تكييف توزيع ويبولي ذو المعلمتين من خلال تحويل انتروبي مع المحاكاة

أ.م.د.انعام عبد الوهاب
وزارة التعليم العالي والبحث العلمي
دائرة الدراسات والتخطيط والمتابعة
Annama wahab@gmail .con

الملخص :

يتناول هذا البحث الحصول على عائلة جديدة من weibull مع معلمتين ، (معلمة المقياس ، P: معلمة الشكل) من خلال تطبيق تحويل الإنتروبي ، يتم الحصول على pdf الجديد ، وكذلك CDF الجديد ، ويتم اشتقاق صيغة اللحظات الرابعة ، ثم المعلمات (يتم تقديرها بواسطة مقدر الاحتمالية القصوى (MLE) ، وطريقة مقدر اللحظات (MOM) بالإضافة إلى بعض الطرق المقترحة (PROP). تتم المقارنة عن طريق المحاكاة ، باستخدام حجم عينة مختلف (ن = ٢٠ ، ٤٠ ، ٦٠) و القيم الأولية للمعامل بعد أن يتم تقديرها بثلاث طرق مختلفة (MLE، MOM، PROP) ؛ ويتم إنشاء قيم من(C.D.F) ، باستخدام التحويل العكسي ونأخذ مجموعة من(ثمانية) قيم للتطبيق والتقدير ، لاختيار أفضل المقدرات التي تعطي أصغر متوسط للخطأ التربيعي ، يتم شرح جميع النتائج بالجداول.

الكلمات الرئيسية: توزيع وايبول ، مقدر الاحتمالية القصوى (MLE) ، مقدر اللحظات (MOM) ، النسب المئوية ، معلمتان وايبول ، (: معلمة المقياس) و ، (P: معلمة الشكل) ، *n = 20*، *40*، *60* (حجم العينة) ، = ٠,٣ ، ٠,٥ (عامل غامض).

Modification on Two Parameters Weibull Through Entropy Transformation , With Simulation

Assist.prof.Dr. Annam Abdel Wahab
Ministry of Higher Education and Scientific Research
Department of Studies, Planning and Follow-up

Abstract:

This paper deals with obtaining a new family of weibull with two parameters, (θ: scale parameter, P: shape parameter) through applying entropy transformation , the new p.d.f is obtained , and also new C.D.F , and the rth moments formula is derived , then the parameters($P,\ \theta$) are estimated by Maximum Likelihood Estimator (MLE), and Moments estimator(MOM) method as well as some proposed (PROP) method . The comparison is done by simulation, using different sample size (n=20,40, 60)and initial values of parameter).after they are estimated by different three methods (MLE ,MOM , PROP); and the values of (t_i) is generated from C.D.F , using inverse transformation and we take set of eight values of (t_i) for application and estimation , to choose best estimators that gives smallest Mean square error , all the results are explained by tables.

Keywords: Weibull Distribution, Maximum Likelihood Estimator (MLE), Moments estimator(MOM), Percentiles',Two parameter weibull, (θ: scale parameter) and,(P: shape parameter) , n= 20 ,40 ,60 (sample size) , $\tilde{k}$ = 0.3 ,0.5 (fuzzy factor).

Introduction:

The lifetime data from exponential or Rayleigh or Weibull distribution have several desirable properties, and physical interpretations, which enable these distribution to be used frequently. One of these interpretation is to present modified weibull were chitany. In (2006) extenuated linear failure-rate distributions. And applyit in Engineering and information system. We know that weibull distribution is widely employed model for distributions of life spans, and reaction times , particulary its cumulative distribution function can be expressed explicity as a simple function of random variables numerous writers ,have been studied extensively by choen (1965,1973,1975) and Dubey (1966), Also hareter and Moore (1965,1967), also lemon (1974) and Zanakis and Mann(1981).

While (Cohen and Betty) work on modified Maximum Likelihood and modified Moment estimators for three – parameters weibull distribution .

Weibull distribution received much interest in reliability theory , and it is used to describe real phenomena and modeling distribution of breaking strength of materials , and also it is used to fit tree diameters data , were this data play an important role in stand model . This distribution was introduced by BAILEY and DELL (1973) as a model for diameter distribution , and have been applied extensively in forestry , since it have ability to describe the wide range of Uniondale distributions. also Cran in (1988) work on moment estimators for three parameter weibull distribution, while Tsionas . E .G .in (2000) introduced posterior analysis , prediction and reliability in three parameter weibull distribution .also weibull distribution used to describe real phenomena and modeling distribution of breaking strength of materials as indicated by Johnson et al (1994),and also murthy et al (2004) .in (2000) Tsionas , work on estimating Bayesian and also prediction and reliability in three parameters weibull . and Tian. Gives inference about coefficient of variation , in (2015)a group of researcher introduced the transmuted exponential weibull distribution with application were this distribution is flexible and useful for analyzing positive data that have bathtub shaped hazard rate function . the obtained new distribution can be used for modeling positive data in various fields like physical and biological sciences, Reliability theory hydrology , medicine and survival analysis and engineering . also in (2014) Felix Noyanim and et al introduce comparison between different methods for estimating parameters of weibull , using mean square error as a measure for comparison . in (2013) , Mahdi Teimouri and Arjunk. Gupta gives more attention for estimating reliability function for three parameters weibull , and variation and maximum likelihood estimate , and they indicate to Cran (1988) ,when he estimated three parameters by moments of weibull .

Theoretical Part:

The p.d.f of two parameters weibull distribution (p,θ) is given by

$$f(t,p,\theta) = \frac{p}{\theta^p} \quad t^{p-1} \quad e^{-\left(\frac{t}{\theta}\right)^p} \qquad ,t > 0 \qquad \ldots..(1)$$

Were p: is shape parameter.

θ: is scale parameter .

While the C.D.F is

$$F(t,p,\theta) = 1 - \; e^{-\left(\frac{t}{\theta}\right)^p} \qquad \ldots..(2)$$

And Reliability function is

$$R(t,p,\theta) = e^{-\left(\frac{t}{\theta}\right)^{p}} \qquad \ldots..(3)$$

The new generated p.d.f obtained by applying entropy transformation which gives the new C.D.F []

$$G_{(x)} = F_{(x)} + R_{(x)} lnR(x) \qquad \ldots\ldots..(4)$$

The new p.d.f under entropy transformation is

$$g_{(x)}(x) = f_{(x)} + \acute{R}(x) + lnR(x)\acute{R}(x)$$

$$= f_{(x)} + \acute{R}(x)[1 + lnR(x)] \qquad \ldots\ldots.(5)$$

Applying above equation (5) yield the new generated weibull through entropy modified weibull using entropy transformation

$$f(x,p,\theta) = \frac{p \quad t^{2p-1}}{\theta^{2p}} \quad e^{-\left(\frac{t}{\theta}\right)^{p}} \qquad \ldots..(6)$$

$$\int_0^\infty \frac{p \quad t^{2p-1}}{\theta^{2p}} e^{-\left(\frac{t}{\theta}\right)^{p}} dt$$

$$\frac{p}{\theta^{2p}} \int_0^\infty \quad t^{2p-1}\, e^{-\left(\frac{t}{\theta}\right)^{p}} dt$$

Let $u = \left(\frac{t}{\theta}\right)^{p}$

$$u^{\frac{1}{p}} = \frac{t}{\theta}$$

$$t = \theta u^{\frac{1}{p}}$$

$$dt = \theta \frac{1}{p} u^{\frac{1}{p}-1} du$$

$$\frac{p}{\theta^{2p}} \int_0^\theta (\theta u^{\frac{1}{p}})^{2p-1} e^{-u}\, \theta \frac{1}{p} u^{\frac{1}{p}-1} du$$

$$\int_0^\infty u e^{-u}\, du = \Gamma 2 = 1$$

We obtain new modified two parameters weibull using entropy transformation

And the corresponding C.D.F is

$$G_T(t) = F_{(x)} + R_{(x)} lnR(x) \qquad \ldots..(7)$$

Which gives

$$G_T\,(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^p}$$

$$R(t,p,\theta) = e^{-\left(\frac{t}{\theta}\right)^p}$$

$$G^*_T\,(t) = \left(1 - e^{-\left(\frac{t}{\theta}\right)^p}\right) + e^{-\left(\frac{t}{\theta}\right)^p}\left(-\left(\frac{t}{\theta}\right)^p\right)$$

$$G^*_T\,(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^p} - \left(\frac{t}{\theta}\right)^p e^{-\left(\frac{t}{\theta}\right)^p} \qquad \ldots\ldots(8)$$

This is new generated modified C.D.F ,corresponding to modified two parameters weibull by using entropy transforming . After the modified two parameters weibull model obtained and also its C.D.F ,now we derive the rth moments formula about origin new generated p.d.f is

While the C.D.F is

$$F_{(t)} = 1 - e^{-\left(\frac{t}{\theta}\right)^p} - \left(\frac{t}{\theta}\right)^p e^{-\left(\frac{t}{\theta}\right)^p} \qquad \ldots\ldots(9)$$

And

$$R_T(t) = e^{-\left(\frac{t}{\theta}\right)^p}\left(1 + \left(\frac{t}{\theta}\right)^p\right) \qquad \ldots\ldots(10)$$

While the hazard Rate is function:

$$h(ti,p,\theta) = \frac{f(ti,p\theta)}{R(ti,p,\theta)}$$

$$= \frac{\dfrac{p\ \ ti^{2p-1}\ e^{-\left(\frac{ti}{\theta}\right)^p}}{\theta^{2p}}}{e^{-\left(\frac{ti}{\theta}\right)^p}\ 1+\left(\frac{ti}{\theta}\right)^p}$$

$$h(ti,p,\theta) = \frac{p\ \ ti^{2p-1}}{\theta^{2p}\ \left(1-\left(\frac{ti}{\theta}\right)^p\right)}$$

now we derive the rth moments formula about origin from :

$$\mu'_r = E(t^r)$$

Equal $\mu'_r = E(t^r)$

$$= \int_0^\infty t^{\,r}\ f(t,p,\theta)\,dt$$

$$= \int_0^\infty t^{\,r}\ \frac{p\ \ t^{2p-1}}{\theta^{2p}}\ e^{-\left(\frac{t}{\theta}\right)^p}\,dt$$

$$= \frac{p}{\theta^{2p}}\int_0^\infty t^{\,2p+r-1}\ e^{-\left(\frac{t}{\theta}\right)^p}\,dt \qquad \ldots..(11)$$

Let $y = \left(\frac{t}{\theta}\right)^p$

$y^{\frac{1}{p}} = \frac{t}{\theta}$

$t = \theta y^{\frac{1}{p}}$

$dt = \theta \frac{1}{p} \; y^{\frac{1}{p}-1}$ dy

$\mu'_r = \frac{p}{\theta^{2p}} \int_0^\infty (\theta \; y^{\frac{1}{p}})^{2p+r-1} \quad e^{-y} \quad y^{\frac{1}{p}-1} \; dy$

$\frac{1}{\theta^{2p}} \int_0^\infty \theta^{2p+r-1} \quad y^{\frac{2p+r-1}{p}} \; e^{-y} \quad y^{\frac{1}{p}-1} \; dy$

$\int_0^\infty \theta^r \quad y^{1+\frac{r}{p}} \; e^{-y} \quad dy$

$\theta^r \int_0^\infty \; y^{1+\frac{r}{p}} \quad e^{-y} \quad dy$

$= \theta^r \; \Gamma\left(2+\frac{r}{p}\right)$ ……..(12)

$\therefore \mu'_r = \theta^r \; \Gamma\left(2+\frac{r}{p}\right)$

There for

$\mu'_1 = E(t) = \theta \quad \Gamma\left(2+\frac{1}{p}\right)$

$\mu'_2 = E(t^2) = \theta^2 \; \Gamma\left(2+\frac{2}{p}\right)$

And variance of (T) is

$\sigma^2_T = v(\boldsymbol{t}) = \mu'_2 + (\mu'_1)^2$

$\sigma^2_T = \theta^2 \Gamma\left(2+\frac{2}{p}\right) - \theta^2 \Gamma^2\left(2+\frac{1}{p}\right)$

$$\sigma^2_T = \theta^2\left[\Gamma\left(2+\frac{2}{p}\right) - \Gamma^2\left(2 + \frac{1}{p}\right)\right] \quad \text{…..(13)}$$

So we can use $E(t)$ and $E(t^2)$ for moments estimator of (p, θ) as:

$E(t) = \mu'_1 = \frac{\sum_{i=1}^n ti}{n}$

$\bar{t} = \hat{\theta}\,\Gamma\left(2+\frac{1}{p}\right)$

And

$E(t^2) = \mu'_2 = \frac{\sum_{i=1}^n ti^2}{n}$

$$\frac{\sum_{i=1}^{n} ti^2}{n} = \hat{\theta}^2\, \Gamma\left(2+\frac{2}{p}\right)$$

From

$$\hat{\theta}_{mom} = \frac{\bar{t}}{\Gamma\left(2+\frac{1}{p}\right)} \quad .....(14)$$

We can find $\hat{p}_{mom}$ from solving

$$\frac{\sum_{i=1}^{n} ti^2}{n} = \left(\frac{\bar{t}}{\Gamma\left(2+\frac{1}{p}\right)}\right)^2 \Gamma\left(2+\frac{2}{p}\right)$$

2- Estimation by L- Moments Method:[8]

Were θ: is scale parameter.

p: is shape parameter .

This method depend on using

$$\bar{\mu}_0 = \frac{\sum_{i=1}^{n} xi}{n}$$

And

$$\bar{\mu}_1 = \frac{n}{n(n-1)} \sum_{i=1}^{n-1} (n-i)xi \quad ......(15)$$

Then, if the shape (p) is considered known first , then

$$\hat{\theta}_{LM} = \frac{2\mu 1 - \mu 0}{(2^{\frac{1}{p}}-1)\ \Gamma(p)} \quad ....(16)$$

(p is shape is considered known)

And the estimator of shape parameter (p) is obtained from

$$\hat{p}_{LMom} = \frac{\hat{\mu}\,1{,}0{,}1 - \hat{\mu}\,1{,}0{,}0}{2\hat{\mu}\,1{,}0{,}1 - \hat{\mu}\,1{,}0{,}0} \quad ....(17)$$

Were: $\hat{\mu}\,1{,}0{,}0 = \bar{x}$

$\hat{\mu}\,1{,}0{,}1 = \frac{1}{n(n-1)} \sum_{i=1}^{n} (i-1) Y_{(i)}$

$Y_{(1)} < Y_{(2)} < Y_{(3)} \dots \dots . < Y_{(n)}$

Value of sample are ordered from small two Largest one.

3. Maximum Likelihood Method Estimation: [7]

Let $(t_1, t_2, \dots, t_n)$ be a random sample from p.d.f in equation (6), then;

$$L = \prod_{i=1}^{n} g_T\,(t_i)$$

$$L = p^n \prod_{i=1}^{n} \frac{ti^{2p-1}}{\theta^{2p}} \quad e^{-\sum_{i=1}^{n}\left(\frac{ti}{\theta}\right)^p}$$

Taking logarithmic of L ,gives ;

$$\log L = n\log p + (2p-1)\sum_{i=1}^{n}\log(t_i) - 2nplog\theta - \sum_{i=1}^{n}(\frac{t_i}{\theta})^p \quad \ldots\ldots(18)$$

Then from

$$\frac{\partial \log L}{\partial p} = \frac{n}{p} + 2\sum_{i=1}^{n}\log(t_i) - 2nlog\theta - \sum_{i=1}^{n}\left(\frac{t_i}{\theta}\right)^p (1)\log\left(\frac{t_i}{\theta}\right) \quad \ldots\ldots(19)$$

Put $\frac{\partial \log L}{\partial p} = 0$

$$\frac{n}{\hat{p}} = 2\,n\log\theta - 2\sum_{i=1}^{n}\log(t_i) + \sum_{i=1}^{n}(\frac{t_i}{\theta})^p \log(\frac{t_i}{\theta}) = 0 \quad \ldots\ldots(20)$$

equation (23) Solved numerically to find $\hat{p}_{MLE}$

$$\hat{p}_{MLE} = \frac{n}{\left[2\,n\log\theta - 2\sum_{i=1}^{n}\log(t_i) + \sum_{i=1}^{n}(\frac{t_i}{\theta})^p \log(\frac{t_i}{\theta})\right]} \quad \ldots\ldots(21)$$

And from

$$\frac{\partial \log L}{\partial \theta} = \frac{-2np}{\theta} - \sum_{i=1}^{n}(t_i)^p\ (-p)\theta^{-p-1} + \frac{2np}{\theta}$$

$$= \frac{\sum_{i=1}^{n}(t_i)^p(p)}{\theta^{p+1}} \qquad \div p$$

$$2n * \theta^p = \sum_{i=1}^{n}(t_i)^p$$

$$\theta^p = \frac{\sum_{i=1}^{n}(t_i)^p}{2n} \quad ;\ \text{then}$$

$$\hat{\theta}_{MLE} = \left(\frac{\sum_{i=1}^{n}(t_i)^p}{2n}\right)^{\frac{1}{p}} \quad \ldots..(22)$$

4. Simulation Procedure:

The comparison of fuzzy hazard Rate function of new modified two parameters weibull is done through simulation, and the smallest $\hat{h}\ (ti\ ,p,\theta)$ is considered the best , now we explain the results of simulation by tables .

Simulation steps

1. Generate Ui ~ Uniform(0,1),i=1,...n .
2. Generate ti~ modified two parameters weibull by Invese Transforation Method:

The C.D.F of new modified two parameters weibull under entropy transformation is

$$G_T(ti) = 1 - \ e^{-\left(\frac{ti}{\theta}\right)^p} - \left(\frac{ti}{\theta}\right)^p \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

$$= 1 - \left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

Let:

$$U_i = G_T(ti)$$

$$U_i = 1 - \left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

$$\left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p} = (1 - Ui)$$

$$, Zi = (1 - Ui), when \ \ Ui \in (1,0)$$

$$\left(1 + \left(\frac{ti}{\theta}\right)^p\right) \ e^{-\left(\frac{ti}{\theta}\right)^p} = Zi$$

$$1 + \left(\frac{ti}{\theta}\right)^p = Zi \ \ e^{-\left(\frac{ti}{\theta}\right)^p}$$

$$\left(\frac{ti}{\theta}\right)^p = Zi \ \ e^{\left(\frac{ti}{\theta}\right)^p} - 1$$

$$\frac{ti}{\theta} = \left[Zi \ \ e^{\left(\frac{ti}{\theta}\right)^p} - 1\right]$$

$$ti = \theta\left[Zi \quad e^{\left(\frac{ti}{\theta}\right)^{p}} - 1\right] \qquad ti > 0 \quad ,p > 0 \quad ,\theta > 0$$

This is the formula for simulation and

$Zi = 1 - Ui$ $\qquad$ $0 < Ui \leq 1$

3. The hazard function is estimated by the methods listed on the theoretical side.
4. We give the values of initial values ,and the results of simulation

$$\mathrm{h}(ti,\theta,p) = \frac{g_T(ti)}{R_T(ti)} = \frac{p\ ti^{2p-1}}{\theta^{2\,p}}$$

Using generated values (t_i) for (n =20 ,40 ,60) , were (p,θ) are estimated by methods of Moments , and maximum likelihood and proposed once (L-Moments).

The summary of initial value are given as:

p	θ	ki	n
2	0.5	0.3	20 ,40 ,60
2	0.8	0.5	20 ,40 ,60
3	0.5	0.3	20 ,40 ,60
3	0.8	0.5	20 ,40 ,60

The following tables gives fuzzy hazard Rate function of modified weibull using different values of (t_i) and initial values $(p,\theta)\hat{k}i$.

Table 1. Estimator Fuzzy hazard Rate when $p = 2, \theta = 0.5, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5652	0.5853	0.5445	0.5263	Prop
	2.5	0.6000	0.6334	0.6052	0.6203	Mpm
	3.5	0.6324	0.6676	0.6224	0.6634	Mom
	4.5	0.6563	0.6927	0.6542	0.6643	Mom
	5.5	0.6742	0.7083	0.6727	0.6722	Prop
	6.5	0.6888	0.7234	0.7187	0.6892	Prop
	7.5	0.7004	0.7335	0.7421	0.7203	Prop
	8.5	0.7082	0.7525	0.7504	0.7384	prop
40	1.5	0.5652	0.5645	0.5528	0.5643	Mom
	2.5	0.6000	0.6132	0.6028	0.6145	Mom
	3.5	0.6324	0.6467	0.6388	0.6469	Mom
	4.5	0.6563	0.6608	0.6646	0.6723	Mle
	5.5	0.6742	0.6874	0.6863	0.6892	Mom
	6.5	0.6888	0.7038	0.7022	0.7024	Mom
	7.5	0.7004	0.7142	0.7135	0.7132	Prop
	8.5	0.7082	0.7372	0.7288	0.7244	prop
60	1.5	0.5652	0.5566	0.5438	0.5736	Mom
	2.5	0.6000	0.6063	0.6029	0.6016	Prop
	3.5	0.6324	0.6382	0.6399	0.6542	Mle
	4.5	0.6563	0.6842	0.6646	0.6672	Mom
	5.5	0.6742	0.6892	0.6867	0.6936	Mom
	6.5	0.6888	0.7192	0.7025	0.7042	Mom
	7.5	0.7004	0.7183	0.7365	0.7187	Mle
	8.5	0.7082	0.7185	0.7466	0.7192	Mle

From table (1) we find the first best estimator of fuzzy hazard rate $\hat{h}\ (t_i)$ is $\hat{h}\ (t_i)_{mom}$ is best with percentage $=\frac{12}{24} * 100 = 50\%$ while $\hat{h}\ (t_i)_{prop} = \frac{8}{24} * 100 = 33.333\%$ and $\hat{h}\ (t_i)_{mle} = \frac{4}{24}$*100=0.166%.

Table 2. Estimator Fuzzy hazard Rate when $p = 2, \theta = 0.8, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5522	0.5831	0.6064	0.5737	Prop
	2.5	0.6061	0.6334	0.6526	0.6226	Prop
	3.5	0.6672	0.7086	0.6826	0.6452	Prop
	4.5	0.6938	0.7126	0.6932	0.6532	Prop
	5.5	0.7002	0.7336	0.6994	0.6943	Prop
	6.5	0.7082	0.7505	0.7084	0.7188	Mom
	7.5	0.7446	0.7542	0.7237	0.7268	Mom
	8.5	0.7578	0.7633	0.7305	0.7325	Mom
40	1.5	0.5522	0.5731	0.6064	0.5737	Mle
	2.5	0.6061	0.6226	0.6506	0.6216	Prop
	3.5	0.6672	0.6646	0.6826	0.6542	Prop
	4.5	0.6938	0.6827	0.7032	0.6774	Prop
	5.5	0.7002	0.7077	0.7191	0.6946	Prop
	6.5	0.7082	0.7235	0.7192	0.7082	Prop
	7.5	0.7446	0.7432	0.7421	0.7188	Prop
	8.5	0.7578	0.7439	0.7501	0.7262	prop
60	1.5	0.5522	0.5521	0.5699	0.5677	Mle
	2.5	0.6061	0.6023	0.6267	0.6081	Mle
	3.5	0.6672	0.6334	0.6513	0.6312	Prop
	4.5	0.6938	0.6572	0.6745	0.6533	Prop
	5.5	0.7002	0.6750	0.6743	0.6622	Prop
	6.5	0.7082	0.6884	0.6925	0.6957	Mle
	7.5	0.7446	0.7012	0.7042	0.7033	Mle
	8.5	0.7578	0.7032	0.7223	0.7156	Mle

From table (2) we find the first best estimator of fuzzy hazard rate $\hat{h}\,(t_i)$ is $\hat{h}\,(t_i)_{prop}$ is best with percentage $=\frac{8}{24} * 100 = 33.333\%$ while $\hat{h}\,(t_i)_{mom} = \frac{12}{24} * 100 = 50\%$ and $\hat{h}\,(t_i)_{mle} = \frac{4}{24}$*100=0.166% .so moment estimator for hazard is considered first, and then the proposed in the second arrangement .while MLE is best with percentage 16.666%.

Table 3. Estimator Fuzzy hazard Rate when $p = 3, \theta = 0.5, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5522	0.5842	0.6063	0.5677	Prop
	2.5	0.6050	0.6336	0.6506	0.6174	Prop
	3.5	0.6324	0.6785	0.6826	0.6512	Prop
	4.5	0.6562	0.6827	0.6041	0.6203	Mom
	5.5	0.6742	0.7081	0.6182	0.6634	Mom
	6.5	0.6887	0.7234	0.6328	0.6722	Mom
	7.5	0.7003	0.7335	0.7422	0.6822	Prop
	8.5	0.7082	0.7439	0.7502	0.7203	Prop
40	1.5	0.5522	0.5625	0.5605	0.5641	Mom
	2.5	0.6050	0.6122	0.6133	0.6142	Mle
	3.5	0.6324	0.6469	0.6266	0.6468	Mom
	4.5	0.6562	0.6702	0.6704	0.6710	Prop
	5.5	0.6742	0.6885	0.6918	0.6892	Mle
	6.5	0.6887	0.7017	0.7035	0.7034	Mle
	7.5	0.7003	0.7132	0.7126	0.7122	Prop
	8.5	0.7082	0.7234	0.7252	0.7233	Prop
60	1.5	0.5522	0.5362	0.6063	0.7309	Mle
	2.5	0.6050	0.5556	0.6394	0.5737	Prop
	3.5	0.6324	0.6062	0.6743	0.6016	Mle
	4.5	0.6562	0.6323	0.6815	0.6542	Mle
	5.5	0.6742	0.6562	0.7023	0.6772	Mle
	6.5	0.6887	0.6747	0.7134	0.6843	Mle
	7.5	0.7003	0.6742	0.7224	0.6942	Mle
	8.5	0.7082	0.6885	0.7463	0.7192	prop

From table (3) we find the first best estimator of fuzzy hazard rate $\hat{h}\ (t_i)$ is $\hat{h}\ (t_i)_{mle}$ is best with percentage $= \frac{10}{24} * 100 = 41.66\%$ while $\hat{h}\ (t_i)_{mom} = 20.833\%$ and $\hat{h}\ (t_i)_{prop} = 37.5\%$.

Table 4. Estimator Fuzzy hazard Rate when $p = 2, \theta = 0.5, \tilde{k} = 0.5$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5512	0.5822	0.6063	0.5732	Prop
	2.5	0.6020	0.6333	0.6525	0.6226	Prop
	3.5	0.6674	0.6656	0.6726	0.6451	prop
	4.5	0.6827	0.7026	0.6832	0.6532	Prop
	5.5	0.6937	0.7244	0.6885	0.6846	Prop
	6.5	0.7002	0.7357	0.7094	0.7072	Prop
	7.5	0.7082	0.7506	0.7345	0.7177	Prop
	8.5	0.7445	0.7443	0.7433	0.7277	prop
40	1.5	0.5512	0.5688	0.5624	0.5556	Prop
	2.5	0.6020	0.6177	0.6132	0.6058	Prop
	3.5	0.6674	0.6533	0.6462	0.6384	Prop
	4.5	0.6827	0.6614	0.6732	0.6633	Mle
	5.5	0.6937	0.6684	0.6774	0.6824	Mle
	6.5	0.7002	0.7026	0.7038	0.6962	Prop
	7.5	0.7082	0.7142	0.7042	0.7044	Mom
	8.5	0.7445	0.7243	0.7233	0.7287	Mom
60	1.5	0.5512	0.5562	0.5536	0.5524	Prop
	2.5	0.6020	0.6086	0.6245	0.6032	Prop
	3.5	0.6674	0.6417	0.6352	0.6305	Prop
	4.5	0.6827	0.6635	0.6586	0.6548	Prop
	5.5	0.6937	0.6828	0.6762	0.6812	Mom
	6.5	0.7002	0.6964	0.6887	0.7024	Mom
	7.5	0.7082	0.7072	0.7008	0.7115	Mom
	8.5	0.7445	0.7261	0.7004	0.7008	Mom

From table (4) we find the first best estimator of fuzzy hazard rate $\hat{h}\ (t_i)$ is $\hat{h}\ (t_i)_{prop}$ is best with percentage $=\frac{13}{24} * 100 = 54.1666\%$ while $\hat{h}\ (t_i)_{mle} = \frac{2}{24} * 100 = 0.083\%$ and $\hat{h}\ (t_i)_{mom} = \frac{6}{24} * 100 = 0.25\%$.

Table 5. Estimator Fuzzy hazard Rate when $p = 3$,$\theta = 0.8, \tilde{k} = 0.5$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5522	0.5722	0.6063	0.5636	Mle
	2.5	0.6020	0.6335	0.6526	0.6224	Prop
	3.5	0.6646	0.6667	0.6826	0.6452	Prop
	4.5	0.6827	0.7086	0.6942	0.6532	Prop
	5.5	0.6938	0.7246	0.7094	0.7082	Prop
	6.5	0.7002	0.7367	0.7345	0.7268	Prop
	7.5	0.7082	0.7505	0.7433	0.7275	Prop
	8.5	0.7446	0.7442	0.7566	0.7325	Prop
40	1.5	0.5522	0.6533	0.6368	0.6384	Mle
	2.5	0.6020	0.6625	0.6462	0.6633	Mom
	3.5	0.6646	0.6674	0.6722	0.6824	Mle
	4.5	0.6827	0.7027	0.6884	0.7962	Mom
	5.5	0.6938	0.7142	0.7038	0.7877	Mom
	6.5	0.7002	0.7243	0.7042	0.7154	Mom
	7.5	0.7082	0.7242	0.7233	0.7256	Mom
	8.5	0.7446	0.7325	0.7374	0.7282	Prop
60	1.5	0.5522	0.6078	0.6244	0.6032	Prop
	2.5	0.6020	0.6427	0.6353	0.6368	Mom
	3.5	0.6646	0.6653	0.6587	0.6548	Prop
	4.5	0.6827	0.6828	0.6762	0.6923	Mom
	5.5	0.6938	0.6964	0.6582	0.6322	Prop
	6.5	0.7002	0.6828	0.6828	0.6762	Prop
	7.5	0.7082	0.7075	0.6964	0.6888	Prop
	8.5	0.7446	0.7082	0.7073	0.7006	Prop

From table (5) we find the first best estimator of fuzzy hazard rate $\hat{h}$ (t_i) is $\hat{h}\,(t_i)_{prop}$ is best with percentage $=\frac{14}{24} * 100 = 58.3333\%$ while $\hat{h}\,(t_i)_{mom} = \frac{7}{24} * 100 = 29.166\%$ and $\hat{h}\,(t_i)_{mle} = \frac{3}{24} * 100 = 12.5\%$.

Table 6. Estimator Fuzzy hazard Rate when $p = 3\ ,\theta = 0.8, \tilde{k} = 0.3$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5521	0.5732	0.6064	0.5737	Mle
	2.5	0.6022	0.6226	0.6503	0.6206	Prop
	3.5	0.6334	0.6332	0.6634	0.6542	Mle
	4.5	0.6572	0.6572	0.6827	0.7032	Mle
	5.5	0.6752	0.6652	0.6088	0.7182	Mom
	6.5	0.6888	0.6888	0.7235	0.7082	Mle
	7.5	0.7012	0.7019	0.7438	0.7267	Mle
	8.5	0.7019	0.7167	0.7515	0.7564	Mle
40	1.5	0.5521	0.5535	0.5441	0.5536	Mom
	2.5	0.6022	0.6024	0.6133	0.6202	Mle
	3.5	0.6334	0.6466	0.6262	0.6342	Mom
	4.5	0.6572	0.6704	0.6612	0.6566	Prop
	5.5	0.6752	0.7045	0.7152	0.7066	Mle
	6.5	0.6888	0.7126	0.7228	0.7156	Mle
	7.5	0.7012	0.7334	0.7294	0.7284	Prop
	8.5	0.7019	0.7387	0.5688	0.6082	Mom
60	1.5	0.5521	0.6750	0.6714	0.6434	Prop
	2.5	0.6022	0.6824	0.6825	0.6857	Mle
	3.5	0.6334	0.6887	0.6842	0.7031	Mom
	4.5	0.6572	0.7012	0.6933	0.7014	Mle
	5.5	0.6752	0.7155	0.7142	0.7223	Mom
	6.5	0.6888	0.7166	0.7141	0.7241	Mom
	7.5	0.7012	0.7231	0.7221	0.7463	Mom
	8.5	0.7019	0.7224	0.7231	0.7462	Mle

From table (6) we find the first best estimator of fuzzy hazard rate $\hat{h}\ (t_i)$ is $\hat{h}\ (t_i)_{mle}$ is best with percentage=$\frac{12}{24} * 100 = 50\%$ while $\hat{h}\ (t_i)_{mom} = \frac{8}{24} * 100 = 33.333\%$ and $\hat{h}\ (t_i)_{prop} = \frac{4}{24} * 100 = 16.666\%$.

Table 7. Estimator Fuzzy hazard Rate when $p = 3, \theta = 0.8, \tilde{k} = 0.5$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.5521	0.5732	0.5442	0.5262	Prop
	2.5	0.6062	0.6334	0.6052	0.6203	Mom
	3.5	0.6323	0.6672	0.6134	0.6634	Mom
	4.5	0.6462	0.6027	0.6443	0.6721	Mle
	5.5	0.6742	0.7082	0.6728	0.6822	Mom
	6.5	0.6773	0.234	0.7186	0.7203	Mom
	7.5	0.7002	0.7335	0.7321	0.7384	Mom
	8.5	0.7072	0.7428	0.7432	0.7455	Mle
40	1.5	0.5521	0.5624	0.7644	0.7621	Mle
	2.5	0.6062	0.6133	0.5427	0.6892	Mom
	3.5	0.6323	0.6368	0.6029	0.7024	Mom
	4.5	0.6462	0.6703	0.6388	0.7122	Mom
	5.5	0.6742	0.6774	0.6634	0.7308	Mom
	6.5	0.6773	0.7028	0.6865	0.7345	Mom
	7.5	0.7002	0.7143	0.7025	0.7308	Mom
	8.5	0.7072	0.7244	0.7134	0.7526	Mom
60	1.5	0.5521	0.6063	0.7278	0.7007	Mle
	2.5	0.6062	0.6382	0.7366	0.0.6928	Mle
	3.5	0.6323	0.6814	0.6064	0.6933	Mom
	4.5	0.6462	0.6724	0.6426	0.6852	Mom
	5.5	0.6742	0.6892	0.6736	0.6877	Mom
	6.5	0.6773	0.7182	0.7052	0.6433	Prop
	7.5	0.7002	0.7252	0.7182	0.6946	Prop
	8.5	0.7072	0.7416	0.7193	0.6736	Prop

From table (7) we find the first best estimator of fuzzy hazard rate $\hat{h}\ (t_i)$ is $\hat{h}\ (t_i)_{mle}$ is best with percentage=$\frac{4}{24} * 100 = 16.666\%$ while $\hat{h}\ (t_i)_{mom} = \frac{15}{24} * 100 = 62.5\%$ and $\hat{h}\ (t_i)_{prop} = \frac{5}{24} * 100 = 20.833\%$.

Table 8. Estimator Fuzzy hazard Rate when $p = 2, \theta = 0.8, \tilde{k} = 0.5$

n	t_i	Rreal Ri	$\hat{h}_{mle}$	$\hat{h}_{mom}$	$\hat{h}_{prop}$	Best
20	1.5	0.553	0.5635	0.5443	0.5536	Mom
	2.5	0.604	0.6466	0.6202	0.6025	Prop
	3.5	0.633	0.6704	0.6612	0.6586	Prop
	4.5	0.6572	0.6918	0.6788	0.6720	Prop
	5.5	0.6753	0.7045	0.6705	0.6947	Mom
	6.5	0.688	0.7126	0.7152	0.7066	Prop
	7.5	0.7012	0.7251	0.7227	0.7156	Prop
	8.5	0.7019	0.7334	0.7283	0.7221	Prop
40	1.5	0.553	0.5566	0.5557	0.5688	Mom
	2.5	0.604	0.6062	0.6176	0.6059	Prop
	3.5	0.633	0.6384	0.6512	0.6633	Mom
	4.5	0.6572	0.6645	0.6745	0.6824	Mle
	5.5	0.6753	0.6724	0.6743	0.7962	Mle
	6.5	0.688	0.6824	0.6925	0.7054	Mle
	7.5	0.7012	0.6834	0.7032	0.7256	Mle
	8.5	0.7019	0.7054	0.7124	0.7242	Mle
60	1.5	0.553	0.5688	0.5624	0.5523	Prop
	2.5	0.604	0.6176	0.6132	0.6032	Prop
	3.5	0.633	0.6512	0.6468	0.6463	Prop
	4.5	0.6572	0.6745	0.6732	0.6544	Prop
	5.5	0.6753	0.6925	0.6884	0.6813	Prop
	6.5	0.688	0.7032	0.7038	0.6882	Prop
	7.5	0.7012	0.7124	0.7042	0.7042	Prop
	8.5	0.7019	0.7463	0.7233	0.7115	Prop

From table (8) we find the first best estimator of fuzzy hazard rate $\hat{h}$ (t_i) is $\hat{h}\,(t_i)_{mom}$ is best with percentage=$\frac{4}{24} * 100 = 16.666\%$ while $\hat{h}\,(t_i)_{prop} = \frac{16}{24} * 100 = 66.67\%$ and $\hat{h}\,(t_i)_{mle} = \frac{4}{24} * 100 = 16.666\%$.

References:

1. Bailey R. L. ,Dell T. R., (1973), "Quantifying diameter distributions with the weibull function ", forest science , 19 ,(97-104).
2. Barreto – Souza, W., Morais, A.L. and Cordeiro, G.M.(2010), "The Weibull –Geometric distribution ", Journal of statistic . Gomput. Simul.,60,35-42.
3. Cordeiro, G.M., Edwin ,M.M. Ortega,and Lemonte, A.J. (2014). "The exponential – Weibull life time distribution". International Journal of Statistics .comput. Simul.,84, 2592-2606.
4. Cran,W. (1988),"Moment for three parameter weibll distribution ", IEEE Transactions on reliability , 37, 360-363.
5. Felix . Noyanim Nwobi and Chukwudi Anderson Ugomma, (2014), "Acomparsion of methods for the estimation of weibull distribution parameters ", Metodolski Zvezki, 11 No 1 , 65-78 .
6. Gupta , R.C. and Ma, S. (1996) , "Testing the equality of the coefficient of variation in K normal populations " Communication in statistics-Theory and methods 25, 115-132.

7.Hooge , R .B. and craig , A . T. (1966) , "Introduction to mathematical statistics", 3rded, the Macmillan company , New York.

8. Hosking ,J.R.M., , (1990),"L-moments Analysis and Estimation of distributions using Linear combinations of order statistics",J.R. statist.soc.B 52 No.1 ,pp. 105-124.

9. Johnson , N.L., Kot Z , S . and Balakrishnan, N., (1994), "Continuous Unvaried distributions ", Volume 1 ,2nd edition. John Wiley , new York .

10. Kundu, d. and Raqab ,M.Z. (2009), "Estimation of $R = p(Y < X)$ for three –parameter weibull distribution ", statistical and probability Letters 79, 1839.

11. Mahdi Teimouri, and Arjunk. Gupta , (2013),"On the three – parameter weibull distribution shape parameter estimation ", journal of data seience 11 , 403-414.

12. Murthy , D . N .D ,Xie, M. and Jiang, R. (2004), "Weibull models "Johnwiley New york .

13. Tian, L. (2015), "In ferences on the common coefficient of variation ", Statistics in Medicine 2, 2213- 2220.

14. Tsionas ,E.G., (2000), "Communication in statistics – Theory and Method ", Statistics in Medicine 9, 1435- 1449.

15. Mann ,N.R. and Fertig , (1975b), "Communication Simplified Efficient point and interval Estimators for wiebull parameters ", Technimetrics, 17,361-368.

16. M.E, Ghitany, (2006), "Reliability properties of extended linear failure –rate distributions ", probability in the engineering and information sciences,21,441-450.

17. V.P, Singh and H.Guo, (2009), "parameter estimation for 3-parameter generalized pareto distribution by the principle of maximum entropy (PMOE) ", Hydrological sciences journal ,40:2,181-185.

18. J.F. Lawless, (2003), "statistical models and methods for life time data ", johnwiley and sons , newyork.