IRAQI
Academic Scientific Journals

University of Kirkuk Journal For Administrative and Economic Science (2024) 14 (2): 178-189

UKJAES

University of Kirkuk Journal For Administrative and Economic Science

ISSN 2222-2995
University of Kirkuk Journal For Administrative and Economic Science

Taher Hindreen A., Salih Kanar K. & Alwan Asraa S. .Forecasting For Silver Closing Price and Modifying Predictions by Using GARCH (1, 1) Wavelet Transformation. *University of Kirkuk Journal For Administrative and Economic Science* (2024) 14 (2):178-189.

Forecasting For Silver Closing Price and Modifying Predictions by Using GARCH (1, 1) Wavelet Transformation

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Abstract: Silver, a precious metal, sees its spot price influenced not only by current supply and demand dynamics but also by investors' expectations regarding future inflation and broader economic conditions. Hence, the primary aim of this research is to forecast the annual closing silver price and refine this forecasting using wavelet transformation. The study intends to scrutinize the time series data of yearly closing silver prices from 1969 to 2022. Employing the modern approach of GARCH (1, 1) methodology, known for its accuracy and adaptability in time series analysis, the research seeks to enhance forecasting precision by incorporating wavelet transformation. Specifically, the focus lies on forecasting yearly closing silver prices using the GARCH (1, 1) model and refining these forecasts through wavelet transformation. The findings reveal that the GARCH (1, 1) model offers the best fit and efficiency, as indicated by minimal metrics such as RMSE, MSE, and AIC. Moreover, upon applying wavelet GARCH (1, 1) to the GARCH (1, 1) model, the adjusted forecasts exhibit reduced RMSE, MSE, and AIC compared to the original GARCH (1, 1) forecasting.

Keywords: ARCH and GARCH method and Wavelet de-noising, Box -jenkins method Time series analysis, silver price.

التنبؤ بسعر الفضة عند إغلاق السوق باستخدام التحويل المويجي لنموذج GARCH(1,1)

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المستخلص: تتأثر أسعار الفضة وهي من المعادن الثمينة، ليس فقط بديناميكيات العرض والطلب الحالية، ولكن أيضًا بتوقعات المستثمرين بشأن التضخم المستقبلي والظروف الاقتصادية الأوسع. وبالتالي، فإن الهدف الأساسي من هذا البحث هو التنبؤ بسعر إغلاق الفضة السنوي وتحسين هذه التنبؤات باستخدام تحويل المويجات. تهدف

الدراسة إلى تحليل بيانات السلاسل الزمنية لأسعار إغلاق الفضة السنوية من عام ١٩٦٩ إلى ٢٠٢٢. باستخدام منهجية GARCH(1,1) الحديثة، المعروفة بدقتها وقابليتها للتكيف في تحليل السلاسل الزمنية، يسعى البحث إلى تحسين دقة التنبؤ من خلال دمج تحويل المويجات. تحديدًا، يركز البحث على التنبؤ بأسعار إغلاق الفضة السنوية باستخدام نموذج GARCH(1,1) ومن ثم تحسين هذه التنبؤات عبر تحويل المويجات. وتكشف النتائج أن نموذج GARCH(1,1) يوفر أفضل ملاءمة وكفاءة، كما هو موضح من خلال الحد الأدنى لمقاييس مثل RMSE وMSE وAIC. عند تطبيق تحويل المويجات على نموذج GARCH(1,1)، تظهر التنبؤات المعدلة انخفاضًا في RMSE وMSE و AIC مقارنةً بتنبؤات نموذج GARCH(1,1) الأصلية.

الكلمات المفتاحية: طريقة ARCH وGARCH وإزالة الضوضاء المويجية، طريقة بوكس-جينكز، تحليل السلاسل الزمنية، سعر الفضة.

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1- Introduction

Like other precious metals, silver has long been seen as a good investment. It has been recognized as a medium of exchange and a store of value for more than 4,000 years. However, after the silver standard was abandoned in 1935, its value as legal tender in developed nations declined. Despite this, some countries continue to create collector coins and bullion with nominal face values, such as the American Silver Eagle. Additionally, statistical tools offer ways to analyze these events, especially because time is crucial in these kinds of investigations. An effective statistical method for predicting annual closing silver prices, which are influenced by fluctuations in the economy, is time series analysis. In this study, wavelet modification is employed to improve the forecasting precision of time-series models utilizing Generalised Autoregressive Conditional Heteroskedasticity (GARCH). The models known as Autoregressive (AR) and Moving Average (MA) are well-known in many fields, such as time series forecasting, biomedical engineering, and spectrum estimation. In particular, the ARMA model is a preferred parametric approach that makes use of input and output data to comprehend physiological system dynamics.

2- Literature Review

Al Wadi et al. (2010) used a variety of approaches to study the use of wavelet ARIMA models as an alternative to conventional ARIMA models for volatility forecasting. They showed that wavelet ARIMA outperforms ARIMA in terms of prediction when it comes to Amman stock forecasting. To do this, each Daubechies wavelet that was produced during the decomposition process has to have ARIMA applied to it separately. Furthermore, to predict volatility, several researchers combined hybrid ARIMA models with feedforward neural networks (FNN) and the maximal overlap discrete wavelet transform (MODWT). When it came to goodness-of-fit in the time series, the hybrid ARIMA-FNN model performed better than ARIMA because it used ARIMA to first identify linear patterns and then FNN to predict residuals and refine prediction errors that came from ARIMA.
Conversely, Saadatmand & Komasi (2024) emphasized the growing significance and worth of surface and groundwater resources on a worldwide scale, attributing this to their limited availability. Comprehending the amounts of surface and groundwater is essential for efficient planning at each interval. Their research enhances the current body of knowledge on predicting stream flow and groundwater levels by introducing a hybrid model that combines wavelet transform (WT) and generalized autoregressive conditional heteroskedasticity (GARCH) algorithms with the M5p model tree (M5pMT). This model is used to forecast the levels of streamflow and sub-surface water in the Silakhor Plain over 17 years from 1997 to 2014. The efficacy of the proposed hybrid model in predicting monthly streamflow and groundwater levels is assessed by comparing it to several benchmark models utilizing several assessment criteria. The results suggest that combining artificial intelligence (AI) models with the GARCH model, which describes errors for the remaining subseries divided by WT, improves the accuracy of predictions. The W-GARCH-M5pMT model exhibits a decrease of around 14% in root mean square error (RMSE) and 2% in mean absolute error (MAE) in comparison to the standalone M5pMT model for streamflow prediction. The accuracy of groundwater prediction is enhanced by W-GARCH-based models. Specifically, for

artificial neural networks (ANN), the accuracy improves from 0.953 to 0.965. For extreme learning machines (ELM), the accuracy improves from 0.955 to 0.976. Lastly, for the M5pMT model, the accuracy improves from 0.95 to 0.994.
In contrast, Ren et al. (2023) examined the prediction of volatility by employing widely recognized deep learning methods such as LSTM. Studies have demonstrated that LSTM, because of its exceptional ability to handle uneven variations, achieves comparable or superior performance compared to SVR, a widely used choice for forecasting volatility. One key advantage of LSTM is its superior computational efficiency, which allows it to be trained using graphics cards. Current research is also exploring the application of artificial neural networks in stacked machine learning models for predicting the level of volatility in the S&P 500 index. These models integrate forecasts from GDB, Random Forest, and SVM using a specific function to generate a single prediction. They demonstrate competitive performance compared to traditional independent models.
The company's goal is to obtain accurate and reliable forecasts for this volatile data type, and each of the previously stated studies has presented new methods for volatility forecasting. Significant noise and complex behavior that is difficult to adjust to are characteristics of instability data; these characteristics are impacted by several outside variables and current market conditions. No model combining wavelet transform, ARIMA, and GARCH for volatility prediction was found even after a thorough review of the literature. This proposed approach analyses the approximate and detailed components obtained from discrete wavelet transform using ARIMA and GARCH.

A. Building a GARCH Model [1]

In time series analysis, the construction of any ARCH or GARCH model requires the following steps:

First Moment Model Construction: The first step in constructing any ARCH or GARCH model is to build a suitable first-moment model. Various methods, including autoregressive integrated moving averages (ARIMA), regression, and transfer function models, can be employed to do this. The objective of this stage is to construct a model that represents the average value of the time series data, given certain conditions. After establishing the initial moment model, the residual series of this model is used in later phases to analyze the current GARCH effects.

Examination for GARCH Effects: Finding GARCH effects in the data comes next after getting the residual series from the first-moment model. Where the variance of the time series data is not constant over time but shows clustering or persistence in volatility, GARCH effects are present. GARCH effects in the residual series are detected using statistical tests, autocorrelation analysis, and visual inspection.

Specification of GARCH Model: A suitable GARCH model has to be specified after GARCH effects are found. Finding the order of the GARCH model the number of error terms and lagged conditional variances in the model is part of this. The order of the GARCH model, marked by 'p' for the autoregressive component and 'q' for the moving average component, is known as GARCH (p, q).

Parameter Estimation: To ascertain the main features of the GARCH model, we must first identify it. Maximum likelihood estimate (MLE) is a widely used technique for doing this. Considering the GARCH model, MLE essentially looks for the parameter configuration that will allow it to observe the real data we have the best. This is determining the best values for a mathematical function that reflects this probability using specialized computer methods.

B. Testing GARCH Effects (Test of heteroscedasticity) [6] [4] [2]

Test for GARCH effects, or heteroskedasticity, is one of the methods used in several methods to find conditional volatility clustering in time series statistics. Engle's Lagrange Multiplier test and the Ljung-Box Portmanteau Q statistics are the two widely utilized techniques.

(1) Ljung-Box Portmanteau Q Statistics [8]

To conduct this test, time series data can be fitted with a GARCH model, and the residuals can then be obtained. Next, square these residuals to get the squared residuals, and figure out the squared residuals' autocorrelation function (ACF). This determines the correlation between the squared residuals at various lags and uses the following formula to compute the Q statistic, also known as the Ljung-Box statistic:

$$Q = n(n+2) \sum_{k=1}^{h} \frac{\hat{p}_k^2}{n-k} \qquad (1)$$

Where n is the sample size, $\hat{p}^2_k$ are the sample autocorrelations of the squared residuals at lag k, and h is the number of lags used in the test. It can be the computed Q statistic compared with the critical values from the chi-square distribution at the significance level that is desired. If the calculated Q statistic is greater than the critical value, it indicates the incidence of autocorrelation in the squared residuals, demonstrating GARCH effects.

C. Volatility [7]

The degree of variability or dispersion of a variable over time is because of instability in time series data. In financial markets, volatility typically refers to the degree of variation in the prices or returns of financial assets such as stocks, bonds, or currencies. However, volatility can also be observed in various other types of time series data, such as economic indicators, weather patterns, or physiological measurements. There are two popular types of volatility:

- **Historical Volatility:** Utilising historical price or return data, this method quantifies the extent to which the price or returns of an asset have varied throughout a designated timeframe.
- **Implied Volatility:** derived from option pricing and represents the volatility predictions of the market going forward.

D. Identification of a GARCH Model [2, 6]

The correct order of a GARCH (Generalised Autoregressive Conditional Heteroskedasticity) model and the lag lengths for the autoregressive and moving average components are determined by the identification of the model. This is the usual flow of the identifying procedure:

- Choosing the Order: A GARCH model can have an order indicated by GARCH (p, q), where 'p' stands for the order of the autoregressive component and 'q' for the order of the effecting average component. The 'p' and 'q' selection depends on the squared residuals' autocorrelation and partial autocorrelation functions that are derived from the model's preliminary estimation.
- Component (p) Autoregressive (AR): The reliance of the present volatility on its previous values is captured by the autoregressive component. We choose the suitable lag length 'p' according to the squared residuals' partial autocorrelation function (PACF). Significant partial autocorrelation at lag 'p' means that volatility autocorrelation requires 'p' lags to be captured.
- The moving average component (q) captures the relationship between the current volatility and prior error terms. The lag length 'q' is determined by examining the autocorrelation function (ACF) of the squared residuals. Autocorrelation at lag 'q' suggests that 'q' lags are required to accurately reflect the autocorrelation in volatility.

- Diagnostic Checking: Another step is a diagnostic check that is performed to evaluate the suitability of the model. This stage is followed by selecting the order of the GARCH model and it involves testing the residuals for autocorrelation, heteroskedasticity, and other violations of model assumptions. If the diagnostic tests indicate model misspecification, adjustments to the model order may be necessary.
- Model Refinement: If the initial GARCH model fails to accurately reflect the volatility dynamics in the data, it may be necessary to make improvements to the model. This can entail modifying the order of the autoregressive and moving average components, either by increasing or lowering them or exploring other specifications such as an asymmetric GARCH model.

E. Likelihood Function of GARCH Models [3,1]

The probability function of a GARCH (Generalised Autoregressive Conditional Heteroskedasticity) model is a crucial element in determining the parameters of the model. The likelihood function represents the probability of witnessing the provided data, given the parameters of the GARCH model. The probability function of a GARCH model is commonly expressed in the following manner:

Let εt represent the residuals of the model, and let σ^2_t represent the conditional variance of the residuals at time t. The GARCH(p, q) model is defined as:

$$\sigma^2_t = \omega + \sum_{i=1}^{p} \alpha_i \ \varepsilon^2_{t-1} + \sum_{j=1}^{p} \beta_j \ \sigma^2_{t-1} \qquad (2)$$

Where:

ω: is the constant term.

α_i and β_j: are the autoregressive and moving average parameters, respectively.

p: is the order of the autoregressive component.

q: is the order of the moving average component.

ε^2_{t-i}: represents the squared residuals at lag i.

σ^2_{t-j}: represents the conditional variance at lag j.

Given the GARCH model specification, the likelihood function $L(\theta|\varepsilon)$, where θ represents the parameters of the model, can be formulated as follows:

$$L(\theta/\varepsilon) = \prod_{t=1}^{T} f\ (\varepsilon_t \mid \theta) \qquad (3)$$

Where:

T: is the total number of observations.

$f(\varepsilon_t|\theta)$: is the conditional probability density function of the residuals given the parameters θ.

For a GARCH model, the residuals ε_t are assumed to follow a normal distribution with mean zero and variance σ^2_t. Therefore, the likelihood function can be expressed as:

$$L(\theta/\varepsilon) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right) \qquad (4)$$

Taking the logarithm of the likelihood function, we get the log-likelihood function $\ell(\theta|\varepsilon)$:

$$\ell(\theta|\varepsilon) = \sum_{t=1}^{T} \left[\frac{-1}{2}\log(2\pi\sigma_t^2) - \frac{\varepsilon_t^2}{2\sigma_t^2}\right] \qquad (5)$$

The goal of parameter estimation in a GARCH model is to find the set of parameters θ that maximizes the log-likelihood function ℓ(θ|ε). This is typically done using optimization techniques, such as the maximum likelihood estimation (MLE) approach. The likelihood function of a GARCH model quantifies the probability of witnessing the provided data, given the model's parameters. It can also be utilized to estimate the parameters by maximizing the log-likelihood function.

F. Existence of the GARCH (1,1) process [5,7,9]

The described process is a specific variant of the GARCH (Generalised Autoregressive Conditional Heteroskedasticity) model, distinguished by the inclusion of one autoregressive term and one moving average term. The GARCH(1,1) process is steady and well-defined under certain conditions. The determination of the existence of a GARCH(1,1) process is as follows:

Model Specification: the GARCH(1,1) model is defined by the following equation for the conditional variance σ^2_t at time t:

$$\sigma^2_t = \omega + \alpha\, \varepsilon_{t-1} + \beta\sigma^2_{t-1} \qquad (6)$$

Where:

ω: is the constant term.
α: is the coefficient of the autoregressive term.
β: is the coefficient of the moving average term.
ε^2_{t-1}: represents the squared residual at the previous time step.
σ^2_{t-1}: represents the conditional variance at the previous time step.

(1) Stationarity Conditions:

- For the GARCH (1,1) process to exist, it must satisfy conditions that ensure stationarity, meaning that the process has stable statistical properties over time.
- A key condition for stationarity in GARCH(1,1) models is that the sum of the autoregressive and moving average coefficients (α+β) must be less than 1 in absolute value: $|\alpha+\beta|<1$
- This condition ensures that the process does not exhibit explosive behavior or diverge to infinity over time

(2) Theoretical Considerations:

- The existence of the GARCH (1, 1) process relies on theoretical considerations that guarantee the convergence of the model.
- The parameters (α and β) need to be chosen such that the process remains stable and well-defined.

(3) Practical Implementation:

- When estimating a GARCH (1,1) model from data, it is essential to ensure that the estimated parameters satisfy the stationarity condition ($|\alpha+\beta|<1$).
- Model estimation techniques typically involve optimization algorithms to find the parameter values that maximize the likelihood function while certifying the constancy of the model.

To sum up, the incidence of a GARCH (1,1) process depends on adequate conditions that ensure the immovability and convergence of the model. Stationarity conditions, such as the requirement that the addition of autoregressive and moving average coefficients is less than 1 in absolute value, are essential for ensuring the existence and well-definedness of the GARCH(1,1) process.

G. Forecasting GARCH (1,1) Model [13,12]

Forecasting in a GARCH (1,1) model involves predicting future values of the conditional variance based on the estimated parameters of the model and past observations of the squared residuals. For forecasting from period t to t+1 the below equation should be used:

$$\sigma^2_{t+1} = \omega + \alpha\,\varepsilon^2_t + \beta\sigma^2_t \qquad (7)$$

Where:
ε^2_t: is the squared residual at t time.
σ^2_t: is the estimated conditional variance at t time.
σ^2_{t+1}: is the forecasted conditional variance at t time.

(1) Haar Wavelet [11]

The Haar wavelet stands out as the most basic form of wavelet. When utilized in its discrete form, these wavelets are associated with a mathematical technique known as the Haar transform. This procedure acts as a foundation, enabling the development of more complex wavelet transforms. To comprehend intricate wavelet transformations better, delving deeper into the Haar transform is essential. Essentially, this wavelet transformation emerges as the optimal option for capturing localized changes and detecting edges. The numerical depiction of the Haar scaling function is as follows:

$$\phi(x) = \begin{cases} 1, & if\ 0 \le x < 1 \\ 0, & otherwise. \end{cases} \qquad (8)$$

In contrast, the description for the Haar mother wavelet is outlined as follows

$$\psi(x) = \phi(2x) - \phi(2x-1) \quad (9)$$

$$\psi(x) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \\ 0, & otherwise. \end{cases} \qquad (10)$$

(2) Haar Wavelet's properties [10];

Properties of the Haar wavelet include its compact support, orthogonality, localization in time and frequency domains, simplicity, and efficiency in detecting abrupt changes or edges in signals.

(3) Level Haar Transform [11]

1st level Haar Transform for $f = (x_1, x_2, x_3, \dots, x_N)$ is given by:

$$f \xrightarrow{H1} (a^1 | d^1) \qquad (11)$$

$$a^1 = \frac{x_1+x_2}{\sqrt{2}}, \frac{x_3+x_4}{\sqrt{2}}, \dots, \frac{x_{N-1}+x_N}{\sqrt{2}} \qquad (12)$$

$$d^1 = \frac{x_1-x_2}{\sqrt{2}}, \frac{x_3-x_4}{\sqrt{2}}, \dots, \frac{x_{N-1}-x_N}{\sqrt{2}} \qquad (13)$$

The list continues so for other levels.

3- Application Part

A. Data Collection

The study's dataset is a series of annually closing silver prices in dollars for the period of 1969 to 2022 that has been taken from the World Bank website, each R-language, Matlab, and Eviews have been used to perform the analysis.

B. Model fitting for the mean equation

The time series data under test must exhibit stationary. Consequently, the Augmented Dickey-Fuller (ADF) test for stationary was employed, as demonstrated in Table 1 below:

Table (1): demonstrates the results of the unique root test

		t-test	p.v
	ADF test value	-3.125222	0.0314
Test critical values:	1% level	-3.577723	
	5% level	-2.925169	
	10% level	-2.600658	

The table indicates that the p.v is below the given critical value of 0.05, indicating that the series has a unique root pattern.

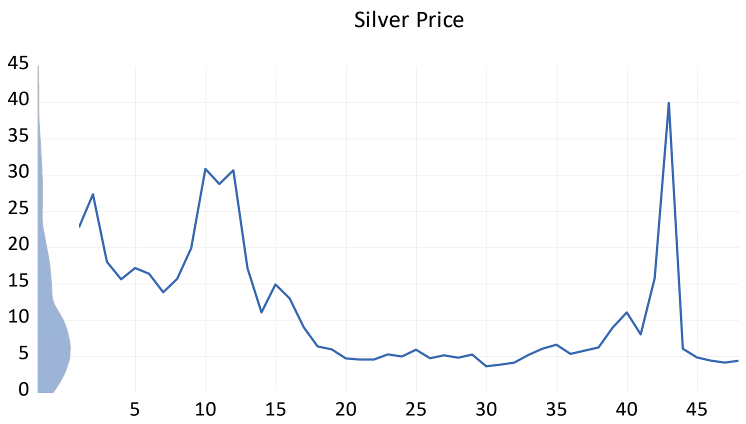


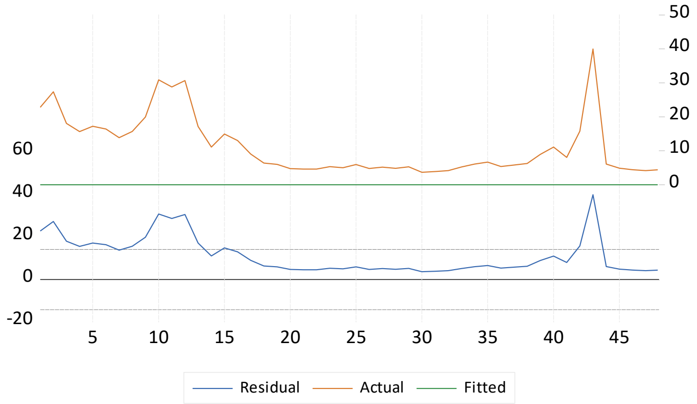
Figure (1): Shows the pattern of the data

Once the series meets the condition of stationary, we proceed to fit the mean equation model, as illustrated below:

Table (2): Represents the fit of the mean equation model

Variable	Coefficient	Std. Error	z-test	p.v
C	-2.801702	0.1471	-19.046	0.000

According to Table (2), since the p-value is below 0.05, it can be inferred that the model is statistically significant.



Figure(2): shows the residuals of the mean equation

The main aim is to determine whether the ARCH's hypothesis influences the equation of the mean. To realize this, the heteroskedasticity test ARCH is utilized, and its consequences are demonstrated in Table (3):

H_0: There is no ARCH effect Vs H1: there is an ARCH effect

Table (3): shows the heteroskedasticity test ARCH

Heteroskedasticity Test: ARCH			
F-statistic	16.6701	Prob. F(1,47)	0.001

The value of p is below five percent which indicates acceptance of the alternative hypothesis by implying the presence of the ARCH effect. Consequently, two critical assumptions for employing the GARCH model, namely series stationarity and ARCH effect in the mean equation model, have been satisfied. Thus, the GARCH model can be utilized for volatility forecasting.

C. Fitting GARCH (1,1)

The GARCH (1,1) model is executed, and its outcomes of fitting are displayed in Table (4) below:

Table (4): demonstrates the fit of the GARCH (1,1) model

Variable	Coefficient	Std. Error	z-test	p.v
	Variance Equation			
C	-2.736143	0.18159	-15.041	0.000
RESID(-1)^2	1.274112	0.25505	4.995	0.001
GARCH(-1)	-0.010430	0. 00273	-3.819	0.013

According to the information that is provided in the table above, it is obvious that the variance equation of estimators is significant. This is detected by the p.v that is less than 5%. To assess whether the GARCH (1,1) model is affected by serial correlation of residuals it is required to conduct tests on the residuals.

H_0: There is no serial correlation of residuals. Vs H_1: There is a serial correlation of residuals.

Table (5): Represents the testing of ACF and PACF

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
. |P. |	. |P. |	1	0.166	0.166	1.4137	0.234
.P| . |	.P| . |	2	-0.114	-0.145	2.0860	0.352
. | . |	. |P. |	3	0.049	0.100	2.2149	0.529
. | . |	. | . |	4	0.055	0.010	2.3781	0.667
. | . |	. | . |	5	-0.007	-0.000	2.3806	0.794
. | . |	. | . |	6	0.014	0.022	2.3913	0.880
.P| . |	.P| . |	7	-0.079	-0.098	2.7533	0.907
. | . |	. |P. |	8	0.040	0.086	2.8491	0.943
. | . |	. | . |	9	0.050	-0.001	3.0054	0.964
. |P. |	. |P. |	10	0.112	0.138	3.7918	0.956

Based on the data presented in the table above, the p.v for the 10 lags exceeds 0.05, leading us to accept the null hypothesis.
The last examination involves the ARCH heteroskedasticity test to determine whether the proposed model is suitable or not. The findings are displayed in Table (6) below:

H_0: ARCH has no effect. Vs H_1: ARCH has an effect.

Table (6): Represents the test of the ARCH effect.

Heteroskedasticity Test: ARCH			
F-Statistic	0.040651	p.v.F(1,47)	0.6821
GARCH(-1)	0.039207	p.v. Chi-Square(1)	0.6932

Up to the data provided in the table above, it is evident that the p.v exceeds 0.05, indicating that we can accept the null hypothesis, H_0.

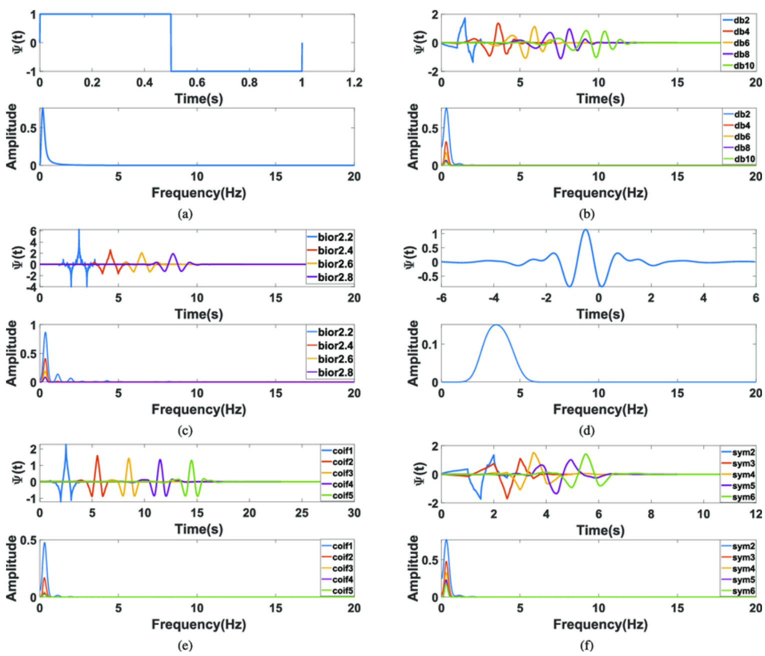


Figure (3): illustrates the plots of the wavelet-transformed data.

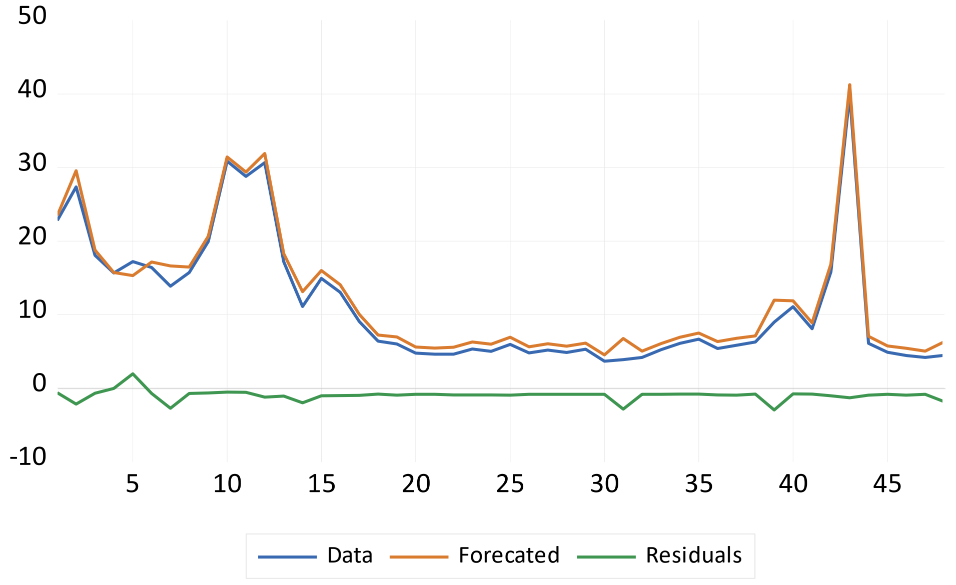


Figure (4): illustrates the graph of real data, predicted, and residuals.

Table (7): demonstrates the evaluation metrics Evaluation Metrics

Models	RMSE	MAP	AIC
GARCH(1,1)	1.368	1.871424	2.746601
Wavelet GARCH (1, 1)	1.0331	1.067296	1.3701

Table (8): represents the forecasted values of Wavelet GARCH (1, 1)

Period	**Forecasted**
54	23.434
55	27.887
56	18.615
57	16.241
58	18.208
59	17.4
60	14.581
61	17.118
62	23.386
63	31.458
64	29.257
65	31.12

4- Conclusions & Recommendations

A. Conclusions

How we forecast volatility has advanced significantly using the Wavelet GARCH model. The advantages of the two methods are combined in it:
The model known as GARCH is particularly good at showing how volatility varies with time.
Wavelet transforms are frequency and time domain data analytic techniques.
These components taken together provide a powerful paradigm for financial data volatility analysis and prediction. Overcoming the shortcomings of earlier models, the Wavelet GARCH model makes it possible to represent volatility patterns at different time scales more adaptable and flexible. This is quite beneficial in the financial markets since the volatility there can be complex and non-linear over a variety of timeframes.

B. Recommendations

The expected value increase of silver makes it a recommended investment to diversify portfolios, according to researchers. Although silver can be a hedge against economic upheaval, managing swings in silver prices requires careful monitoring of economic data such as inflation and interest rates. Tools for technical analysis can point up patterns that can guide wise investing choices. The Wavelet GARCH (1,1) model and deep learning algorithms are also compared in this work for silver price prediction.

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