

Studying the effect of conductiong bodies of revolution (BoR) shape on computing radar cross section (RCS)

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Abstract:

Integral equation (IE) formulation is the most popular method to treating the electromagnetic (EM) scattering and radiation problems together with method of moments (MoM) as a numerical solution.

In this paper an EM scattering problem from the complex body of revolution (BoR) is study to investigate the effect of the BoR shape on the radar cross section (RCS) for two different models, the results was compared and discussed.

1- **Introduction:** Through the past forty years, when the level of target radar cross section RCS is necessary to define the exploring efficiency of the first radar system , the numerical methods has been developed to treat the electromagnetic of scattering and radiation problems from surfaces and radar cross section measurement^[1,2]. RCS measurement of military pad (e.g. submarine and aircraft carrier) was of great importance in that period which should be taken into account in the basic design of the total antenna, therefore the prediction of RCS is the important part in the process of designing antenna of new system^[3] by the virtue of the speed progress in the measurement and radar technologies in addition to the signal processing technology and digital computer^[4,5].

The electromagnetic scattering from three-dimensions perfect conducting bodies of as a subject of great importance to several researchers for many years, where it leads to several practical applications in addition to the improvement of great number of analytical methods and typical technique to compute the electromagnetic scattering for different bodies^[6,7]. The surface integral equation is considered as the most suitable method for the numerical solution because it represents the general method through the shorthand of three-dimensional problem to two dimensions by the arrangement of the problem in respect of unknown functions defined according to the body surface and not in respect of unknown volumetric function. If the body is conductor, the scattering problem is formulated according to current J_s generated on the conductor body surface S ^[8]. To solve such problems, method of moment MoM is used because of its superiority in giving efficient results in addition of its ability to contain all radiation conditions for orderly and unorderedly bodies ^[9]. Furthermore, Galerkin method is used to be the most

suitable for choosing the weight function of the solution especially after the development of the high speed computer of great precision^[10].

1-1 Scattering Problems:

The electromagnetic problems are of several types like scattering problems and radiation problems. The simplest definition of scattering problems is if the (EM) wave is incident $\bar{E}^i$ on some body (perfect conductor or dielectric), this wave will generate surface current density (electrical $\bar{J}_s$ for perfect conductor, electrical $\bar{J}_s$ and magnetic $\bar{M}_s$ for dielectric). If a magnetic wave is incident on perfect conducting body, the boundary condition is:

$$\hat{n} \times \bar{E}_{\tan}^i = -\hat{n} \times \bar{E}_{\tan}^s \quad \dots\dots\dots (1)$$

Where $\bar{E}^i$ denoted the known impressed field and $\bar{E}^s$ the scattered field due to currents on the body surface, while the subscript (tan) denotes tangential component on S.

The scattered field can be expressed in terms of a magnetic vector potential $\bar{A}$ and electric scalar potential ϕ as:

$$\bar{E}^s(\bar{r}) = -j\omega\bar{A}(\bar{r}) - \nabla\phi(\bar{r}) \quad \dots\dots\dots (2)$$

Fig.(1) shows a body of revolution, generated by revolving a planar curve, around z-axis with it's coordinate explaining source point and field point, $\hat{t}$ and $\hat{\phi}$ represents vector units of the tangential and azimuthal components which satisfy that ($\hat{t} \times \hat{\phi} = \hat{n}$). After compensating (2) in (1), we get:

$$\hat{n} \times \bar{E}_{\tan}^i = \hat{n} \times \left[j\omega\mu \int_s \bar{J}(\bar{r})G(\bar{r},\bar{r}')ds - \frac{\nabla}{j\omega\epsilon} \int_s \nabla' \cdot \bar{J}(\bar{r})G(\bar{r},\bar{r}')ds \right]_{\tan} \quad \dots\dots\dots (3)$$

Where ω, μ, ε are the angular frequency, magnetic permeability and permittivity. $G(\vec{r}, \vec{r}')$ is Green's Function for free-space which can be defined as^[7, 11]:

$$G(\vec{r}, \vec{r}') = \frac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|} \quad \dots\dots\dots (4)$$

Equation (3) can now be stated succinctly as:

$$L(\vec{J}) = \vec{E}_{\tan}^i \quad \dots\dots\dots (5)$$

Where L is the integro-differential operator. Hence,

$$L(\vec{J}) = j\omega\mu \int_s \vec{J} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|} ds - \frac{\nabla}{j\omega\varepsilon} \int_s \nabla' \cdot \vec{J} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|} ds \quad \dots\dots\dots (6)$$

The solution of this equation is obtained numerically by using method of moments, which is closely related to *Galerkin's* method^[12] with procedure as follow.

A set of expansion functions J_j is next defined, and the current on S

approximated by:

$$\vec{J} = \sum_j I_j \vec{J}_j \quad \dots\dots\dots (7)$$

where I_j are constant to be determined. From equations (5) and (7), and by using a set of testing functions W_i is defined, and the inner product formula can be written as:

$$\sum_j I_j \langle \vec{W}_i, L(\vec{J}_j) \rangle = \langle \vec{W}_i, \vec{E}_{\tan}^i \rangle \quad (i=1,2,3,\dots) \quad \dots\dots\dots (8)$$

we now define the generalized network matrices:

$$[\xi] = [\langle \vec{W}_i, L(\vec{J}_j) \rangle] \quad \dots\dots\dots (9)$$

and:

$$[\mathfrak{R}] = \left[\left\langle \bar{W}_i, \bar{E}_{\tan}^i \right\rangle \right] \dots\dots\dots (10)$$

$$[I] = [I_j] \dots\dots\dots (11)$$

and rewrite eq. (8) as:

$$[\xi][I] = [\mathfrak{R}] \dots\dots\dots (12)$$

To determine the coefficient I_j of the current expansion from $[I] = [Y][\mathfrak{R}]$, where $[Y] = [\xi]^{-1}$ is the generalized admittance matrix.

1-2 Excitation Matrix $[\mathfrak{R}]$:

Consider the linear measurement of the field from the current $\bar{J}_s$ on a body S can be expressed as linear functional^[12], that is

$$measurement = \iint_S \bar{E}^r \cdot \bar{J} ds \dots\dots\dots (13)$$

where $\bar{E}^r$ is a known function. For a moment method solution, the current is given by the eq.(7) and (13) is reduced to:

$$measurement = [R][I] \dots\dots\dots (14)$$

Now one can rewrite in partitioned form as :

$$measurement = \begin{bmatrix} R_n^t & R_n^\phi \end{bmatrix} \begin{bmatrix} Y_n^{tt} \\ Y_n^{\phi t} \end{bmatrix} \begin{bmatrix} Y_n^{t\phi} \\ Y_n^{\phi\phi} \end{bmatrix} \begin{bmatrix} \mathfrak{R}_n^t \\ \mathfrak{R}_n^\phi \end{bmatrix} \dots\dots\dots (15)$$

where the sub matrices $[Y]$ are obtained after entire $[\xi]$ matrix is inverted. It has been shown that the radiation field from currents $\bar{J}$ on S is given by:

$$\bar{E} \cdot \hat{u} = -\frac{j\omega\mu}{4\pi r} e^{-jk r} [R][I]$$

..... (16)

Where $\hat{u}$ is unit vector which specifies the incident wave polarization, and $\bar{E}^r$ can be defined as a plane wave as the following:

$$\bar{E}^r = \bar{E}_\theta^r + \bar{E}_\phi^r = \hat{u}_\theta^r e^{-j\bar{k} \cdot \bar{r}} + \hat{u}_\phi^r e^{-j\bar{k} \cdot \bar{r}}$$

..... (17)

The values of $(R_n^{t\theta})_i, (R_n^{\phi\theta})_i, (R_n^{t\phi})_i, (R_n^{\phi\phi})_i$ can be found out from equation (10) as:

$$(R_n^{\alpha\theta})_i = \iint_S \bar{J}_{ni}^\alpha \cdot \bar{E}_\theta^r ds \quad \theta - \text{polarized}$$

..... (18)

$$(R_n^{\alpha\phi})_i = \iint_S \bar{J}_{ni}^\alpha \cdot \bar{E}_\phi^r ds \quad \phi - \text{polarized}$$

..... (19)

Where α is t or ϕ . It is important to mention that $(R_n^{t\theta})_i, (R_n^{\phi\phi})_i$ are even function in n while $(R_n^{\phi\theta})_i, (R_n^{t\phi})_i$ are odd in n . It should be noted here the excitation matrix $[\mathfrak{R}]$ differs from the measurement matrix $[R]$ only by the sign of n . If the incident wave is supposed to be plane wave in the direction of axis of symmetry ($\theta = 0^\circ$ or 180°), it is possible to write the following relation as:

$$(\mathfrak{R}_n^{pq})_i = (R_{-n}^{pq})_i$$

..... (20)

Where pq represents $t\theta, t\phi, \phi\theta, \phi\phi$.

1-4 Radar Cross Section (RCS):

Radar scattering problem consists of a plane wave incident on a scattering body, plus measurement of far zone scattered field. Fig.(2) illustrates geometry of the problem for conducting bodies of revolution.

These components for scattering and incident field are related by the scattering matrix $[S]$ of the body according to:

$$\begin{bmatrix} \bar{E}_\theta^s \\ \bar{E}_\phi^s \end{bmatrix} = \frac{e^{-jkr}}{r} \begin{bmatrix} S^{\theta\theta} & S^{\theta\phi} \\ S^{\phi\theta} & S^{\phi\phi} \end{bmatrix} \begin{bmatrix} \bar{E}_\theta^i \\ \bar{E}_\phi^i \end{bmatrix}$$

..... (21)

The element of $[S]$ can be expressed as a summation over the modal components.

$$S^{pq} = \sum_n S_n^{pq}$$

..... (22)

where pq denotes $\theta\theta, \theta\phi, \phi\theta, \phi\phi$ or the field is given by (16) and hence.

$$\begin{bmatrix} \bar{E}_\theta^s \\ \bar{E}_\phi^s \end{bmatrix} = -\frac{j\omega\mu}{4\pi r} e^{-jkr} \begin{bmatrix} (R_n^{t\theta})_i & (R_n^{\phi\theta})_i \\ (R_n^{t\phi})_i & (R_n^{\phi\phi})_i \end{bmatrix} \begin{bmatrix} (I_n^t)_i \\ (I_n^\phi)_i \end{bmatrix}$$

..... (23)

From which it is possible to obtain.

$$S_n^{pq} = -\frac{j\omega\mu}{4\pi} \begin{bmatrix} (R_n^{tp})_i & (R_n^{\phi p})_i \\ (Y_n^{\phi t})_{ij} & (Y_n^{\phi\phi})_{ij} \end{bmatrix} \begin{bmatrix} (Y_n^{tt})_{ij} & (Y_n^{t\phi})_{ij} \\ (Y_n^{\phi t})_{ij} & (Y_n^{\phi\phi})_{ij} \end{bmatrix} \begin{bmatrix} (\mathfrak{R}_n^{tq})_i \\ (\mathfrak{R}_n^{\phi q})_i \end{bmatrix}$$

..... (24)

where p represents the received polarization and q is the incident polarization. The elements of $(\mathfrak{R}_n)$ are the same of (R_n) formulas but $(n \rightarrow -n)$ and $(\theta_r \rightarrow \theta_t)$. Radar scattering data are often presented in terms of radar cross section, defined as:

$$\sigma^{pq} = 4\pi r^2 \left| \frac{\overline{E}_p^s}{\overline{E}_q^i} \right|^2$$

..... (25)

As it is mentioned previously, σ depends on the received polarization p and incident polarization q , so it is possible to write equation (21) as:

$$\frac{\overline{E}_p^s}{\overline{E}_q^i} = \frac{e^{-jkr}}{r} S^{pq}$$

It follows from equation (21) and (25) that for a given polarization:

$$\sigma^{pq} = 4\pi |S^{pq}|^2$$

..... (26)

which is represented the basic equation for computing RCS. The elements of scattering matrix of equation (21) in the principle planes $\phi_r = 0^\circ$ and $\phi_r = \pi/2$.

$$S^{\theta\theta} = 2S_1^{\theta\theta} \cos(\phi_r)$$

$$S^{\phi\theta} = 2jS_1^{\phi\theta} \sin(\phi_r)$$

..... (27)

where $S_1^{\phi\theta}, S_1^{\theta\theta}$ are $n=1$ modal solutions evaluated at $(\phi_r = 0^\circ)^{[12]}$.

2- Numerical Results:

To prove the mathematical and practical treatment correctness which has been taken into account in this paper, the radar cross section (RCS) is computed for a simple body of revolution, sphere; where this example is important to satisfying the mathematical analysis correctness for its revolution. Now we take the *Yehuda et. al.*^[8] example of perfect conductor sphere of radius 0.2λ which is computed by method of moment (point-matching). When comparing

Yehuda et. al. results with our where we use method of moment (*Galerkin*), we find great correspondence between the two as shown in fig.(3).

To know the effect of body volume on (RCS), we prefer to take the same previous example (perfect conductor sphere) but with larger radius 1.0λ so we find great correspondence in result correctness with the *Yehuda et. al.* results as shown in fig.(4).

Now we take the complex geometrical shape and an example to study mixing process between many uniform bodies, as it in the studied examples *Yehuda et. al. and Mogus, G. Andreassen*^[1] that is composed sphere of two parts (radius 0.2λ) with cylinder (length 1.1λ) which compose capsule shape fig.(5).

To study the incensement of complexity elements in geometrical shapes for (BoR) *Hashim, A.*^[13] example is taken as shown in fig.(6) .

2-1 Applications:

What is introduced is respected as clear and confirmed proof of programmatic mathematical analytical process validity that is used in this paper. This mentioned method is preferred to apply on two models resemble two military rocket approximately; one of them is naval and the other is air. Because of the military secrecy of certain data about this rocket and the complexity to obtain them, it is impossible to know the rocket real dimensions, their radar impress or their real physical elements. We depend her on the possible approximation of rocket shape, where the total length of any one of them is 1.5λ , it is possible to compare our results with the results of previous studies of the same length and know the total basic impact of the changes in the samples shape as the following:

2-1-1 First Sample

This sample is studied as approximation for rocket shape as shown in fig.(8-a), which is approximated to fig. (8-b) to become body of revolution.

$$\begin{array}{llll} R=1.5 \lambda & A0= 1.500 * a & L1= 6.000 * L & b1= 1.500 * a \\ & A1= 0.500 * a & L2= 6.000 * L & b2= 1.250 * a \\ =14*L+2. & A2= 0.500 * a & L3= 1.500 * L & b3= 1.750 * a \\ 5*a & & L4= 0.500 * L & b4= 1.250 * a \\ A=0.18 \lambda & & & \end{array}$$

If L , a value is chosen as reference rate by which it is possible to compare our results with previous results, by virtue of if these value are changed, a clear change will appear in the results and these value are confirmed to be taken as reference for change and possibilities which we can study and know the rocket dimension impact with the range 1.5λ .

The important factor of this paper is to obtain the sample radar impress or what is called body RCS at these chosen dimensions by using equation (26). This is made within the range($\phi_r = 0 \rightarrow 2\pi$) to know which level of polarization has the great impact on the process of finding radar cross section area, so the result will be as shown in fig.(9) .

From the figure, we can notice that the radar cross section area is effected only at the vertical and horizontal levels, while it is decreased gradually between these two levels. In addition, we notice summit recurrence at these levels and this clarify the study of the two levels ($\phi_r = 0^\circ$) and ($\phi_r = \pi/2$) only to avoid the extension in computing as shown in fig. (10).

In respect of the effected factor on the RCS that is clarified previously, we find that the target geometrical shape is considered as the first factor. If the rocket head is displaced with conic instead of half-sphere but its total length is stayed the same 1.5λ , we find that there is an impact on the RCS value when we notice decrease in its value at H and E levels as shown in figs.(11) and (12) .

2-1-2- Second sample:

We take up this sample as an approximation of the rocket shape employed by the US Air Force as shown in fig (13). The major dimension which is taken into account here of this rocket are the following:

$$\begin{array}{llll} R=1.5\lambda & L1= 1.000 * L & b1= 1.000 * a & A1= 1.000 * a \\ & L2= 1.500 * L & b2= 2.000 * a & A2= 1.000 * a \\ =24*L+13* & L3= 3.500 * L & b3= 3.000 * a & A3= 2.000 * a \\ a & L4= 5.500 * L & b4= 4.000 * a & A4= 2.000 * a \\ a=0.025 \lambda & L5= 12.500 * L & b5= 5.000 * a & A5= 5.000 * a \\ & & b6= 6.000 * a & A6= 2.000 * a \end{array}$$

If equation (26) is used, we can obtain the body RCS at the previously chosen dimension and the result will be as shown in fig.(14).

In the previous case itself, we will notice from the diagram that the RCS area is affected only at horizontal and vertical levels where it is decreased gradually between these two levels and the summit recurrence at these levels. From these point, we can study the levels ($\phi_r = \pi / 2$) and ($\phi_r = 0^\circ$) only as shown in fig.(15).

Now, if the rocket head is displaced with half-sphere shape instead of conic with the remain of rocket total length 1.5λ as shown in fig. (16).

When comparison is made between the same samples but one of them is of conic head and the other is of sphere

head, we find that there is a great correspondence between them because of the small sample head in comparison with the first sample is shown in fig.(17), in addition to the small diagram that doesn't show all changes specialties.

3- **Conclusions:**

The results obtained in this paper RCS give a clarified extensive idea that the shape of any body in space has a great and effective impact in computing RCS area. This give the reason behind the vasted interest of the space shuttle designers in vehicle industry, specially the military one, in order to be complex to know it through received radar. For each body in space, there are shape and value for RCS which is different completely from any other body; this symbolizes the reason behind the name "radar impress".

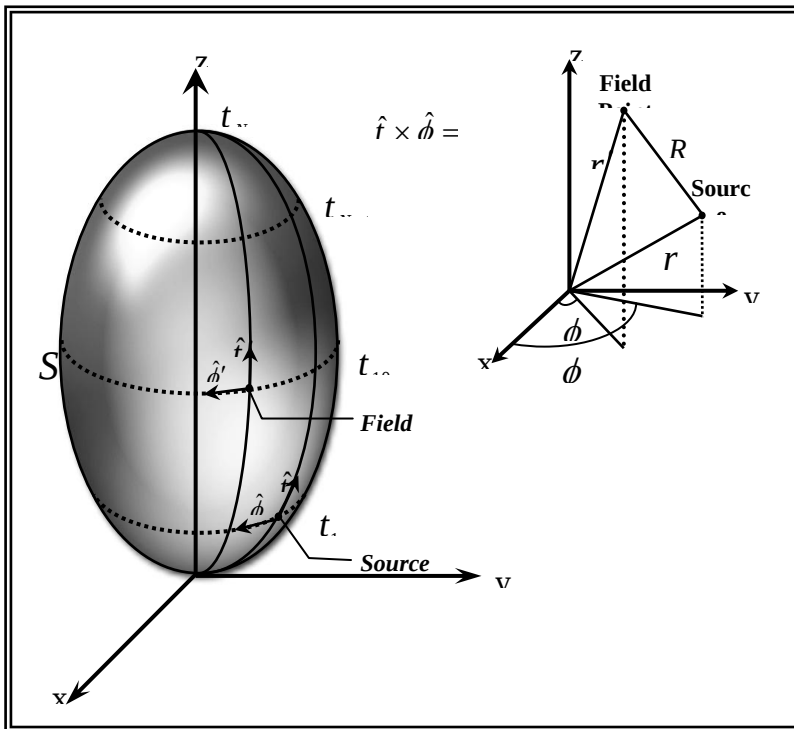


Fig.(1): System coordinate for body of revolution.

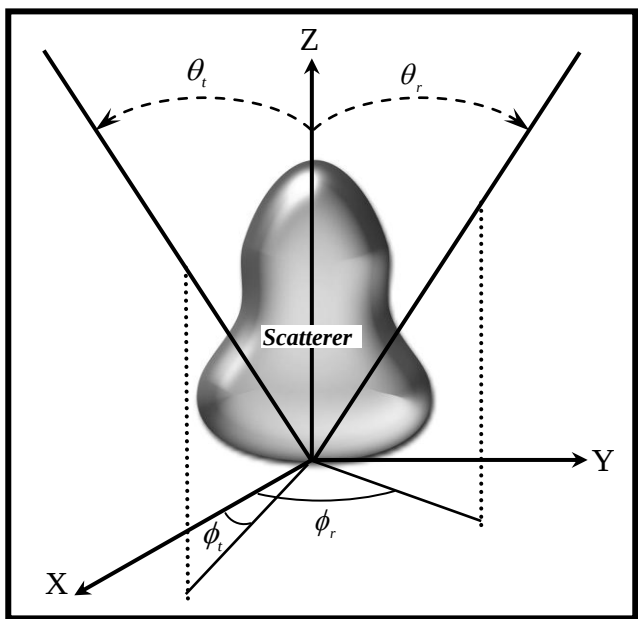
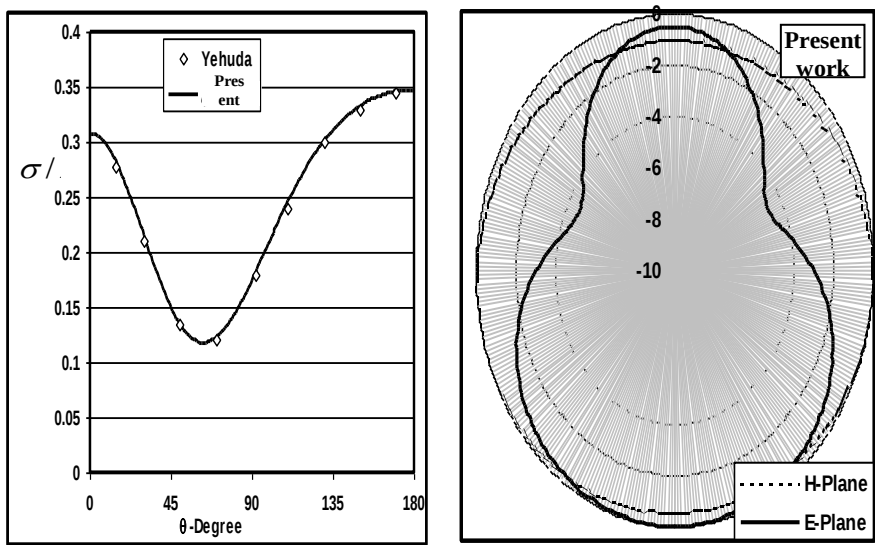


Fig.(2): Scattering plane wave by conducting body of revolution.



ig.(3): The radar cross section(RCS) of perfect conductor sphere of radius 0.2λ

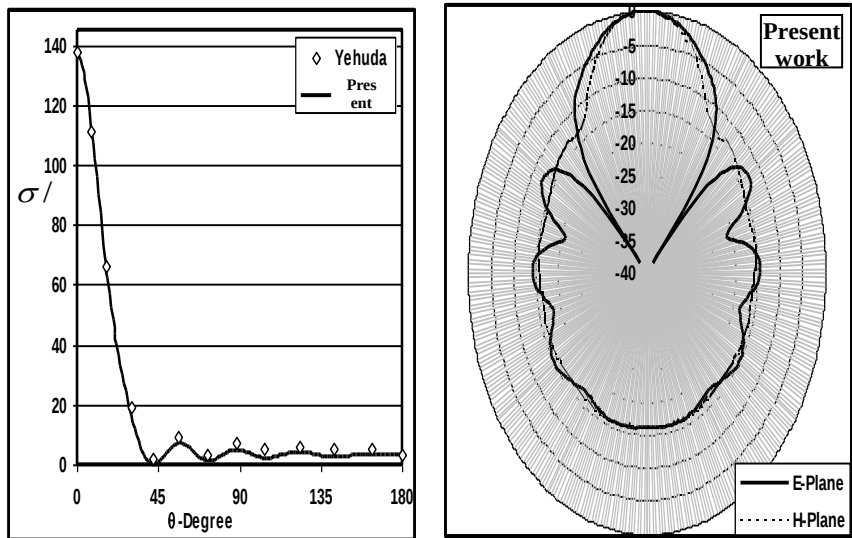


Fig.(4): The radar cross section (RCS) of perfect conductor sphere of radius 1.0λ

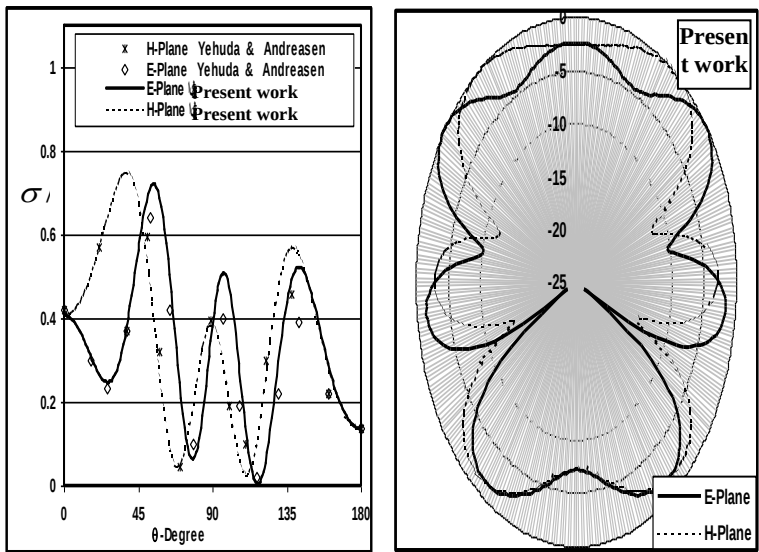


Fig.(5): The radar cross section(RCS) of perfect conductor capsule shape.

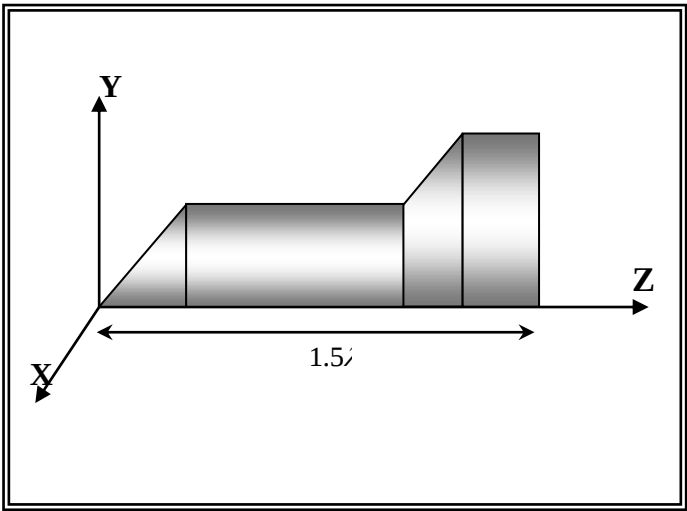


Fig.(6): The parts of sample used by Hashim, A. .

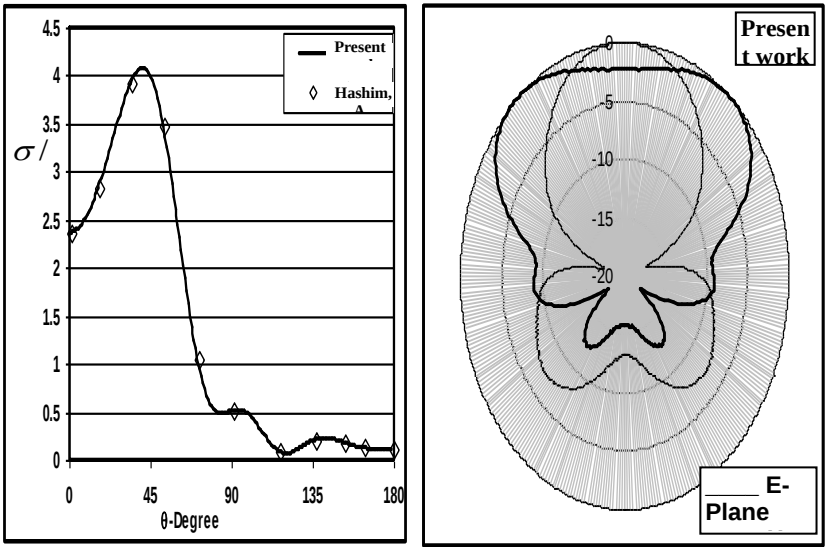


Fig.(7): The radar cross section(RCS) of the sample used by Hashim, A. .

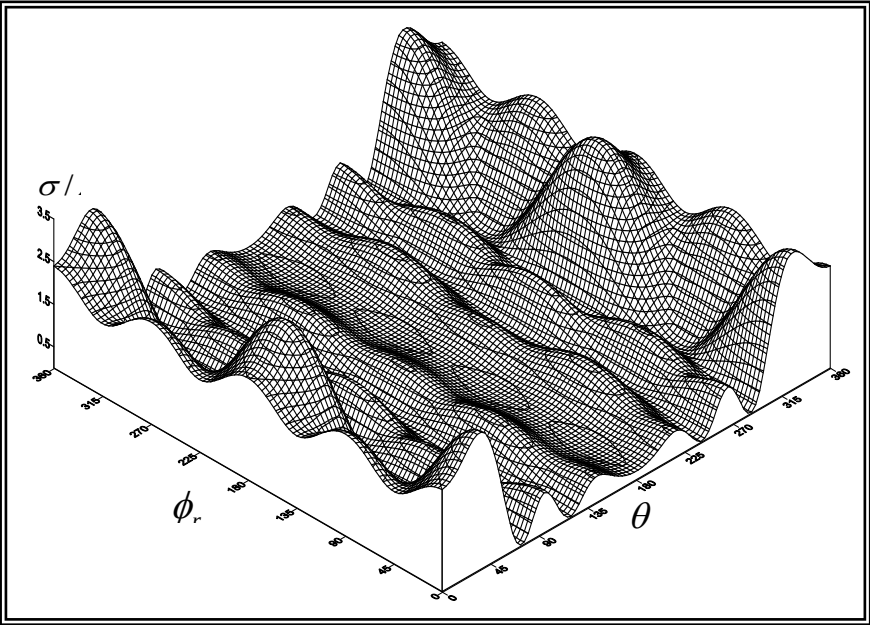
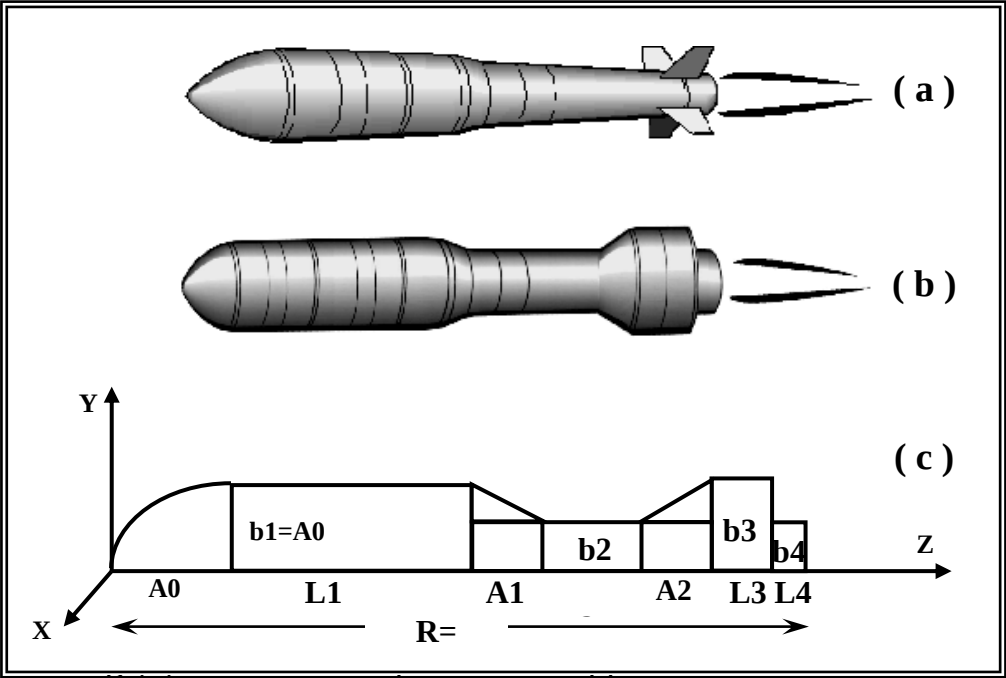


Fig.(9): RCS values for 3-dim. body at different angles.

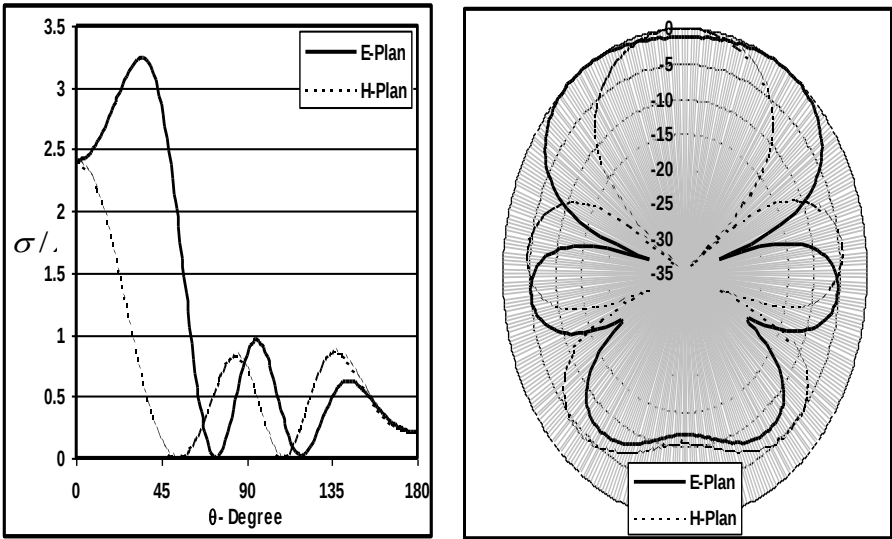


Fig.(10): RCS shape at H and E planes .

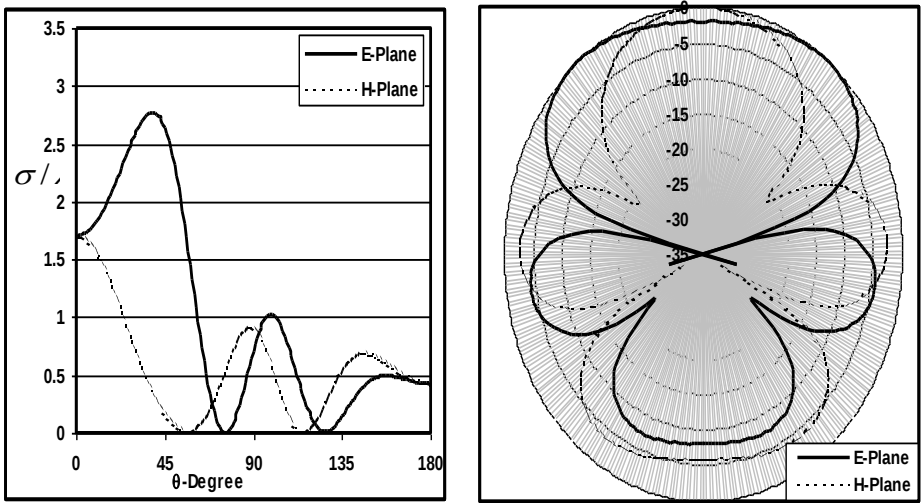


Fig.(11): RCS shape for the same sample but with conic head instead of sphere at the H and E levels.

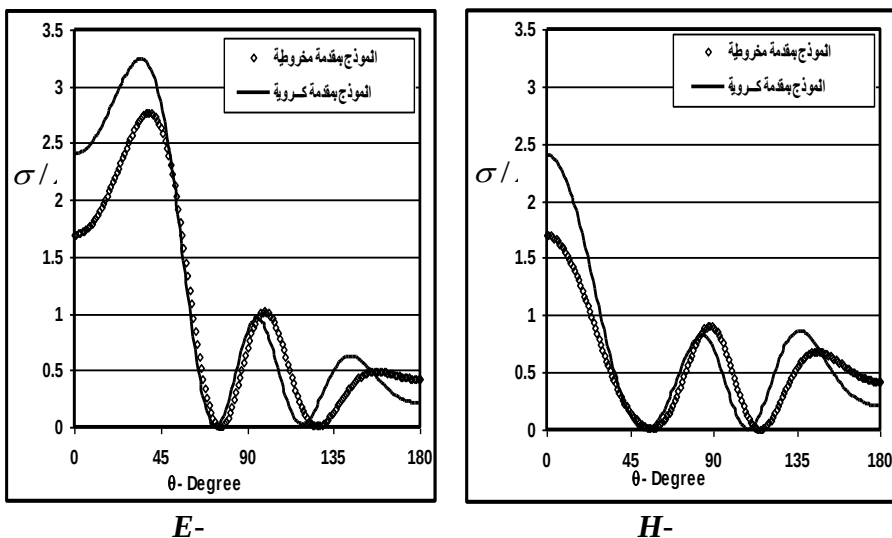


Fig.(12): Comparison between RCS shape for sphere-head rocket and conic-head rocket at H and E levels.

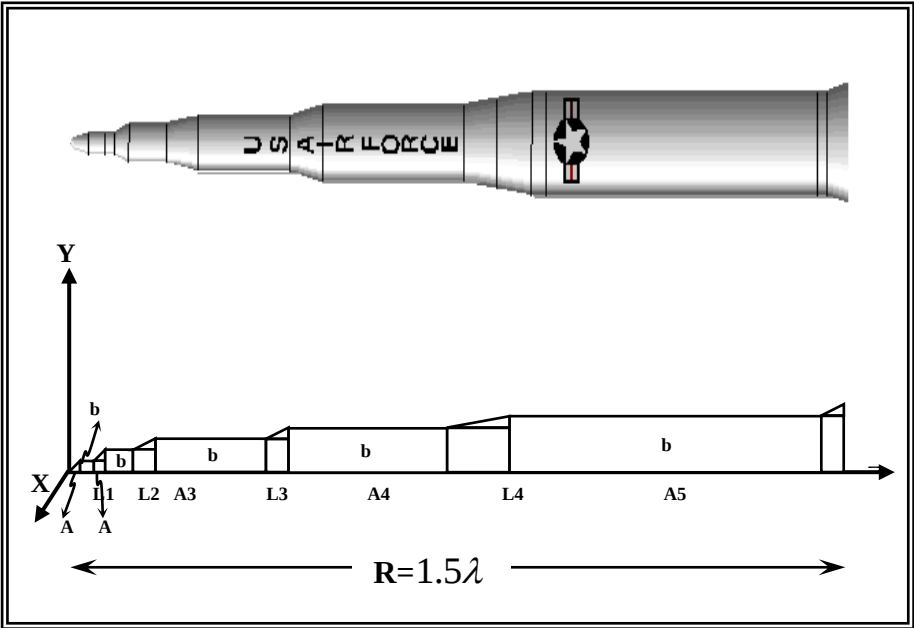


Fig.(13): Diagram shows the second sample parts.

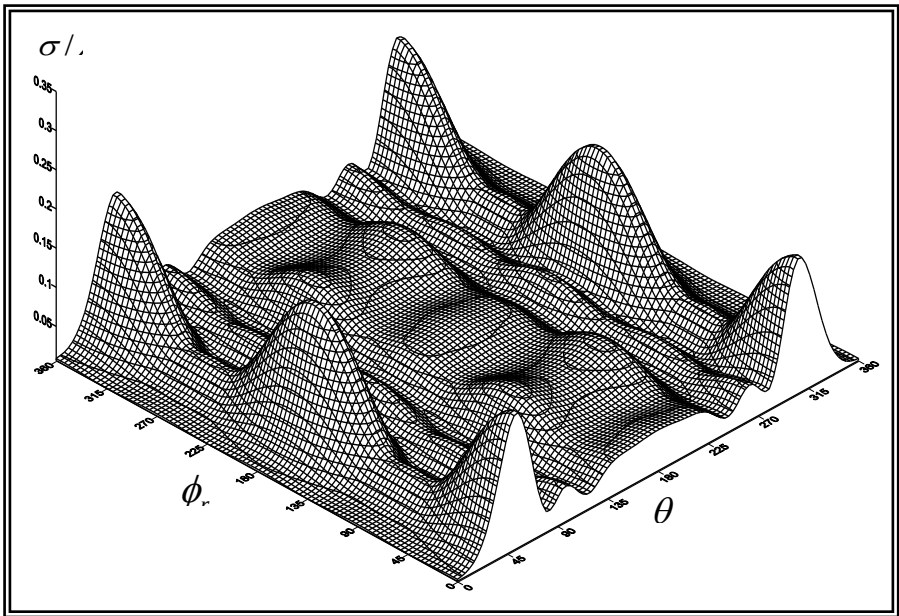


Fig.(14): RCS values for 3-dim. body at different angles.

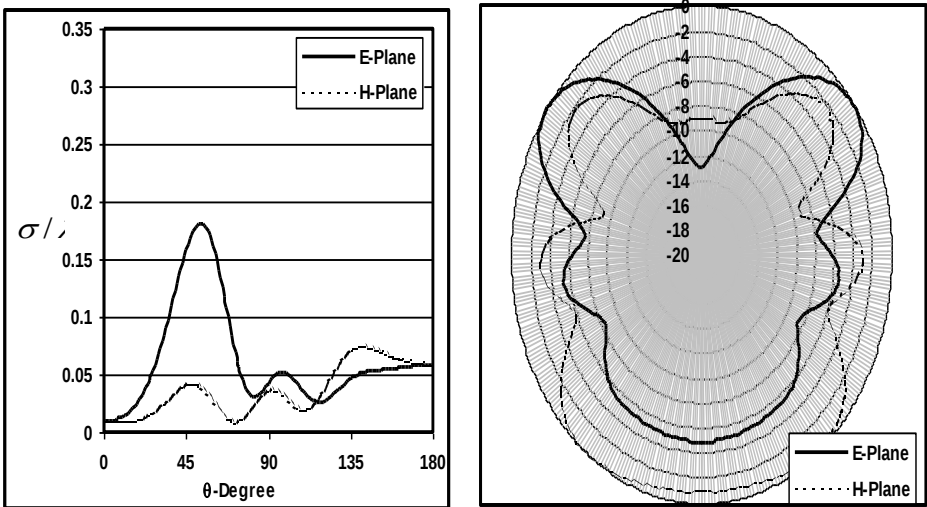


Fig.(15): RCS shape at H and E planes

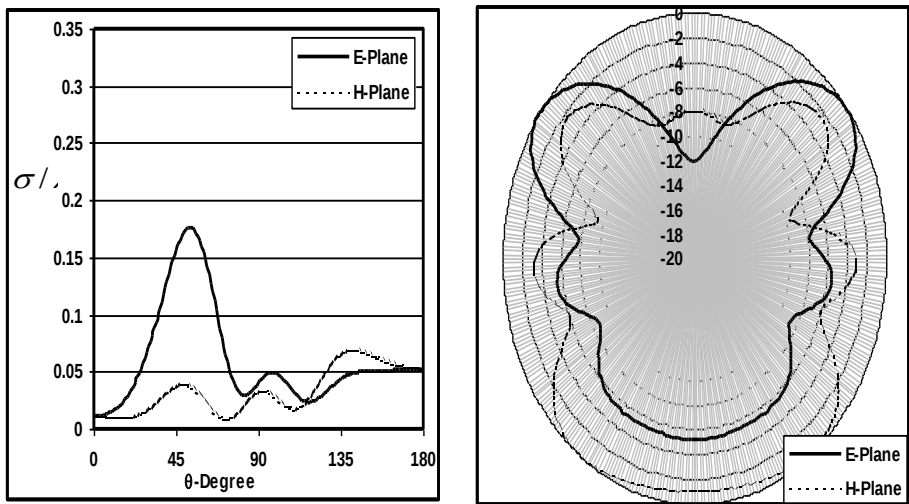


Fig.(16): RCS shape for the same sample but with sphere head at *H* and *E* levels.

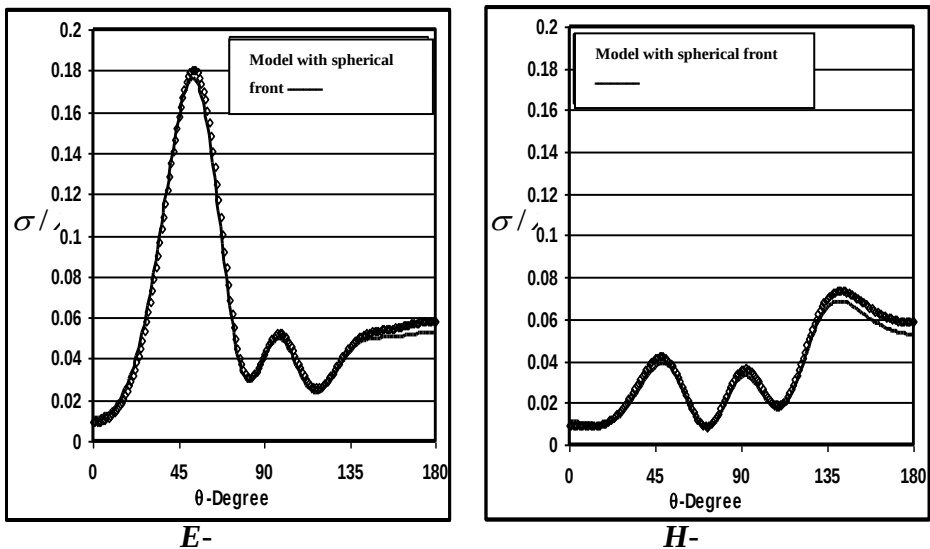


Fig.(17): comparison between RCS shape for the rocket of conic head with one of sphere head at *H* and *E* levels.

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دراسة تأثير الجسم الموصل المتناظر محوريا على حساب مساحة المقطع الرادارية .

أ.د.زكي عبدالله أحمد.

رائد عبد الجبار يوسف.

كلية العلوم - جامعة البصرة .

أ.م.د.أحمد هاشم عبود.

كلية الطب - جامعة ميسان .

الخلاصة:

صيغة المعادلة التكاملية من أكثر الطرق شيوعا لمعالجة مسائل الاستطارة والإشعاع الكهرومغناطيسيين مع طريقة العزوم كطريقة عددية. في هذا البحث مسالة الاستطارة الكهرومغناطيسية من الجسام المعقدة المتناظرة محوريا قد درست لتقصي تأثير شكل هذا الجسم على مساحة المقطع الرادارية لنموذجين مختلفين، كما تم مقارنة النتائج ومناقشتها.