OPTIMIZATION THE EFFECT OF COLUMN CROSS SECTION SHAPE AND DIMENSIONS ON THECRITICAL BUCKLING LOAD +

Sabah Khammass Hussein *

Abstract:

The critical buckling load of column for different cross section shapes is analyzed. The constant cross sectional area of all shapes is assumed constant. The theoretical equation of buckling load is compared with the finite element model using ANSYS11 program. A good resultsare obtained as comparing with the theoretical. The results indicate that the critical buckling load depends on the shape and dimensions of column section. It is observed that the higher values of critical buckling load are found in the U-section while the Z- section gave lower values. Increasing the thickness of cross section resulted in decreasingthe critical buckling load. Approximately, it has been observed that the increasing the width of column section resulted in increasing the critical buckling load.

Keywords:**Critical buckling load, design of experiments, different cross section, FEM, Euler buckling.**

تحسين تأثير شكل مقطع العمود وأبعاده على حمل الانبعاج الحرج

د.صباح خماس حسين

المستخلص:

تم تحليل حمل الانبعاج الحرج للعمود ذواشكال مساحة مقطع مختلفة . مساحة كل المقاطع أفترضت ثابتة. المعادلة النظرية لحمل الانبعاج قُورنت مع نموذج العناصر المحددة بأستخدام برنامج ال (ANSYS11). حصلنا على نتائج جيدة مقارنة مع النظري. النتائج أشارت بأن حمل الانبعاج الحرج يعتمد على شكل وأبعاد العمود. لوحظ أن القيم العالية للحمل الانبعاج الحرج وجدت في مقطع (U) بينما مقطع ال (Z) أعطى أقل القيم. زيادة سمك المقطع أعطى نقصان في حمل الانبعاج الحرج. وبشكل تقريبي لاحظنا أن الزيادة في عرض مقطع العمود اعطى زيادة في حمل الانبعاج الحرج.

Introduction:

Euler is the first who studied for engineering applications the problem of buckling arising in a simple, monolithic beam loaded axially by a concentrated load. He observed from two

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*Assist. Prof. /Engineering Technical College – Baghdad,Middle Technical University

solutions of the problem due to Timoshenko theory; the elastic foundation increases the critical buckling load of the beam [1].

The deflection theoretical model and the critical load capacity of beam are derived for this combined loading condition. The Finite Element Analysis is utilized to apply the axial load on the beam at various configuration locations and it is shown that this application location determines the buckling behavior and the critical load of the buckling of the I-beam [2].

It was observed that the tapered I-beams can carry a maximum bending moment at a single location while in the rest of the member the moment carrying capacity is considerably lower. Modification factors of the elastic critical moment with reference to the mean cross-section are to be given for various taper ratios. The presented approach can be very easily applied for the design of tapered beams against lateral-torsional buckling [3].

The lateral-torsional buckling of composite strip and I-beams is studied. The analytical model is thereby reduced to a single fourth-order differential equation and boundary conditions. With certain exceptions regarding the load offset parameter; the analytical formula provides results that agree quite well with the numerical solution of the exact equations as long as all the small parameters remain small[4].

Experimental and numerical investigation of coped I-beams with stiffeners in the coped region was conducted. The test and numerical results showed that the horizontal stiffeners at the cope displaced laterally due to the gross web distortion. It was found that when the ratio of cope depth to beam depth ratio (d/D) ≥ 0.3, both horizontal and vertical stiffeners are required in order to prevent local web buckling at the region of cope [5].

Theidentifying buckling load of a beam-column method is presented based on a technique of Multi-segment Integration technique. This method is applied to a number of problems to ascertain its soundness and accuracy. The obtained results fromfinite difference method are compared in order to determine the efficiency of this method [6].

The lateral buckling of a simply supported beam subjected to a mid-span concentrated load is discussed. A closed form solution of the equation system cannot be, in general, obtained. Therefore, one has to resort to approximate analytical solutions. Hence, an analytic approximate technique for solving the above system of differential equations is successfully employed [7].

Theoretical part:

Consider a load which applied on the upper end of (pin-pin) column as shown in figure (1). The critical buckling load depends on the material properties and column dimensions. According to "Euler column formula", the critical buckling load is given by[8, 9]:

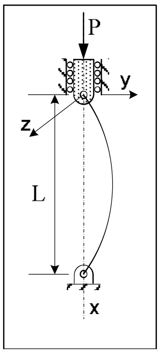


Figure (1) Buckling of pin-pin ends column.

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} \qquad (1)$$

Where:
E: modules of elasticity (N/m^2).
I: second moment of inertia (m^4).
L: column length (m).
P_{cr} : critical vertical load (N).

The value of (k) depend on the end fixing; where, k=1 in the case of both ends pinned hinged or free rotate, k= 0.5 for both ends fixed and k=0.669 in the case of one end fixed and the other pinned. In this work, the value of k=1.

The column sections used are; hollow rectangular, channel, L, T, I, and Z- section.

The cross sectional area of column shown in figure (1) leis in the (*y-z*) plane. According to (Euler formula), the moment of inertia must be the minimum value (about y or z-axis). This is true for column section have a zero moment of inertia about the axis of area plane (*Iyz=0*). The sections (L and Z) have a non-zero value of moment of inertia. According to these phenomena, the minimum moment of inertia will have three chances (about y or z or yz-axis). Table (1) represents the buckling axis for the column sections used in this work.

Table (1) the buckling axis for each section

Column section	*Iyz*	Buckling axis
Hollow rectangular, channel , T and I	0	y or z
L and Z	$A\bar{y}\bar{z}$	y or z or yz

The values of principle moment of inertia are given by:

$$I_Y = \frac{I_y+I_z}{2} + \sqrt{\left(\frac{I_y-I_z}{2}\right)^2 + {I_{yz}}^2} \qquad (2)$$

$$I_Z = \frac{I_y+I_z}{2} - \sqrt{\left(\frac{I_y-I_z}{2}\right)^2 + {I_{yz}}^2} \qquad (3)$$

Where:
I_y, I_z: second moment of inertia about the axis (y &z-axis), (m^4).
I_Y, I_Z: principle moment of inertia about the principle axis (Y &Z-axis), (m^4).
A: Cross sectional area (m^2).
$\bar{y}\bar{z}$: Centroid of area (m).

Material properties and dimensions of column:

The material used is of structural carbon steel having the following properties, table (2):

Table (2) mechanical properties

Yield stress (MPa.)	Ultimate stress (MPa.)	Modulus of elasticity (GPa.)	Poisson's ratio	Density Kg/m^3
250	400	200	0.29	7800

In order to remain the cost of each column, the length and area of cross section must be the same. This means that the weight and volume of each column will be constant. From this condition, the research is started. It's assumed that the cross sectional area and length for each section used are (100 mm^2, 2000 mm) respectively. The condition is that the column dimensions must be satisfied in order to ensure the applicable of (Euler formula). In which, the buckling must be within the yield region. This condition is as follow:

$$S > L/k = 120$$

Where:
S: slenderness ratio
k: radius of gyration (m).

In order to verification the above two conditions, the following column section dimensions are chosen, table (3).The dimensions of each cross section are indicated in figure (2).

Table (3) dimensions of cross sectional area

Section	Hollow rectangular			Channel			L and T			I and Z		
Thickness (mm)	1	2	3	1	2	3	1	2	3	1	2	3
Width (W1, mm)	5	5	5	5	5	5	5	5	5	5	5	5
	10	10	10	10	10	10	10	10	10	10	10	10
	15	15	15	15	15	15	15	15	15	15	15	15
	20	20		20	20		20	20	20	20	20	
	25	25		25			25	25	25	25		
	30			30			30	30	30	30		
	35			35			35	35		35		
	40			40			40	40		40		
	45						45	45				

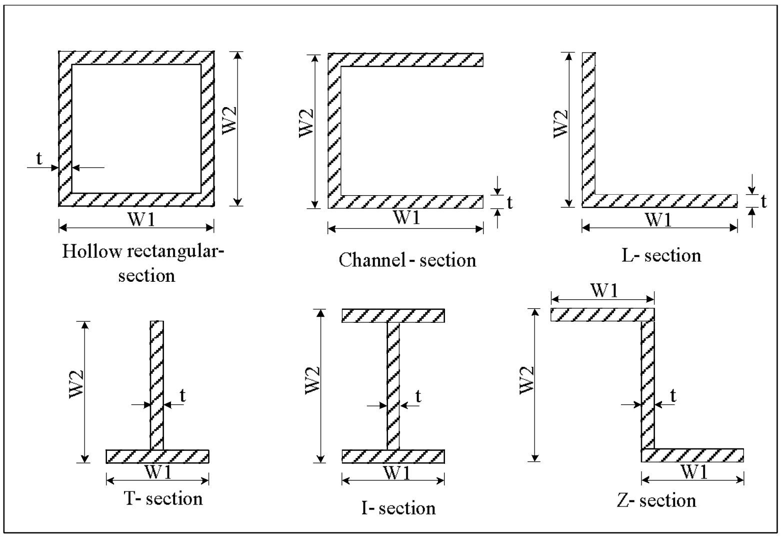


Figure (2) dimensions of columnsections

Finite element method:

ANSYS11 program is used to create the finite element model with element type (structural column, 3D finite strain, 3node 189). Each end of columnis simply supported which represents the boundary conditions as shown in figure (3). A half- length of column is used to analyze the problem. One end is considered as a fixed end (fixed all degree of freedom) and the other end is simply.The load isapplied at node (11).The number of elements used is ten elements with (11 node).

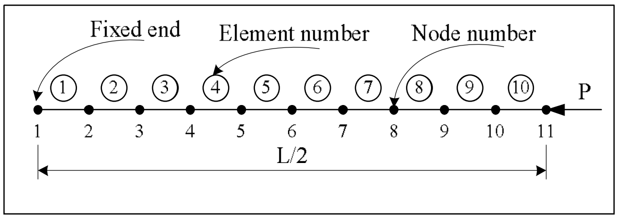


Figure (3) Schematic Graph of column FEM model.

The meshed model, boundary conditions and the applied load locations for each section are shown in figure (4).

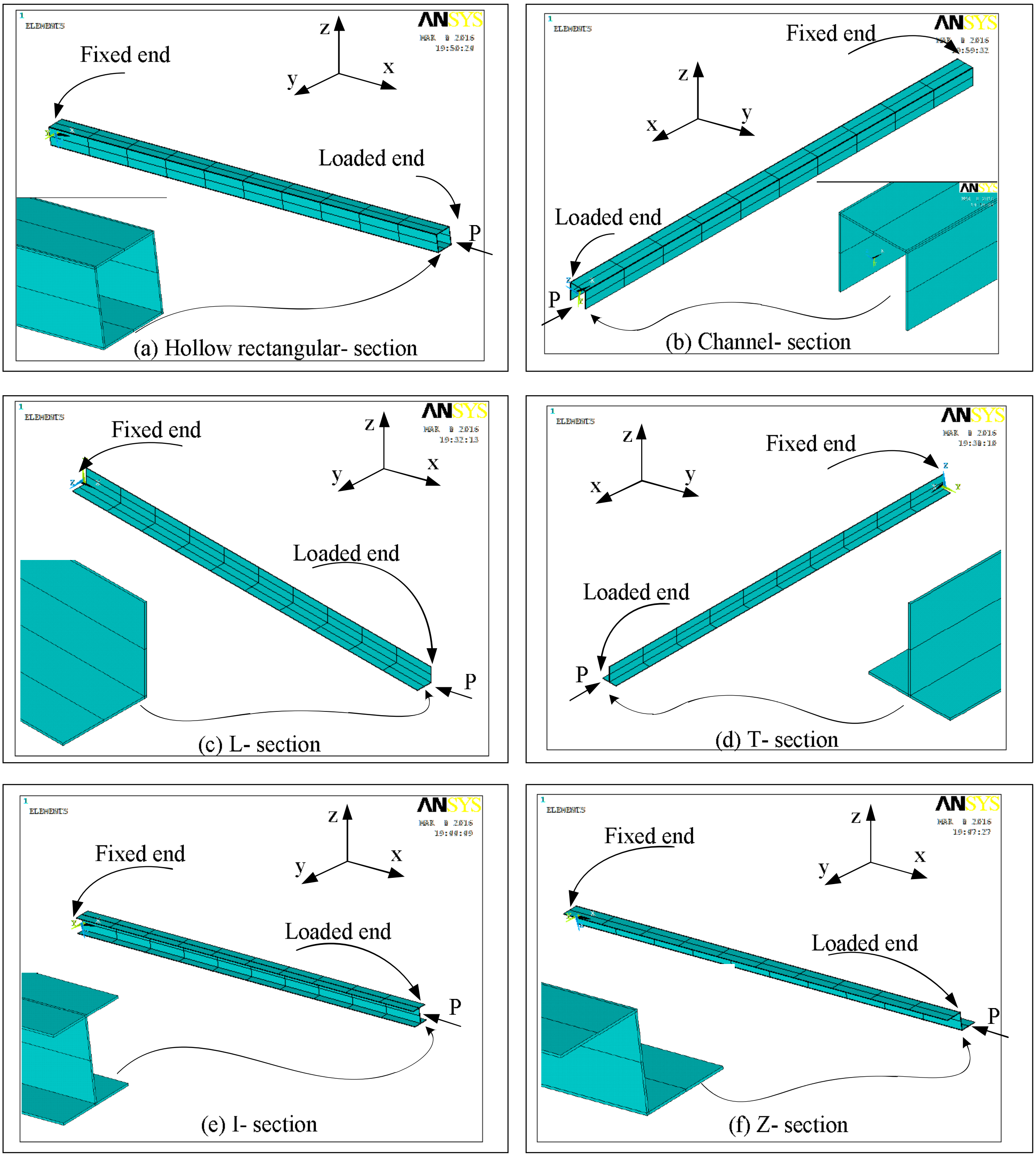


Figure (4) model, boundary conditions and the applied load location for each section

The ANSYS solution has been done initially with static and buckling solution. The resulted data are presented as a critical buckling load.

Model and parameters of design of experiment:

In this work, a factorial model is used. The *k*th response of this model at each *i*th and *j*th level for the factors A and B respectively is given by the following model [10]:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk} \qquad (4)$$

Where:
μ: the overall mean
α_i: levels of factor A
β_j: levels of factor B
i: runs from 1 to α
j: runs from 1 to β
k: runs from 1 to n
n: experiment units
ε_{ijk}: the independent and normally distributed with mean zero and variance
α: mean effect of factors

The term ($_{ij}$) in the above equation called the interaction effect of factors A and B through *i* and *j* runs.

The interaction effects must have a zero sum in the *i*th row and *j*th column. Hence, the response value is given as follows:

$$Ey_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} \qquad (5)$$

Results:

In this section, the critical buckling load for each column section is calculated according to (Euler column formula)and compared with the FEM results (ANSYS11), figures (5-10).ANSYS software is based on the Timoshenko theory which includes the residual stresses and geometric imperfection. In Euler buckling theory, the minimum stresses and geometric imperfection are neglected, so the Euler results being small than those of ANSYS results. Hence, a good agreement has been found between the theoretical and FEM with a small error value (error =0.1-0.3%). The results have been analyzed using design of experiments. In which, the separately and combined effect of width and thickness on the critical buckling load is studied.

Theoretical results:

The critical buckling force increase uniformly until reach the maximum value and then decrease for each thickness value for hollow rectangular section, figure (5). A thinner section gave a higher critical buckling load along all values of width (W1). On the other hand, the Increasing in (*Pcr.*) when the width (W1) is increasedcan be consoled to the fact the minimum moment of inertia is increased with increasing width. The maximum value of (*Pcr=4840N*) is observed at the minimum thickness (t=1mm) and (W1=25mm).

In the I-section, figure (6), the critical buckling load increase gradually with increasing the width for each value of thickness.Hence, the maximum critical buckling load is observed at the final value of width. Where *($Pcr)_{max}$= 4685N, 1319N and 836.5N*) at (*t=1mm,W1=40mm;t=2mm W1=20mm and t=3mm W1=15mm*) respectively. For width range (W1< 20mm), the thicker section presents a good buckling property.

For L-section, figure (7), the critical buckling load (*Pcr.*) approximately is the samefor (t=2& 3mm) with a width range (W1< 15mm). The critical buckling load (*Pcr.*) decreased with increasing the width (W1> 20mm) for (t=3mm) and (W1> 30mm) for (t=2mm). the critical buckling load increase gradually with increasing the width for (t=1mm) until reach the maximum value at the final width (*Pcr=2642N*).

It was found that the behavior of critical buckling load for T-section, figure (8), nearly the same that for the previous section (L). This can be consoled to the fact that the minimum moment of inertia about the bulked axis is approximately the same. The higher ranges of critical buckling load are observed in the T-section, hence, this section have a minimum moment of inertia about the buckled axis as comparing with the other sections.

The critical buckling force somewhere is the same for each thickness of U-section, figure (9). The maximum value is observed in (W1=35mm, t=1 mm), ($(Pcr)_{max}$= *6522N*).

In the final section (Z-section), figure (10), the higher buckling load values are found is found in the thinner section (t=1 mm).

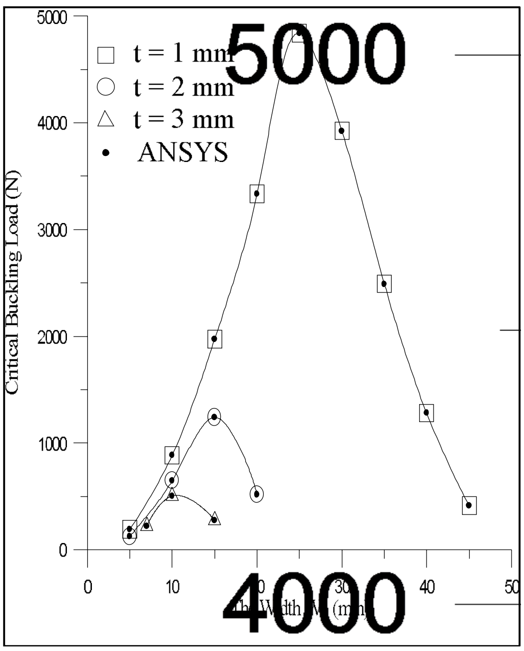


Figure (5) critical buckling load for Hollow rectangular- section

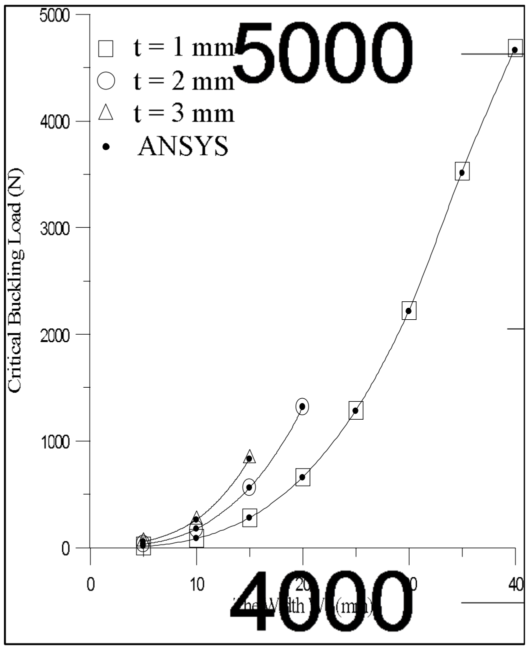


Figure (6) critical buckling load for I- section

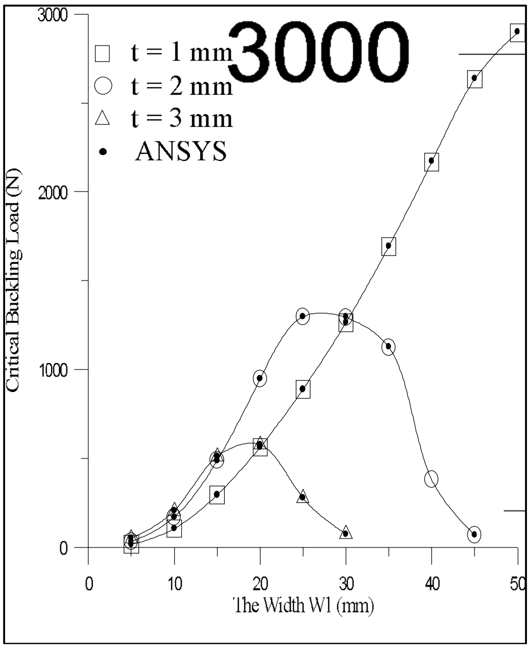


Figure (7) critical buckling load for L- section

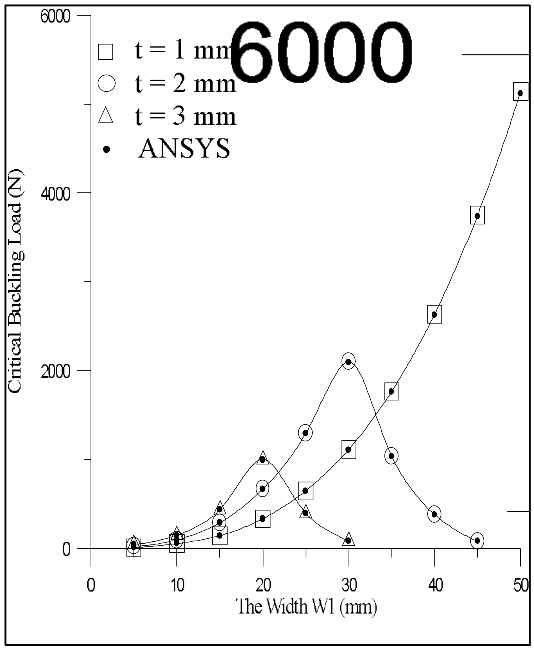


Figure (8) critical buckling load for T- section

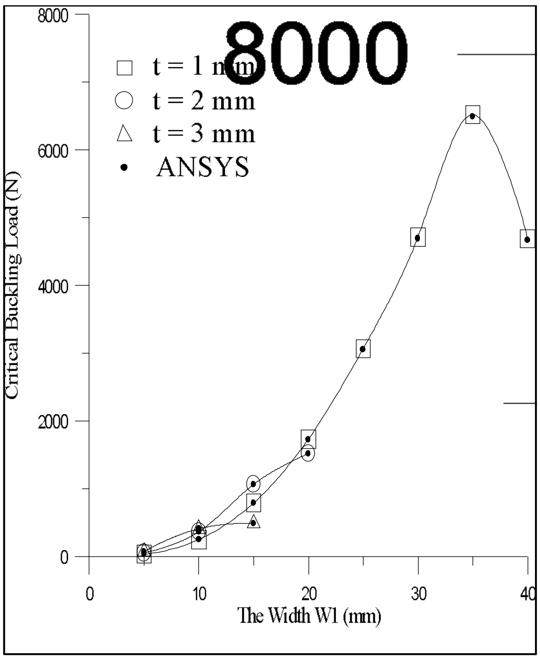


Figure (9) critical buckling load for U- section

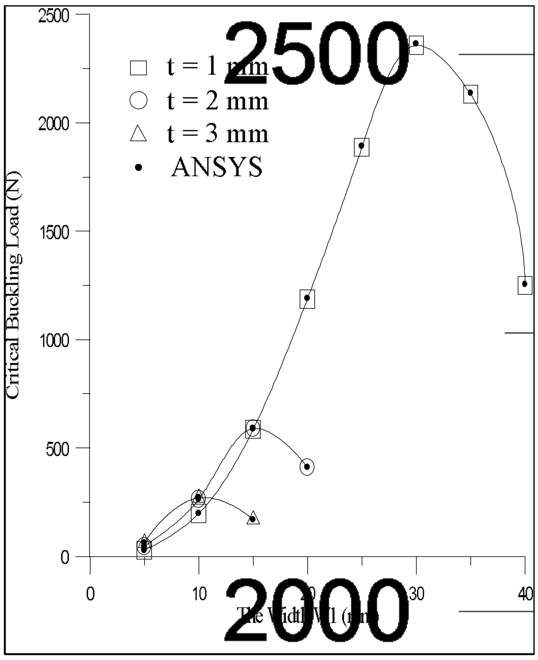


Figure (10) critical buckling load for Z- section

A common comparison for the critical buckling load of all sections is shown in figures (11, 12 and 13) for thickness (t=1, 2 and 3 mm respectively). It's clearly observed that the maximum critical buckling load is found in the U-section for each thickness values.

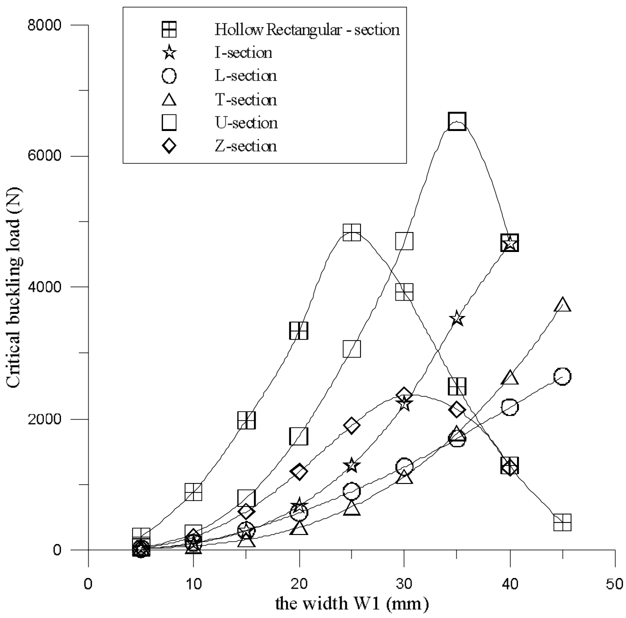


Figure (11) critical buckling load, t=1mm

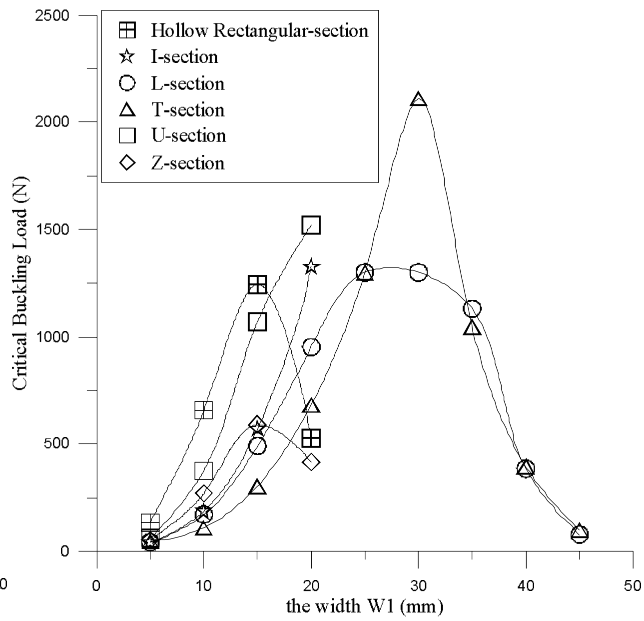


Figure (12) critical buckling load, t=2mm

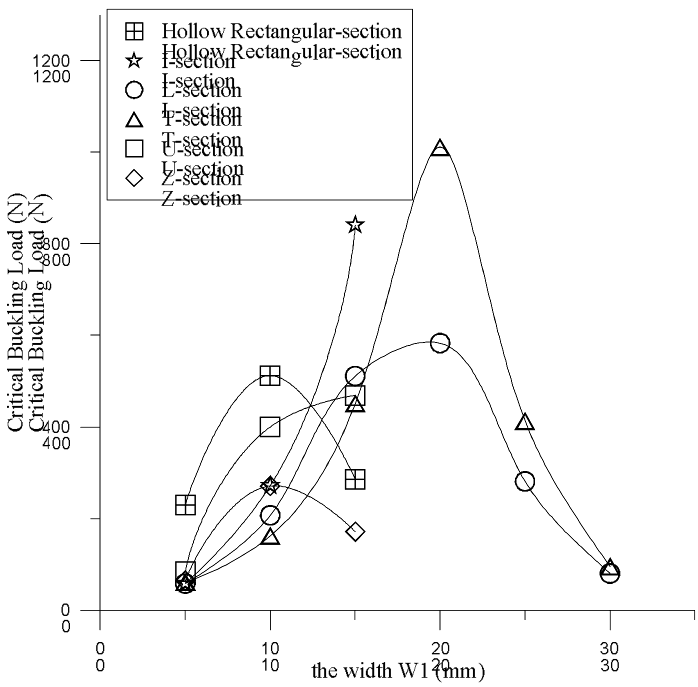
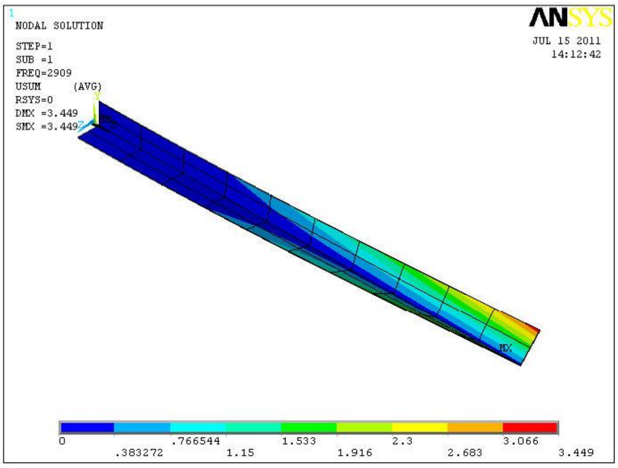
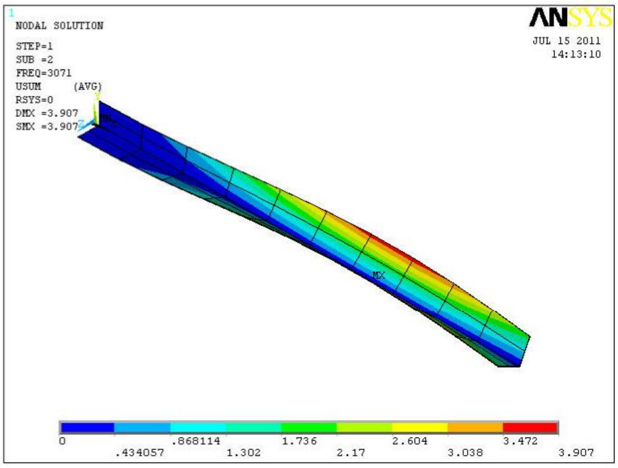


Figure (13) critical buckling load, t=3mm

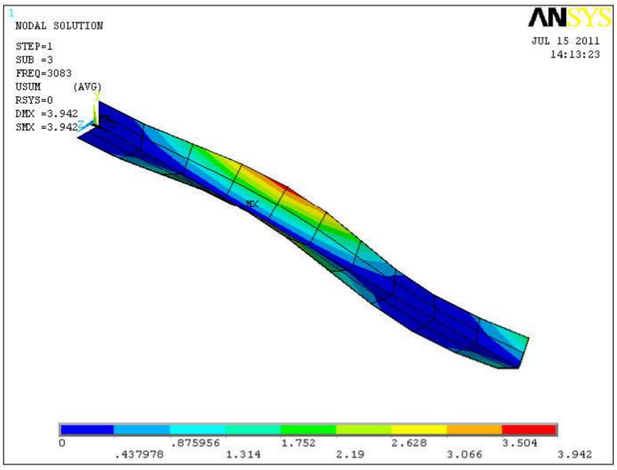
The deformed and four mode shape of the buckled L-section is taken here as an example to illustrate the ANSYS results, figure (14).



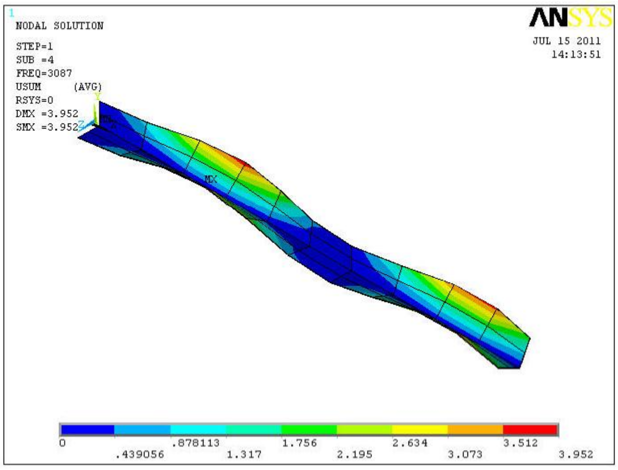
(a) Mode shape1



(b) Mode shape 2



(c) Mode shape 3



(d) Mode shape 4

Figure(14) the deformed L-column with four mode shapes.

Design of experiment results:

In order to summarize the effect of column section type, thickness and width of section on the critical buckling load, the design of experiment method (DOE) is used with the aid of (MINITAB program). As well as, this method give an impression about the optimization design and interactive effect of the used variable.

A fractional factorial design with general full factorial method is used to predict the most effect factor on the buckling load for all cases mentioned in table (3).

The input parameters of thickness and width are un-coded, while, the section type is coded and given in the following table (4).

Table (4) column section codes used in MINTAB program

Section type	Hollow rectangular section	I- section	L- section	T- section	U- section	Z- section
code	Box	I	L	T	U	Z

The effect plot, figure (15), indicate that the maximum critical buckling load is occurred in the U-section with a minimum thickness (t=1mm) and a width (W1= 35 mm). The weakness buckling property is noted in the two sections (L and Z). Increasing thickness reduce the buckling load, while, increasing the width of column section until (W1=35 mm) increase the buckling load. The main factor that affect on the buckling load is the thickness of section, hence, increasing this factor results in decreasing the buckling load. These phenomena can be consoled to the fact that the moment of inertia decrease with increasing the thickness.

As it has been observed from the contour plot, figure (16), the maximum buckling load is occurred with a narrow range of width and thickness as it indicated in this figure. The minimum buckling load is occurred through a higher area.

The interaction effect of column section, width and thickness is shown in figure (17). Initially, take the interactions between thickness and width; it has been observed that minimum thickness gives a good buckling load and this effect be clearly for the width range (W1 >20 mm). The interaction between the column section and width as well as thickness state that the U-section is the best used section along a wide range of column thickness and width. The minimum width and maximum thickness present the weakness buckling load for all column section.

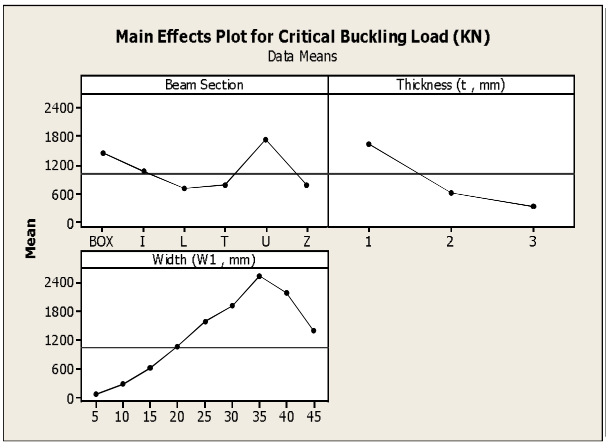


Figure (15) main effect plot for critical buckling load

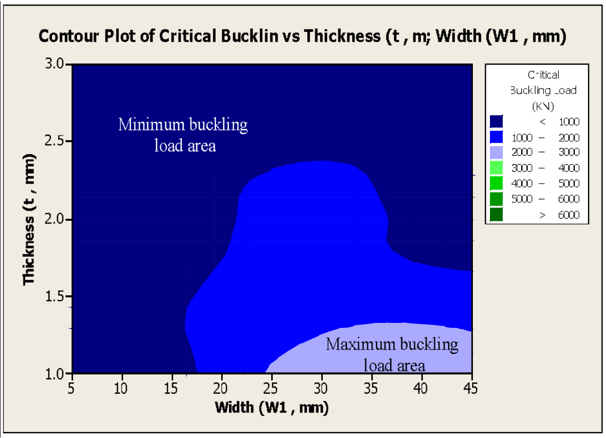


Figure (16) contour plot for critical buckling load

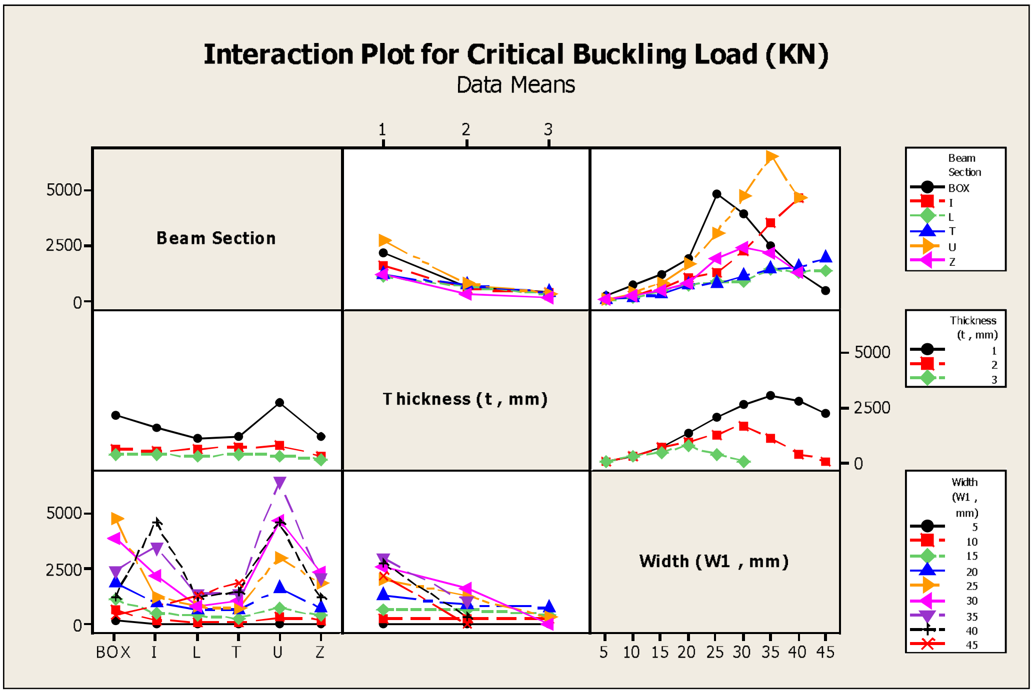


Figure (17) interaction plot for critical buckling load

Conclusion:

1- The critical buckling load is affected by the change in the column section shape and dimensions for the same cross section area value.
2- The critical buckling load increase through a limit range of column width and then decrease over a wide range of thickness for all column section except the (I-section) where the load increase continuously.
3- The higher values of critical buckling load are found in the (U- column section).
4- The lower values of critical buckling load are found in the (Z- column section).
5- Approximately, increasing the width of column section increase the critical buckling load.
6- Reducing the thickness of column section results in increase the critical buckling load.

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