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Bayesian Estimation for Semiparametric Logistic Regression

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***Abstract* :** Logistic regression for binary response variable is often used many fields of study for instance clinical applications humanitarian and social issues. In this paper, Bayesian estimation approach is introduce to estimate the unknown link function and the coefficient vector in the semiparametric logistic regression. The normal distribution prior was considered to the coefficient vector and Gaussian process prior was set for the unknown link function. Bayesian hierarchical model was developed for the single index logistic regression model. MCMC algorithm was adopted for posterior inference. To compare our proposed method BSLR with the existing methods real data and two simulation examples are considered. We have conclude that our proposed method do well.

Keywords - Bayesian logistic regression, binary regression, Gaussian process, MCMC, single index model.

INTRODUCTION: Regression methods have become an integral component of any data analysis concerned with describing the relationship between a response variable and one or more explanatory variables. Regression analysis achieves most of the goals of scientific research, whereas its methods represent the main part for any data analysis that aims to study and explain the relationship between the dependent variable and the independent variables. However, it is unable to describe and explain the relationships between the explanatory variables and the response variable when the later has binary value, however the nature of the dependent variable is required to be a continuous quantity and not a classification (Lea,1997). This type of dependent variable is common in the studies of a large number of humanitarian and social issues (Lea, 1997 ,Poston, 2004). This is why the need has arisen for developing new statistical methods that have the power of linear regression in reaching the best equations and dealing with them. At the same time, it recovers the problem of the inability to apply the usual linear regression analysis models in the case of dependent variables that have a binary value (Lea ,1997). Quite often the outcome variable is discrete, taking on two possible values. Binary can have only two possible outcomes which we will denote as 1 and 0. Two relevant binary regression models, logit (logistic) and probit regression when the dependent variable is a binary response and take two values: 0 and 1

$$y = \begin{cases} 0 & if \quad no \\ 1 & if \quad yes \end{cases}$$

y is a response variable distributed as Bernoulli with probability of success P.

A problem with the regression model is that the predicted probabilities will not be limited between 0 and 1. Binary regression model is defined as p_i= F(x′β), i=1,...,N where β is a k X I vector of unknown parameters, x′=(x_{i1}………x_{ik}) is a vector of covariates, and F(.) is a known cumulative distribution function (cdf) linking function. The linear structure of ($x'\beta$) is the logit model when F is the logistic cdf, where the predicted probabilities are limited between 0 and 1. Whereas the probit model is obtained if F is the standard Gaussian cdf. The predicted probabilities are limited between 0 and 1.

Sometimes the explanatory variables are non-linear, which led researchers to find another method that deals with the nonlinear effect of these variables or nonparametric regression. It was proposed by the researcher (Jacob) in 1942.

It is also noticeable that nonparametric regression suffers from some problems, including the problem of dimensions (the curse of dimensionality), which occurs when the number of independent variables increases. This led to the emergence of some modern methods in regression models, for instance semiparametric regression models (regression for data analysis that combines parameterized compounds with nonparametric compounds that contribute to some degree for obtaining a practical ability to avoid the problem of dimensional curse). Single index model this model was study by Hardle & Stoker in (1989) and Ichimura (1993) which allows the mean response to be a nonparametric function of linear combinations for predictive variables. This model is one of the most common semi-parametric models and is widely used in applied medical and economic sciences and this model was named because all explanatory variables are summarized under one linear index $m(x'\beta)$. There are some statistical analyses when the information is not available about the sample under study. Therefore, researchers resort to making assumptions that the parameters to be assessed about random variables that require obtaining prior information about them are taken based on previous experiences about the studied phenomenon by formulating them in the form of a prior probability

distribution (prior distribution, which is combined with the probability observation distribution to obtain the posterior distribution, which contains all information about the parameters to be evaluated; this is called 'the Bayesian'. Bayesian single index were introduced in Antoniadis et al. (2004).
In this paper, Bayesian hierarchical model was developed for logistic regression based on single index and MCMC algorithm was adopted for posterior inference.

2. Bayesian Semi Parametric Logistic Regression and prior assumption

Logistic regression model is one of the nonlinear regression models, in which the relationship between the binary response variable (y), and the independent explanatory variables (x_1, x_2, ...x_k) is nonlinear. The logistic regression model is based on the basic assumption that the dependent variable (y) is binary take either of the two values (1, 0), either success with probability (P_i) or failure with probability (1-P_i).
A logistic regression model, can be expressed by the formula:

$$p(y) = P_i^{yi}(1 - P_i)^{1-yi} \quad (1)$$

$$P_i = \frac{e^{\beta_0+\beta_1 x_{i1}+\beta_2 x_{i2}+\cdots+\beta_k x_{ik}}}{1 + e^{\beta_0+\beta_1 x_{i1}+\beta_2 x_{i2}+\cdots+\beta_k x_{ik}}}$$

$$1 - P_{i=} \frac{1}{1 + e^{\beta_0+\beta_1 x_{i1}+\beta_2 x_{i2}+\cdots+\beta_k x_{ik}}}$$

Where y binary response variable(1,0), $x_i, i = 1,2, \dots, k$ explanatory variables P_i the probability of the response occurring when y=1, $1 - P_i$ The possibility of lack of response when y=0, $\beta_0, \beta_1, \dots, \beta_k$ unknown parameters of the logistic regression model.
Bayesian inference provides a useful way to combine expert knowledge (prior belief) with data to arrive at some posterior belief. Bayesian inference is conducted through the use of Bayes' theorem (Press, 1989; Bernardo and Smith, 2000; Lee, 2004; Greenberg, 2008; Ntzoufras, 2009). Press (1989) explains that when has a prior belief (called a prior distribution) before observes the data, Bayes' theorem gives a mathematical procedure for updating the prior belief to arrive at a posterior distribution.
The semi-parametric regression model has gained widespread popularity in recent times, due to its advantage in integrating parametric regression models with non-parametric regression models at the same time.
The semiparametric logistic regression based on single index regression model is :

$$P_i = p(y = 1) = \frac{exp\ \ (m(x_i'\boldsymbol{\beta})}{1 + exp\ \ (m(x_i'\boldsymbol{\beta})} \quad (2)$$

where β parameters vector (Parametric part) and $m(.)$ is an unknown link function (non-parametric part) and

$$1 - P_i = p(y = 0) = \frac{1}{1 + exp\ \ (m(x_i'\boldsymbol{\beta})}$$

The likelihood function is the probability density function of the data which is seen as a function of the parameter treating the observed data as fixed quantities. For a given sample size n, the likelihood function is given as:

$$L(y|m, \boldsymbol{\beta}) = \prod_{i=1}^{n} f(y_i)$$

$$L(y|m, \boldsymbol{\beta}) = \prod_{i=1}^{n} P_i^{y_i}(1 - P_i)^{1-y_i}$$

Therefore, the likelihood function is of the form:

$$L(y|m, \boldsymbol{\beta}) = \prod_{i=1}^{n} (\frac{exp\ \ (m(x_i'\boldsymbol{\beta})}{1 + exp\ \ (m(x_i'\boldsymbol{\beta})})^{y_i} (\frac{1}{1 + exp\ \ (m(x_i'\boldsymbol{\beta})})^{1-y_i} \quad (3)$$

As in (Choi et al. (2011)) and (Gramacy and Lian (2012)), the Gaussian prior distribution is considered as a prior for the unknown nonparametric link function $m(.)$. More specially, the previous distribution of $m(.)$ is GP, with zero mean and square exponential covariance function is written as follows:

$$m \sim GP\big(0, E(.,.)\big), \qquad where \qquad E(x, x') = \partial \frac{(x - x')^2}{w}$$

where ∂ and w are Hyperparameters. Writing this out in the single-index model framework using the observed covariatesx_i , we have,

$$\pi(m_n|\beta, \partial) = \; det[E_n]^{-1/2} exp\left\{-\frac{m_n{}'\, E_n{}^{-1}\, m_n}{2}\right\}$$

E_n is the covariance matrix with dimension (n x n) and elements E (.,.) given in Equation

$$E\big(x_i\,, x_j\,\big) = \partial\; exp\{-\big(x_i - x_j\big)'\beta\beta'\big(x_i - x_j\big)\}$$

Follow Gramacy and Lian (2012), and when we use the Gaussian process as a prior distribution to the nonparimatric link function, then $\frac{\beta}{\sqrt{w}}$ is identifiable without the necessity for the constraint $\|\beta\| = 1$. Therefore we will instead of $\frac{\beta}{\sqrt{w}}$ by β and the covariance function is reformulated as follows:

$$E\big(x_i\,, x_j\,\big) = \partial \exp\left\{-\big(\, x_i{}'\boldsymbol{\beta} - \; x_j{}'\boldsymbol{\beta}\big)^2\right\} \tag{4}$$

the inverse gamma distribution is set as a hyper prior for which implies that $\partial \sim Inv.Gamma(a_{\partial,}\, b_\partial)$ where $a_{\partial,}$and b_∂ are the hyperparameters.

Follow Wilhelmsen *et al*. (2009) and Ziemba (2005) we will set a normal distribution as prior for the model parameters vector $\boldsymbol{\beta} \sim N(0, \sigma^2)$**,** the inverse gamma distribution is set as a prior for $\sigma^2 \sim Inv.Gamma(a_{\sigma,}\, b_\sigma)$.

3. Hierarchical model and MCMC algorithm

The hierarchic model for the Bayesian single index logistic regression can be written as follows:

$$\boldsymbol{f}(\boldsymbol{y}\,|\boldsymbol{m}, \boldsymbol{\beta}, \partial, \sigma^2) = \prod_{i=1}^{n} \Big(\frac{exp\;(m(x_i'\beta)}{1 + \; exp(m(x_i'\beta)}\Big)^{yi} \; \Big(1 - \frac{exp(m(x_i'\beta)}{1 + \; exp(m(x_i'\beta)}\Big)^{1-yi}$$

$$m|\beta, \partial \sim GP\big(0, E(.,.)\big),$$

$$\beta|\sigma^2 \sim N(0, \sigma^2) \tag{5}$$

$$\partial \sim Inv.Gamma(a_{\partial,}\, b_\partial)$$

$$\sigma^2 \sim Inv.Gamma(a_{\sigma,}\, b_\sigma)$$

Based on the Bayesian hierarchical model (5), the full conditional posterior distributions are as follows:

$$\begin{aligned} p(\boldsymbol{m}, \boldsymbol{x_i}, \boldsymbol{\beta}, \partial, \sigma^2|\boldsymbol{y}) & \\ \propto & \prod_{i=1}^{n} \left(\frac{exp\;(m(x_i'\beta)}{1 + \; exp(m(x_i'\beta)}\right)^{yi} \; \Big(1 - \frac{exp(m(x_i'\beta)}{1 + \; exp(m(x_i'\beta)}\Big)^{1-yi} \\ & \times det[E_n]^{-1/2} exp\left\{-\frac{m_n{}'\, E_n{}^{-1}\, m_n}{2}\right\} \times \prod\nolimits_{j=1}^{p} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(\beta)^2}{2\sigma^2}\} \times \left(\frac{1}{\sigma^2}\right)^{a_{\sigma,}+1} \exp\{\frac{b_\sigma}{\sigma^2}\} \\ & \times \left(\frac{1}{\partial}\right)^{a_{\partial,}+1} \exp\{\frac{b_\partial}{\partial}\} \end{aligned}$$

the conditional posterior distributions for all parameters can easily derived for the Bayesian single index logistic regression:

- Sample the link function $\boldsymbol{m}|\ \boldsymbol{x_i}, \boldsymbol{\beta}, \partial, \sigma^2, \boldsymbol{y}$ from the following conditional posterior distribution:

$$\boldsymbol{\pi}(\boldsymbol{m}|\ \boldsymbol{x_i}, \boldsymbol{\beta}, \partial) \propto \prod_{i=1}^{n} \left(\frac{exp\ (m(x_i'\beta)}{1+\ exp(m(x_i'\beta)} \right)^{yi} (1 - \frac{exp(m(x_i'\beta)}{1+\ exp(m(x_i'\beta)})^{1-yi} \times det[E_n]^{-1/2} exp\left\{-\frac{{m_n}'\ {E_n}^{-1}\ m_n}{2}\right\}$$

- Sample the parameters vector $\boldsymbol{\beta}|\boldsymbol{m},\ \boldsymbol{x_i}, \partial, \sigma^2, \boldsymbol{y}$ from the following conditional posterior distribution:

$$\boldsymbol{\pi}(\boldsymbol{\beta}|\boldsymbol{m}, \sigma^2) \propto \prod_{i=1}^{n} \left(\frac{exp\ (m(x_i'\beta)}{1+\ exp(m(x_i'\beta)} \right)^{yi} (1 - \frac{exp(m(x_i'\beta)}{1+\ exp(m(x_i'\beta)})^{1-yi} \times det[E_n]^{-1/2} exp\left\{-\frac{{m_n}'\ {E_n}^{-1}\ m_n}{2}\right\} \times \prod_{j=1}^{p} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(\beta)^2}{2\sigma^2}\}$$

- The conditional distribution of $\sigma^2|\boldsymbol{m},\ \boldsymbol{x_i}, \partial, \boldsymbol{\beta}, \boldsymbol{y}$ is the Inverse Gamma distribution

$$\pi(\sigma^2|\beta) \propto \prod_{j=1}^{p} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(\beta)^2}{2\sigma^2}\} \times \left(\frac{1}{\sigma^2}\right)^{a_{\sigma,}+1} \exp\{-\frac{b_\sigma}{\sigma^2}\}$$

  So that we will sample $\sigma^2|\boldsymbol{m},\ \boldsymbol{x_i}, \partial, \boldsymbol{\beta}, \boldsymbol{y}$ from the following conditional posterior distribution $IG(a_\sigma + \frac{1}{2}, \frac{(\beta)^2}{2} + b_\sigma)$
- We will sample ∂ from the following posterior conditional distribution

$$\pi(\partial|m_n) \propto det[E_n]^{-1/2} exp\left\{-\frac{{m_n}'\ {E_n}^{-1}\ m_n}{2}\right\} \times \left(\frac{1}{\partial}\right)^{a_{\partial,}+1} \exp\{\frac{b_\partial}{\partial}\}$$

An efficient Gibbs sampler algorithm is used to sample σ^2, whereas a Metropolis-Hastings algorithm is used to sample $\boldsymbol{\beta_\tau}$, $\boldsymbol{m_n}$ and ∂. We set the initial values for the hyperparameters $a_\sigma, b_\sigma, a_\partial$ and b_∂ as (0.1).

4. Simulation study

In this section simulation examples are considered to evaluation the performance of our proposed method Bayesian semiparametric logistic regression BSLR. We have compare our proposed method to some existing methods Bayesian Logistic regression BLR, Bayesian Probit regression BPR these function are reported in MCMCpack R package and Bayesian Binary Quantile regression BBQR was reported in bayesQR R package. We have use two simulation examples as same as examples that use by (Hu et al. (2013), Alshaybawee et al. (2016), Zhao and Lian (2015), Alkenani and Yu (2013), Lv et al. (2014) and Kuruwita (2015)). We have constructed an R code to implement MCMC algorithm. MCMC algorithm are run 20000 iteration and remove the first 4000 as burn in.

Example 1

In this example three sample size is considered (N=50, 150 and 250) and the following model used to generate our data:

$$y^* = g(t) + 0.1\varepsilon, \qquad g(t) = sin\left\{\frac{\pi(t-H)}{M-H}\right\}, \qquad y_i = \begin{cases} 1 & if \quad y_i^* \geq 0 \\ 0 & otherwise \end{cases}$$

Where $t = \boldsymbol{x}_i^T\boldsymbol{\beta}$, $i = 1,2,\ldots\ldots,10$, distributed as uniform [0,1], $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots.,\beta_{10})^T = \frac{1}{\sqrt{3}}(1,1,0,0,1,0,0,0,0,0)^T$, $H = \frac{\sqrt{3}}{2} - \frac{1.645}{\sqrt{12}}$, $M = \frac{\sqrt{3}}{2} + \frac{1.645}{\sqrt{12}}$ and the error term ε_i is distributed as standard normal $N(0,1)$. The result of this study based on 100 replications for each sample size.

Table (1) show the standard division to the estimates parameters that estimate by the proposed and existing methods BSLR , BLR,BPR and BBQR at three samples size 50, 150 and 250. We can see that the proposed method BSLR have get the smallest values over all the samples that mean this method more consistent compare to the other methods. BBQR method has get small values of SD when the sample small (N=50) but these values are increase when the sample size increase. The other two methods BLR and BPR have high values of SD when the sample is small whereas these values decrease when N=150 and 250. Over all samples SD values of BPR method smaller than the values of BLR method.

Table 1. The average SD of the parameter estimates of BSLR , BLR,BPR and BBQR methods for the samples (Simulated Example 1)

N	Methods	$SD.\beta_1$	$SD.\beta_2$	$SD.\beta_3$	$SD.\beta_4$	$SD.\beta_5$	$SD.\beta_6$	$SD.\beta_7$	$SD.\beta_8$	$SD.\beta_9$	$SD.\beta_{10}$
50	**BSLR**	**0.265**	**0.187**	**0.326**	**0.112**	**0.355**	**0.429**	**0.178**	**0.320**	**0.340**	**0.290**
	BLR	**72.386**	**2.169**	**17.054**	**65.886**	**21.926**	**27.912**	**6.215**	**47.994**	**52.386**	**69.736**
	BPR	**63.711**	**2.054**	**16.229**	**58.741**	**19.437**	**22.861**	**5.273**	**45.004**	**44.314**	**63.871**
	BBQR	**4.442**	**3.316**	**3.984**	**1.85**	**1.856**	**1.319**	**2.095**	**0.436**	**2.291**	**1.644**
150	**BSLR**	**0.250**	**0.131**	**0.351**	**0.252**	**0.295**	**0.222**	**0.208**	**0.345**	**0.177**	**0.374**
	BLR	**1.463**	**0.998**	**0.949**	**0.494**	**1.207**	**0.491**	**1.170**	**2.173**	**1.047**	**1.158**
	BPR	**1.232**	**0.756**	**0.791**	**0.387**	**1.171**	**0.478**	**1.098**	**1.702**	**0.897**	**1.133**
	BBQR	**1.838**	**1.706**	**1.990**	**0.995**	**1.364**	**0.958**	**1.877**	**2.140**	**0.786**	**1.635**
250	**BSLR**	**0.087**	**0.03**	**0.183**	**0.163**	**0.262**	**0.217**	**0.323**	**0.151**	**0.191**	**0.211**
	BLR	**0.133**	**0.221**	**0.299**	**0.307**	**0.315**	**0.173**	**0.590**	**0.317**	**0.286**	**0.265**
	BPR	**0.117**	**0.218**	**0.256**	**0.238**	**0.307**	**0.135**	**0.492**	**0.276**	**0.239**	**0.241**
	BBQR	**0.506**	**0.822**	**0.737**	**0.503**	**1.349**	**0.198**	**1.141**	**0.776**	**0.919**	**1.094**

In Table (2), we summarize the MSE and MAE values for all methods and at the three samples size. The proposed method has the smallest MSE and MAE compare to the other three method at all samples. The MSE and MAE for BBQR method are smaller than the other two method in the case of N=50 and 150, but it exceeded the rest methods when the sample size was increased to 250. As same as SD the BPR method get MSE and MAE smaller than BLR method. In Figure (1) we are showing MSE and MAE for all methods and at the samples size.

Table 2. The values of MSE and MAE of BSLR, BLR, BPR and BBQR methods for each sample (Simulated Example 1)

N	Methods	MSE	MAE
50	**BSLR**	**0.9881**	**0.5182**
	BLR	**26.7304**	**15.8206**
	BPR	**23.5322**	**11.8523**
	BBQR	**20.7445**	**4.0955**
150	**BSLR**	**0.2102**	**0.4257**
	BLR	**8.7532**	**2.6755**
	BPR	**6.6657**	**1.9875**
	BBQR	**2.8176**	**1.3443**
250	**BSLR**	**1.3153**	**1.1052**
	BLR	**1.8629**	**1.1580**
	BPR	**1.6641**	**1.0516**
	BBQR	**2.7126**	**2.3382**

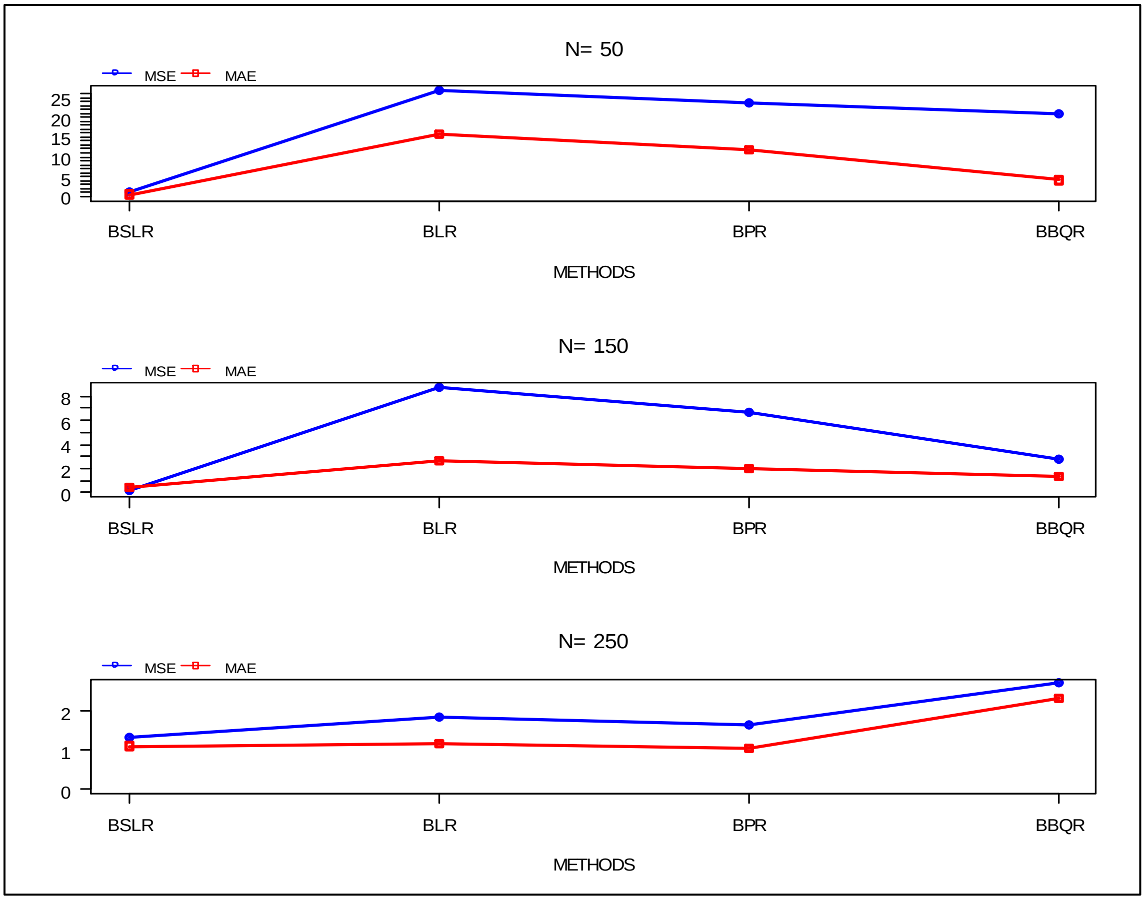

Figure 1. show the MSE and MAE for BSLR, BLR, BPR and BBQR methods at three samples size (Example 1).

Example 2:

In this example, data are generated with three samples size (N=50, 100 and 250) from the following model

$$y_i^* = g(t) + \sqrt{(\sin(t)+1)}\,\varepsilon, \quad where\ g(t) = 10\sin(0.75t), \quad y_i = \begin{cases} 1 & if \quad y_i^* \geq 0 \\ 0 & otherwise \end{cases}$$

Where $t = \boldsymbol{x}_i^T\beta$ $(i = 1,2,\ldots,6)$ are i.i.d. generated from a normal distribution $N[0,(1/4)^2]$, $\boldsymbol{\beta} = \frac{1}{\sqrt{5}}(0,1,0,0,1,0)^T$, and standard normal distribution use for the error term.

Table (3) show the standard division to the parameters for all method under study BSLR , BLR,BPR and BBQR at the samples size 50, 150 and 250. It is clearly that the proposed method BSLR get the smallest values of SD over all methods and at all samples size. As same as Example 1 we find that the BBQR method has gotten small values of SD compare to the other two methods BLR and BPR when the sample size is 50. When the samples size are increase SD values for BBQR method are increase and exceed the BLR and BPR SD. On the contrary, BLR and BPR methods have high values of SD when the sample size is small whereas these values decrease when increase the samples size. SD values of BPR method are smaller than the values of BLR method in most cases.

Table 3. The average SD of the parameter estimates of BSLR , BLR,BPR and BBQR methods for the samples (Simulated Example 2)

N	Methods	$SD.\beta_1$	$SD.\beta_2$	$SD.\beta_3$	$SD.\beta_4$	$SD.\beta_5$	$SD.\beta_6$
50	**BSLR**	0.239	0.279	0.313	0.049	0.225	0.385
	BLR	2.467	3.115	6.554	6.857	5.817	8.601
	BPR	2.137	2.851	5.608	6.442	5.197	6.958
	BBQR	1.799	3.471	4.889	5.213	5.106	6.162
150	**BSLR**	0.199	0.159	0.219	0.406	0.378	0.127
	BLR	2.395	2.841	2.482	1.934	3.039	2.468

	BPR	2.007	2.531	2.322	1.583	2.725	2.276
	BBQR	1.811	3.201	2.686	1.546	3.654	2.216
250	**BSLR**	0.36	0.081	0.267	0.308	0.269	0.403
	BLR	1.839	2.414	1.707	1.574	2.538	2.697
	BPR	1.244	2.003	1.257	1.137	2.378	2.321
	BBQR	1.83	3.051	1.391	1.534	3.009	3.218

We reported the values of the MSE and MAE In Table (4) for all methods in this study. The MSE and MAE values for the proposed method are the smallest compare to the other three existing methods at all samples. BBQR method get small MSE and MAE compare to BLR and BPR methods in the case of N=50. MSE and MAE for these three methods are almost close when the sample size is 150 whereas these values of BBQR exceeded the other when N=250. MSE and MAE for BPR method are smaller than BLR method in most case. Figure (1) show MSE and MAE for all methods and at the samples size.

Table 4. The values of MSE and MAE of BSLR , BLR,BPR and BBQR methods for each sample (Simulated Example 2)

N	Methods	MSE	MAE
50	**BSLR**	0.5163	0.5336
	BLR	0.9814	0.8391
	BPR	0.8564	0.7639
	BBQR	0.7468	0.6195
150	**BSLR**	0.4073	0.4166
	BLR	0.8804	0.6549
	BPR	0.8473	0.6017
	BBQR	0.8169	0.6703
250	**BSLR**	0.4204	0.4315
	BLR	0.6415	0.6410
	BPR	0.5911	0.5278
	BBQR	0.6991	0.6710

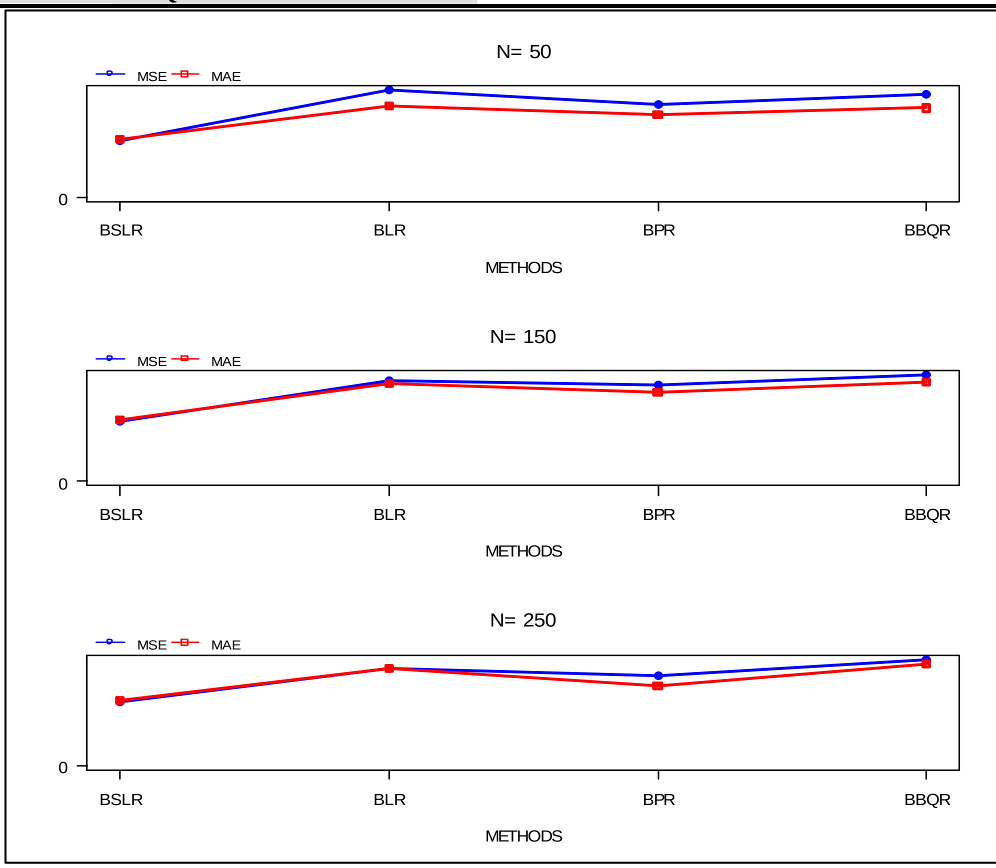

Figure 2. show the MSE and MAE for BSLR, BLR, BPR and BBQR methods at three samples size (Example 2).

Real data

In this section birth weight dataset is considered, these dataset was reported in MASS package of R. This data was collected by Baystate Medical Center, Springfield, Mass during 1986. The dataset are consist of 189 observations and 10 variables. In this study we have considered six variables (low) show the birth weight less than 2500 g, as response variable. The following variables are used as independent variables (age: the age of mother iv years, lwt: mother's weight in pounds, race: the race of mother (1= white, 2= black, 3=other), smoke: mother smoke status during pregnancy, bwt: baby weight in grams).

Table 5. The values of MSE and MAE of BSLR , BLR,BPR and BBQR methods for real data.

Methods	MSE	MAE
BSLR	4.0133	2.8045
BLR	27.3736	16.2894
BPR	26.4522	13.1995
BBQR	7.1999	5.8721

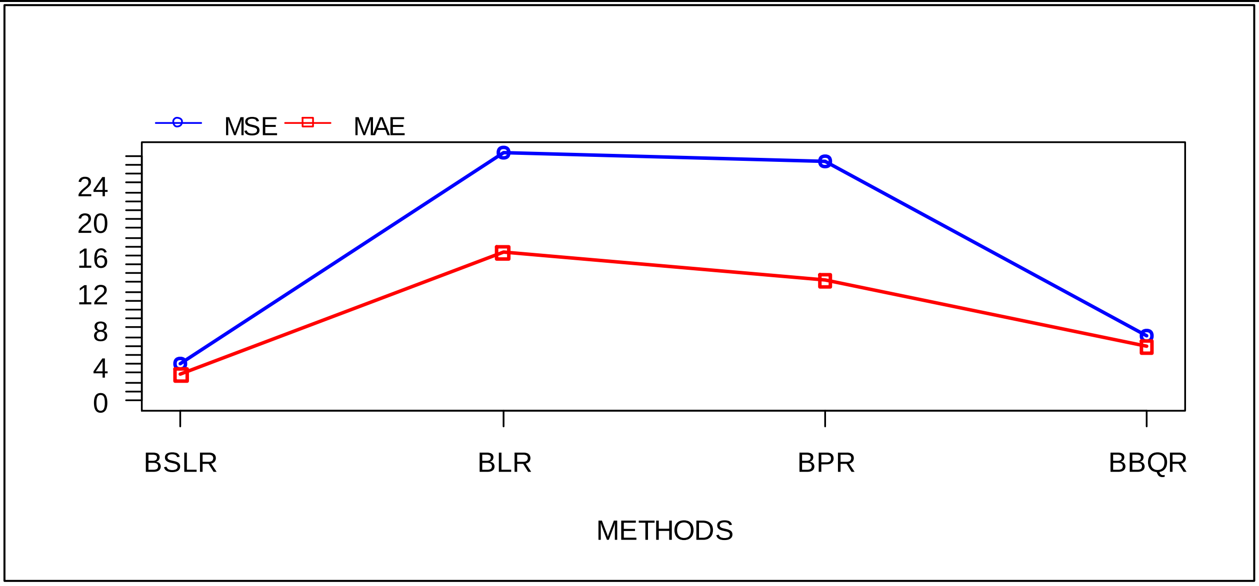

Figure 3. show the MSE and MAE for BSLR, BLR, BPR and BBQR methods for real data.

In Table (5) and Figure (3) we summarize the MSE and MAE values for our proposed method and also the existing methods. The results show that the proposed method BSLR get the small MSE and MAE compare to all other methods in this study. MSE and MAE values for BBQR method are smaller than that for BLR and BPR. The largest amount of MSE and MAE were for BPR methods.

Conclusion

In this study, Bayesian estimation approach is introduce to estimate the unknown link function and the coefficient vector in the semiparametric logistic regression. The normal distribution prior was considered to the coefficient vector and Gaussian process prior was set for the unknown link function. Bayesian hierarchical model was developed for the single index logistic regression model. MCMC algorithm was adopted for posterior inference.

To compare our proposed method BSLR with the existing methods BLR, BPR and BBQR real data and two simulation examples are considered. The results that reported at the tables and figures above indicate that the proposed method is doing better than the other methods at all. The BLR get the largest SD, MSE and MAE values in most simulation and real data. BBQR is doing better than BLR and BPR in small samples size. We concluded that the proposed method BSLR performs better than other methods.

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