H-Compact Space

Assistant Teacher Hussein Kadhum Asker

University of Kufa/College of Mathematics and Computer Science

Mathematics Department

Abstract:

Using the concept of H-open set, we introduce the concept of H-compact space. Several properties of H-compact spaces are proved , among these are:

1. H-closed subset of H-compact is H-compact.
2. The continuoues image of H-compact is H-compact.
3. H-compact subset of H-Hausdorff is H-closed.

1- Introduction:

In [1], the concept of H- topology is introduced, any member of the H-topology is called H-open set. Using the concept of H-open set, we introduce the concept of H-compact space.

2- Basic definitions:

In this section, we recall and introduce the basic definitions needed in this work. First, we recall the definition of H-topological space.

2-1 Definition:[3]

Let X be a non empty set, let τ be a collection of subsets of X which satisfies the following conditions

i. $X \in \tau$
ii. $E \subseteq X$ implies $X\text{-}E=E^c \in \tau$
iii. The union of any number of members of τ also belongs to τ.

Then τ is called an H-Topology on X and the pair (X,τ) is called H-Topological Space (H-Space).

2-2 Definition:

Let (X, τ) be an H-Topological space, every $W \in \iota$ is called H-open and W^c is called H-closed.

2-3 Definition:

Let (X, τ) be an H-topological space, let $f=\{W_\alpha : \alpha \in \Omega\}$ be a family of H-open sets, f is called an H-open cover of X if

$$X = \bigcup_{\alpha\in\Omega} W_{\alpha_i}$$

2-4 Definition:

Let (X, τ) be an H-topological space, X is called H-compact if every H-open cover of X has a finite sub cover.

If $A \subseteq X$, we say that A is H-compact set if every H-open cover of A by H-open sets in X has a finite sub cover.

3- Main Results:

In this section, we state and prove several properties of H-compact spaces.

3-1 Theorem:

let (X, τ) be H-compact and let S $\subseteq$ X be H-closed set, then S is an H-compact set.

Proof: let f={w_α : $\alpha \in \Omega$} be an H-open cover of S consider $f^*= f \cup \{S^c\}$ then f^* is an H-open cover of X. but X is H-compact then $\exists$ α_1 , … , α_n such that $X = \bigcup_{i=1}^{n} W_{\alpha_i} \cup S^c$ then S $\subseteq$ $\bigcup_{i=1i}^{n} W_{\alpha_i}$ so S is H-compact.

Before, we state the next result, we introduce the following definition.

3-2 Definition:

Let f: (X, τ) →(Y, α) be a function, we say that f is H-continuous if W H-open in Y implies $f^{-1}(W)$ H-open in X.

3-3 Theorem:

let f: (X, τ) →(Y, α) be H-continuous and onto, if X is H-compact, then Y is also H-compact.

Proof: let f={W_α : $\alpha \in \Omega$} be an H-open cover of Y now X= $f^{-1}(Y)=f^{-1} \bigcup_{\alpha\in\Omega} W_\alpha$ so $X = \bigcup_{\alpha\in\Omega} f^{-1}(W_\alpha)$ but $f^{-1}(W_\alpha)$ is H-open in X so $\{f^{-1}(W_\alpha)/\alpha \in \Omega\}$ is an H-open cover of X since X is H-compact then $\exists$ α_1,…, α_n such that $X = \bigcup_{i=1}^{n} f^{-1}(W_{\alpha_i})$ now

$$Y = f(X) = f\left(\bigcup_{i=1}^{n} f^{-1}(W_{\alpha_i})\right) = \bigcup_{i=1}^{n} f\left(f^{-1}(W_{\alpha_i})\right) = \bigcup_{i=1}^{n} W_{\alpha_i}$$

which means that Y is H-compact.

Before we state the next theorem we introduce the following definition**.**

3-4 Definition:

let (X, τ) be an H-topological space we say that X is H-Hausdorff if given a, b $\in$ X, a≠b then $\exists$ W_1, W_2 H-open in X such that a $\in W_1$, b $\in W_2$, $W_1 \cap W_2 = \varnothing$.

3.5 Theorem:

Every H-compact subset of H-hausdorff space is H-closed

Proof: let S be H-compact in X we will prove that S^c is H-open. Let p$\in S^c$, let X $\in$ S, since X is H-husdorff then $\exists$ V_x, W_x such that p$\in V_x$, x$\in W_x$, $V_x \cap W_x=\varnothing$

Consider f={W_x : x $\in$ S} so f is an H-open cover of S

So $\exists$ x_1,…, x_n such that $S \subseteq \bigcup_{i=1}^{n} W_{x_i}$

consider V_{x1},…,V_{xn} , $p \in \bigcap_{i=1}^{n} V_{x_i}$ let $V = \bigcap_{i=1}^{n} V_{x_i}$ so p $\in$V and V is H-open so S^c is H-open and S is H-closed.

3.6 Theorem:

let (X, τ) be an H-topological space, let A and B be two H-compact spaces of X, then A$\cup$B is also H-compact.

Proof: let f be an H-open cover of A∪B, since A ⊆ A∪B, B ⊆ A∪B, so f is an H-open cover of A, but A is H-compact, then exists a finite sub cover f_1 of A. Similarly there exists a finite sub cover f_2 of B. Now $f_3 = f_1 \cup f_2$ will be a finite sub cover of A∪B, this mean that A∪B is H-compact.

4- Comparison between |Topological Space and H-topological Space:

Let (X, τ) be a topological space, it is known that the intersection of any number of open sets may not be open, for example: take X=R, $\tau = \tau_u$=the usual topology on R then $\bigcap_{n=1}^{\infty}\left(\frac{-1}{n},\frac{1}{n}\right) = \{0\}$ which is not open.

Now, Let (X, τ) be an H-topological space, we notice that the intersection of any number of H-open sets is also H-open. So, we can say that every H-topological space is an H-Alexandroff space (Remember that (X, τ) is called an Alexandroff space if the intersection of any number of open sets is open [1].

Finally, we introduce a new concept, namely H-limit point as fellows:

Let (X, τ) be an H-topological space, let A ⊆ X, p∈X, we say that p is an H-limit point of A if for all W H_open, p∈W, (W-{p})∩A≠∅.

In our future work, we will talk in details abut this concept**.**

References:

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الفضاء المتراص - H

المدرس المساعد حسين كاظم عسكر

جامعة الكوفة /كلية الرياضيات وعلوم الحاسبات/قسم الرياضيات

المستخلص:

في هذا البحث استخدمنا مفهوم المجموعة المفتوحة - **H** في عرض مفهوم جديد وهو الفضاء المتراص - **H**. وقد برهنا عدة قضايا على هذا المفهوم، من هذه القضايا:

١. المجموعة المغلقة – **H** الجزئية من الفضاء المتراص – **H** هي فضاء متراص - **H**.

٢. الصورة المستمرة للفضاء المتراص – **H** هي فضاء متراص - **H**.

٣. المتراص – H الجزئي من H-Hausdorff هو مجموعة مغلقة – H.