A STUDY OF A TRANSMITTED FORCE FROM HARMONIC MOVING LOAD ON A BEAM SUPPORTED BY NONLINEAR VIBRATION ISOLATOR [+]

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Abstract:

The force transmitted from Bernoulli–Euler beam, resting on a nonlinear vibration isolator at each end and subjected to a moving harmonic load, has been investigated in this paper .

The vibration isolator consists of nonlinear spring and nonlinear damper. Assumed-mode method is used to obtain the nonlinear equations of motion. The effect of the nonlinear characteristics of spring and damper on transmissibility ratio is discussed. Also the effect of velocity of the moving load is explained.

The simulation results show that the increasing of the nonlinear parameter for each spring and damper leads to reduce transmissibility ratio. Results have confirmed that it is not necessary to install nonlinear spring with nonlinear damper in order to reduce transmissibility ratio more. The vibration isolator which includes linear spring and nonlinear damper may give good isolation. For the velocity effect, the results show the velocity values of moving load have clear impact on the transmissibility ratio especially at resonance state where the increasing in velocity leads to reduce transmissibility ratio.

Key words: **transmissibility ratio, harmonic moving load, nonlinear vibration isolator.**

دراسة القوة المنتقلة من حمل موجي متحرك على عتبة مستندة على عازل اهتزازات لا خطي

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المستخلص:

القوة المنتقلة من عتبة برنولي–اويلر مستندة على عازل اهتزازات لاخطي عند كل نهاية ومعرضة لحمل موجي متحرك فحصت في هذا البحث. يتكون عازل الاهتزازات اللاخطي من نابض لاخطي ومخمد لاخطي. استخدمت طريقة فرض النسق للحصول على معادلات الحركة اللاخطية. ولقد تم مناقشة تاثير خصائص اللاخطية للنابض والمخمد على نسبة النقل وتم شرح تاثير سرعة الحمل المتحرك.

وضحت نتائج المحاكاة ان الزيادة في قيم المعاملات اللاخطية لكل من النابض والمخمد تقلل نسبة النقل . كما اكدت النتائج انه ليس من الضروري تركيب نابض لاخطي مع مخمد لاخطي لخفض نسبة النقل اكثر. عازل الاهتزازات الذي يتكون من نابض خطي ومخمد لاخطي يعطي عزل جيد. بخصوص تاثير السرعة فقد وضحت النتائج ان قيم السرعة لها تاثير كبير على نسبة النقل خاصة عند حالة الرنين حيث ان زيادة السرعة تقلل من نسبة النقل.

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Nomenclature:

A=cross section of area(m^2)
c_1= linear parameter of damper (Ns/m)
c_2= nonlinear parameter of damper (Ns^3/m^3)
E= modules of elasticity (Pa)
I= moment of inertia (m^4)
F_o= force amplitude (N)
F_T= transmitted force(N)
k_1= linear parameter of spring (N/m)
k_2= nonlinear parameter of spring (N/m^3)
k_{ij}= stiffness element (N/m)
L= length of panel (m)
m_{ij}= mass element (kg)
Q_i=generalized force
q_i= generalized coordinate
t= time (sec)
TR= transmissibility ratio
V= velocity of moving load (m/sec)
x_o= position of damper and spring (m)
x_p= position of moving load (m)
w=beam displacement in y-direction (m)
$\beta_i L$ =weighted frequency
ρ= density (kg/m^3)
ω_f =forced frequency (rad/sec)
ψ_i =coordinate function

Introduction:

The value of the force that is transferred to foundation of the vibrated structure and machines such as engines, motors, and pumps is very important case in structural dynamics. This problem is an unavoidable difficulty but it can be reduced using the vibration absorber. This reduces the possibility that a resonance condition will occur, which can cause rapid catastrophic failure. Properly implemented, a dynamic absorber will neutralize the undesirable vibration, which would otherwise reduce service life or cause mechanical damage. Most available researches in this filed study the force transmissibility and design the vibration absorbers based on rigid model (mass- spring- damper model)[1 to 5].

The problem of force transmissibility becomes difficult when the vibrated load is moving on the structure like, bridges, guide ways, cranes, cableways, rails, roadways, runways and pipelines. The inertia, velocity and acceleration of the moving object become very important parameters which have effect on the structural dynamics. Therefore; force transmissibility calculations based on the rigid model has become useless .

The moving load problem in itself has been the subject of many research efforts since the beginning of the last century. There are thousands of references that cover this topic, for example, see references [6 to 10].

Various kinds of moving load problems associated with structural dynamics have been presented in excellent monograph by Fryba [6]. On other hand reference [7] mentions a variety of references that deal with various problems of moving load by different techniques .

The response of a beam on elastic foundation subjected to moving concentrated loads is studied by some researchers like [11 to 14], but these studies do not address the subject of force transmissibility. Rustighi et al. [15] said that there are no studies that aim to find the force transmitted through supports, probably because of the limited literature available on structural force transmissibility.

The representation of a structure by the beam is commonly used in this field because many members can be modeled as beams under moving loads in the design of machining processes.

In this paper, transmissibility ratio due to harmonic moving load on the beam which is supported at each end by nonlinear spring and nonlinear damper is studied. Also, the effect of the nonlinear characteristics of spring and damper on transmissibility ratio is discussed.

Mathematical model:

The model that is taken in present analysis includes nonlinear flexible support of Euler-Bernoulli Beam subjected to a harmonic moving load force as shown in figure (1).

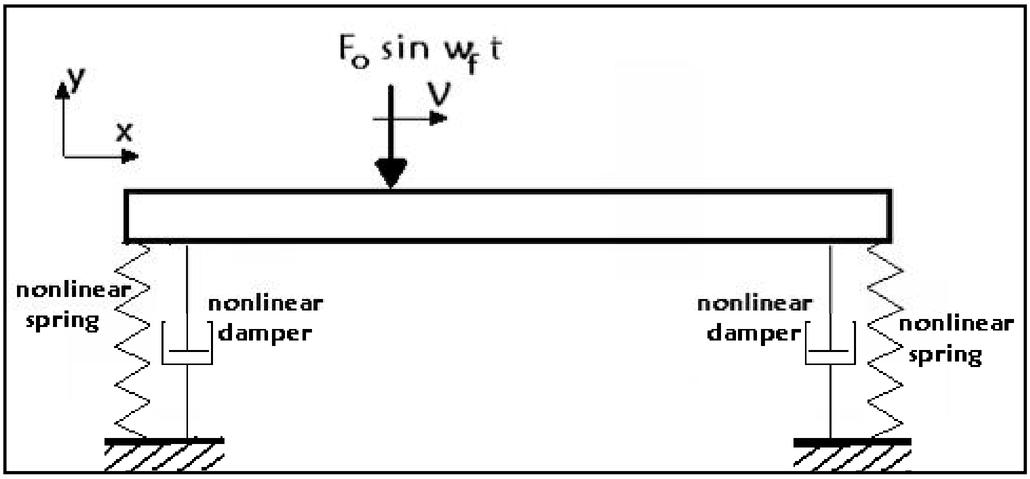


Figure (1): Schematics of flexible support beam subjected to a harmonic moving load

The vibration isolator at both ends of beam consists of cubic nonlinear spring and damper. Therefore the force produced by the nonlinear spring and nonlinear damper obey the following relation:

for spring $$F_s = k_1 w(t, x_p) + k_2 w(t, x_p)^3$$

and for damper $$F_c = c_1 \dot{w}(t, x_p) + c_2 \dot{w}(t, x_p)^3$$

The kinetic energy of the system can be obtained as follows:

$$KE = \frac{1}{2}\rho A \int_0^L \left(\frac{\partial w}{\partial t}\right)^2 dx \quad \text{......} (1)$$

The damping energy *DE* and the potential energy *PE* are as follows

$$DE = \frac{1}{2}c_1 \dot{w}(0)^2 + \frac{1}{4}c_2 \dot{w}(0)^4 + \frac{1}{2}c_1 \dot{w}(L)^2 + \frac{1}{4}c_2 \dot{w}(L)^4 \quad \text{......} (2)$$

$$PE = EI \int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{1}{2}k_1 w(0)^2 + \frac{1}{4}k_2 w(0)^4 + \frac{1}{2}k_1 w(L)^2 + \frac{1}{4}k_2 w(L)^4 \quad \text{......} (3)$$

The assumed –mode method is used in the derivation of the equations of motion for the beam. This method depends on assuming suitable solution to the displacements of the problem.
The presence of springs at both ends of the beam requires solving free vibration problem mathematically first to find the characteristic equation, weighted frequencies ($\beta_i L$), and mode shape.

The characteristic equation is taken from Jokonien K. work [16] and then is solved numerically for each value of linear stiffness spring k_1 to find the weighted frequencies.
In the present beam problem the displacements are assumed to be of the form

$$w(x,t)=\psi_1(x)q_1(t)+\psi_2(x)q_2(t)+\psi_3(x)q_3(t)+\dots\dots\psi_i(x)q_i(t) \quad \dots\dots\dots\dots\dots\dots (4)$$

where $q_i(t)$ is generalized coordinates, and $\psi_i(x)$ is the admissible beam functions which satisfy present beam boundary conditions and is defined as follows [17]:

$$\psi_i(x)=\sin(\beta_i x)+\mu\cos(\beta_i x)+\nu\sinh(\beta_i x)+\mu\cosh(\beta_i x) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

where

$$\nu=\frac{\sin(\beta_i L)-\eta(\cos(\beta_i L)-\cos(\beta_i L))}{\sinh(\beta_i L)-\eta(\cosh(\beta_i L)-\cos(\beta_i L))} \quad , \quad \mu=\eta(1-\nu) \quad , \quad \eta=\frac{(\beta_i L)^3}{2k_1}$$

In this analysis, the displacements take the first two modes as follows

$$w(x,t)=\psi_1(x)q_1(t)+\psi_2(x)q_2(t) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6)$$

Substituting equation (6) into Equations (1,2&3) and applying Lagrange's equation the following two differential equations of flexible ends beam motion are obtained

$$\begin{aligned}
& m_{11}\ddot{q}_1+m_{12}\ddot{q}_2+k_{11}q_1+k_{12}q_2+c_1a_1\dot{q}_1+c_1a_2\dot{q}_1+c_1b_1\dot{q}_2+c_1b_2\dot{q}_2+\\
& k_1a_1q_1+k_1a_2q_1+k_1b_1q_2+k_1b_2q_2+c_2[d_1\dot{q}_1^3+d_2\dot{q}_1^3+3e_1\dot{q}_1^2\dot{q}_2+\\
& 3e_2\dot{q}_1^2\dot{q}_2+3f_1\dot{q}_1\dot{q}_2^2+3f_2\dot{q}_1\dot{q}_2^2+g_1\dot{q}_2^3+g_2\dot{q}_2^3]+k_2[d_1q_1^3+d_2q_1^3+\\
& 3e_1q_1^2q_2+3e_2q_1^2q_2+3f_1q_1q_2^2+3f_2q_1q_2^2+g_1q_2^3+g_2q_2^3]=Q_1(t)\\
& m_{21}\ddot{q}_1+m_{22}\ddot{q}_2+k_{21}q_1+k_{22}q_2+c_1h_1\dot{q}_2+c_1h_2\dot{q}_2+c_1b_1\dot{q}_1+c_1b_2\dot{q}_1+\\
& k_1h_1q_2+k_1h_2q_2+k_1b_1q_1+k_1b_2q_1+c_2[i_1\dot{q}_2^3+i_2\dot{q}_2^3+3e_1\dot{q}_1^3+3e_2\dot{q}_1^3+ \qquad \dots\dots\dots (7)\\
& 3f_1\dot{q}_1^2\dot{q}_2+3f_2\dot{q}_1^2\dot{q}_2+g_1\dot{q}_1\dot{q}_2^2+g_2\dot{q}_1\dot{q}_2^2]+k_2[i_1q_2^3+i_2q_1^3+3e_1q_1^3+\\
& 3e_2q_1^3+3f_1q_1^2q_2+3f_2q_1^3q_2+g_1q_1q_2^2+g_2q_1q_2^2]=Q_2(t)
\end{aligned}$$

where

$$Q_1(t)=Fo\sin(\omega_f)\psi_1(x_o)$$
$$Q_2(t)=Fo\sin(\omega_f)\psi_2(x_o)$$

The value of x_o is varied with time depending on the value of moving load velocity (V) and is calculated as follows $\quad x_o=Vt \quad \le L$

where t is the total traveling time of the load moving from the left end of the beam to the right end.
The mathematical expression of the constants in the equations of motion (7) is listed in appendix (A)
The transmitted force to the base acting at each end through the nonlinear spring and nonlinear damper is

$$\begin{aligned} F_T(t)\big|_{x=0} &= c_1 \dot{w}(0) + c_2 \dot{w}(0)^3 + k_1 w(0) + k_2 w(0)^3 \\ F_T(t)\big|_{x=L} &= c_1 \dot{w}(L) + c_2 \dot{w}(L)^3 + k_1 w(L) + k_2 w(L)^3 \end{aligned} \quad \text{...} (8)$$

The transmissibility ratio, TR in the present analysis is calculated as

$$TR\big|_{x=0} = \frac{\max(F_T\big|_{x=0})}{Fo} \quad \text{and} \quad TR\big|_{x=L} = \frac{\max(F_T\big|_{x=L})}{Fo} \quad \text{...} (9)$$

Parameter values employed in the calculation are listed in table (1).

Table (1): System parameters

A	16e-4m^2	L	10m
E	200e9 N/m^2	$\beta_1 L$	3.027
I	2.133e-7 m^4	$\beta_2 L$	6.085
F_o	1000N	ρ	8000 kg/m^3

Validity of Present Model:

In this section, the comparison with related published works will be performed. Fahim J.[18] studied the effects of elastic support (linear spring and damper) on the beam dynamic performance at different values of moving force velocities. The sensitivity analysis of root mean square (RMS) beam deflection values is carried out, for different spring and damping properties.
In order to illustrate as how K1 and C1 values affect the beam dynamic performance, the root mean square (RMS) values of the deflection are calculated and presented in Table (3). The results of present work are seen that a good agreement with that results of Fahim J.[18] .

Table (2): RMS Values of the beam deflections vs. velocity for different values of spring and damper coefficient

Case No.	K1 (N/m)	C1 (Nsec/m)	Velocities (m/sec)	RMS Values of the beam deflections(m)	
				Ref. [18]	Present work
1	4×10^{10}	4×10^{7}	50	0.009	0.0103
2	2×10^{9}	3×10^{5}	75	0.012	0.0128
3	8×10^{8}	2×10^{4}	100	0.0142	0.0151
4	3×10^{8}	8×10^{3}	125	0.0161	0.0168

Simulation:

The proposed nonlinear flexible support of beam subjected to a harmonic moving load is verified with computer simulation using the fourth-order Runge-Kutta integration method which is programmed in Matlab language. The time step is chosen to be 0.0001sec to ensure the stability of nonlinear equation of motion solution.

In this work, the effect of nonlinear spring, nonlinear damper and moving load velocity on the transmissibility ratio is studied as follows:

1- Effect of nonlinear spring:

The effect of nonlinear parameter of nonlinear spring on the transmissibility ratio, TR without damper is studied. Figure (2) shows how the value of k2 affects transmissibility ratio at resonance state. The increasing of k2 leads to reduce TR at resonance in comparison with the case of k2=0. The reduction in TR is equal to 12% at k2=106 N/m3, while the reduction becomes 44% when k2 equals k2=5□106 N/m3. This indicates that excessive increase in k2 may be useful.

Figure (3) shows the large effect of damper coefficient (c1=30 N s/m) on reduction of TR value in the absence of the effect of nonlinear spring (k2=0). Good reduction in TR appears and equals 65%. Also, figure (3) shows how the response of TR changes when the value of k2 is changing. It is seen that the value of k2 is not effective on reduction TR with existence of linear damper.

2- Effect of nonlinear damper:

In the absence of the spring nonlinear effect; non-linear coefficient of the damper capable to reduce TR more compared with the use of only linear damper, figure (4). It is found that the damping coefficient of linear (c1=30Ns/m) is able to reduce the TR 65% compared with the use of linear spring only .

When nonlinear damper coefficient is used with (c2=30Ns3/m3), the reduction in TR becomes 80%.When the value of c2 increased to 60Ns3/m3, it is found that the reduction in TR becomes 95%, hence the damping coefficient has been doubled. Figure (5) shows the effect of interaction between the nonlinear spring and nonlinear damper. It is obvious that the effect of k2 is not effective in this case.

3-Effect of velocity:

The velocity effect of the moving load on TR is investigated at different velocities as shown in figures (6, 7&8).

Generally, results show that the increase of velocity at resonance state will decrease the TR at different kinds of spring and damper coefficient (linear and nonlinear). At high velocities, TR may reduce below one at resonance state and a good isolation can be achieved.

When the velocity of moving load is changed, figure (9) shows the TR response behavior at resonance. Important note can be obtained from figures (6-9); any increase in velocity is able to suppress the amplitude at resonance to a reasonable value thus avoiding the failure in the system.

Conclusions:

The transmissibility ratio due to harmonic moving load on the beam which is supported at each end by nonlinear spring and nonlinear damper is investigated. Also, the effect of the nonlinear characteristics of spring, damper and velocity on transmissibility ratio is discussed.

From simulation results can be drawn these conclusions:

1- The first mode is enough for continues system to be used in calculation of the transmitted forces.
2- An increase in nonlinear spring coefficient k_2 causes less transmissibility ratio at resonance state. The large increasing k_2 is not useful.
3- The k_2 has no effect on the performance of nonlinear damper.
4- The velocity values of moving load have clear impact on the transmissibility ratio especially in resonance state where the increasing in velocity leads to reduce transmissibility ratio
5- The best selection of vibration absorber includes linear spring and nonlinear damper because the high reduction in TR (at same value of velocity) comes from installation of them.
6- A good design of nonlinear dampers makes designers dispense of the active vibration controllers which are very costly.

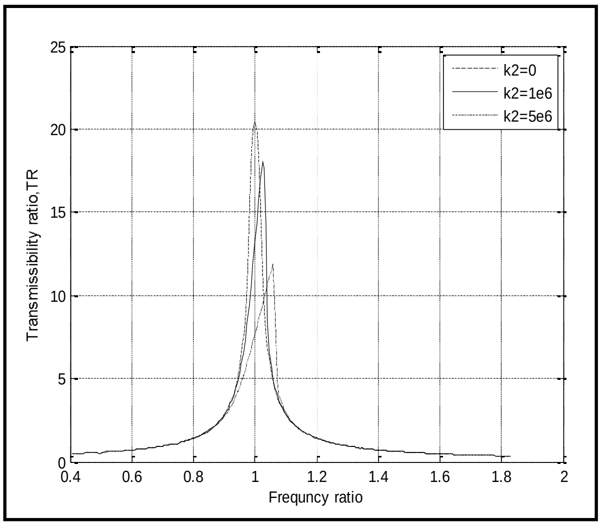


Figure (2): Effect of k_2 on the TR, (c_1=0,c_2=0)

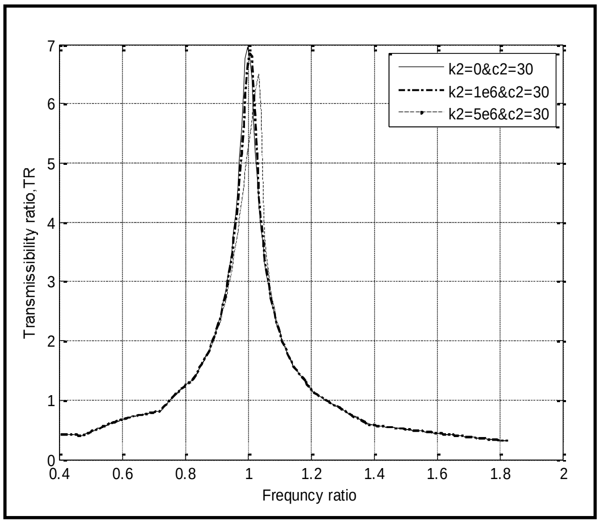
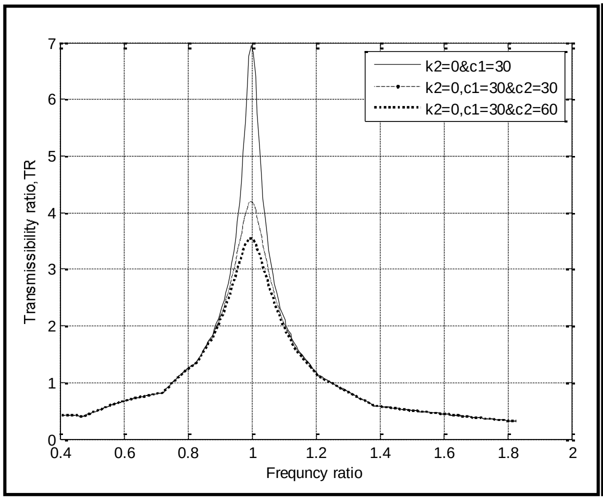


Figure (3): Effect of k_2 on the TR, (c_1=30,c_2=0)



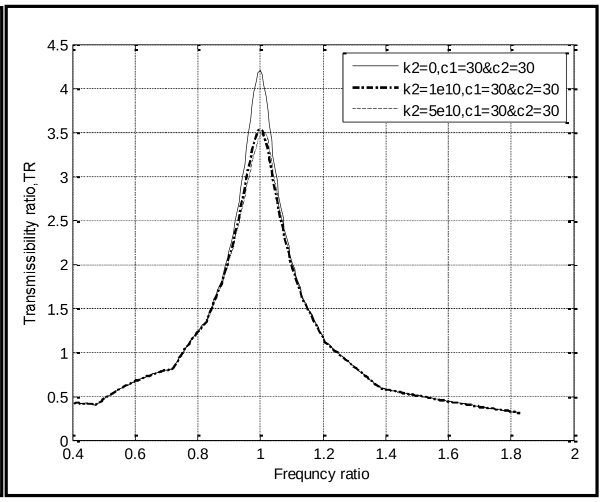


Figure (4): Effect of c_2 on the TR, (k_2=0 ,c_1=30)

Figure (5): Effect of k_2 on the TR, c_1=30,c_2=30

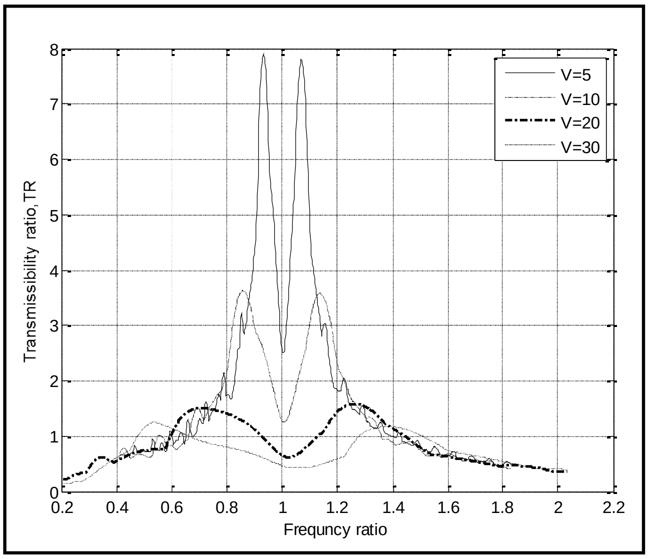


Figure (6): Effect of velocity of moving load on the TR (k_2=0 , c_1=0,c_2=0)

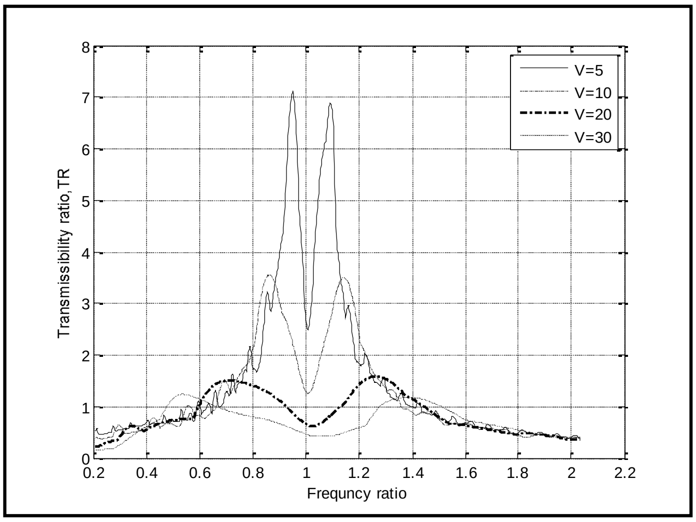


Figure (7): Effect of velocity of moving load on the TR (k_2=1e5 , c_1=0,c_2=0)

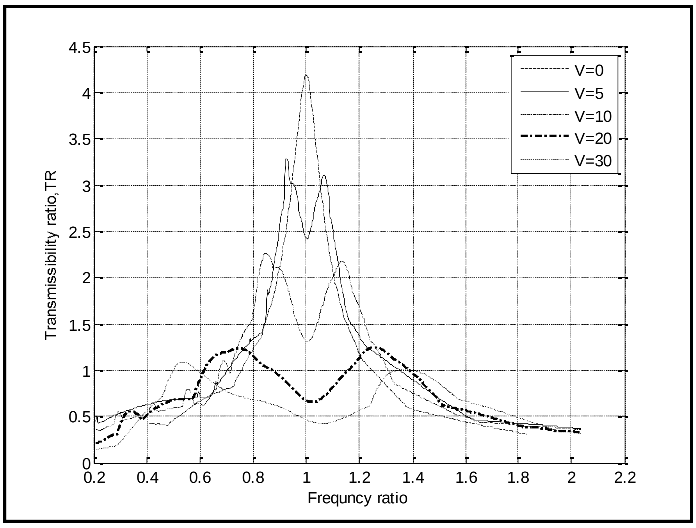
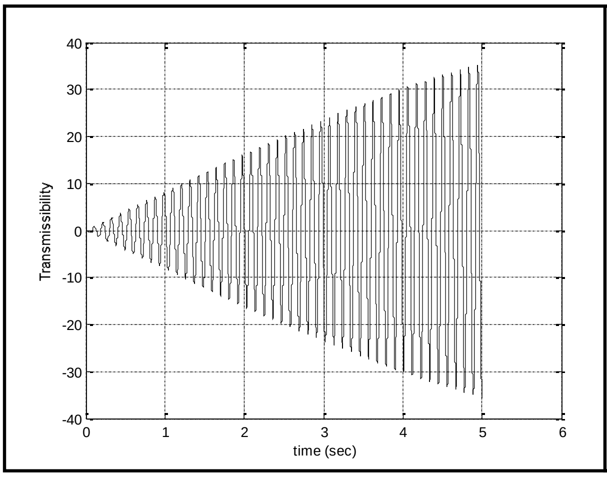
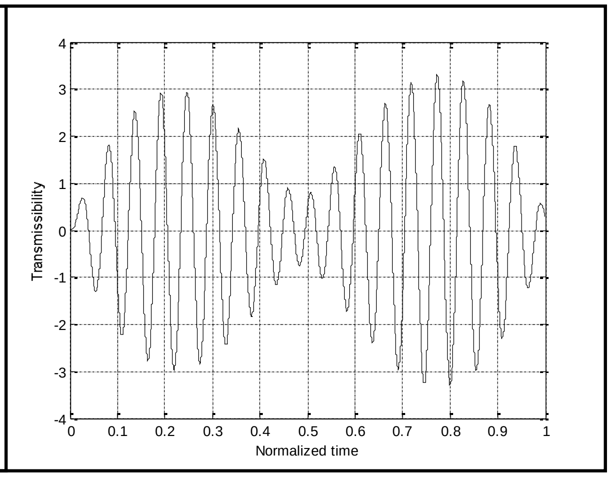
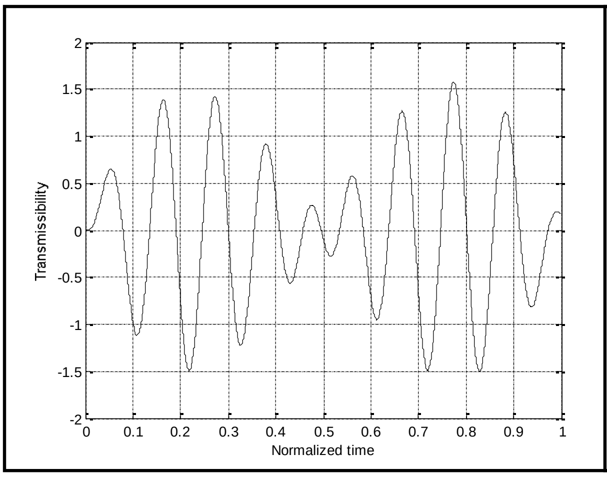


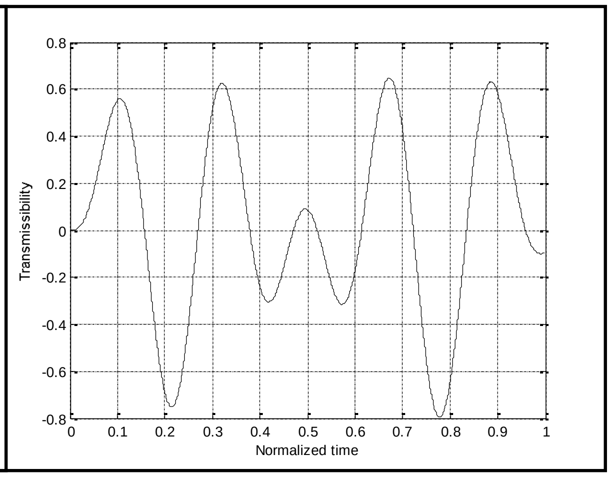
Figure (8): Effect of velocity of moving load on the TR (k_2=0 , c_1=30,c_2=30)





V=0 m/sec **V=5 m/sec**





V=10 m/sec **V=20 m/sec**

Figure (9): Effect of velocity of moving load on the TR response

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Appendix (A):

The constants of the equations of motion ()

$$m_{ij} = \rho A\int_0^L m\psi_i\psi_j\,dx$$

$$k_{ij} = EI\int_0^L \frac{\partial^2\psi_i}{\partial x^2}\frac{\partial^2\psi_j}{\partial x^2}\,dx$$

$$a_1 = \psi_1(0)^2$$

$$a_2 = \psi_1(L)^2$$

$$b_1 = \psi_1(0)\psi_2(0)$$

$$b_2 = \psi_1(L)\psi_2(L)$$

$$d_1 = \psi_1(0)^4$$

$$d_2 = \psi_1(L)^4$$

$$e_1 = \psi_1(0)^3\psi_2(0)$$

$$e_2 = \psi_1(L)^3\psi_2(L)$$

$$f_1 = \psi_1(0)^2\psi_2(0)^2$$

$$f_2 = \psi_1(L)^2\psi_2(L)^2$$

$$g_1 = \psi_1(0)\psi_2(0)^3$$

$$g_2 = \psi_1(L)\psi_2(L)^3$$

$$h_1 = \psi_2(0)^2$$

$$h_2 = \psi_2(L)^2$$

$$i_1 = \psi_2(0)^4$$

$$i_2 = \psi_2(L)^4$$