The Rational valued characters of the Group ($Q_{2m}\times C_2$) when $m=2^h$, $h\in Z^+$

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1. Abstract

The main purpose of this paper is to find the rational valued characters table of the group ($Q_{2m}\times C_2$) ,when $m=2^h$, $h\in Z^+$, which is denoted by $\equiv^*(Q_{2^{h+1}}\times C_2)$,where Q_{2m} is denoted to Quaternion group ,and C_2 is the cyclic group of order 2.

Moreover we have found the general form of the rational valued characters table of the group ($Q_{2^{h+1}}\times C_2$).

2. Introduction

Let G be a finite group ,two elements of G are said to be *Γ-conjugate* if the cyclic subgroups they generate are conjugate in G; this Γ-relation defines an equivalence relation on G.Its classes are called Γ- classes

The Z-valued class function on the group *G,* which is constant on the *Γ- classes* forms a finitely generated abelian group *cf(G,Z)* of a rank equal to the number of Γ- classes

The intersection of *cf(G,Z)* with the group of all generalized characters of *G, R(G)* is a normal subgroup of *cf(G,Z)* denoted by $\overline{R}(G)$, Each element in $\overline{R}(G)$ can be written as $u_1\theta_1+u_2\theta_2+\dots\dots+u_l\theta_l$, where l is the number of *Γ-classes* ,

$u_1,u_2,\dots,u_l\in Z$ and $\theta_i=\sum_{\sigma\in Gal(Q(\chi_i)/Q)}\sigma(\chi_i)$, where χ_i is an irreducible character of the group G and σ is any element in Galios group $Gal\ (Q(\chi_i)/Q)$.

Let $\equiv^*(G)$ denotes the $l\times l$ matrix which the rows corresponds to the θ_i's and columns correspond to the *Γ- classes* of G .In 1995 N. R. Mahamood [3] studied the factor group $cf(Q_{2m},Z)/\overline{R}(Q_{2m})$. The aim of this paper is to find $\equiv^*(Q_{2^{h+1}}\times C_2)$ and determine general $(2h+8\times 2h+8)$ matrix form of the rational valued characters table of the group($Q_{2^{h+1}}\times C_2$).

3. Preliminaries

The Generalized Quaternion Group Q_{2m}(3.1) [3]

For each positive integer m, The generalized Quaternion Group Q_{2m} of order 4m with two generators x and y satisfies

$Q_{2m}=\{x^h y^k, 0\le h\le 2m-1, k=0,1\}$

which has the following properties

$\{x^{2m}=y^4=I\ , yx^m y^{-1}=x^{-m}\}$

The character table of the quaternion group Q_{2m} when $m=2^h$, $h\in Z^+$ (3.2) [3]

There are two types of irreducible characters .one of them is the character of the linear representations R_1, R_2, R_3 and R_4 which are denoted by ψ_1, ψ_2, ψ_3 and ψ_4.

respectively as in the following table:

	x^k	$x^k y$
ψ_1	1	1
ψ_2	1	-1
ψ_3	(-1)k	(-1)k
Ψ_4	(-1)k	(-1)k+1

Table (1)

Where $0 \le k \le 2m-1$.

The rest characters of irreducible representations T_h of degree 2 are denoted by χ_h such that :

$$\chi_h(x^k) = \omega^{hk}+\omega^{-hk} = e^{\pi ihk/m} + e^{-\pi ihk/m} = 2\cos(\pi hk/m)$$

We are denoted to $\omega^{hk}+\omega^{-hk}$ by V_{hk} ,Thus $V_{hk} = V_{2m-hk}$, $V_m=-2$, $V_{2M}=2$ .also we will write $V_{J(hk)}$ such that

$J(hk) = \min\{hk \pmod{2m}, 2m-hk \pmod{2m}\}$,

in the character table of the quaternion group Q_{2m} when $m=2^h$, where $V_{J(hk)}= 2\cos(\pi J(hk)/m)$,

$\chi_h(x^k y) = 0$

Where $0 \le k \le 2m-1$, $1 \le h \le m-1$

and $\omega = e^{2\pi i/2m}$.

So there are m+3 irreducible characters of Q_{2m}. Then , the general form of the characters table of Q_{2m} when $m=2^h$, $h\in Z^+$ is given in the table (2) .

Theorem (3.3)[1]

Let $T_1:G_1\rightarrow GL(n,K)$ and $T_2:G_2\rightarrow GL(m,K)$ are two irreducible representation of the group G_1 and G_2 with characters χ_1 and χ_2 respectively, then $T_1 \otimes T_2$ is irreducible representation of the group $G_1\times G_2$ with the character $\chi_1.\chi_2$.

The Group $Q_{2m}\times C_2$ (3.4)

The direct product group $Q_{2m}\times C_2$,where C_2 is a cyclic group of order 2 then

$$|Q_{2m}\times C_2| = 8m .$$

Since , the irreducible representations of the group $Q_{2m}\times C_2$ are the tensor products of those of Q_{2m} and those of C_2.The group C_2 has two irreducible representations, their characters σ_1 and σ_2 are given in the table(2):

$\equiv(C_2) =$

CL_α	1	r
$\lvert CL_\alpha \rvert$	1	1
σ_1	1	1
σ_2	1	-1

According to Theorem (3.3), each irreducible character χ_i of Q_{2m} defines two irreducible characters χ_{i1},χ_{i2} such that $\chi_{i1}=\chi_i\sigma_1$, $\chi_{i2} = \chi_i\sigma_2$ of $Q_{2m}\times C_2$.

Then $\equiv(Q_{2m}\times C_2) = \equiv(Q_{2m}) \otimes \equiv(C_2)$

Example (3.5)

To find characters table of $Q_{16}\times C_2$.

From (3.3) we have the character table of Q_{16} as Table (3),Where $V_i = 2\cos(\pi i/8)$, $V_{2m}=2$, $V_m=-2$,$V_4=2\cos(4\pi/8)=0$.

And

$\equiv(C_2) =$

CL_α	1	R
$\lvert CL_\alpha \rvert$	1	1
σ_1	1	1
σ_2	1	-1

By Theorem (3.3), the characters table of $Q_{16}\times C_2$ can be written as follows:

$\equiv(Q_{16}\times C_2) = \equiv(Q_{16}) \otimes \equiv(C_2)$,

Then $\equiv(Q_{16}\times C_2)$ is given in the table (4) .

Where $V_i = 2\cos(\pi i/8)$, $V_{2m}=2$, $V_m=-2$,$V_4=2\cos(4\pi/8)=0$.

4.The main results

Proposition(4.1)[2]

The rational valued characters

$$\theta_i = \sum_{\sigma\in Gal(Q(\chi_i)/Q)} \sigma(\chi_i) \text{ form basis for } \overline{R}(G),$$

where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ-classes of G.

The rational character table of the quaternion group Q_{2m} when $m=2^h$, $h\in Z^+$ (4.2)[3]

the rational characters table of Q_{2m} when $m=2^h$, $h\in Z^+$ is given in the following table (after change order the rows and the columns) :

$\equiv^*(Q_{2^{h+1}})=$

1	**1**	**1**	**1**	**1**	.	.	.	**1**	**1**	**1**
-1	**1**	**-1**	**1**	**1**	.	.	.	**1**	**1**	**1**
-1	**-1**	**1**	**1**	**1**	.	.	.	**1**	**1**	**1**
1	**-1**	**-1**	**1**	**1**	.	.	.	**1**	**1**	**1**
0	**0**	**0**	**-2**	**2**	.	.	.	**2**	**2**	**2**
0	**0**	**0**	**0**	**-4**	.	.	.	**4**	**4**	**4**
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$				$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$		$\ddots$		$\ddots$		$\vdots$	$\vdots$	$\vdots$
0	**0**	**0**	.	.	.	.	**0**	-2^{h-1}	2^{h-1}	2^{h-1}
0	**0**	**0**	.	.	.	.	**0**	**0**	-2^h	2^h

Table (5)

Example (4.3)

To construct the rational valued characters table of $Q_{16}\times C_2$,when have to do the following: From Example(3.5) we have the characters table of $Q_{16}\times C_2$.

By the definition of $Q_{2m}\times C_2$:

$$\equiv(Q_{16}\times C_2)= (\equiv Q_{16}) \otimes (\equiv C_2)$$

To calculate the rational valued character table of $Q_{16}\times C_2$,

$\theta_{11}=\psi_{11}$, $\theta_{12}=\psi_{12}$, $\theta_{21}=\psi_{41}$, $\theta_{22}=\psi_{42}$, $\theta_{31}=\psi_{21}$, $\theta_{32}=\psi_{22}$, $\theta_{41}=\psi_{31}$, $\theta_{42}=\psi_{32}$,

$\theta_{51}=\chi_{41}$, $\theta_{52}=\chi_{42}$.

The elements of $Gal(\chi_{1i})/Q$, are :

$\{\sigma_{1i}, \sigma_{3i}, \sigma_{5i}, \sigma_{7i}\}$

Where $\sigma_{1i}(\chi_{1i})=\chi_{1i}$, $\sigma_{3i}(\chi_{1i})=\chi_{3i}$, $\sigma_{5i}(\chi_{1i})=\chi_{5i}$, $\sigma_{7i}(\chi_{1i})=\chi_{7i}$ and i=1,2.

By proposition (4.1)

1- (I) if i=1

$\theta_{71}=\sigma_{11}(\chi_{11})+\sigma_{31}(\chi_{11})+\sigma_{51}(\chi_{11})+\sigma_{71}(\chi_{11})$

θ_{71} ([1,1])=2+2+2+2=8

θ_{71} ([1,r])=2+2+2+2=8

θ_{71} ($[x^8,1]$)= (-2)+ (-2)+ (-2)+ (-2)=-8

θ_{71} ($[x^8,r]$)= (-2)+ (-2)+ (-2)+ (-2)=-8

θ_{71}([x,1])=$V_1+V_3+V_5+V_7$=0

θ_{71} ([x,r])= $V_1+V_3+V_5+V_7$=0

θ_{71} ($[x^2,1]$)= $V_2+V_6+V_6+V_2$=0

θ_{71} ($[x^2,r]$)= $V_2+V_6+V_6+V_2$=0

θ_{71} ($[x^4,1]$)= 0+0+0+0=0

θ_{71} ($[x^4,r]$)= 0+0+0+0=0

θ_{71}([y,1])=0

θ_{71} ([y, r])=0

θ_{71} ([xy,1])=0

θ_{71} ([xy, r])=0

1- (II) if i=2

$\theta_{72}=\sigma_{12}(\chi_{12})+\sigma_{32}(\chi_{12})+\sigma_{52}(\chi_{12})+\sigma_{72}(\chi_{12})$

θ_{72} ([1,1])=2+2+2+2=8

θ_{72} ([1,r])= (-2)+ (-2)+ (-2)+ (-2)=-8

θ_{72} ($[x^8,1]$)= (-2)+ (-2)+ (-2)+ (-2)=-8

θ_{72} ($[x^8,r]$)= 2+2+2+2=8

θ_{72} ([x,1])=$V_1+V_3+V_5+V_7$=0

θ_{72} ([x,r])= $(-V_1)+(-V_3)+(-V_5)+(-V_7)$=0

θ_{72} ($[x^2,1]$)= $V_2+V_6+V_6+V_2$=0

θ_{72} ($[x^2,r]$)= $(-V_2)+(-V_6)+(-V_6)+(-V_2)$=0

θ_{72} ($[x^4,1]$)= 0+0+0+0=0

θ_{72} ($[x^4,r]$)= 0+0+0+0=0

θ_{72} ([y,1])=0

θ_{72}([y, r])=0

θ_{72} ([xy,1])=0

θ_{72}([xy, r])=0

Also $\sigma_{1i}(\chi_{2i})=\chi_{2i}$, $\sigma_{3i}(\chi_{2i})=\chi_{6i}$, $\sigma_{5i}(\chi_{1i})=\chi_{6i}$, $\sigma_{7i}(\chi_{1i})=\chi_{2i}$ and i=1,2**.**

By proposition (4.1) then $\theta_{4i}=\chi_{2i}+\chi_{6i}$

2- (I) if i=1

$\theta_{61}=\chi_{21}+\chi_{61}$

θ_{61}([1,1])=2+2=4

θ_{61}([1,r])= 2+2=4

θ_{61} ($[x^8,1]$)= 2+2=4

θ_{61} ($[x^8,r]$)= 2+2=4

θ_{61} ([x,1])= V_2+V_6=0

θ_{61} ([x,r])= V_2+V_6=0

θ_{61} ($[x^2,1]$)= 0+0=0

θ_{61} ($[x^2,r]$)= 0+0=0

θ_{61} ($[x^4,1]$)= (-2)+ (-2)= -4

θ_{61} ($[x^4,r]$)= (-2)+ (-2)= -4

θ_{61} ([y,1])=0

θ_{61}([y, r])=0

θ_{61} ([xy,1])=0

θ_{61}([xy, r])=0

2- (II) if i=2

$\theta_{62}=\chi_{22}+\chi_{62}$

θ_{62} ([1,1])= 2+2=4

θ_{62} ([1,r])= (-2)+ (-2)= -4

θ_{62} ($[x^8,1]$)= 2+2=4

θ_{62} ($[x^8,r]$)= (-2)+ (-2)= -4

θ_{62} ([x,1])= V_2+V_6=0

θ_{62} ([x,r])= $(-V_2)+(-V_6)$=0

θ_{62} ($[x^2,1]$)= 0+0=0

θ_{62} ($[x^2,r]$)= 0+0=0

θ_{62} ($[x^4,1]$)= (-2)+ (-2)= -4

θ_{62} ($[x^4,r]$)= 2+2=4

θ_{62} ([y,1])=0

θ_{62}([y, r])=0

θ_{62} ([xy,1])=0

θ_{62}([xy, r])=0

The elements $[x,1],[x^3,1],[x^5,1],[x^7,1]$ are in the same г-conjugate and$[x, r]$, $[x^3, r]$, $[x^5, r]$,$[x^7, r]$ are in the same г-conjugate and$[x^2,1]$,$[x^6,1]$ are in the same г-conjugate and $[x^2, r]$,$[x^6,r]$are in the same г-conjugate as Table (6) .

Theorem (4.4)
The rational valued characters table of the group $Q_{2m}\times C_2$ when $m=2^h$, $h\in Z^+$ is given as follows:

$$\equiv^*(Q_{2m}\times C_2) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_2)$$

Proof:-
Since

$\equiv^*(C_2)=$

	r_1'	r_2'
θ_1'	1	1
θ_2'	1	-1

Then ,

$$\chi_1'(r_1') = \chi_1'(r_2') = \theta_1'(r_1') = \theta_1'(r_2') = 1$$
$$\chi_2'(r_1') = \theta_2'(r_1') = 1, \chi_2'(r_2') = \theta_2'(r_2') = -1.$$

From the definition of $Q_{2m}\times C_2$, (theorem (3.4)),

$$\equiv Q_{2m}\times C_2 = (\equiv Q_{2m}) \otimes (\equiv C_2)$$

each element in $Q_{2m}\times C_2$

$$r_{ns} = r_n.r_s' \quad \forall r_n \in Q_{2m}, \ r_s' \in C_2$$

$n=1,2,3,\ldots,4m$, $s=1,2$
and each irreducible character of $Q_{2m}\times C_2$ is

$$\chi_{ij} = \chi_i \chi_j'$$

Where χ_i is an irreducible character of Q_{2m} and χ_j' is an irreducible character of C_2 , then

$$\chi_{ij}(r_{ns}) = \begin{cases} \chi_i(r_n) & if\ j=1 \quad and\, s=1,2 \\ \chi_i(r_n) & if\ j=2 \quad and\, s=1 \\ -\chi_i(r_n) & if\ j=2 \quad and\, s=2 \end{cases}$$

From proposition (4.1)

$$\theta_{ij} = \sum_{\sigma\in Gal(Q(\chi_{ij})/Q)} \sigma(\chi_{ij})$$

Where θ_{ij} is the rational valued of character table of $Q_{2m}\times C_2$
Then ,

$$\theta_{ij}(r_{ns}) = \sum_{\sigma\in Gal(Q(\chi_{ij}(r_{ns}))/Q)} \sigma(\chi_{ij}(r_{ns}))$$

(I) if j=1 and s=1,2

$$\theta_{ij}(r_{ns}) = \sum_{\sigma\in Gal(Q(\chi_i(r_n))/Q)} \sigma(\chi_i(r_n)) = \theta_i(r_n).1 = \theta_i(r_n).\theta_j'(r_s')$$

Where θ_i is the rational valued character of Q_{2m}.
(II) (a) if j=2 and s=1

$$\theta_{ij}(r_{ns}) = \sum_{\sigma\in Gal(Q(\chi_i(r_n))/Q)} \sigma(\chi_i(r_n)) = \theta_i(r_n).1 = \theta_i(r_n).\theta_j'(r_s')$$

(b) if j=2 and s=2

$$\theta_{ij}(r_{ns}) = \sum_{\sigma\in Gal(Q(\chi_i(r_n))/Q)} \sigma(-\chi_i(r_n)) = -\sum_{\sigma\in Gal(Q(\chi_i(r_n))/Q)} \sigma(\chi_i(r_n))$$
$$= \sum_{\sigma\in Gal(Q(\chi_i(r_n))/Q)} \sigma(\chi_i(r_n)).-1 = \theta_i(r_n).-1 = \theta_i(r_n).\theta_j'(r_s')$$

From (I) and (II) we have $\theta_{ij} = \theta_i.\theta_j'$

Then $\equiv^*(Q_{2m}\times C_2) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_2)$

The rational character table of the quaternion group ($Q_{2m}\times C_2$) when $m=2^h$, $h\in Z^+$ (4.5)

From Theorem (4.4) and form of $\equiv^*(Q_{2^{h+1}})$= in then table (5) then The rational character table of the quaternion group ($Q_{2m}\times C_2$) when $m=2^h$, $h\in Z^+$ is given in the general ($2h+8\times 2h+8$) matrix form $\equiv^*(Q_{2^{h+1}}\times C_2)$ as Table (7) .

Example (4.3)
By using Theorem (4.4) the rational valued characters table of $Q_{16}\times C_2$, Since h=3 ,it is same to the table Example (4.3), (after change order the rows and the columns) as Table (8) .

CL_α	[I]	$[x^2]$	$[x^4]$	….	$[x^{m-2}]$	$[x^m]$	$[x]$	$[x^3]$	….	$[x^{m-1}]$	$[y]$	$[xy]$
$\|CL_\alpha\|$	1	2	2	….	2	1	2	2	….	2	m	m
ψ_1	1	1	1	….	1	1	1	1	….	1	1	1
ψ_4	1	1	1	….	1	-1	-1	-1	….	-1	-1	1
χ_2	2	$V_{J(4)}$	$V_{J(8)}$	….	$V_{J(2m-4)}$	2	V_2	$V_{J(6)}$	….	$V_{J(2m-2)}$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
χ_{m-2}	2	$V_{J(2m-4)}$	$V_{J(4m-8)}$	….	$V_{J((m-2)(m-2))}$	2	$V_{J(m-2)}$	$V_{J(3m-6)}$	….	$V_{J((m-2)(m-1))}$	0	0
χ_1	2	V_2	$V_{J(4)}$	….	$V_{J(m-2)}$	-2	V_1	$V_{J(3)}$	….	$V_{J(m-1)}$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
χ_{m-1}	2	$V_{J(2m-2)}$	$V_{J(4m-4)}$	….	$V_{J((m-1)(m-2))}$	-2	$V_{J(m-1)}$	$V_{J(3m-3)}$	….	$V_{J((m-1)(m-1))}$	0	0
ψ_2	1	1	1	….	1	1	1	1	….	1	-1	-1
ψ_3	1	1	1	….	1	-1	-1	-1	….	-1	1	-1

Table (2)

CL_α	[1]	$[x^2]$	$[x^4]$	$[x^6]$	$[x^8]$	$[x]$	$[x^3]$	$[x^5]$	$[x^7]$	$[y]$	$[xy]$
$\|CL_\alpha\|$	1	2	2	2	1	2	2	2	2	6	6
ψ_1	1	1	1	1	1	1	1	1	1	1	1
ψ_4	1	1	1	1	1	-1	-1	-1	-1	-1	1
χ_2	2	0	-2	0	2	V_2	V_6	V_6	V_2	0	0
χ_4	2	-2	2	2-	2	0	0	0	0	0	0
χ_6	2	0	-2	0	2	V_6	V_2	V_2	V_6	0	0
χ_1	2	V_2	0	V_6	-2	V_1	V_3	V_5	V_7	0	0
χ_3	2	V_6	0	V_2	-2	V_3	V_7	V_1	V_5	0	0
χ_5	2	V_6	0	V_2	-2	V_5	V_1	V_7	V_3	0	0
X_7	2	V_2	0	V_6	-2	V_7	V_5	V_3	V_1	0	0
ψ_2	1	1	1	1	1	1	1	1	1	-1	-1
ψ_3	1	1	1	1	1	-1	-1	-1	-1	1	-1

Table (3)

CL_α	[1,1]	[1,r]	$[x^2,1]$	$[x^2,r]$	$[x^4,1]$	$[x^4,r]$	$[x^6,1]$	$[x^6,r]$	$[x^8,1]$	$[x^8,r]$	[x,1]	[x,r]	$[x^3,1]$	$[x^3,r]$	$[x^5,1]$	$[x^5,r]$	$[x^7,1]$	$[x^7,r]$	[y,1]	[y,r]	[xy, 1]	[xy, r]
$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	2	1	1	2	2	2	2	2	2	2	2	6	6	6	6
ψ_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ψ_{12}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
ψ_{41}	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1
ψ_{42}	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1
χ_{21}	2	2	0	0	-2	-2	0	0	2	2	V_2	V_2	V_6	V_6	V_6	V_6	V_2	V_2	0	0	0	0
χ_{22}	2	-2	0	0	-2	2	0	0	2	-2	V_2	$- V_2$	V_6	$-V_6$	V_6	$-V_6$	V_2	$- V_2$	0	0	0	0
χ_{41}	2	2	-2	-2	2	2	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	2	-2	-2	2	2	- 2	-2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	2	2	0	0	-2	-2	0	0	2	2	V_6	V_6	V_2	V_2	V_2	V_2	V_6	V_6	0	0	0	0
χ_{62}	2	-2	0	0	-2	2	0	0	2	-2	V_6	$-V_6$	V_2	$-V_2$	V_2	$-V_2$	V_6	$-V_6$	0	0	0	0
χ_{11}	2	2	V_2	V_2	0	0	V_6	V_6	-2	-2	V_1	V_1	V_3	V_3	V_5	V_5	V_7	V_7	0	0	0	0
χ_{12}	2	-2	V_2	$- V_2$	0	0	V_6	$-V_6$	-2	2	V_1	$- V_1$	V_3	$-V_3$	V_5	$- V_5$	V_7	$-V_7$	0	0	0	0
χ_{31}	2	2	V_6	V_6	0	0	V_2	V_2	-2	-2	V_3	V_3	V_7	V_7	V_1	V_1	V_5	V_5	0	0	0	0
χ_{32}	2	-2	V_6	$-V_6$	0	0	V_2	$-V_2$	-2	2	V_3	$-V_3$	V_7	$-V_7$	V_1	V_1	V_5	$-V_5$	0	0	0	0
χ_{51}	2	2	V_6	V_6	0	0	V_2	V_2	-2	-2	V_5	V_5	V_1	V_1	V_7	V_7	V_3	V_3	0	0	0	0
χ_{52}	2	-2	V_6	$-V_6$	0	0	V_2	$- V_2$	-2	2	V_5	$- V_5$	V_1	$- V_1$	V_7	$- V_7$	V_3	$- V_3$	0	0	0	0
χ_{71}	2	2	V_2	V_2	0	0	V_6	V_6	-2	-2	V_7	V_7	V_5	V_5	V_3	V_3	V_1	V_1	0	0	0	0
χ_{72}	2	-2	V_2	$- V_2$	0	0	V_6	$-V_6$	-2	2	V_7	$- V_7$	V_5	$-V_5$	V_3	$- V_3$	V_1	$- V_1$	0	0	0	0
ψ_{21}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
ψ_{22}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
ψ_{31}	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1
ψ_{32}	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1

Table (4)

CL_α	[1,1]	[1,r]	$[x^2,1]$	$[x^2,r]$	$[x^4,1]$	$[x^4,r]$	$[x^8,1]$	$[x^8,r]$	[x,1]	[x,r]	[y,1]	[y,r]	[xy, 1]	[xy, r]
$\lvert CL_\alpha \rvert$	1	1	4	4	2	2	1	1	2	2	8	8	8	8
θ_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
θ_{12}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
θ_{21}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1
θ_{22}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	-1
θ_{31}	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
θ_{32}	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
θ_{41}	1	1	1	1	1	1	1	1	-1	-1	1	1	-1	-1
θ_{42}	1	-1	1	-1	1	-1	1	-1	-1	1	1	-1	-1	1
θ_{51}	2	2	-2	-2	2	2	2	2	0	0	0	0	0	0
θ_{52}	2	-2	-2	2	2	- 2	2	-2	0	0	0	0	0	0
θ_{61}	4	4	0	0	-4	-4	4	4	0	0	0	0	0	0
θ_{62}	4	-4	0	0	-4	4	4	-4	0	0	0	0	0	0
θ_{71}	8	8	0	0	0	0	-8	-8	0	0	0	0	0	0
θ_{72}	8	-8	0	0	0	0	-8	8	0	0	0	0	0	0

Table (6)

$$\begin{array}{ccccccccc|ccccccccc}
1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\
-1 & 1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 \\
-1 & -1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 \\
0 & 0 & 0 & -2 & 2 & \cdots & 2 & 2 & 2 & 0 & 0 & 0 & -2 & 2 & \cdots & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & -4 & \cdots & 4 & 4 & 4 & 0 & 0 & 0 & 0 & -4 & \cdots & 4 & 4 & 4 \\
\vdots & \vdots & \vdots & \ddots & \ddots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \cdots & 0 & -2^{h-1} & 2^{h-1} & 2^{h-1} & 0 & 0 & 0 & \cdots & \cdots & 0 & -2^{h-1} & 2^{h-1} & 2^{h-1} \\
0 & 0 & 0 & \cdots & \cdots & 0 & 0 & -2^{h} & 2^{h} & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & -2^{h} & 2^{h} \\
\hline
1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & \cdots & -1 & -1 & -1 \\
-1 & -1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & \cdots & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & \cdots & -1 & -1 & -1 \\
0 & 0 & 0 & -2 & 2 & \cdots & 2 & 2 & 2 & 0 & 0 & 0 & 2 & -2 & \cdots & -2 & -2 & -2 \\
0 & 0 & 0 & 0 & -4 & \cdots & 4 & 4 & 4 & 0 & 0 & 0 & 0 & 4 & \cdots & -4 & -4 & -4 \\
\vdots & \vdots & \vdots & \ddots & \ddots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \cdots & 0 & -2^{h-1} & 2^{h-1} & 2^{h-1} & 0 & 0 & 0 & \cdots & \cdots & 0 & 2^{h-1} & -2^{h-1} & -2^{h-1} \\
0 & 0 & 0 & \cdots & \cdots & 0 & 0 & -2^{h} & 2^{h} & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 2^{h} & -2^{h}
\end{array}$$

Table (7)

CL_α	[y,1]	[xy, 1]	[x,1]	$[x^2,1]$	$[x^4,1]$	$[x^8,1]$	[1,1]	[y,r]	[xy, r]	[x,r]	$[x^2,r]$	$[x^4,r]$	$[x^8,r]$	[1,r]
$\|CL_\alpha\|$	8	8	2	4	4	1	1	8	8	2	2	2	1	1
θ_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
θ_{21}	-1	1	-1	1	1	1	1	-1	1	-1	1	1	1	1
θ_{31}	-1	-1	1	1	1	1	1	-1	-1	1	1	1	1	1
θ_{41}	1	-1	-1	1	1	1	1	1	-1	-1	1	1	1	1
θ_{51}	0	0	0	-2	2	2	2	0	0	0	-2	2	2	2
θ_{61}	0	0	0	0	-4	4	4	0	0	0	0	-4	4	4
θ_{71}	0	0	0	0	0	-8	8	0	0	0	0	0	-8	8
θ_{12}	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
θ_{22}	-1	1	-1	1	1	1	1	1	-1	1	-1	-1	-1	-1
θ_{32}	-1	-1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
θ_{42}	1	-1	-1	1	1	1	1	-1	1	1	-1	-1	-1	-1
θ_{52}	0	0	0	-2	2	2	2	0	0	0	2	- 2	-2	-2
θ_{62}	0	0	0	0	-4	4	4	0	0	0	0	4	-4	-4
θ_{72}	0	0	0	0	0	-8	8	0	0	0	0	0	8	-8

Table (8)

References

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