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Using Coloring Method to assign the required tasks with application

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Abstract: This research studies (required tasks distribution) using coloring method which is considered as means of Graph theory, a real issue has been processed which is a tasks distribution in one of marketing centre on five employees in this centre. The study includes two topics, the first is theoretical in coloring method and the second is practical to find solution for this issue according to this method. A solution to the problem is found that satisfies the conditions of the coloring method and achieves the objective of the issue.

Keywords: Coloring method, edges, Graphic chart, Graph theory, nodes.

استخدام طريقة التلوين في توزيع المهام المطلوبة مع التطبيق

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المستخلص: تناول هذا البحث موضوع (مسألة توزيع المهام المطلوبة) باستخدام طريقة التلوين التي تعتبر احدى وسائل نظرية الرسم البياني وتم معالجة مسالة واقعية وهي توزيع المهام في احدى مراكز التسوق على خمسة موظفين في هذا المركز . تتضمن الدراسة جانبين الأول نظري في طريقة التلوين. والثاني جانب تطبيقي لإيجاد حل لهذه المسالة وفق هذه الطريقة . وقد تم ايجاد حل للمسالة يحقق شروط طريقة التلوين ويحقق الهدف من المسالة.

الكلمات المفتاحية: طريقة التلوين، الحافات، المخطط البياني ، العقد.

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1- Introduction

Graph theory and its methods are considered one of the easiest ways to solve many problems, and one of the methods of this theory is the Colouring method. This method is used to solve some problems, and among these problems is the issue of distributing tasks according to a specific timetable. In this study, a solution to this problem was reached using this method, as one of the marketing centres, namely the Toddy Mall Centre, was chosen by selecting five employees in this centre and identifying the tasks of each employee. These tasks include four tasks that they rotate during the centre's official working hours, so that each of them works during a specific time that does not overlap with the work time of others, according to the schedule that arranges these duties.

2- The Aime

It is to find a solution to the issue of distributing tasks in one of the marketing centers to a number of employees in this centre according to the time schedule allocated for work.

3- The theoretical aspect

First - graph theory
Graph theory studies the properties of graphs. They are mathematical constructs used to model binary relationships between objects within a given set. Where the diagram can be considered a group of objects called vertices, one of which is a vertex, connected to each other by edges or edges, or sometimes called edges arcs, that can be directed, that is, with or without a direction. The representation of this diagram is on paper with a set of points representing the vertices connected by lines called the edges of the diagram. Graph theory studies the properties of diagrams [2].
The rudiments of this theory were created by the Swiss scientist Leonhard Eular in 1736, and Euler solved the problem known as the Seven Bridges of Kungsberg. How much in the following figure [4].

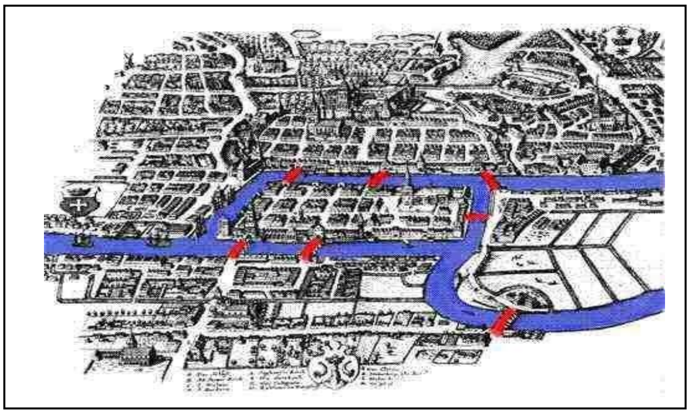

Figure (1): A drawing of the Kongsberg Bridge in Russia

In the city of Kongsberg, Russia, located on the Pregulia River, there were two large islands, connected to each other and to the mainland by seven bridges. The dilemma is to answer the following question: Is it possible to find a route that passes through the seven bridges, once, no more or less, with each bridge, and then return to the starting point? The answer to this question is negative, because this diagram does not contain any Orelian circuit. This solution is considered the first theorem in graph theory, and specifically in planar graph theory. As in the following figure: -

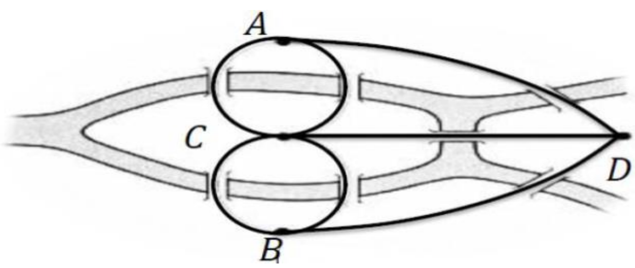

Figure (2)

Euler found that this problem is one of the problems that cannot be solved, and he adopted the method of diagrams in arriving at this result, as he assumed that both banks of the river and the two islands are points representing nods, or called vertexes, and he considered the bridges to be the lines that connect the nods, and representing arcs, or called edges,

Node degree : The degree of a node in a directed graph is the number of edges (connections) connected to this node, and there are two types of node degree in a directed graph:
First - the entry degree (in degree), which is the number of edges entering this node and coming from another node.
Secondly - the out degree, which is the number of edges extending from this node to another node.
In order to understand the analysis of the Kongsberg Bridges issue, a problem diagram is drawn up to explain the problem, the components of this issue, how to analyze it, and try to find a solution to it [3], as in Figure No. (3).

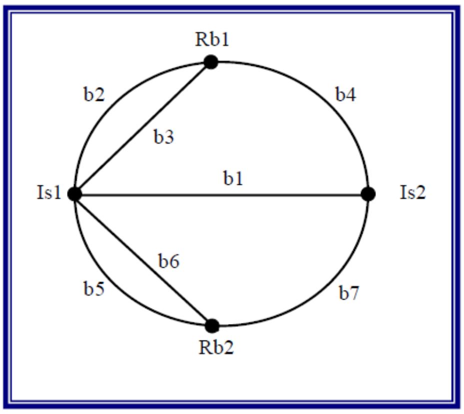

Figure (3): Schematic of the Kongsberg Bridge problem

We note that Rb1, Rb2 represent the two banks of the river and that Is1, ls2 represent the two islands, while b7,…, b1 represent the seven bridges and they represent the arches. After understanding the problem, Euler came up with the theory that says:
The problem can be solved by walking around the city and crossing each bridge only once in two cases:
A. If there are only two nodes with an odd degree.nodes degree odd two
B. If there is no node of odd degree zero, meaning that all nodes are of even degree.
Other than that, the problem is impossible to solve, and you cannot walk around the city without crossing a bridge more than once.

Then determine the course of action in cases where the problem can be solved, as follows:

(1) If the diagram has two odd-degree nodes, the walk will start from the first odd-degree end and end at the odd-degree end.

(2) If there is no node in the diagram of an odd degree, meaning that all the nodes are of an even degree, then the walk will start from one of these nodes and end at the same node. 200 years passed before the first book on graph theory appeared. In 1936, the scientist König published a book that was translated into English in 1990, entitled:

("Theorie der endlichen und unendlichen Graphen", Teubner, Leipzig, 1936)

It is in German and means in Arabic (infinite and finite graph). [9] The period of two hundred years included some few attempts in the field of graph theory, such as finding a solution to the problem of the four colors within the coloring method. After 1936, many attempts began to write and research this theory.

At present, graph theory contributes to finding solutions to many problems, including:

(A) Chains and Cycles: A directed path is a directed path with all its vertices different, since Vo=Vn and its formula is $(v_0,e_1,v_1,e_2.....e_n,v_n)$. A circuit is a directed circuit with all its vertices different, since Vo ≠ Vn and its formula is $(v_0,e_1,v_1,e_2.....e_n,v_n)$.

(B) The tree is a simple, non-cyclic diagram devoid of loops. It is a partial diagram T of a connected diagram G. The properties of the tree are:
First: There is one path between every two nodes in T.
Second: The number of nodes is n and the number of arcs is n-1.
Third: T is a plan and every arc in it is an isthmus.
Fourth: T is a connected diagram that does not contain any loops.
Fifth: If we connect any two adjacent nodes in T, we get a diagram that contains only one loop.

(C) Graph Coloring: Map coloring is finding the smallest number of colors that can be used to color a map, as every two adjacent regions are colored in a different color from the second. It has been found that four colors are sufficient to color a map.

(D) Network Analysis: A network is a group of nodes and a group of arcs that connect pairs of nodes. Nodes are symbolized either by sequential letters or in equal numbers, and by arcs with the names of the nodes that connect them. An example of networks are road networks, transportation pipelines, railways, and transportation lines. All of these networks are concerned with transporting a specific commodity.

Network analysis includes:

A. Transport networks: concerned with transporting a specific product from multiple sources to different places at the lowest possible cost.
It is used in analyzing transportation networks to determine the flow of vehicles (or individuals), such as walking and riding cars, to model multi-road trips.

B. Business networks, projects, and project program evaluation and review:
The cycle of large projects, which consists of a huge number of activities, is a complex network in terms of planning, scheduling and control. Difficulties arise in the issue when these activities are implemented according to a specific technical sequence. There are two types of activities:

First - real activities
Second - Fictitious activities (as it is not permissible to represent more than one activity with one number, and the fictitious activity is used to avoid this situation)

C. Network Flow:
In the case of the connection of the source node and the destination node through an intermediate store (nodes) connected by means of transportation, the means take certain capacities, as there is a minimum and maximum limit for the permissible flow on these means of transportation, and it is assumed that it is not stored in any intermediate store, but rather transferred directly to another center. Thus, the goal is to find the highest amount that the source can reach the destination.

D. Assignment Networks: How to distribute the best productive economic resources to the various tasks to be accomplished. It is considered one of the important matters in distributing the available resources to specific tasks so that the cost of allocation is as low as possible and solves frequently recurring problems in administrative applications such as organizing individuals, production, machines, marketing, etc.

E. Minimum Spanning Tree Network: It is a connected statement drawn to find a branching tree whose edges in that tree measure as least as possible. The edge measurement is interpreted as the length of the path it represents or the time to complete a work from one stage to another.

F. Shortest Path Networks: These are shortest path networks that aim to find optimal paths between network nodes so that the cost of routing is reduced to a minimum.
In addition to what was mentioned, there are also many issues that graph theory solves, such as:
(1) Problems of tree matching in organic chemistry
(2) Electrical network analysis issues
(3) Issues of reducing intersection accidents in factories
(4) Issues of organizing timetables
(5) Issues of games and matches
(6) Network analysis issues.
(7) The issue of the traveling merchant.
(8) The issue of the Chinese postman.

Second - Coloring Method or Four Colors Method

In this research, one of the graph coloring methods will be used, which is one of the oldest problems in graph theory (graph theory). The origins of the coloring problem go back to the year 1852, when the scientist De Morgan sent a letter to a friend of William Hamilton telling him: One of his students noticed that when coloring the administrative map of England, four colors are sufficient to color it so that no two adjacent regions have the same color. This is what is called the Four Colors Problem [1]. If he considers that each region represents a node in the chart and that the arcs are expressed by Access between regions: Every two nodes are connected by an arc, which means that the two regions are adjacent.

The coloring method is one of the methods of graph theory, as the striped shapes that represent the components of the problems for which a solution is to be found can be found by dyeing the nodes or arcs with a number of colors. For example, (What is the minimum number of colors that can be used to color the map of the country?) Arabic such that there are no neighboring countries with the same two colors) Such a question led to the creation of the four-color problem [6]. If it is considered that each country represents a node in the diagram and that the arcs express the neighborhood between the countries, then every two nodes connected by an arc means that The two countries are neighbors, and thus the solution begins, as will be explained later.

The distribution of required tasks can be addressed according to graph theory using the four-color method [10]. Coloring the diagram will include either coloring the nodes or coloring the arcs, and we can learn about which type of coloring is used in the method of solving it by identifying the components of the problem for which a solution is to be found and converting it into the form of a diagram. For example, in the problem of finding the least number of colors, it is represented by coloring the map of the countries of the Arab world. So that there are no two neighboring countries that have the same color, we can assume that the countries represent the nodes in the diagram, so the coloring in this case will be on the nodes and not the arcs, since the issue is that there are no two neighboring nodes that have the same color. As for the issue of distributing tasks according to the timetable [5]. The distribution of tasks among individuals in the mall business will be represented by nodes in the diagram, while the time periods are represented by arcs. Therefore, the coloring will be on the arcs because the problem is to find the least number of time periods distributed over the days of the week, and since the arcs represent time periods, the problem of solving the least number of periods is Finding the least number of colors such that no two arcs located on the same node have the same color [4] [9]. In order to achieve this problem on intervals, the following conditions must be met:

(A) Each task in the mall is done once during each period of the day.
(B) Each employee only works on the same task once a day.
(C) There should be no intersection between workers' work schedules in the same place.

Naturally, there are algorithms specific to each type of coloring and each of its cases, as in the case of organizing working hours for tasks for the employees under investigation, the algorithm that is applied to it is the one that says { If the diagram G is a simple diagram devoid of circuits, then the coloring number is x(G) =K }, where (color number: is the smallest number of colors that the chart needs for its arcs).
(Circuits: It is a partial diagram in which every node is of the second order), and the path (P of vn, v0) is said to be a circuit if V0 = Vn, and it is said to be a simple circuit if all its nodes are different,

that is, it is in the following formula (v_0, e_1 ,v_1,e_2,.....e_n,v_n) since the nodes (v_0,v_1,....v_{n_1}) are all different.

Since the diagrams are developed on the basis of the components of the problem under investigation, the diagram that can be formed from the components of the timetables [4] is of the type of bipartite multigraph. Suppose we have a simple bipartite diagram, let it be G, and this diagram has two groups of nodes. They are opposite each other. The first group is (X= x1,…, Shown in Figure (1):

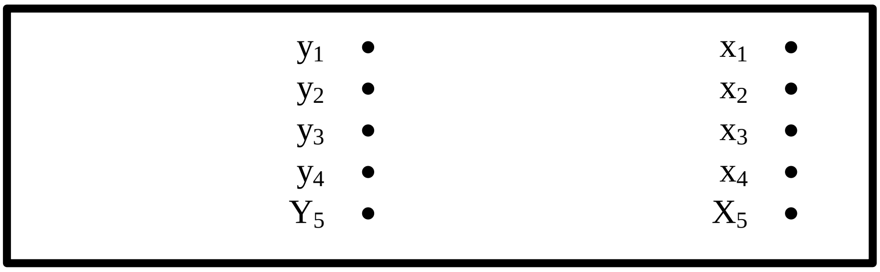

Figure (4): A diagram of two opposite groups of employees and tasks

4 . The practical side

In order to carry out a real application of this issue, one of the malls in the city of Mosul was chosen, which is Toddy Mall, and five employees were selected to work in it to rotate in four tasks, which are the job of (cashier, arranging goods on shelves, sales, and stores), as shown in Figure (5).

X	Y
• Yaman	• The cashier
• Maria	• Arrange the shelves
• Mustafa	• Sales
• Lillian	• Store management
•Ahmed	

Figure (5)

One of the requirements of the plan is to set a number of conditions, which include not repeatedly giving the employee the same task on the same day and not repeating the task in the same period. We can put a matrix of tasks, which is the condition for completing the application, and we will solve it according to the following steps:

- **Steps to solve the practical side:**

A. The first step:
We formulate the distribution of tasks required in (Today Mall) according to the work system in the form of the following task matrix:

Table (1)

P	Y_1	Y_2	Y_3	Y_4
X_1	1	0	1	0
X_2	1	0	0	1
X_3	1	١	1	0
X_4	٠	0	1	1
X_5	٠	1	0	1

We have five employees and four tasks, and the work system in the mall was as follows:
(1) That employee x_1 performs the three tasks during the day: (y_1, y_3).
(2) That employee (x_2) performs the three tasks during the day, which are (y_1, y_4).
(3) That employee x_3 performs the three tasks during the day: (y_1, y_2, y_3).
(4) That employee (x_4) performs the three tasks during the day, which are (y_3, y_4).
(5) That employee x_5 performs the three tasks during the day, which are (y_2, y_4).

B. The second step:

Finding the smallest number of available periods required by the schedule to distribute employees to tasks that meet the conditions. Therefore, the second group must be formed as shown in Figure (3):
And by applying the following (Euler's formula):
$N - E + F = 2$ … (1)
N: number of nodes
E: Number of edges (arches)
F: The number of faces (and in this study, time periods) available is four
Planar graphs contain faces as well as edges and nodes. Faces are the areas between edges. This study is one way to find a solution to the tables. The faces represent the available periods, and the four colors represent the four-color method.
Since the number of nodes in this study is nine, and by applying Euler's formula, we find that the number of sides is eleven, as follows:

$E = N + F – 2 = 9 +4 – 2 = 11$ … (2)

We find the minimum number of periods required by the schedule to distribute employees to tasks in a way that fulfills the conditions. Therefore, we first construct the following binary group:

We symbolize each arc with the symbol e, so the set of arcs is ei = e1, e2,...e11, as in Figure No. (6)

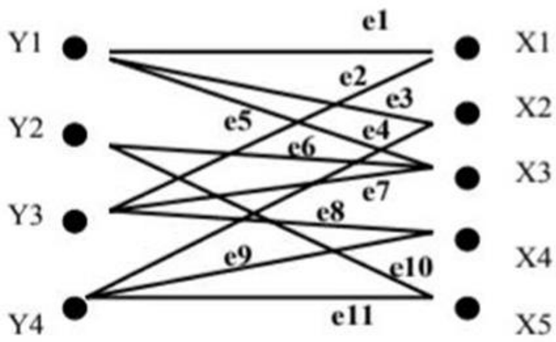

Figure (6)

Where Xi: represents employees

Yi: represents tasks

C. The third step:

Building the matrix consisting of the adjacency of the arcs to each other at the same nodes, which is the adjacency matrix (or the projection matrix, meaning the arcs fall on the same nodes). Using the coloring algorithm, we have two sets of X and Y, which are the nodes. We get The matrix is according to the following algorithm:

$$.e_{ij} = \begin{cases} 1 & \text{If the two edges lie on the same node} \\ 0 & \text{If they are otherwise} \end{cases} \quad \dots \quad (٣)$$

The elements of this matrix are calculated according to equation (3) to give the relationship between the rows and columns, represented by nodes, whether they are adjacent or otherwise.

Table (2) Adjacency matrix

	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11
e1	0	1	1	0	1	0	0	0	0	0	0
e2	1	0	0	0	0	0	1	1	0	0	0
e3	1	0	0	1	1	0	0	0	0	0	0
e4	0	0	1	0	0	0	0	0	1	0	1
e5	1	0	1	0	0	1	1	0	0	0	0
e6	0	0	0	0	1	0	1	0	0	1	1
e7	0	1	0	0	1	1	0	1	0	0	0
e8	0	1	0	0	0	0	1	0	1	0	0
e9	0	0	0	1	0	0	0	1	0	0	1
e10	0	0	0	0	0	1	0	0	0	0	1
e11	0	0	0	1	0	0	0	0	1	1	0

D. Fourth step:

In order to apply the four-color theorem, which fulfills the conditions of not having two adjacent nodes of the same color and reaching the minimum number of colors with which the nodes are dyed.

[7] And that the number of nodes does not exceed four colors, and there is proof for this case that says (when we have the plan G immersed in the surface S, and we assume that G has individual nodes whose number exceeds four nodes, and if the plan G sufficiently fulfills the level condition, then the plan G has Can be dyed to four colours.

Since the time periods have eleven arcs, we assume that each arc represents a node, so we get 11 nodes. We can apply the four-color theory to them, and the minimum number of colors is four colors, provided that we do not have two adjacent nodes that have the same color. The adjacency here is that the two nodes fall on both ends of the same arc and are Two nodes are adjacent when there is an arc between them, where the colors of the nodes are as in Figure No. (7) as follows:

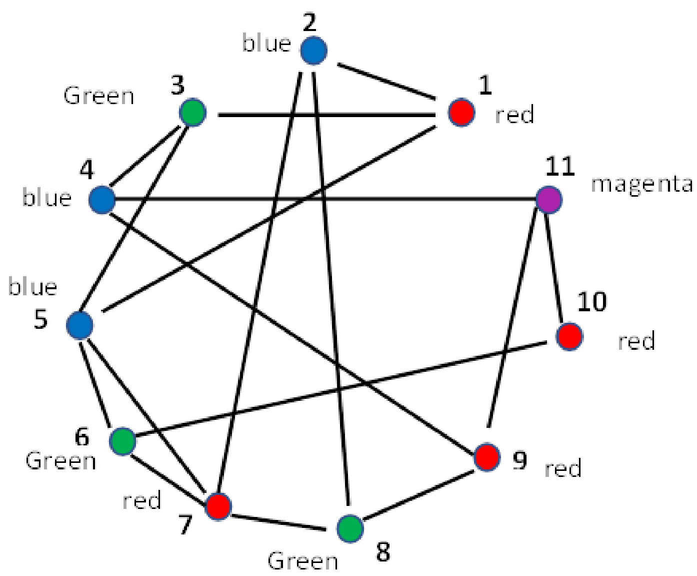

Figure (7)

Through Figure (7), we found the smallest number of nodes, which represent the arcs that we assumed, which are the time periods to cover the conditions of employees in the markets. We performed the substitution process by assuming that the arcs represent the nodes in order to apply the four-color problem, and we obtain the smallest number of colors, which is four colors for the set of nodes. Which applies the condition that no two adjacent nodes have the same color.
Then we find a chart that represents the required schedule, and through the four periods, we arrive at the number of daily periods that the schedule needs to distribute employees to tasks in the markets.

E. The fifth step:

(In this step, we will re-represent the nodes and arcs according to the first steps of the problem, that is, we will invert the diagram so that the nodes are in the place of the axes and the arcs are in the place of the nodes, in order to arrive at a final solution to the problem, as follows)

:actions and occupancies represent the contract and the parentheses represent the occupancies in the periods, as in the solution as follows:

(Assuming that the first period has straight arcs of green color, the second period has straight arcs of red color, the third period has straight arcs of purple color, and the fourth period has straight arcs of blue color) According to the employee condition matrix, Table No. (1), the diagram Which distributes employees to jobs according to periods is as follows :

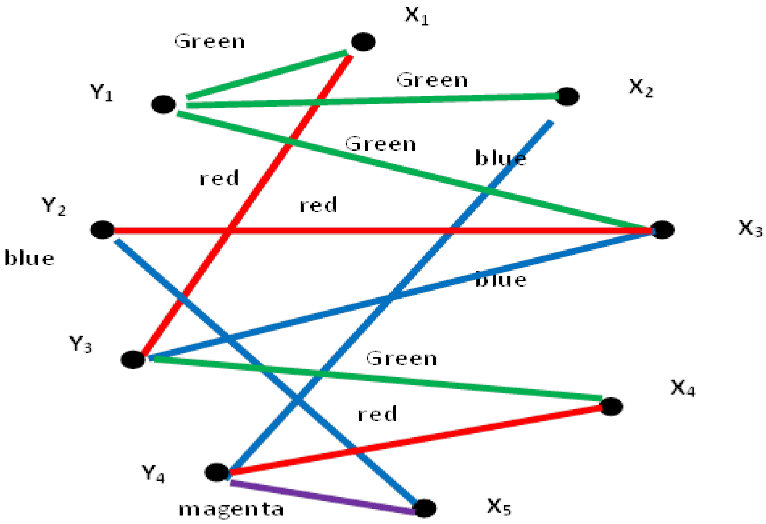

Figure (8)

We draw a diagram on the condition that no two arcs are adjacent to each other in the same node, and according to the time periods in which each period takes a specific color for its arcs.

In the first period, the number of employees is exhausted with one task for each one until the capacity of the period is exhausted, which here is four tasks, then moving to the second period in the same way, and so on until all the tasks that must be completed are completed according to the registration condition matrix. Accordingly, the initial solution to the time period schedule is as follows:

Table (3)

P	1 Green	2 Red	3 Blue	4 Magenta
X1	Y1	Y3	-	-
X2	Y1	-	Y4	-
X3	Y1	Y2	Y3	-
X4	Y3	Y4	-	-
X5	-	-	Y2	Y4

Thus, the schedule is according to the plan, and since what is required is that an employee not repeat more than one task in the same period or that the task not be repeated for more than one employee in the same period [8]. The above plan does not meet the condition of non-repetition because the employee repeats the same task in the same

Therefore, this solution must be improved by making permutations of the scheme as follows :

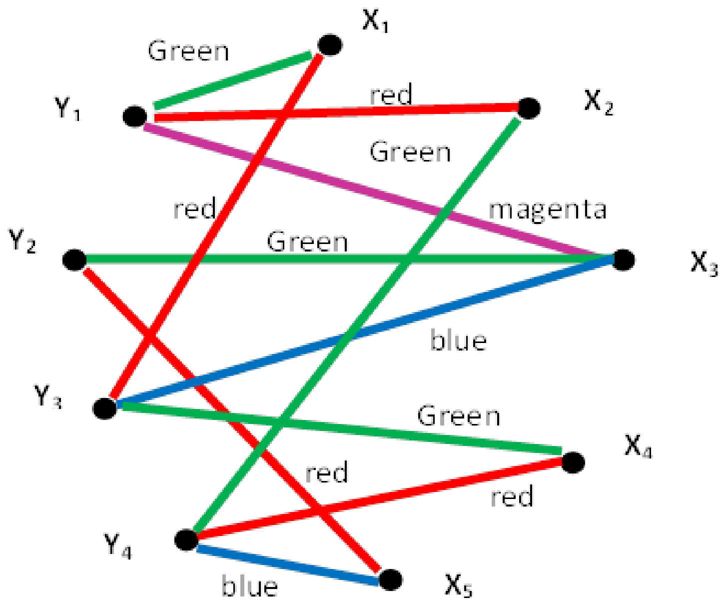

Figure (9)

Where permutations are made based on the basics of achieving the conditions that were set for the problem, and this is what color theory and graph theory provide in terms of flexibility for the researcher [6], so that we can make many changes to the diagram in a way that achieves the conditions of the studied issue. The mechanism for performing permutations is to distribute tasks to employees. For periods, it is an average of one task per employee.

According to the sequence, the number of tasks for each period is exhausted, then moving to the next period, and completing the distribution of tasks to employees for the periods, which is at a rate of one task for each employee. Thus, each employee participates once in each period, and repetitions occurred in our initial solution to the table regarding tasks. Therefore, it was necessary to make permutations on the basis of changing the locations of tasks in the table between employees and periods, in order to achieve the condition of no repetition within one period. We obtained the resulting table that distributes employees to tasks according to the periods specified for the conditions of the problem, and it also matches the employee condition matrix.

Table (4)

P	**1 Green**	**2 Red**	**3 Blue**	**4 Magenta**
X1	Y1	Y3	-	-
X2	Y٤	Y1	-	-
X3	Y2	-	Y3	Y1
X4	Y3	Y4	-	-
X5	-	Y2	Y4	-

5. Results and discussion

A. In the first period, the first task was distributed to the first employee, the second task to the third employee, the third task to the fourth employee, and the fourth task to the second employee.
B. In the second period, the first task was distributed to the second employee, the second task to the fifth employee, the third task to the first employee, and the fourth task to the fourth employee.
C. In the third period there are two tasks, the third task is for the third employee, and the fourth task is for the fifth employee.
D. In the fourth period, one task remained and was given to the third employee.
E. The employees were distributed among the tasks in all available periods, so that there are no employees working on the same task in the same period, and this fulfills the conditions.

6. Recommendations

A. The use of the coloring method (four colors) and its means provides management with a clear tool to see the components of the plan, which consist of the available employees and tasks, as well as the available time period and the number of tasks required to be distributed.
B. The coloring method provides simple means to find optimal solutions in setting timetables for tasks and according to the required conditions.
C. The coloring method facilitates complex problems and finding the necessary permutations when needed because it provides a general outline of the problem.
D. It makes it easy for management to monitor any emergency situation that may occur on the schedule and easily take the necessary measures and find a solution to it in the event that any employee is absent for any reason.

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