# On strongly faintly M-$\theta$-i-continuous functions in Bi-Supra Topological Space

**Taha H. Jasim[1] , Reem O. Rasheed[2]**

[1] *Department of Mathematics, college of computer science and Mathematics, University of Tikrit , Tikrit , Iraq*
[2] *Department of Mathematics , college of Education for pure sciences , University of Tikrit , Tikrit , Iraq*

**Abstract**.
In this paper we introduces a new definition , called i- open and via this definition we introduce class of topological concepts($\mu$-$\theta$-i-open set, $\mu$-$\theta$-i-closed, strong faintly $\mu$-$\theta$-continuity, strong $\mu$-$\theta$-continuity )and we generalized these concepts in bi -supra topological space .At last many important theorems in strongly faintly M-$\theta$-i-continuous functions are investigated. And study the relationships among these functions and other forms are discussed.

**Key Words and Phrases:** bi-supra topological space ,strongly $\mu$-$\theta$- i-continuity, strongly faintly $\mu$- i -continuity.

## 1. Introduction

Nasef and Noiri [1] introduce three classes of strong forms of faintly continuity namely: strongly faint semicontinuity, strongly faint precontinuity and strongly faint β-continuity. Recently Nasef [2] defined strong forms of faint continuity under the terminologies strongly faint α-continuity and strongly faint γ-continuity. In this paper using $\mu$-$\theta$- i -open sets, strongly faintly $\mu$-$\theta$- i -continuity is introduced and studied in bi-supra topological spaces (Let X be non-empty set , let $\mathcal{S}$o(X) be the set of all semi open subset of X non-empty set , let $\mathcal{S}$o(X) be the set of all semi open subset of X (for short $\mathcal{ST}$) and let $\mathcal{P}$o(X) be the set of all pre open subset of X(for short $\mathcal{PT}$), then we say that (X,$\mathcal{ST}$,$\mathcal{PT}$) is a bi-supra topological space, when each of (X,$\mathcal{ST}$) and (X,$\mathcal{PT}$) is a supra topological space)[3] Moreover, basic properties and preservation theorems of strongly faintly $\mu$-$\theta$- i - continuous functions are investigated and relationships between strongly faintly $\mu$-$\theta$- i - continuous functions and graphs are investigated.

## 2. Preliminaries.

Throughout this paper (X ,$\mathcal{T}_x$) and (Y, $\mathcal{T}_y$) (Simply, X and Y) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure of subset A of X, the interior of A and the complement of A is denoted by cl(A), int(A) and $A^c$ or X\A respectively [3] A subset A of a space X is said to be regular open if it is the interior of its closure, i.e, A = int(cl(A)). The complement of a regular- open set is referred to as a regular –closed set. A union of regular-open sets is called δ-open [4] The complement of a δ-open set is referred to as a δ-closed set. A subset A of a space(X, $\mathcal{T}_x$ )is called θ-open set [5] if there exists an open set U containing x such that U⊆cl(U) ⊆ A. The set of all θ-interior points of A is said to be the θ-interior set and denoted by θ-int (A). A subset A of X is called θ-open if A = θ- int (A) The family of all θ- open sets in bi-supra topological space (X, , $\mathcal{T}_x$ ) forms a supra topology $\mathcal{T}_x$ on X. A subset A of a space (X, $\mathcal{T}_x$ ) is called semi-open [6] (resp, α-open [7], pre-open [8] β-open [9], γ- open [10]) if A⊆cl(int(A)(resp , A ⊆ int(cl(int(A)) A ⊆ int(cl(A)) A ⊆cl(int(cl(A)), cl(int(A))) . The complement of semi-open (resp. α-open , pre-open ,β-open , γ-open) set is called semi-closed (resp. α-closed , pre- closed ,β- closed , γ-closed [11] ).

A of a space (X, $\mathcal{T}_x$ ) is called semi- θ-open (resp. α-θ -open [12], pre- θ -open ,β- θ -open ,γ- θ – open[12] )if and only if for each x ∈ A there exists G∈ semi-open (resp , α-open ,pre-open , β-open , γ-open ) such that semi-CL(G) ⊆A (resp. α-CL(G) ⊆A, pre- CL(G) ⊆A, β-CL(G) ⊆A, γ- CL(G) ⊆A).

The union of all θ-open (resp,semi- θ-open, δ-open, ) sets contained in A is called the θ-interior (resp,semi- θ-interior [2], δ-interior ) of A and it is denoted by θ- int(A) (resp, semi- θ-int(A), δ-int (A)). The intersection of all θ-closed (resp.,semi- θ- closed, δ- closed,) sets containing A is called the θ- closure (resp.,semi- θ- closure, δ- closure [13] ) of A and it is denoted by θ- cl(A) (resp, semi- θ-cl(A), δ-cl(A)). We recall the following definitions and results, which are useful in the sequel

**Definition 2.1[14]:** Let (X ,$\mathcal{T}_x$) be a topological space. Then a subset A of X is said to be: (i) an M-open set, if A ⊆ cl( $int_\theta$ (A)) [ int($cl_\delta$(A)),
(ii) an M-closed set if int( $cl_\theta$(A)) \ cl($int_\delta$(A)) ⊆A.

**Definition 2.2[14]**: Let (X ,$\mathcal{T}_x$) be a topological space and A ⊆ X.Then:
(i) the M-interior of A is the union of all M-open sets contained in A and is denoted by M-int(A),
(ii) the M-closure of A is the intersection of all M-closed sets containing A and is denoted by M-cl(A)

**Definition 2.3[15]:** A function f : (X, $\mathcal{T}_x$ )→ (Y , $\mathcal{T}_y$ ) is said to be:
(i) M-continuous [14] if $f^{-1}$ (U) ∈ M-open in X , for each U∈ $\mathcal{T}_y$
(ii) pre-M-open [16] if, f(U) ∈ M-open in Y , for each U∈ M- open in X.
(iii) pre-M-closed [11] if, f(U) ∈ M -closed in Y , for each U∈ M- closed in X.

**Definition 2.4**: A function f : (X,$\mathcal{T}_x$ )→ (Y , $\mathcal{T}_y$ ) is said to be strongly faintly continuous [17] (resp. strongly faintly semicontinuous, strongly faintly pre continuous [18], strongly faintly β-continuous [18], strongly faintly α-continuous [1], strongly faintly γ- continuous[2]) i f for each x ∈ X

and each open (resp.semi- open, pre -open, β-open, α-open, γ-open) set V of Y containing f(x), there exists a θ-open set U of X containing x such that f(U) ⊆ V.

## 3 . A new type of bi-supra topological *space*

**Definitions 3.1:** Let (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )be a bi-supra topological space, and let G be a subset of X. Then G is said to be i- open set if G=(AUB) UØ where A∈$\mathcal{ST}$ and B∈$\mathcal{PT}$. The Complement of i- open set is called i- closed set.

**Definitions 3.2:** A subset A of a space (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ ) is called θ-i-open set if there exists an i- open set U containing x such that U ⊆i- cl(U) ⊆ A.

**Definitions 3.3:** The set of all θ-i-interior points of A is said to be the θ-i-interior set and denoted by θ-i-int (A). A subset A of X is called θ-i-open if A = θ-i- int (A).

**Definitions 3.4:** A of a space (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )is called semi- θ-i-open (resp. α- θ -i-open , pre- θ -i-open , β- θ -i-open , γ- θ -i-open )if and only if for each x ∈ A there exists G in semi-i-open (resp. , α-i-open , pre-i-open ,β-i-open , γ-i-open ) such that semi-i-CL(G) ⊆A (resp. α-i CL(G) ⊆A -, pre-i- CL(G) ⊆A , β-i CL(G) ⊆A , γ-i- CL(G) ⊆A).

**Definitions 3.5**: A subset A of a space (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )is called M-i-open if A ⊆ i- cl(θ-i int(A)) ∪ i- int(δ –i-cl(A)). The complement of semi-i-open (resp. , α-i-open , pre-i-open ,β-i-open , γ-i-open ,semi- θ-i-open, θ-i-open, M-i-open) set is called semi-i-closed (resp., α-i-closed , pre-i- closed ,β-i-closed , γ-i- closed ,semi- θ-i- closed, θ-i- closed, M-i- closed) . The union of all M-i-open (resp. θ-i-open,semi- θ-i-open, δ-i-open, ) sets contained in A is called the M-i-interior (resp. θ-i-interior ,semi- θ- i-interior , δ-i-interior ) of A and it is denoted by M-i-int(A) (resp. . θ-i- int(A), semi. θ-i-int(A), δ-i-int (A)). The intersection of all M-i-closed (resp. . θ-i-closed ,semi- θ-i- closed, δ-i- closed,) sets containing A is called the M-i-closure (resp. θ-i- closure ,semi-θ-i- closure, δ-i- closure ) of A and it is denoted by M-i-cl(A) (resp. θ-i- cl(A), semi. θ-i-cl(A), δ-i-cl(A)).

**Definition 3.6:** A subset A of a space (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ ) is called M-θ-i- open if and only if for each x ∈ A there exists G in M-i-open in X such that M-i-cl(G) ⊆A.

Also , clearly every M-θ-i- open (resp. M-θ-i- closed) set is M-i- open (rsp. M-i- closed) set. And every θ-i- open (resp. θ-i- closed) set is M-i- open (rsp. M-i-closed).

**Example 3.1.** Let X = {a, b, c} and $\mathcal{T}_x$ = { Ø, {a}, {a, c}, {b, c}, X}
{ Ø,{b, c}, {b}, {a}, X} , $\mathcal{ST}_x$= { Ø, {a}, {a, c}, {b, c}, X}= $\mathcal{T}_x^c$
$\mathcal{PT}_x$ = { Ø, {a},{c}, {a, b} {a, c}, {b, c}, X}
i-open in X = { Ø, {a},{c}, {a, b} {a, c}, {b, c}, X}, i-closed in X = { Ø, {b, c} {a, b},{c},{b} ,{a}, X}
= { Ø, {a, b}, {b, c}, X} in X θ-i- open
= { Ø, {c}, {a}, X} in X θ-i-closed
Regular –i-open in X= { Ø, {a},{c}, {a, b} , {b, c}, X}
δ-i-open in X= { Ø, {a},{c}, {a, b} {a, c}, {b, c}, X}
δ-i-closed in X= { Ø,{b, c} {a, b},{b},{c} ,{a} , X}
M-i-open in X= { Ø, {a},{c}, {a, b} ,{a,c}, {b, c}, X}
M-i-closed in X = { Ø, {b, c} {a, b},{c},{b},{a} ,X}
M-θ-i- open in X= { Ø, {a, b} {b, c} ,X}
Then {a},{c} are M-i-open set but not M-θ-i- open set in X .

**Definition 3.7:**The M-θ-i-closure of a subset A in bi-supra topological space (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ ) is denoted by M-θ-i-cl(A) and is defined to be the set of all points x of X such that for each G in M-θ-i- open in X, M-θ-i-cl(G) ∩ A ≠ Ø.

**Definition 3.8:** A subset A in bi-supra topological space (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ ) is said to be M-θ-i-closed if M-θ-i-cl(A) = A. The complement of a M-θ-i- closed set is called a M-θ-i -open set.

We recall the following definitions and results, which are useful in the sequel.

**Definition 3.9:** A function f : (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y, $\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is said to be strongly faintly -$\theta-$ i-continuous (resp. . strongly faintly semi-$\theta-i-$ continuous, strongly faintly pre θ-i-continuous, strongly faintly β- θ-i--continuous , strongly faintly α- θ-i-continuous , strongly faintly γ-θ-i-continuous) i f for each x ∈ X and each θ-i-open (resp.semi θ-i-open, pre θ-i-open, β- θ-i-open, α- θ-i-open, γ- θ-i-open) set V of Y containing f(x), there exists a θ-i-open set U of X containing x such that f(U) ⊆ V.

**Definition 3.10:** A function f : (X,$\mathcal{T}_x$ )→ (Y , $\mathcal{T}_y$ ) (resp, f : (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is said to be:
(i) θ -continuous [19](resp. θ-i –continuous), if $f^{-1}$ (V) is θ-open (resp. θ-i-open) in X for every open (resp. i- open )set V of Y.
(ii) quasi θ-continuous [20](resp. quasi θ-i-continuous) , if $f^{-1}$(V) is θ-open(resp. θ-i-open)set in X for every θ-open (resp. θ-i-open )set V of Y.
(iii)faintly continuous [15](resp. faintly i-continuous) if $f^{-1}$(V) is open(resp. i-open)set in X for every θ-open (resp. θ-i-open) set V of Y.

**Definition 3.11:** A function f : (X, $\mathcal{T}_x$ )→ (Y , $\mathcal{T}_y$ )(resp, f : (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ )) is said to be:
(i) M-continuous [14] (resp, M-i-continuous) if $f^{-1}$ (U) ∈ M-open(resp, M-i-open ) set in X , for each U∈ Y
(ii) pre-M-open [16](resp, pre-M- θ-i –open) if, f(U) ∈ M-open (resp, M- θ-i-open ) set in Y , for each U∈ M-open(resp, M- θ -i-open ) set in X.
(iii) pre-M-closed [16] (resp, pre-M- θ-i –closed) if, f(U) ∈ M -closed (resp, pre-M- θ-i –closed) set in Y , for each U∈ M- closed(resp, pre-M- θ-i –closed)set in X.

**Lemma 3.1**:For topological space (X,$\mathcal{T}_x$ ) (resp,bi-supra topological space (X,$\mathcal{ST}_x$ ,$\mathcal{PT}_x$ ) and A ⊆ X, then the following statements are hold:
(i) If A ⊆ Fi, Fi is an M-closed [15](resp, M-i-closed )set of X, then A ⊆ M-cl(A) ⊆ Fi,(resp.M -i-cl(A) ⊆ Fi).

(ii) If Gi ⊆ A, Gi is an M-open[14] (resp . M-i-open) set of X, then Gi ⊆ M-int(A) ⊆ A(resp. Gi ⊆ M-i-int(A) ⊆ A) .

**Proposition 3.1:** Let (X,$\mathcal{T}_x$ ) topological space (resp. (X,$\mathcal{ST}_x$ ,$\mathcal{PT}_x$ ) be bi-supra topological space ) and A ⊆ X. Then . the following statements are hold:

(i) θ-$F_r$ (A) = θ-cl (A) \ θ-int (A) )[9] (resp. θ-i-$F_r$ (A) = θ-i cl (A) \( θ-i int (A) )

(ii) M-$F_r$ (A) = M-cl(A) \ M-int(A) [13] (resp.M-i-$F_r$ (A) = M-i-cl(A) \( M-i-int(A))

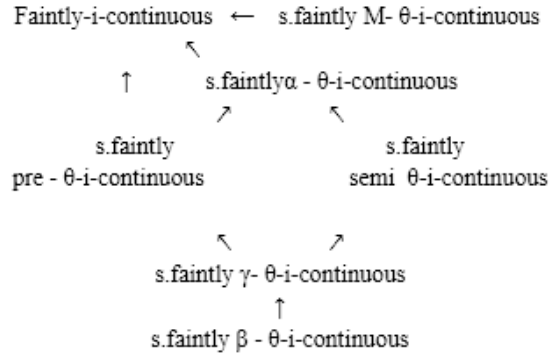
(iii) M- b(A) = A\M-int(A) [11] (resp. M-i- b(A) = A\M-i-int(A)).

The set of θ-boundary (resp. θ-i-boundary ,M-i-boundary, M-i-border) of A is denoted by θ-$F_r$ (A) (resp. θ-i-$F_r$ (A),M-i-$F_r$ (A), M-i- b(A)).

**4. strongly faintly M-θ-i –continuous functions.**

**Definition 4.1:** A function f : (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is said to be strongly faintly M- θ-i -continuous if for each x ∈ X and each M- θ-i -open set V of Y containing f(x), there exists a θ-i-open set U of X containing x such that f(U) ⊆ V

**Remark 4.1 :** The implication between some types of definitions (**3.9**) and (**4.1**) are given by the following diagram.




The proof of implication form definition directly .However, none of these implications is reversible as shown by the following examples and well- known facts.

**Example 4.1 :** Let X=Y = {a, b, c}
and $\mathcal{T}_x$ = { ∅, {a},{b}, {a, b}, {a, c}, X}
{ ∅,{b, c},{a,c}, {c}, {b}, X} = $\mathcal{T}_x^c$
$\mathcal{ST}_x$= { ∅, {a},{b}, {a, b}, {a, c}, X} , $\mathcal{PT}_x$ = { ∅, {b}, {a, b} {a, c}, X}
i-open in X = { ∅, {a},{b}, {a, b} {a, c}, X}
i-closed in X = { ∅, {b, c} {a, c},{c},{b} , X}, θ-i-open in X={ ∅, {a, c}, X}
with $\mathcal{T}_Y$ = { ∅, {a, c}, Y}
{ ∅,{b}, Y} = $\mathcal{T}_Y$ $^c$
$\mathcal{ST}_Y$= { ∅, {a, c}, Y} , $\mathcal{PT}_Y$ = { ∅, {b}, {a, b} {b, c}, Y}
i-open in Y = { ∅,{b}, {a, b} {a, c},{b,c}, Y} , i-closed in Y = { ∅, {a, c} , {c},{b},{a} , Y}
semi-i-open in Y={ ∅,{b}, {a, b} {a, c},{b,c}, Y}
semi-i-closed in Y= { ∅ , {a, c} { c},{b},{a}, Y} , semi- θ-i- open in Y={ ∅ , {a, c}, Y}
γ-i-open in Y={ ∅ ,{a},{b},{a,b} {a, c},{b,c} Y}
γ-i-closed in Y={ ∅ ,{b,c},{a,c},{c } {b},{a} Y}

define the function f : (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) by the identity function . then f is strongly faintly semi- θ-i-continuous but not strongly faintly γ - θ-i-continuous.

**Example 4.2:** Let R be the set of real numbers $\mathcal{T}_x$ the indiscrete topology for R and $\mathcal{T}_Y$ the discrete topology for R.
Then the identity function f : (X,$\mathcal{T}_x$ )→ (X , $\mathcal{T}_y$ )(resp, f : (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ )) is strongly faintly pre-continuous but not strongly faintly γ -continuous [9] (resp, strongly faintly pre- θ-i-continuous but not strongly faintly γ - θ-i-continuous).

**Example 4.3:** Let $\mathcal{T}$x be the uasul topology for R and $\mathcal{T}_Y$ ={ R,∅ , [0,1]∪ (1,2) ∩ $Q$} where Q denotes the set of the set of rational numbers. Then the identity function .
f : (X,$\mathcal{T}_x$ )→ (X , $\mathcal{T}_y$ )(resp, f : (R,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (R,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ )) is strongly faintly γ - continuous but not strongly faintly β –continuous[20] (resp, strongly faintly γ - θ-i-continuous but not strongly faintly β - θ-i-continuous.

**Example 4.4:** (1):in ([1], example 3.2) showed a strongly faintly semi-continuity (resp,strongly faintly semi- θ-i-continuity) which is not a strongly faintly pre-continuity. (resp,strongly faintly pre- θ-i-continuity).
(2): using example 3.2 of [8] , this easily observed that a strongly faintly α -continuity(resp, faintly α - θ-i-continuity) but not strongly faintly M –continuity (resp, not strongly faintly M - θ-i-continuity) .

**Theorem 4.1**: For a function f : (X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y, $\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) the following statements are equivalent:
(i) f is strongly faintly M - θ-i-continuous.
(ii) f: (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y, $\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is faintly-i-continuous.
(iii) $f^{-1}$ (V) is θ-i-open in X for every M - θ-i -open set V of Y (iv) $f^{-1}$ (F) is θ-i-closed in X for every M - θ-i -closed subset F of Y.
Proof. (i)⇒(iii): Let V be an M - θ-i -open set of Y and x ∈ $f^{-1}$ (V). Since f(x) ∈ V and f is strongly faint M - θ-i -continuous, there exists a θ-i-open set U of X containing x such that f(U) ⊂ V. It follows that x ∈ U ⊂ $f^{-1}$ (V).
Hence $f^{-1}$ (V) is θ-i-open in X.
(iii)⇒(i): Let x ∈ X and V be an M - θ-i -open set of Y containing f(x). By (iii), $f^{-1}$ (V) is a θ-i-open set containing x. Take U = $f^{-1}$ (V). Then f(U)⊆ V.
This shows that f is strongly faint M - θ-i -continuous.
(iii)⇒(iv): Let V be any M - θ-i -closed set of Y. Since Y \ V is an M - θ-i -open set, by (iii), it follows that $f^{-1}$ (Y\V) = X\$f^{-1}$ (V) is θ-i-open. This shows that $f^{-1}$ (V) is θ-i-closed in X.
(iv)⇒(iii): Let V be an M - θ-i -open set of Y. Then Y\V is M - θ-i -closed in Y. By(iv), $f^{-1}$ (Y\V) = X\$f^{-1}$ (V) is θ-i-closed and thus $f^{-1}$ (V) is θ-i-open.(i)⇔(ii): Clear.
Now we intuduce a very important theorem to explam the equivalence implication.

**Theorem 4.2:** For a function f : $(X,\mathcal{ST}_x, \mathcal{PT}_x)\to(Y, \mathcal{ST}_y, \mathcal{PT}_y)$, the following statements are equivalent:

(i) f is strongly faintly M- θ-i -continuous,

(ii) For each x∈ X and each M- θ-i -open V of f(x) in Y, there exists an θ-i -open set U of x in X such that f (U) ⊆ V,

(iii) $f^{-1}$ (F) is θ-i -closed in X, for every M- θ-i-closed set F of Y,

(iv) θ-i -cl($f^{-1}$ (B)) ⊆ $f^{-1}$ (M-θ-i cl (B)), for each B ⊆ Y,

(v) f(θ-i -cl(A)) ⊆M- θ-i cl (f(A)), for each A ⊆ X,

(vi) $f^{-1}$ (M-θ-i- int(B)) ⊆ θ-i -int($f^{-1}$ (B)), for each B ⊆ Y,

(vii) θ-i –b ($f^{-1}$ (B)) ⊆ $f^{-1}$ (M- θ-i- b(B)) , for each B ⊆ Y,

(viii) θ-i -Fr($f^{-1}$ (B)) ⊆ $f^{-1}$ (M-θ-i -Fr(B)), for each B ⊆ Y.

**Proof.** (i)→(ii). Let x∈ X and V ⊆ Y be A M- θ-i-open set containing f(x). Then x∈ $f^{-1}$ (V). Hence by hypothesis, $f^{-1}$ (V) is θ-i-open set of X containing x. We put U = $f^{-1}$ (V).

Then x∈ U and f(U) ⊆ V.

(ii)→(iii). Let F ⊆ Y be M-θ-i-closed. Then Y\F is M-θ-i-open and x∈ $f^{-1}$ (Y\F). Then f(x) ∈ Y\F. Hence by hypothesis, there exists an θ-i-open set U containing x such that f(U) ⊆ Y\F, this implies that, x∈ U ⊆ $f^{-1}$ (Y\F). Therefore, $f^{-1}$ (Y\F) = X\ $f^{-1}$ (F) which is θ-i-open in X. Therefore, $f^{-1}$ (F) is θ-i closed.

(iii)→(i). Let V ⊆ Y be a M-θ-i-open set. Then Y\V is M-θ-i-closed in Y. By hypothesis, $f^{-1}$ (Y\V) = X\$f^{-1}$ (V) is θ-i -closed and hence $f^{-1}$ (V) is θ-i-open. Therefore, f is strongly faintly M- θ-i -continuous.

(i)→ (iv). Since B ⊆ M-θ-i cl(B) ⊆ Y which is a M-θ-i-closed set. Then by hypothesis, $f^{-1}$ (clθ(B)) is θ-i -closed in X . Hence by Lemma 2.1, θ-i-cl($f^{-1}$ (B)) ⊆ f -1(M-θ-i cl(B)) for each B ⊆ Y.

(iv) →(v). Let A ⊆ X. Then f(A) ⊆Y, hence by hypothesis, θ-i -cl(A) ⊆ θ-i -cl($f^{-1}$ (f(A))) ⊆ $f^{-1}$ (M-θ-i cl(f(A))). Therefore, f(θ-i -cl(A)) ⊆ f $f^{-1}$ (M-θ-i -cl(f(A))) ⊆M- θ-i- cl(f(A)), (v) →(i). Let V ⊆ Y be a M-θ-i-closed set. Then, $f^{-1}$ (V) ⊆ X . Hence, by hypothesis, f(θ-i -cl($f^{-1}$V))) ⊆ M- θ-i -cl(f($f^{-1}$ (V))) ⊆ M- θ-i -cl(V) = V. Thus θ-i -cl($f^{-1}$ (V)) ⊆ $f^{-1}$ (V) and hence $f^{-1}$ (V) ∈ θ-i –closed in X .Hence, f is strongly faintly M- θ-i -continuous,

(i) → (vi). SinceM -θ-i - int(B) ⊆ B ⊆ Y is M- θ-i-open. Then by hypothesis, $f^{-1}$ (M- θ-i -int(B)) is an θ-i -open set in X. Hence, by Lemma 2.1, $f^{-1}$ (M- θ-i -int(B)) ⊆ θ-i -int($f^{-1}$ (B)), for each B ⊆ Y.

(vi) → (i). Let V ⊆ Y be a M-θ-i-open set. Then by assumption, $f^{-1}$(V) = $f^{-1}$ (M- θ-i -int(V)) ⊆ θ-i -int($f^{-1}$ (V)) . Hence, $f^{-1}$ (V) is θ-i -open in X. Therefore, f is strongly faintly M- θ-i continuous.-

(vi) → (vii). Let V ⊆ Y. Then by hypothesis, $f^{-1}$ (M- θ-i -int(V)) ⊆ θ-i -int($f^{-1}$ (V)) and so $f^{-1}$ (V) \ θ-i -int($f^{-1}$ (V)) ⊆ $f^{-1}$ (V) \ $f^{-1}$ (M- θ-i -int(V)) = $f^{-1}$ (V\M- θ-i - int(V)). By Proposition (3.1), θ-i -b($f^{-1}$ (V)) ⊆ $f^{-1}$ ( M- θ-i-b(V)).

(vii) → (vi). Let V ⊆ Y. Then by hypothesis, $f^{-1}$ (V) \ θ-i -int(f (V)) ⊆ $f^{-1}$ (V) \ $f^{-1}$ (M- θ-i int(V)). Therefore, $f^{-1}$ (M- θ-i -int(V)) ⊆ θ-i -int($f^{-1}$ (V)).

(vi) → (viii). Let B ⊆ Y. Then by (vi), $f^{-1}$ (M- θ-i –int(B)) ⊆ θ-i -int($f^{-1}$ (B)) . Hence by (iv), θ-i -cl($f^{-1}$ (B)) \ θ-i -int($f^{-1}$-(B)) ⊆ $f^{-1}$ (M- θ-i -cl(B)) \ $f^{-1}$ (M- θ-i -int(B)) . So, by Proposition (3.1), θ-i -Fr($f^{-1}$ (B)) ⊆ $f^{-1}$ (M- θ-i -Fr(B)), for each B ⊆ Y.

(viii)→(vi). Let B ⊆ Y. Then by Proposition(3.1), θ-i -Fr($f^{-1}$ (B)) = θ-i -cl($f^{-1}$ (B)) \ θ-i -int($f^{-1}$ (B)) ⊆ $f^{-1}$ (M- θ-i -cl(B)) \ $f^{-1}$ (M- θ-i -int(B)) this implies that $f^{-1}$ (M- θ-i -int(B)) ⊆ θ-i -int( $f^{-1}$ (B)), for each B ⊆ Y.

**Definition 4.2:** A function f : $(X, \mathcal{ST}_x, \mathcal{PT}_x)\to(Y, \mathcal{ST}_y, \mathcal{PT}_y)$ is said to be strongly M- θ-i-continuous , if for each x∈ $X$ and each i-open set V of Y containing f(x) , there exists U∈ M-i- open in X such that f(M-i -cl(U))⊆ $V$.

**Proposition 4.1:** If a function f : $(X,\mathcal{ST}_x, \mathcal{PT}_x)\to(Y,\mathcal{ST}_y, \mathcal{PT}_y)$ is strongly faintly M- θ-i-continuous then f is strongly M- θ-i-continuous .

**Remark 4.2:** the converse of the above proposition is not true as shown by the following example.

**Example 4.5:** Let X=Y = {a, b, c} with topologies

$\mathcal{T}$x = { ∅, {a},{b}, {a, b}, {a, c}, X}

{ ∅,{b, c},{a,c}, {c}, {b}, X} = $\mathcal{T}_x^c$

$\mathcal{ST}_x$= { ∅, {a},{b}, {a, b}, {a, c}, X} , $\mathcal{PT}_x$ = { ∅, {b}, {a, b} {a, c}, X}

i-open in X = { ∅, {a},{b}, {a, b} {a, c}, X}i-closed in X = { ∅, {b, c} {a, c},{c} ,{b} X}

θ-i- open in X={ ∅,{a, c},X} , θ-i- closed in X={ ∅,{b},X}

regular –i-open in X = { ∅, {b},{c}, {a, c}, X}

δ-i-open in X= { ∅, {b},{c}, {a, c},{b,c}, X}, δ-i-closed in X= { ∅,{a, c},{a,b},{a},{b}, X}

M-i- open in X= { ∅, {b},{c}, {a, b} {a, c},{b,c}, X},

M-i- closed in X= { ∅, {a},{b}, {a, b} {a, c},{b,c}, X},

And let $\mathcal{T}_Y$ = { ∅, {a, c}, Y},

{ ∅,{b}, Y} = $\mathcal{T}_y^c$

$\mathcal{ST}_Y$= { ∅, {a, c},Y}, $\mathcal{PT}_Y$ = { ∅, {b}, {a,b} {b, c}, Y}

i-open in Y = { ∅,{b}, {a, b} {a, c},{b,c},Y }, i-closed in Y = { ∅,{a, c} { c},{b},{a},Y },

θ-i- open in Y={ ∅,{a, c},Y}, θ-i- closed in Y={ ∅,{b},Y},

regular –i-open in Y = { ∅,{b}, {a, c}, Y}

δ-i-open in Y= { ∅, {b}, {a, c}, Y}, δ-i-closed in Y= { ∅, {b}, {a, c}, Y},

M-i- open in Y= { ∅,{b}, {a, b} {a, c}, Y}

M-i- closed in Y= { ∅,{a},{b},{c}, {a, b} {a, c}, {b,c},Y}

M- θ-i- open in Y={ ∅, {a,b},{a, c},Y}

define the function f : $(X,\mathcal{ST}_x, \mathcal{PT}_x)\to(Y,\mathcal{ST}_y, \mathcal{PT}_y)$ by the identity function . then f is strongly M- θ-i-continuous but not strongly faintly M- θ-i-continuous.

**Proposition 4.1:** If f : (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is strongly faintly M- θ-i-continuous then:
(i) f is quasi θ-i-continuous
(ii) f is faintly i- continuous.

**Proof.(i):** Let x∈X and V ⊆ Y be M- θ-i-open containing f(x). then there exist an θ-i-open set U such that f(U) ⊆ V .since every M- θ-i-open is θ-i-open set if $f^{-1}$ (V) ∈ θ-i-open in X for every V∈ θ-i-open in Y .then f is quasi θ-i-continuous .
(ii) similar (i)

**Definition 4.3:** A function f : (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is called weakly-M- θ-i -continuous if, for each x∈X and each i-open set V of Y containing f(x), there exists M- θ-i –open in X such that f(U) ⊆ i-cl(V).

**Theorem 4.4:** the following statements are hold for function
f : (X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y, $\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) and g: (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ )→ (Z,$\mathcal{ST}_z$ ,$\mathcal{PT}_z$ )
(i) If, f is quasi θ-i-continuous and g is strongly faintly M- θ-i-continuous, then
g o f is strongly faintly M- θ-i-continuous,
(ii) If, f is strongly faintly M- θ-i-continuous and g is weakly-M θ-i -continuous, then g o f is θ-i –continuous.

**Proof.** (i) Let V ⊆ Z be M- θ-i-open set and g be strongly faintly M- θ-i-continuous,, then $g^{-1}$(V) ∈ θ-i-open in Y. But f is quasi θ-i-continuous, then $(gof)^{-1}$(V) ∈ θ-i-open in Y .Hence, g o f is strongly faintly M- θ-i-continuous.
**(ii)** similar (i).

**Theorem 4.5:.**For two function f :(X,$\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y, $\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) and g: (Y, $\mathcal{ST}_y$ , $\mathcal{PT}_y$ )→ (Z,$\mathcal{ST}_z$ ,$\mathcal{PT}_z$ ) the following properties are hold:
(i) If, g is a surjective pre-M- θ-i -open and g o f is strongly faintly M- θ-i-continuous. then f is strongly faintly M- θ-i-continuous.
(ii) If, g is a surjective pre-M- θ-i -closed and g o f is strongly faintly M- θ-i-continuous. then f is strongly faintly M- θ-i-continuous.

**Proof.** (i) Let V⊆ Z be a M- θ-i -open set. Since, g o f is strongly faintly M- θ-i-continuous., then$(gof)^{-1}$ (V) is θ-i –open in X . But, g is surjective pre-M- θ –i-open, then $g^{-1}$(V) is M- θ-i -open set in Y. Therefore ,f is faintly M- θ-i-continuous
(ii) Obvious

**Definition 4.4:** A function f :(X, $\mathcal{ST}_x$ , $\mathcal{PT}_x$ )→ (Y,$\mathcal{ST}_y$ , $\mathcal{PT}_y$ ) is called :
(i) M-θ-i-open if f(V) ∈ θ-i-open in Y for each V ∈ M- θ-i-open in X,
(ii) M-θ-i-closed if f(V) ∈ θ-i-open in Y for each V ∈ M- θ-i-closed in X.

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# حول الدوال القوية الضعيفة من النمط i- $\theta$ -M في الفضاء ثنائي التبولوجي الفوقي

**طه حميد جاسم[1] ، ريم عمران رشيد[2]**

*[1]قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق*

*[2]قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق*

**الملخص**

في هذا البحث قدمنا تعريفا جديدا أسمينا i-open ومن خلال هذا التعريف قدمنا صف من المفاهيم التبولوجيه ( مجموعه مفتوحة من النمط **i-** $\theta$ **-M** , مجموعه مغلقة من النمط **i-** $\theta$ **-M** , الدوال القوية الضعيفة من النمط $\theta$ **-M** , الدوال القوية من النمط $\theta$ **-M** ) وعممنا هذه المفاهيم في الفضاء ثنائي التبولوجي الفوقي وأجراء عدة مبرهنات مهمة في هذا الموضوع قد برهنت ودرسنا العلاقات بين تلك الدوال وافترضنا أشكال أخرى.