On Artin Cokernel of the Group D_{nh} When n is an Odd Number

حول النواة المشارك-آرتن لزمرة ثنائي السطوح D_{nh} عندما n عدد فردي

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Abstract

The group of all Z-valued characters of G over the group of induced unit characters from all cyclic subgroups of G forms a finite abelian group, called ***Artin Cokernal of G*** ,denoted by AC(G)= $\overline{R}(G)/T(G)$

The problem of finding the cyclic decomposition of Artin cokernel $AC(D_{nh})$ has been considered in this paper when n is an odd number , we find that if $n = p_1^{\alpha_1}.p_2^{\alpha_2}...p_m^{\alpha_m}$, where $p_1,p_2,\ldots,p_m$ are distinct primes and not equal to 2 , then

$$AC(D_{nh}) = \bigoplus_{i=1}^{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))-1} C_2$$

$$= \bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

And we give the general form of Artin's characters table $Ar(D_{nh})$when n is an odd number.

المستخلص

ان زمرة كل الشواخص العمومية ذات القيم الصحيحة للزمرة G على زمرة الشواخص المحتثة من الشواخص الأحادية للزمر الجزئية الدائرية من الزمرة G تكون زمرة ابيلية منتهية و تسمى النواة المشارك – آرتن للزمرة G, ويرمز لها بالرمز $AC(G) = \overline{R}(G)/T(G)$

إن مسألة إيجاد التجزئة الدائرية لزمرة القسمة AC(G) قد اعتبرت في هذه الرسالة للزمرة D_{nh} عندما n عدد فردي , فقد وجدنا إذا كانت

$$n = p_1^{\alpha_1}.p_2^{\alpha_2}....p_m^{\alpha_m}$$

حيث إن $p_1,p_2,\ldots,p_m$ أعداد أولية مختلفة لا تساوي 2 فان :

$$AC(D_{nh}) = \bigoplus_{i=1}^{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))-1} C_2$$

$$= \bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

وكذلك وجدنا الصيغة العامة لجدول شواخص آرتن $Ar(D_{nh})$ عندما يكون n عدد فردي

Introduction :

The abelian group of all Z-valued characters of a finite group G under the operation of pointwise addition over the group of induced unit characters form all cyclic subgroups of the group G (Artin characters) , $\overline{R}$ (G)/ T(G)) form a finite abelian group which is called Artin Cokernel of the group G ,denoted by AC(G) .The problem of determining the cyclic decomposition of AC(G) seem to be untouched .In this work, G is considered to be the dihedral group D_{nh} when n is an Odd number .To do this work we must do the following steps

1- we must know the rational valued characters table of the group D_{nh} , $\equiv^*(D_{nh})$.
2- we must find Artin characters table of the group D_nh , Ar(D_nh) .
3- we must find the matrix which expresses the Artin characters of the group D_{nh} in terms of rational valued characters , $M(D_{nh}) = Ar(D_{nh}) . (\equiv^* D_{nh})^{-1}$
4- From (3) we must find the invariant factors matrix $M(D_{nh})$.
5- From (4) we can find the cyclic decomposition of $AC(D_{nh})$.

The exponent of AC(G) is called the Artin exponent of the group G, denoted by A (G) .
In 1968 T.Y Lam [15] defined AC(G) and he studied AC(G) , when G is acyclic group.
In 1970 K . Yamacchi [11]studied 2-parts of A(G) . In 1976 G.David [4] studied A(G) of arbitrary characters of cyclic subgroups .In 1996 K.Knwabuez [10] studied A(G) of p-group .
In 2000 H.R.Yassien [6] studied the cyclic decomposition of AC(G) when G is an elementary abelian group .In 2002 H.H.Abbass [5] found $\equiv^*(D_n)$.In 2006 A.S.Abed [2] found $Ar(C_n)$ when C_n is the cyclic group of order n .In this paper , we find Ar (D_{nh}) and we study $AC(D_{nh})$ of the non abelian group D_{nh} ,when is an odd number.

1.Some Basic Concepts:-

In this section , we shall give basic concepts , notations and theorems about matrix representation ,characters and Artin characters , which will be used in the next sections .

Definition (1.1):[2]
The general Linear group GL(n,F) is a multiplicative group of all non-singular $n\times n$ matrices over the field F .

Definition (1.2):[3]
A matrix representation of a group G is a homomorphism of G into GL(n,F), n is called the degree of matrix representation T . In particular ,T is called a unit representation (principal) if T(g)=1,for all $g\in G$.

Definition (1.3):[3]
The trace of an $n\times n$ matrix A is the sum of the main diagonal elements, denoted by tr(A)

Definition (1.4):[3]

Let T be a matrix representation of degree n of a finite group G over the field F .The character χ of degree n of T is the mapping $\chi : G \rightarrow F$ defined by $\chi(g)= tr(T(g))$ for all $g \in G$. In particular , the character of the principal representation ($\chi(g) = 1$, for all $g \in G$). is called the principal character .

Definition (1.5):[3]

Two elements g and h in the group G are said to be conjugate if $h= x g x^{-1}$, for some $x \in G$. The relation of conjugacy is an equivalence relation on G. The equivalence classes determined by this relation are referred to as the conjugate classes and CL_g , $g \in G$ is the conjugate class of the element g .

Definition (1.6):[3]

The centeralizer of x in G is the subgroup
$C_G(x) = \{a \in G : a x a^{-1}= x \}$.

Definition (1.7):[3]

Let H be a subgroup of G and ϕ be a character of H ,the induced character on G is given by

$$\phi^{\uparrow G}(g) = \frac{1}{|H|}\sum_{x \in G}\phi^{\circ}(xgx^{-1})$$
, where $g \in G$ and ϕ° is defined by

$$\phi^{\circ}(h) = \begin{cases} \phi(h) & if \quad h \in H \\ 0 & if \quad h \notin H \end{cases}$$

Theorem (1.8):[6]

Let H be a cyclic subgroup of G and $h_1, h_2, ..., h_m$ are chosen representatives for the m-conjugate classes of H contained in CL_g , $g \in G$, then

$$\phi^{\uparrow G}(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\phi(h_i) & \text{if } h_i \in H \cap CL_g \\ \\ 0 & if \ H \cap CL_g = \Phi \end{cases}$$

Definition (1.9):[6]

Let G be a finite group, any character induced from the principal character of cyclic subgroup of G is called Artin character of G .

Definition (1.10):[9]

Two elements of the group G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G ,this defines an equivalence relation on G .Its classes are called Γ-classes .

Proposition (1.11):[15]

The number of all distinct Artin characters on a group G is equal to the number of Γ-classes on G .

Definition (1.12):[2]

The information about Artin characters of a finite group G is displayed in a table called Artin characters table of G , denoted by Ar(G) which is $l \times l$ matrix whose columns are Γ-classes and rows the values of all Artin characters on G ,where l is the number of Γ-classes .

Definition (1.13):[3]

A rational valued character θ of G is a character whose values are in the set of integers Z,which is $\theta(g) \in Z$,for all $g \in G$.

Proposition (1.14):[12]

The number of all distinct rational valued characters of a finite group G equals to the number of Γ-classes on G .

Definition (1.15):[12]

The information about rational valued characters of a finite group G is displayed in a table called the rational valued characters table of G ,denoted by $\equiv^{*}(G)$ which is $l \times l$ matrix whose columns are Γ-classes and rows are the values of all rational valued characters of G ,where l is the number of Γ-classes

Theorem [Artin] (1.16):[9]

Every rational valued character of a finite group G can be written as a Linear combination of Artin's characters with coefficient rational numbers .

2 . The Factor Group AC(G) :-

The definition of the factor group AC (G) was introduced by T.Y Lam [15] in 1967 . The applications of the factor group AC(G) not only in the mathematics but also in physics and chemistry .

In this section we shall study AC(G) , dihedral group D_n and $\equiv^{*}(D_n)$, when n is an odd number

Definition (2.1):[15]

Let $\overline{R}$ (G) be the group of Z- valued generalized characters of G under the operation pointwise addition and T(G) is the normal subgroup of $\overline{R}$ (G) generated by Artin's characters.
The abelian factor group $\overline{R}$ (G)/T(G) is called Artin's Cokernel of G , denoted by AC(G) .

Definition (2.2):[12]

Let M be a matrix with entries in a principle domain R . A K-minor of M is the determinant of $K \times K$ Submatrix preserving row and column order .

Definition (2.3):[12]

A K-th determinant divisor of M is the greatest common divisor (g.c.d) of all K-minor ,denoted by $D_K(M)$.

Theorem (2.4):[12]

Let M be an $n \times n$ matrix with entries in a principle domain R ,then there exist matrices P and W such that

1- P and W are invertables .
2- P. M.W = D.
3- D is a diagonal matrix .
4- If we denote D_{jj} by d_j then there exists a natural number m ; $0 \le m \le n$ such that $j > m$ implies $d_j = 0$ and $j \le m$ implies $d_j \neq 0$ and $1 \le j \le m$ implies d_j / d_{j+1} .

Definition (2.5):[12]

Let M be a matrix with entries in a principal domain R , and equivalent to matrix $D=\{ d_1,d_2,..,d_m ,0,0 \dots,0\}$, Such that d_j/d_{j+1} for $1 \leq j \leq m$, D is called the invariant factor matrix of M and $d_1,d_2,..,d_m$ the invariant factors of M .

Remark (2.6) :-

According to the Artin theorem (1.16) there exists an invertible matrix M(G) with entries in the field of rational Q such that

$\equiv^*(G) = M^{-1}(G) . Ar(G)$

and this implies

$M(G) = Ar(G).(\equiv^*(G))^{-1}$

By theorem (2.4) there exists two matrices P(G) and W(G) such that

$P(G) . M(G) . W(G) = diag =\{ d_1,d_2,..,d_l \}=D(G)$, where $d_j= \pm D_j(M(G))/ D_{j-1}(M(G))$ and l is the number of Γ-classes .

Theorem (2.7):[6]

$$AC(G) = \bigoplus_{j=1}^{l} C_{d_j}$$

where $d_j= \pm D_j(M(G))/ D_{j+1}(M(G))$, l is the number of all distinct Γ-classes and C_{d_j} is cyclic subgroup of order d_j .

Theorem(2.8)): [14]

If n is an odd number such that $n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha_m}$, where $p_1, p_2, \cdots, p_m$ are distinct primes , then :

$$AC(D_n)= \bigoplus_{i=1}^{(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-1} C_2$$

Proposition (2.9):[12]

Let P be a prime number, then the rational valued characters table of cyclic group$C_{p^s}=\langle r\rangle$ is given by

$\equiv^*(C_{p^s}) =$

Γ-Classes	[1]	$[r^{P^{S-1}}]$	$[r^{P^{S-2}}]$	$[r^{P^{S-3}}]$	…	$[r^{P^2}]$	$[r^P]$	[r]
θ_1	$p^{S-1}(p-1)$	$-p^{S-1}$	0	0	…	0	0	0
θ_2	$p^{S-2}(p-1)$	$p^{S-2}(p-1)$	$-p^{S-2}$	0	…	0	0	0
θ_3	$p^{S-3}(p-1)$	$p^{S-3}(p-1)$	$p^{S-3}(p-1)$	$-p^{S-3}$	…	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
θ_{s-1}	p(p-1)	p(p-1)	p(p-1)	p(p-1)	…	p(p-1)	-p	0
θ_s	p-1	p-1	p-1	p-1	…	p-1	p-1	-1
θ_{s+1}	1	1	1	1	…	1	1	1

Remark (2.10) :- In general if $n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha m}$ where $p_1 , p_2 ,..., p_m$ are distinct primes ,then

$\equiv^* (C_n) = \equiv^* (C_{p_1}^{\alpha 1}) \otimes \equiv^* (C_{p_2}^{\alpha 2}) \otimes \cdots\cdots \otimes \equiv^* (C_{p_m}^{\alpha m})$ where $\otimes$ is the tensor product .

Definition (2.12):[9]

The dihedral group D_n is a certain non-abelian group of order 2n ,it is usually thought agroup of transformations of Euclidean plane of regular n-polygon consisting of rotation (r^k) (about the origin)with angle 2πk/n and reflections sr^k (a cross lines through the origin).

In general it can be written as
$D_n=\{S^i r^k : 0 \le k \le n-1, 0 \le j \le 1\}$,where $r^n=1, S^2=1$, $S r^k S = r^{-k}$.
The cyclic group of order n , $C_n=<r>$ is a normal subgroup of D_n.

Definition (2.12) [9]

The group D_{nh} is the direct product group $\mathbf{D_n \times C_2}$,where $\mathbf{C_2}$ is a cyclic group of order 2 consisting of elements $\{1, r'\}$with $(r')^2=1$. It is of order 4n .

Proposition *(2.13): [5]*

The rational valued characters table of D_n when n is an odd number is given as follows:

$\equiv^*(D_n)=$

	Γ- classes of C_n	[S]
θ_1	$\equiv^*(C_n)$	0
⋮		⋮
θ_{S-1}		0
θ_S	1 1 1 … 1 1	1
θ_{S+1}	1 1 1 … 1 1	-1

Table (2.6)

Where S is the number of Γ- classes of C_n .

Theorem(2.14) : [13]

The rational valued characters table of the group D_{nh} when n is an odd number is given as follows:

$\equiv^*(D_{nh}) = \equiv^*(D_n) \otimes \equiv^*(C_2$

Theorem (2.15):[2]

Let p be a prime number, then Ar (C_{p^s})=

Γ-Classes	[1]	$[r^{p^{s-1}}]$	$[r^{p^{s-2}}]$	$[r^{p^{s-3}}]$	…	[r]
φ'_1	p^S	0	0	0	…	0
φ'_2	p^{S-1}	p^{S-1}	0	0	…	0
φ'_3	p^{S-2}	p^{S-2}	p^{S-2}	0	…	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
φ'_s	p	p	p	p	…	0
φ'_{s+1}	1	1	1	1	…	1

Remark (2.16) :-

Let n be any positive integer and

$n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha m}$ where $p_1, p_2, \dots, p_m$ are distinct primes ,then

Ar(Cn)= $Ar(C_{p1^{\alpha 1}}) \otimes Ar(C_{p2^{\alpha 2}}) \otimes \ldots \otimes Ar(C_{pm^{\alpha m}})$

Where $\otimes$ is the tensor product .

Proposition (2.17):[13]

If P be a prime number and S is a positive integer ,then

$$M\left(C_{p^s}\right)=\begin{bmatrix}1&1&1&\cdots&1&1\\0&1&1&\cdots&1&1\\0&0&1&\cdots&1&1\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&1&1\\0&0&0&\cdots&0&1\end{bmatrix},\ P\left(C_{p^s}\right)=\begin{bmatrix}1&-1&0&0&\cdots&0&0\\0&1&-1&0&\cdots&0&0\\0&0&1&-1&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&0&\cdots&1&-1\\0&0&0&0&\cdots&0&1\end{bmatrix}.$$

And $W(C_{p^s}) = I_{S+1}$ where I_{S+1} is the identity matrix .

Remark (2.18) :

1. In general if $n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha_m}$ such that $p_1, p_2, \cdots, p_m$ are distinct primes and α_i any positive integers for all i = 1, 2, … , m ; then $C_n = C_{p_1^{\alpha 1}} \times C_{p_2^{\alpha 2}} \times \ldots \times C_{p_m^{\alpha m}}$ .and

$M(C_n) = M(C_{p_1^{\alpha 1}}) \otimes M(C_{p_2^{\alpha 2}}) \otimes \ldots \otimes M(C_{p_m^{\alpha m}})$.

So, we can write $M(C_n)$ as:

$$M(C_n) = \begin{bmatrix} & & & & & 1 \\ & & R(C_n) & & & 1 \\ & & & & & \vdots \\ & & & & & 1 \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix}$$

Where $R(C_n)$ is the matrix obtained by omitting the last row {0, 0,...,0, 1} and the last column {1,1,…,1} from the tensor product, $M(C_{p_1^{\alpha 1}}) \otimes M(C_{p_2^{\alpha 2}}) \otimes \cdots . \otimes M(C_{p_m^{\alpha m}})$. $M(C_n)$ is, $(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_m+1) \times (\alpha_1+1)(\alpha_2+1)\ldots(\alpha_m+1)$ square matrix.

2.

α- $P(C_n) = P(C_{p_1^{\alpha 1}}) \otimes P(C_{p_2^{\alpha 2}}) \otimes \ldots \otimes P(C_{p_m^{\alpha m}})$.

β- $W(C_n) = W(C_{p_1^{\alpha 1}}) \otimes W(C_{p_2^{\alpha 2}}) \otimes \ldots \otimes W(C_{p_m^{\alpha m}})$.

3. The Main Results

In this section we give the general form of Artin characters table of the group D_{nh} and the cyclic decomposition of the factor group $AC(D_{nh})$ when n is an odd number .

Theorem(3.1):

The Artin characters table of the group D_{nh} when n is an odd number is given as follows :

$Ar(D_{nh})$ =

Γ-Classes	$[1,1']$	$[1,r']$	Γ-Classes of $C_n \times C_2$					$[S,1]$	$[S,r']$
$\lvert CL_\alpha \rvert$	1	1	2	2		$\cdots$	2	n	n
$\lvert C_{D_{nh}}(CL_\alpha)\rvert$	4n	4n	2n	2n		$\cdots$	2n	4	4
$\Phi_{(1,1)}$	$2Ar(C_n) \otimes Ar(C_2)$							0	0
$\Phi_{(1,2)}$									
$\vdots$								$\vdots$	$\vdots$
$\Phi_{(l,1)}$									
$\Phi_{(l,2)}$								0	0
$\Phi_{(l+1,1)}$	2n	0	0		$\cdots$		0	2	0
$\Phi_{(l+1,2)}$	2n	0	0		$\cdots$		0	0	2

Table(3.1)

where l is the number of Γ-classes of C_n and C_2=< r' >={ $1'$, r' }.

Proof:-

By theorem(2.15)

Ar (C_2) =

Γ- classes	$[1']$	$[r']$
$\lvert CL_\alpha \rvert$	1	1
$\lvert C_2(CL_\alpha)\rvert$	2	2
φ'_1	2	0
φ'_2	1	1

Table (3.2)

Each cyclic subgroup of the group D_{nh} is either a cyclic subgroup of $C_n \times C_2$ or $<(S,r')>$ or$<(S,1')>$.

If H is a cyclic subgroup of $C_n \times C_2$, then :

H=$H_i \times <1'>$ or $H_i \times <r'> = H_i \times C_2$ for all $1 \le i \le l$ where l is the number of Γ- classes of C_n

If H= $H_i \times <1'>$ and g $\in D_{nh}$

If g $\notin$ H then by theorem(1.8)

$\Phi_{(1,i)}(g)=0$ for all $0 \le i \le l$ [since H $\cap$ CL(g) = ϕ]

If g $\in$ H then either g=$(1,1')$ or $\exists S, 0 < S < n$ such that g=$(r^S,1')$

When g=$(1,1')$, then :

$\Phi_{(I,,1)}(g) = \dfrac{|C_{D_{nh}}(g)|}{|C_H(g)|}\varphi(g)$ [since H$\cap$CL(g)={$(1,1')$}] where φ is the principle character (i.e

ϕ(g)=1 $\forall$ g$\in D_{nh}$)

$$= \frac{4n}{|H_i|\cdot|<1'>|}\cdot 1 = \frac{4n}{|H_i|} = 2\cdot\frac{n}{|H_i|}\cdot 1\cdot 2 = 2\frac{|C_{C_n}(1)|}{|C_{H_i}(1)|}\cdot\varphi(1)\cdot\varphi'(1') = 2\cdot\varphi_i(1)\cdot\varphi'(1')$$

For g=$(r^S,1')$ then

$\Phi_{(i,1)}(g) = \dfrac{|C_{D_{nh}}(g)|}{|C_H(g)|}\sum_{i=1}^{2}\varphi'(g)$ [since H $\cap$ CL(g) = {$(r^S,1'),(r^{-S},1')$}]

$$= \frac{2n}{|H_i \times <1'>|}\cdot(1+1)$$

$$= \frac{2n}{|H_i|\cdot|<r'>|}\cdot 2 = \frac{2n}{|H_i|}\cdot 2$$

$$= 2\cdot\frac{n}{|H_i(r^S)|}\cdot 1\cdot 2 = 2\frac{|C_{C_n}(r^S)|}{|C_{H_i}(r^S)|}\cdot\varphi(r^S)\cdot\varphi'(1') = 2\cdot\varphi_i(r^S)\cdot\varphi_1'(1')$$

If H= $H_i \times <r'> = H_i \times C_2$

let g $\in D_{nh}$

if g $\notin$ H then

$\Phi_{(i,2)}(g)=0$ for all $1 \le i \le l$ [since H $\cap$ CL(g) = ϕ]

If g $\in$ H then either g=$(1,1')$ or g=$(1,r')$ or $\exists S, 0 < S < n$ such that g=(r^S,r')

When g=$(1,1')$

$\Phi_{(i,2)}$(g) = $\dfrac{|C_{D_{nh}}(g)|}{|C_H(g)|}\varphi(g)$ [since H $\cap$ CL(g) = {$(1,1')$}]

$$=\frac{4n}{|H_i\times C_2|}=\frac{4n}{2|H_i|}=\frac{2n}{|H_i|}=2\,\frac{|C_{C_n}(1)|}{|C_{H_i}(1)|}\varphi(1)1=2\cdot\varphi_i(1)\cdot\varphi_2'(1)$$

For g=$(1,r')$

$\Phi_{(i,2)}$(g) = $\dfrac{|C_{D_{nh}}(g)|}{|C_H(g)|}\varphi(g)$ [since H $\cap$ CL(g) = {$(1,r')$}]

$$=\frac{4n}{|H_i\times C_2|}$$

$$=\frac{4n}{2|H_i|}=\frac{2n}{|H_i|}=2\,\frac{|C_{C_n}(1)|}{|C_{H_i}(1)|}1\,1=2\cdot\varphi_i(1)\cdot\varphi_2'(r')$$

,and if g=(r^S,r') then

$\Phi_{(i,2)}$(g) = $\dfrac{|C_{D_{nh}}(g)|}{|C_H(g)|}\sum_{i=1}^{2}\varphi'(g)$ [since H $\cap$ CL(g) = {$(r^S,r'),(r^{-S},r')$}]

$$=\frac{2n}{|H_i\times C_2|}(1+1)=\frac{4n}{2|H_i|}=\frac{2n}{|H_i|}=2\,\frac{|C_{C_n}(r^s)|}{|C_{H_i}(r^s)|}\varphi(r^s)\varphi_2'(r')=2\cdot\varphi_i(r^s)\varphi_2'(r').$$

If H=<$(S,1')$> = { $(1,1'),(S,1')$ }

$\Phi_{(l+1,1)}$ $((1,1'))$=$\dfrac{|C_{D_{nh}}((1,1'))|}{|C_H((1,1'))|}\varphi(g)=\dfrac{4n}{2}=2n$

$\Phi_{(l+1,1)}$ $((S,1'))$=$\dfrac{|C_{D_{nh}}((S,1'))|}{|C_H((S,1'))|}\varphi(g)$ [since H $\cap$ CL($(S,1')$) = {$(S,1')$}]

$$=\frac{4}{2}=2$$

otherwise

$\Phi_{(l+1,1)}$ (g)= 0 for all g $\in D_{nh}$,[since g $\notin$ H]

If H=<(S,r')> = { $(1,1'),(S,r')$ }

$\Phi_{(l+1,2)}\left((1,1')\right)=\dfrac{\left|C_{D_{nh}}((1,1'))\right|}{\left|C_H((1,1'))\right|}\varphi\left((1,1')\right)$ [since H $\cap$ CL$((1,1'))$ = $\{(1,1')\}$]

$$=\frac{4n}{2}\cdot 1=2n$$

$\Phi_{(l+1,2)}\left((S,r')\right)=\dfrac{\left|C_{D_{nh}}((S,r'))\right|}{\left|C_H((S,r'))\right|}\varphi\left((S,r')\right)$

$$=\frac{4}{2}\cdot 1=2$$

Otherwise $\Phi_{(l+1,2)}(g)=0$ for all g $\in D_{nh}$ since H $\cap$ CL(g) = ϕ ■

<u>Example (3.2)</u>:

To find Ar(D_{2197h}) by using theorem(3.1)

Ar(D_{2197h})= Ar(D_{13^3h})= 2Ar(C_{13^3})$\otimes$Ar(C_2) ,by using theorem(2.15) we get

Ar (C_{13^3}) =

Γ- classes	[1]	$[r^{13^2}]$	$[r^{13}]$	$[r]$
$\lvert CL_\alpha\rvert$	1	1	1	1
$\lvert C_{C_{13^3}}(CL_\alpha)\rvert$	13^3	13^3	13^3	13^3
ϕ_1	13^3	0	0	0
ϕ_2	13^2	13^2	0	0
ϕ_3	13	13	13	0
ϕ_4	1	1	1	1

Table (3.3)

Ar (C_2) =

Γ- classes	$[1']$	$[r']$
$\lvert CL_\alpha\rvert$	1	1
$\lvert C_2(CL_\alpha)\rvert$	2	2
ϕ_1	2	0
ϕ^2	1	1

Table (3.4)

So we get $Ar(D_{2197h})$

Γ- classes	$[1\ ,1']$	$[1\ ,r']$	$[r^{13^2},1']$	$[r^{13^2},r']$	$[r^{13}\ ,1']$	$[r^{13}\ ,r']$	$[r\ ,1']$	$[r\ ,r']$	$[S\ \ ,1']$	$[S\ \ ,r']$
$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	2	2197	2197
$\lvert C_{C_3}(CL_\alpha) \rvert$	8788	8788	4394	4394	4394	4394	4394	4394	4	4
$\acute{\phi}_{(1,1)}$	8788	0	0	0	0	0	0	0	0	0
$\acute{\phi}_{(1,2)}$	4394	4394	0	0	0	0	0	0	0	0
$\acute{\phi}_{(2,1)}$	676	0	676	0	0	0	0	0	0	0
$\acute{\phi}_{(2,2)}$	338	338	338	338	0	0	0	0	0	0
$\acute{\phi}_{(3,1)}$	52	0	52	0	52	0	0	0	0	0
$\acute{\phi}_{(3,2)}$	26	26	26	26	26	26	0	0	0	0
$\acute{\phi}_{(4,1)}$	4	0	4	0	4	0	4	0	0	0
$\acute{\phi}_{(4,2)}$	2	2	2	2	2	2	2	2	0	0
$\acute{\phi}_{(5,1)}$	4394	0	0	0	0	0	0	0	2	0
$\acute{\phi}_{(5,2)}$	4394	0	0	0	0	0	0	0	0	2

Table(3.5)

Proposition (3.3):

If $n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha_m}$ where $p_1, p_2, \cdots, p_m$ are distinct primes and $p_i \neq 2$ for all $1 \le i \le m$ and α_i any positive integers, then :

$$M(D_{nh}) = \left[\begin{array}{cccccccc} & & & & 1 & 1 & 1 & 1 \\ & & & & 1 & 0 & 1 & 0 \\ & & & & 1 & 1 & 1 & 1 \\ & 2R(C_n)\times M\left(C_2\right) & & & & & & \\ & & & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & \cdots & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & \cdots & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

which is $2\left[\left(\alpha_1+1\right)\cdot\left(\alpha_2+1\right)\cdots\left(\alpha_m+1\right)+1\right]\times 2\left[\left(\alpha_1+1\right)\cdot\left(\alpha_2+1\right)\cdots\left(\alpha_m+1\right)+1\right]$ square matrix .

Proof:-

By theorem(3.1) we obtain the Artin characters table Ar(D_{nh}) and from theorem(2.14) we find the rational valued characters table $\overset{*}{\equiv}$(D_{nh}) .Thus by the definition of M(G) we can find the matrix M(D_{nh}) :

$$M(D_{nh})= Ar(D_{nh}).\ (\overset{*}{\equiv}(\ D_{nh}))^{-1} = \begin{bmatrix} & & & & 1&1&1&1\\ & & & & 1&0&1&0\\ & & & & 1&1&1&1\\ & 2R(C_n)\times M(C_2) & & & \vdots&\vdots&\vdots&\vdots\\ & & & & & & & \\ & & & & 1&0&1&0\\ 0&0&\cdots&0&1&1&1&1\\ 0&0&\cdots&0&1&0&1&0\\ 1&1&\cdots&1&1&1&0&0\\ 1&1&\cdots&1&1&0&0&1 \end{bmatrix}$$

Which is $2\left[(\alpha_1+1)\cdot(\alpha_2+1)\cdots+1\right]\times 2\left[(\alpha_1+1)\cdot(\alpha_2+1)\cdots+1\right]$ square matrix .

Example (3,4) :

To find $M(D_{1331\ h})= M(D_{11^3\ h})$ we must find

$$R(C_{11^3}) = \begin{bmatrix} 1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1 \end{bmatrix}$$

Which is 3×3 square matrix .

and $M(C_2)=\begin{bmatrix} 1 & 1\\ 0 & 1 \end{bmatrix}$

Hence

$$M(D_{11}{}^{3}{}_{h})=\left[\begin{array}{cccccccccc}
 & 2R(C11^{3})\otimes M(C_{2}) & & & & & 1 & 1 & 1 & 1 \\
 & & & & & & 1 & 0 & 1 & 0 \\
 & & & & & & 1 & 1 & 1 & 1 \\
 & & & & & & 1 & 0 & 1 & 0 \\
 & & & & & & 1 & 1 & 1 & 1 \\
 & & & & & & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1
\end{array}\right]=\begin{bmatrix}
2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\
0 & 2 & 0 & 2 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 0 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 2 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 2 & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1
\end{bmatrix}$$

Which is 10×10 square matrix

Proposition (3.5):

If $n = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots p_m^{\alpha_m}$ such that $p_1, p_2, \ldots, p_m$ are distinct primes different from 2 and α_i any positive integers, then

$$P(D_{nh}) = \begin{bmatrix} & & & & & & 0 & 0 \\ & & & & & & 0 & 0 \\ & & P(C_n) \otimes P(C_2) & & & & & \\ & & & & & & \vdots & \vdots \\ & & & & & & 0 & 0 \\ & & & & & & -1 & -1 \\ & & & & & & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$W(D_{nh}) = \begin{bmatrix} & & & & & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ & & I_k & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \\ -1 & -1 & \cdots & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & \cdots & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Where $k = 2[(\alpha_1+1)\cdot(\alpha_2+1)\cdot(\alpha_3+1)\cdots(\alpha_m+1)-1] \times 2[(\alpha_1+1)\cdot(\alpha_2+1)\cdot(\alpha_3+1)\cdots(\alpha_m+1)-1]$

They are $2\left[(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)+1\right] \times 2\left[(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)+1\right]$ square matrix .

Proof :

By using theorem(2.4) and taking the form of $M(D_{nh})$ from proposition(3.3) and the above forms of $P(D_{nh})$ and $W(D_{nh})$ then we
Have

$$P(D_{nh}).\ M(D_{nh}).\ W(D_{nh})=\begin{bmatrix} 2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \end{bmatrix}$$

$=\text{diag}\{2,2,2,\ldots..,-2,1,1,1\}=D(D_{nh})$

Which is $2\left[(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)+1\right]\times 2\left[(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)+1\right]$ square matrix .

Example (3.6):

Consider the group D_{27869h} , then we can find the matrices $P(D_{27869h})$, $W(D_{27869h})$ immediately by using proposition(3.5) and by proposition(3.3) we find $M(D_{27869h})$.

To find $P(D_{27869h})= P(D_{31^2.29h})$

By remark(2.18-2) $P(C_{31^2.29}) = P(C_{31^2})\otimes P(C_{29})$

Then $P(C_{31^2})\otimes P(C_{29})\otimes P(C_2)=\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\otimes\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\otimes\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By proposition (3.5)

$$P(D_{27869h})=\left[\begin{array}{cccccccccccccc}
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
\multicolumn{12}{c}{P(C_{31^2\cdot 29})\otimes P(C_2)} & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & -1 & -1 \\
 & & & & & & & & & & & & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]$$

Then $P(D_{27869h}).\ M(D_{27869h}).\ W(D_{27869h})=$

$$\begin{bmatrix}
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \bullet \begin{bmatrix}
2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 2 & 0 & 2 & 0 & 2 & 0 \\
0 & 0 & 2 & 2 & 0 & 0 & 2 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}$$

$$\left.\begin{matrix}2&2&2&1&1&1&1\\2&0&2&1&0&1&0\\2&0&0&1&1&1&1\\2&0&0&1&0&1&0\\2&2&2&1&1&1&1\\2&0&2&1&0&1&0\\2&0&0&1&1&1&1\\2&0&0&1&0&1&0\\0&2&2&1&1&1&1\\0&0&2&1&0&1&0\\0&0&0&1&1&1&1\\0&0&0&1&0&1&0\\1&1&1&1&1&0&0\\1&1&1&1&0&0&1\end{matrix}\right] \bullet \begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0&0&0\\-1&-1&-1&-1&-1&-1&-1&-1&-1&-1&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&-1&0&1&0\\1&1&1&1&1&1&1&1&1&1&-1&1&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0&1\end{bmatrix} =$$

$$\begin{bmatrix}2&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&2&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&2&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&2&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&2&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&2&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&2&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&2&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&2&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&2&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&-2&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&1\end{bmatrix}$$

=diag{2,2,2,2,2,2,2,2,2,2,-2,1,1,1}

Which is $2\left[(2+1)\cdot(1+1)+1\right]\times 2\left[(2+1)\cdot(1+1)+1\right]=14\times 14$ square matrix .

Theorem (3.7):

If $n = p_1^{\alpha 1}\cdot p_2^{\alpha 2}\cdots p_m^{\alpha_m}$ where $p_1 , p_2 ,\cdots, p_m$ are distinct primes , $p_i \neq 2$ and α_i any positive integers for all i ,$1\le i \le m$, then the cyclic decomposition AC(D_{nh}) is :

$$AC(D_{nh})=\bigoplus_{i=1}^{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))-1} C_2$$

$$=\bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

Proof :-

From proposition (3.5) we have

$P(D_{nh})\,.M(D_{nh})\,.W(D_{nh})=\mathrm{diag}\{2,2,2,\ldots,-2,1,1,1\}=\{d_1,d_2,\ldots,$

$d_{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1)-1)}, d_{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))-1}, d_{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))}$,

$d_{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))+1}\ d_{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))+2}\}$.

By theorem (2.7) we get

$$AC(D_{nh})=\bigoplus_{i=1}^{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))-1} C_{d_i}$$

$$=\bigoplus_{i=1}^{2((\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_m+1))-1} C_2$$

From theorem(2.8) we have :

$$AC(D_{nh})=\bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

Example (3.8):

To find the cyclic decomposition of the groups $AC(D_{29791h})$, $AC(D_{25054231h})$ and $AC(D_{576247313h})$.

by using above theorem :

1- $AC(D_{29791h}) = AC(D_{31^3 h}) = \bigoplus_{i=1}^{2(3+1)-1} C_2 = \bigoplus_{i=1}^{7} C_2 = \bigoplus_{i=1}^{2} AC(D_{31^3}) \oplus C_2$.

$$AC(D_{25054231h}) = AC(D_{31^3.29^2\ h}) = \bigoplus_{i=1}^{2((3+1)\cdot(2+1))-1} C_2 = \bigoplus_{i=1}^{23} C_2$$

$$= \bigoplus_{i=1}^{2} AC(D_{31^3.29^2}) \oplus C_2 .$$

2- $$AC(D_{576247313h}) = AC(D_{31^3.29^2.23\ h}) = \bigoplus_{i=1}^{2((3+1)\cdot(2+1)\cdot(1+1))-1} C_2 = \bigoplus_{i=1}^{47} C_2$$

$$= \bigoplus_{i=1}^{2} AC(D_{31^3.29^2.23}) \oplus C_2 .$$

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