in Ecology and the Environment*, 1(8): 412–420. 1
4. Fadel Hamid Hadi Alhusseini. "New Bayesian Lasso Composite Quantile Regression." American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS) 29.1 (2017): 138-152.
5. Fan, J., & Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. Journal of the American statistical Association, 96(456), 1348-1360.
6. Hashem, H., Vinciotti, V., Alhamzawi, R., and Yu, K. (2015). "Quantile regression with group lasso for classification." Advances in Data Analysis and Classification, 1
7. Koenker, R. (2004). Quantile regression for longitudinal data. *Journal of Multivariate Analysis*, *91*(1), 74-89.
8. Koenker, R. and G. J. Bassett (1978). Regression quantiles. *Econometrica 46*, 33–50.
9. Koenker, R. and V. D'Orey (1987). Algorithm AS 229: Computing regression quantiles.
10. Koenker, R., & Geling, O. (2001). Reappraising medfly longevity: a quantile regression survival analysis. *Journal of the American Statistical Association*,*96*(454), 458-468.
11. Kostov, P., & Davidova, S. (2013). A quantile regression analysis of the effect of farmers' attitudes and perceptions on market participation. *Journal of Agricultural Economics*, *64*(1), 112-132.
12. Li, Q., Xi, R., & Lin, N. (2010). Bayesian regularized quantile regression. *Bayesian Analysis*, *5*(3), 533-556.
13. Li, Y., & Zhu, J. (2008). L 1-norm quantile regression. Journal of Computational and Graphical Statistics, 17(1), 163-185.
14. Mallick, H. and Yi, N. (2014). "A new Bayesian lasso." *STATISTICS AND ITS IN-*
15. Park, T. and Casella, G. (2008). The Bayesian lasso. Journal of the American Statistical Association 103, 681–686.
16. reference growth charts." *Statistics in medicine*, 25(8): 1369–1382. 1
17. Reich, B. J., H. D. Bondell, and H. J. Wang (2010). Flexible Bayesian quantile regression for independent and clustered data. *Biostatistics 11*, 337–352.*TERFACE*, 7(4): 571–582. 2, 3, 4
18. Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society. Series B (Methodological), 267-288.
19. Wang, H., G. Li, and G. Jiang (2007). Robust regression shrinkage and consistent variable selection through the LAD-Lasso. Journal of Business & Economic Statistics 25, 347–355.
20. Wei, Y., Pere, A., Koenker, R., and He, X. (2006). "Quantile regression methods for
21. Yu, K., W. Dang, H. Zhu, and R. Al Hamzawi (2013). Comment on article by spokoiny,wang and hardle. *Journal of Statistical Planning and Inference 7* (143), 1140–1144.
22. Zhao, Z. and Z. Xiao (2014). Efficient regressions via optimally combining quantile information. *Econometric theory 30* (06), 1272–1314.
23. Zou, H. (2006). The adaptive lasso and its oracle properties. *Journal of the American statistical association*, *101*(476), 1418-1429.
24. Zou, H., & Hastie, T. (2005). Regularization and variable selection via the elastic net. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 67(2), 301-320.