NATURAL CONVECTION HEAT TRANSFER OF LIQUIDS IN A RECTANGULAR ENCLOSURE [+]

Ayad M. Salman [*] **Mohammed A. Fayad** [*]

Abstract:

This research is focused on the numerical investigation of a steady laminar natural convection heat transfer of liquid (water) with Prandtl number (Pr = 6.2) in a 2-D rectangular enclosure and Rayleigh number, Ra in range from 10^3 to 10^5 are taken into consideration for various aspect ratios, AR of 0.5, 1.0 and 2.0.
The enclosure object with heated left side wall, while the right side was cold, the top and bottom walls was adiabatic. The theoretical study involved a numerical solution of the Navier-Stokes and energy equations by using finite difference method. The stream function – formulation was used in the mathematical model. The numerical solution is capable of calculating the velocity, stream function, vorticity and temperature fields of the computational domain. A computer program in (FORTRAN 90) was used to carry out the numerical solution and used the Tecplot7 program to draw the results of program.

The numerical results showed that the main process of heat transfer is conduction for Rayleigh number less than $Ra \leq 10^3$, and convection for Rayleigh number grater than $Ra > 10^3$. For $Ra=10^5$ and AR=1, show a bicellular flow in stream lines contour because the velocity nearby cold wall is larger than those which are faraway from cold wall. In temperature contour shows plume region be clearer. Average Nusselt number is increased with aspect ratio increasing for Ra ranging with (10^3-10^5) because of increasing of heat transfer area.
In order to validate the numerical model, the results of variation local Nusselt number and a relation between average Nusselt number and Ra number, are compared with previous works. For square enclosure filled by air (Pr=0.72). There are agreement in results.
Keywords: Natural convection, stream-vorticity, liquids, finite difference, rectangular enclosure.

انتقال الحرارة بالحمل الحر للسوائل في حيز مستطيل مغلق

اياد محمد محمود محمد علي فياض

المستخلص:

يركز البحث الحالي على التنبؤ العددي لانتقال الحرارة بالحمل الحر الطباقي المستقر لسائل ذو رقم براندتل يساوي 6.2 خلال حيز مغلق ثنائي الأبعاد لمدى رقم رالي 10^3 إلى 10^5 ولنسب ارتفاع الى العـــــرض للمـــــدى 0.5 ، 1 و 2.

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الحيز المستخدم هو ثنائي الأبعاد مع تسخين الجانب الأيسر من الحيز، في حين أن الجانب الأيمن كـــان بـــاردا، وكانت الجدران العلوية والسفلية معزولة. الدراسة تضمنت حل معادلات نافير–ستوك والطاقة باستخدام طريقة الفروق المحددة. استخدمت صياغة الدوامية – دالة الانسياب في النموذج الرياضي وللحل العددي القابلية على حساب السرعة، دالة الانسياب، الدوامية ودرجة الحرارة خلال المجال المدروس. تم بناء برنامج حاسوبي بلغة فورتران 90 لتنفيذ الحل العددي.

وأظهرت النتائج العددية أن العملية الرئيسية لانتقال الحرارة هي بالتوصيل لرقم رالي أقل من 10^3، ويكون انتقال الحرارة بالحمل لارقام رالي اكبر من 10^3. كما بينت النتائج ان دالة الانسياب تظهر بشكل خلية منفردة ما عدا حالـــة رقم رالي يساوي 10^5 حيث تظهر بشكل ثنائي الخلايا عندما AR=1 ان السرعة قرب الجدار البارد هي اكبـــر مـــن السرع البعيدة عن الجدار البارد. مخطط توزيع درجات الحرارة يبين منطقة الريشة بشكل واضح. معدل رقـــم نـــسلت يزداد بزيادة النسبة الباعية لارقام رالي (10^5-10^3) بسبب زيادة مساحة انتقال الحرارة.

لتأكيد النتائج العددية فقد تم مقارنتها مع النتائج العددية للبحوث السابقة، وتتم مقارنة نتائج تغير رقم نسلت الموقعية والعلاقة بين رقم نسلت ورقم رالي لحيز مربع مملوء بالهواء (لرقم براندتل=0.72)، حيث كان التوافق بـــين النتـــائج ممتاز مما يؤكد صحة النموذج الحسابي الحالي.

Nomenclature :

Symbols	Meaning	Units
AR	Aspect ratio	——
C_p	Specific heat at constant pressure	J/kg.K
E_{max}	Maximum error	——
g	Gravitational acceleration	m/s^2
H	Enclosure height	m
k	Thermal conductivity	W/m.K
W	Enclosure Width	m
Nu	Nussel number	——
$\overline{Nu}$	Average Nussel number	——
p	pressure	Pa
Pr	Prandtl number	——
Ra	Rayleih number	——
r, r_Ψ, r_Ω, r_θ	Relaxation parameter for stream function, vorticity, and temperature respectively	——
T	Temperature	K
u	Velocity component in x-direction	m/s
U	Dimensionless Velocity component in x-direction	——
v	Velocity component in y-direction	m/s
V	Dimensionless Velocity component in y-direction	——
x, y	Cartesian space coordinates	m
X,Y	Dimensionless Cartesian space coordinates	——
Greek Symbol		
β	Coefficient of thermal expansion	1/K
ϕ	General dependent variable	——
μ	Molecular dynamic viscosity	$N.s/m^2$
ν	Kinematic viscosity	m^2/s
ω	Vorticity	1/s
Ω	Dimensionless vorticity	——
ρ	Density of fluid	kg/m^3
ψ	stream function	m^2/s
Ψ	Dimensionless stream function	——
θ	Dimensionless temperature	——

Subscript Symbols		
C, H	Related to cold and hot side respectively	——
(i,j)	Grid nodes in (x,y) direction	——

Introduction:

Convection in nature occurs in two different forms, the so-called natural convection and forced convection. Natural convection is the motion that results from the interaction of gravity with density differences within a fluid. The differences may result from gradients in temperature, concentration or composition **[1]**. In many engineering applications and naturally occurring processes, natural convection plays an important role as a dominating mechanism. Natural convection in enclosures has been widely used in many thermal applications such as in solar collectors, cooling devices for electronic instruments, building insulation, energy storage devices, and many others. Air and water is much used as a heat transfer fluid because of its availability and good thermal properties: it has conductivity, a large thermal capacity, low viscosity, and presents no hazard **[2]**.

Natural convection in rectangular enclosure has been studied for many years. Some of studies made to study laminar natural convection in rectangular enclosure.

Vasseur and Robillard (1980) **[3]** investigated the inversion of flow patterns caused by the maximum density of water at 4 °C enclosed in rectangular cavities.

Robillard and Vasseur (1981) **[4]** studied the maximum density effect and supercooling of two-dimensional transient laminar natural convection heat transfer of water in a rectangular cavity with a convective boundary and found that temperature the maximum density existing between the temperature 273 K and 285 K.

Aktas et al. (2004) **[5]** studied the generation and propagation of thermoacoustic waves in water, were investigated by solving the fully compressible form of the Navier–Stokes equations. A square enclosure filled with water is considered as the computational domain. Thermally induced pressure waves are generated by rapidly heating the left wall.

Basak et al. (2006) **[6]** investigated numerically the steady laminar natural convection flow in a square cavity with uniformly and non-uniformly heated bottom wall, and adiabatic top wall maintaining constant temperature of cold vertical walls has been performed. The numerical procedure adopted in the present study yields consistent performance over a wide range of parameters (Rayleigh number Ra, 103 ≤ Ra ≤ 105 and Prandtl number Pr, (0.7 ≤ Pr ≤ 10) with respect to continuous and discontinuous Dirichlet boundary conditions.

Sivasankaran and Kandaswamy (2007) **[7]** performed a numerical study to analyze the combined temperature and species gradients induced buoyancy-driven natural convection flow of cold water, near its density extremum, contained in a rectangular partitioned enclosure with isothermal side walls and insulated top and bottom.

Diaz and Winston (2008) **[8]** made numerical model based on finite differences to develop solve the mass, momentum, and energy equations, solution for the case of a coordinate transformation is used to map the parabolic shape into a rectangular domain where the governing equations are solved.

A study of an enclosure having uniform temperature condition at the vertical walls for various aspect ratios, AR = H/W of 0.5, 1.0 and 2.0. Based on the numerical predictions, the effects of aspect ratio on the heat transfer and flow patterns of the water are investigated.

The purpose is to determine the influence of Rayleigh number Ra on the heat transfer and flow patterns of the liquid, for Prandtl number, Pr of 6.2 and Rayleigh number, Ra ranging from 10^3 to 10^5 are considered for the enclosure, where the top and bottom walls are assumed to be adiabatic.

Mathematical Model :

The basic equations of the mass and momentum conservations (Navier-Stokes equations) in terms of primitive variables for an incompressible, viscous fluid. Will be used this formulation contains the velocity and the pressure of the fluid which are the original unknowns. The difficulties arise due to the satisfaction of the continuity equation and missing pressure equation. Thus, in most of the numerical procedures these equations are transformed to stream function-vorticity and velocity-vorticity formulations. A schematic diagram of the physical situation is presented in figure (1). In this item, the main equations for the laminar natural convection through two-dimensional enclosure with appropriate boundary conditions and the working fluid is water.

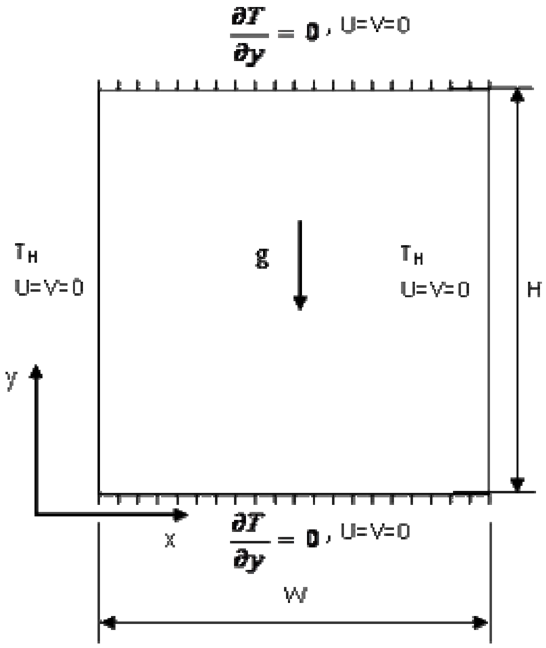


Figure (1): show schematic diagram of physical configuration.

Following assumptions have been made: two-dimensional problem, with no viscous dissipation, the gravity acts in the vertical direction, the fluid properties are constant and radiation heat exchange is neglect. At steady state conditions using above assumption, the governing equations as given below **[8]**:
Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

x-momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

y-momentum equation:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta(T-T_C) \tag{3}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\nu}{\Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where g is the gravitational acceleration (m/sec^2), β is coefficient of thermal expansion and Pr is Prandtl number ($\Pr=\mu C_p/k$). The governing equations given above, i.e., equation (1) in equation (4), are given in terms of the so-called primitive variables, i.e., u, v, p, and T. The solution procedure discussed here is based on equations involving the stream function, the vorticity, and the temperature, as variables. The stream function and vorticity are defined by:

$$u=\frac{\partial\psi}{\partial y},\ \mathrm{v}=-\frac{\partial\psi}{\partial \mathrm{x}} \tag{5}$$

$$\omega=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{6}$$

The vorticity equation is obtained by eliminating the pressure between the two momentum equations, i.e., by taking the y-derivative of equation (2) and subtracting from it the x-derivative of equation (3). The equation of vorticity (conservation of angular momentum) becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}=\nu\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)+\beta g\frac{\partial T}{\partial x} \tag{7}$$

In terms of stream function, the equation become

$$\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)=-\omega \tag{8}$$

While in terms of the stream function the energy equation becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\nu}{\Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{9}$$

Before considering the numerical solution to the above set of equations, it is convenient to rewrite the equations in terms of dimensionless variables. The following dimensionless variables will be used here:

$$\left.\begin{aligned}
&\Psi=\frac{\psi\Pr}{\nu},\ \Omega=\frac{\omega \mathrm{W}^2\Pr}{\nu}\\
&X=\frac{x}{W},\ \mathrm{Y}=\frac{\mathrm{y}}{\mathrm{W}},\ AR=\frac{H}{W}\\
&\theta=\frac{T-T_c}{T_h-T_c}\\
&Ra=\frac{g\beta(T_H-T_C)W^3}{\alpha\nu}=\frac{g\beta(T_H-T_C)W^3}{\nu^2}\Pr
\end{aligned}\right\} \tag{10}$$

In terms of these variables, the stream function, vorticity and energy equations respectively becomes

$$\left(\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}\right)=-\Omega \tag{11}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\Omega}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\Omega}{\partial Y}=\Pr\left(\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right)+Ra\frac{\partial\theta}{\partial X} \tag{12}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y}=\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \tag{13}$$

Boundary Conditions:

The boundary conditions of velocity and temperature fields are shown in figure (1) presented as **[9]**:

$$\left.\begin{aligned}
&X=0: \mathrm{u}=\mathrm{v}=\Psi=0,\theta=1,-\Omega=\frac{\partial^2\Psi}{\partial X^2}\\
&\mathrm{X}=1: \mathrm{u}=\mathrm{v}=\Psi=0,\theta=0,-\Omega=\frac{\partial^2\Psi}{\partial X^2}\\
&\mathrm{Y}=0: \mathrm{u}=\mathrm{v}=\Psi=0,\frac{\partial\theta}{\partial Y}=0,-\Omega=\frac{\partial^2\Psi}{\partial Y^2}\\
&\mathrm{Y}=1: \mathrm{u}=\mathrm{v}=\Psi=0,\frac{\partial\theta}{\partial Y}=0,-\Omega=\frac{\partial^2\Psi}{\partial Y^2}
\end{aligned}\right\} \tag{14}$$

The boundary conditions of velocity and temperature fields are shown in figure (1) can be expressed in table (1) as follows:

Table (1): illustrated boundary conditions of problem.

X	Y	θ	$\frac{\partial\theta}{\partial X}$	(U, V, ψ)	ω
0	All Y	1	-	0	$-\frac{\partial^2\psi}{\partial X^2}$
1	All Y	0	-	0	$-\frac{\partial^2\psi}{\partial X^2}$
All X	0	-	0	0	$-\frac{\partial^2\psi}{\partial Y^2}$
All X	1	-	0	0	$-\frac{\partial^2\psi}{\partial Y^2}$

Nusselt Number Calculation :

The local heat transfer rate along the heated wall is obtained from the heat balance that gives an expression for local Nusselt number as **[10]**

$$Nu=-\left.\frac{\partial\theta}{\partial X}\right|_{\mathrm{X}=0} \quad ; \quad 0\le Y\le 1 \tag{15}$$

In difference form this can be written as

$$Nu=\frac{25\theta_{1,j}-48\theta_{2,j}+36\theta_{3,j}-16\theta_{4,j}+3\theta_{5,j}}{12\Delta X} \tag{16}$$

The average value for Nusselt number is defined by

$$\overline{Nu} = \frac{1}{H}\int_0^H Nu dY \tag{17}$$

This equation can be readily evaluated using Simpson's rule.

Solution Procedure :

The governing dimensionless differential equations are discretized to a central finite difference scheme is used. The computational scheme, based on (Gauss-Seidel method) Successive Over Relaxation, SOR, is arranged to solve the three equations for the n^{th} iteration step. The initial values over the field for temperature θ, vorticity ω and stream function ψ are assumed zero to all internal nodes are taken as initial starting values. Over-relaxation is actually used so the "updated" values of general variable $\phi_{i,j}$ internal node are actually taken as:

$$\phi_{i,j}^{new} = \phi_{i,j}^{old} + r\left(\phi_{i,j}^{calculated} - \phi_{i,j}^{old}\right) \tag{18}$$

Where the subscripts i and j refer to a grid node, ϕ is a general dependent variable (θ, ω, or ψ). The relaxation parameters, $\gamma_\theta = \gamma_\omega = 1$ and $\gamma_\psi = 1.6$ give stable numerical computation Ra ≤ 10^4.

The criterion for convergence is examined according to a realistic condition for each state variable at each node as:

$$\frac{\left|\phi_{i,j}^{new} - \phi_{i,j}^{old}\right|}{\left|\phi_{i,j}^{old}\right|} \leq E\max \tag{19}$$

Where the subscripts i and j refer to a grid node, ϕ is a general dependent variable (θ, ω, or ψ) and E_{max} is a small quantity of error set to 10^{-5} for Ra ≤ 10^5.

Finer uniform grid (41×41) for Ra=10^3, (51×51) for Ra=10^4 and (61×61) for Ra=10^5 at aspect ratio (AR=1). The iterative routine, the selected grid and the relaxation parameters are verified at different Rayleigh numbers because of the solution become independent on no. of grids. The number of iterations was 10000 for Ra=10^3, 18000 for 10^4 and 31000 for 10^5.

A computer program in (Fortran 90) was built to execute the numerical algorithm which is mentioned above; it is general for a natural convection in 2-D rectangular enclosure and the results drawn by using Tecplot7 program.

Results and discussion :

In this section the results of the computational algorithm for laminar natural convection in two-dimensional rectangular enclosure will be discussed. All the results are calculated for liquid as working fluid (Pr=6.2), while Ra numbers ranging within (10^3≤Ra≤10^5).

Figure (2) show development of stream function and isotherm lines contours respectively for Ra numbers ranging within (10^3≤Ra≤10^5) at AR=0.5. The mechanism of flow fields occurs when the liquid near the hot wall is heated causing the density to be decreased and the liquid will be start to move upward nearby the hot wall towards the cold wall as shown in stream function contours. For isotherm lines contour, heat transfer by conduction which extended, it considered to be start of convection region. Also single cellular flow regime appears.

Figure (3) show development of stream function and isotherm lines contours respectively for Ra numbers ranging within (10^3≤Ra≤10^5) at AR=1.0. The stream lines will be closer to the cold wall, this means that buoyancy force increase cause an accelerate to flow, the isotherm lines shows transition region from conduction to convection until plume region is appear at Ra≥10^4. Also shown a bicellular flow in stream lines contour because the velocity nearby cold wall is larger than those which are faraway from cold wall. Also show the mechanism of flow occurs when the liquid near hot wall is heated causing the density to be density increasing and the liquid metal will be start to move nearby the hot wall towards the cold wall. From the stream function plots it can be seen that all three configurations contain a single rotating vortex which takes the shape of the cavity.

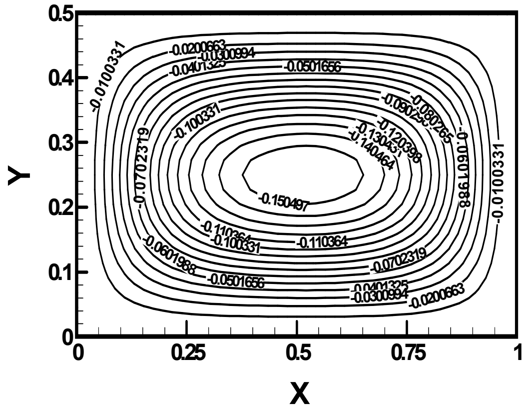
Figure (4) show development of stream function and isotherm lines contours respectively for Ra numbers ranging within ($10^3 \leq Ra \leq 10^5$) at AR=2.0. the opposite behavior shown in Fig 2.

Figure (5) shows the distribution of dimensionless axial and vertical component of velocity, at axial and vertical center line of rectangular enclosure respectively, i.e. X=0.5 and Y=0.5, we showed the maximum value of velocity at $Ra=10^5$ and minimum values (near to zero) at $Ra=10^3$ because conduction region (heat transfer mode is controlled). Also the same behavior in temperature profile in figure (6).

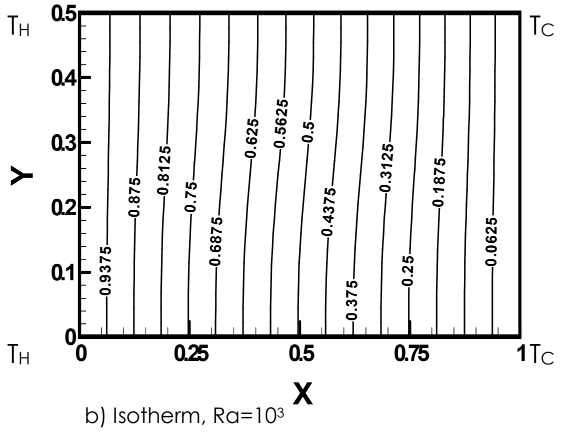
Figure (7) show the variation of local Nusselt number along vertical hot wall for arranging ($Ra=10^3$ and 10^4), shows the temperature is increases with increasing of Ra. Nusselt number increasing near the hot wall and decrease near the cold wall. As convection is enhanced by increasing Ra number, the boundary layer become thinner and the velocity increases. Fig 10 show the variation of average Nusselt number with Ra number for ranging ($10^3 \leq Ra \leq 10^5$) this relation is logarithm relation. The critical $Ra_c = 10^3$ mark in a natural way the demarcation point between the conduction mode and the onset of the natural convection mode.

Figure (8) show the relation between average Nusselt number and Ra number for aspect ratio (0.5-1), it seen average Nusselt number increase with increased aspect ratio because the area of heat transfer will be increased. And the same behavior in Fig 9 shows the relation between average Nusselt number and aspect ratio for Ra ranging with (10^3-10^5).

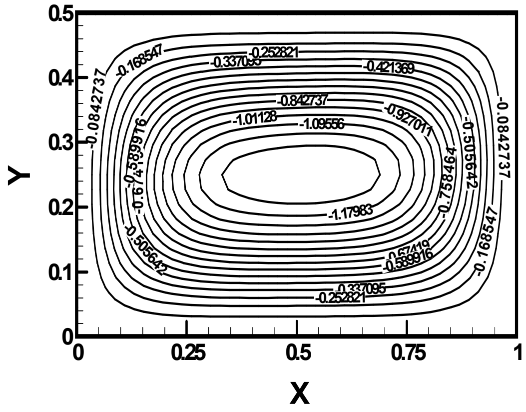
In order to validate the numerical model, the results of variation local Nusselt number and a relation between average Nusselt number and Ra number, are compared with previous works. For square enclosure filled by air (Pr=0.72) the results are compared with (Lo et al. 2007) **[11]**. There are agreement in results and found excellent agreement which validate the present computational model as shown in figures (10) and (11) respectively.



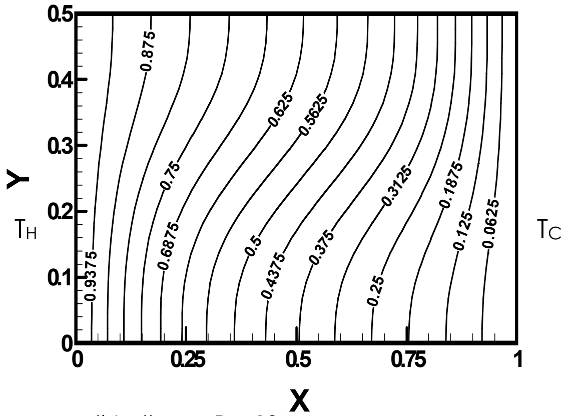
a) Stream Function, Ra=10^3



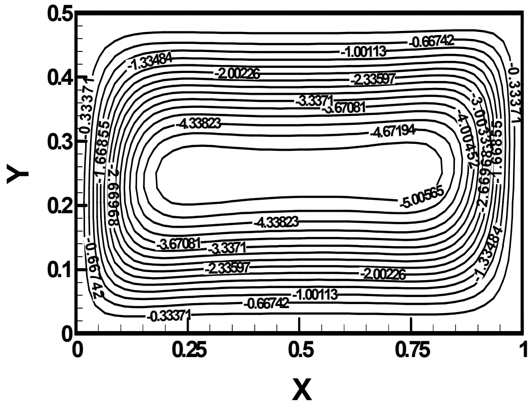
b) Isotherm, Ra=10^3



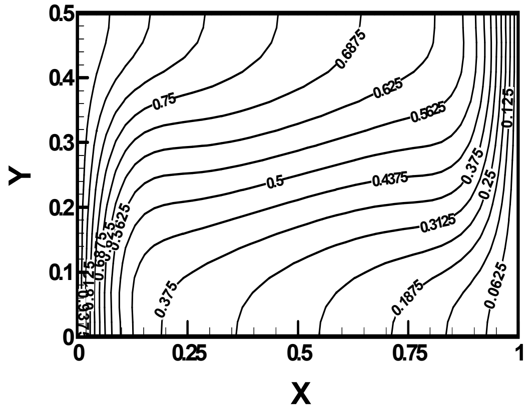
c) Stream Function, Ra=10^4



d) Isotherm, Ra=10^4

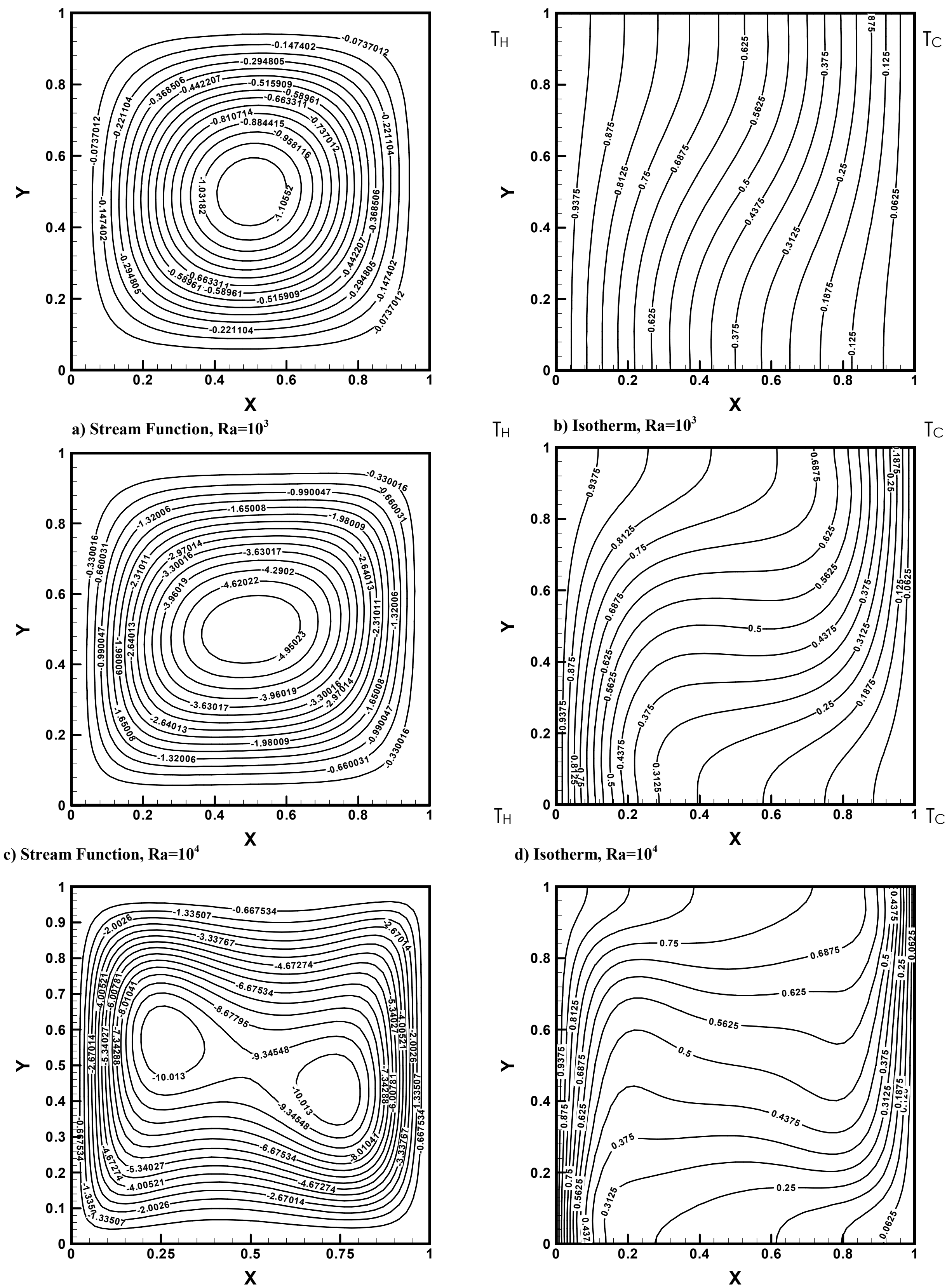


e) Stream Function, Ra=10^5



f) Isotherm, Ra=10^5

Fig (2): illustrated development of stream function and isotherm with Ra No. for AR=0.5.



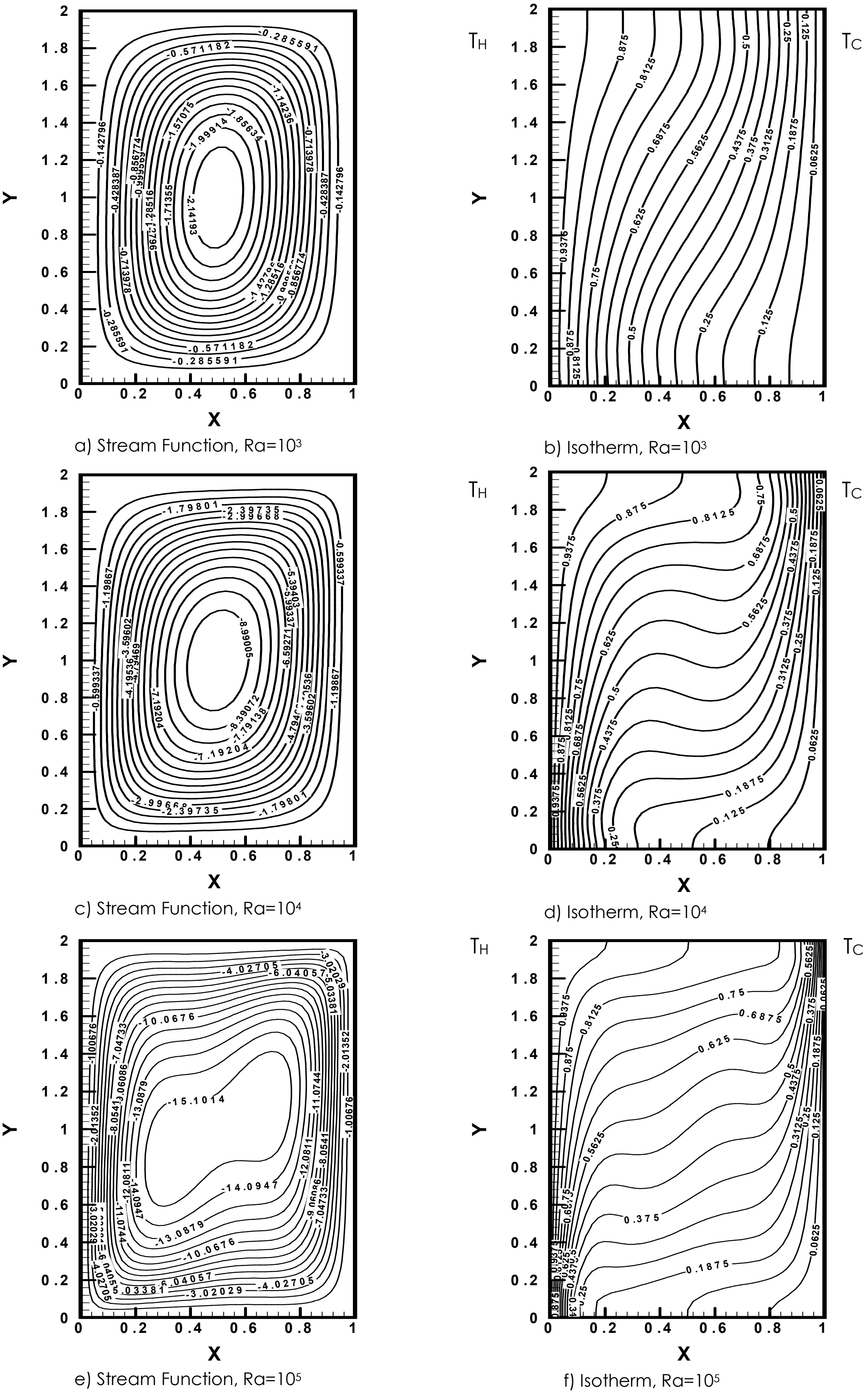
a) Stream Function, Ra=10^3

b) Isotherm, Ra=10^3

c) Stream Function, Ra=10^4

d) Isotherm, Ra=10^4

Fig (3): illustrated development of stream function and isotherm with Ra No. for AR=1.0.



a) Stream Function, Ra=10^3

b) Isotherm, Ra=10^3

c) Stream Function, Ra=10^4

d) Isotherm, Ra=10^4

e) Stream Function, Ra=10^5

f) Isotherm, Ra=10^5

Fig (4): illustrated development of stream function and isotherm with Ra No. for AR=1.0.

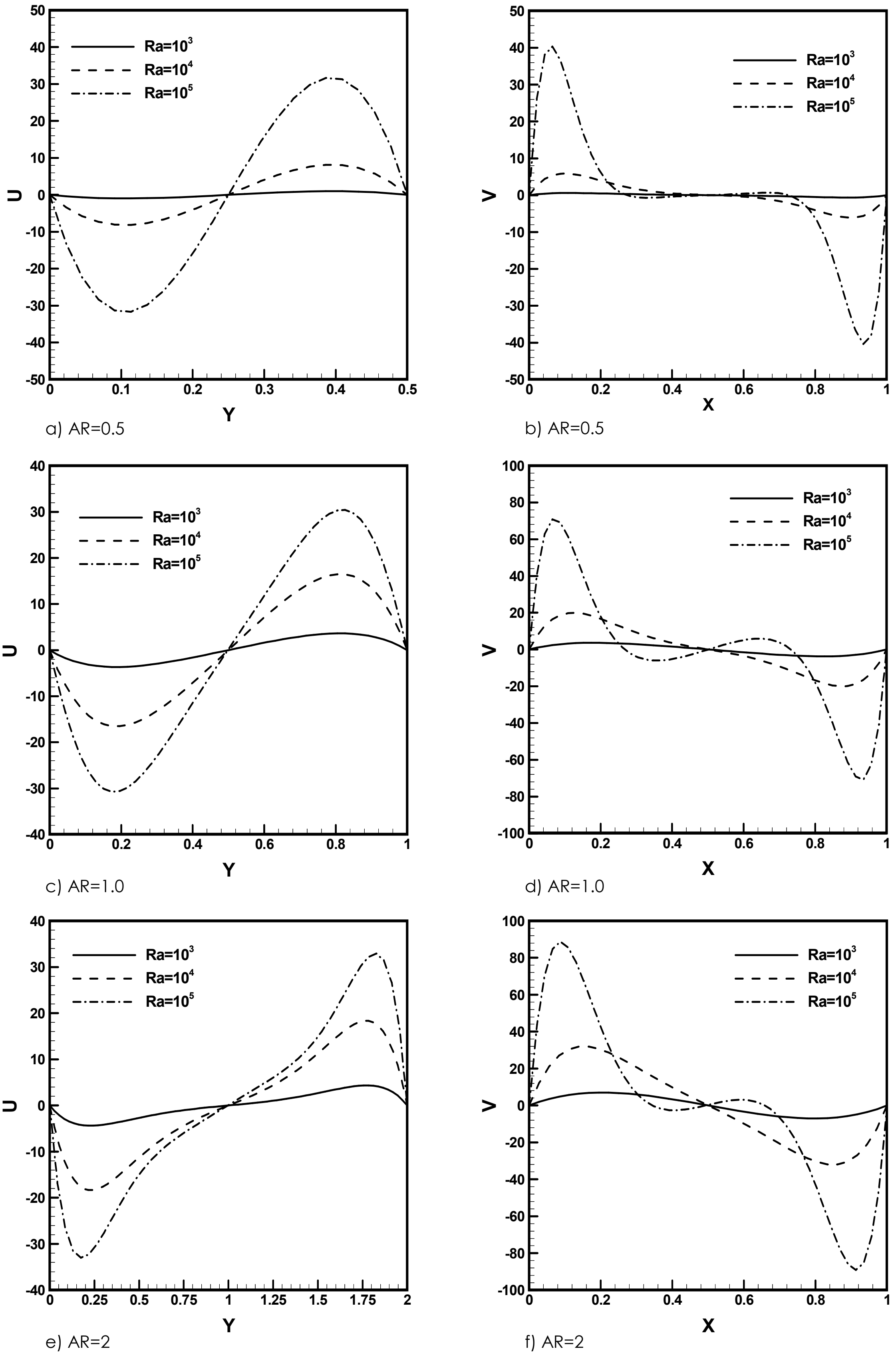
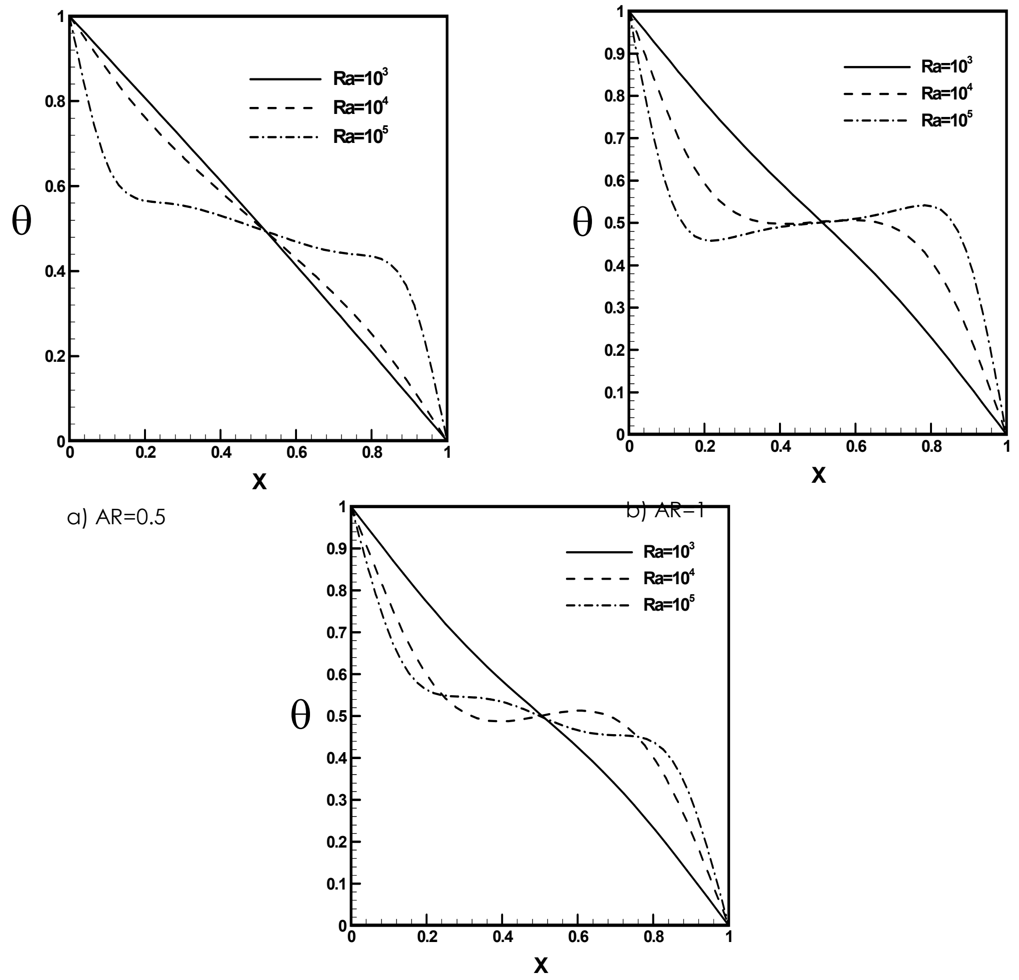


Fig (5): illustrated centerline velocity profile for $0.5 \leq AR \leq 2$ within $10^3 \leq Ra \leq 10^5$.



a) AR=0.5

b) AR=1

c) AR=2.0

Fig (6): illustrated centerline temperature profile for 0.5≤AR≤2 within 10^3≤Ra≤10^5.

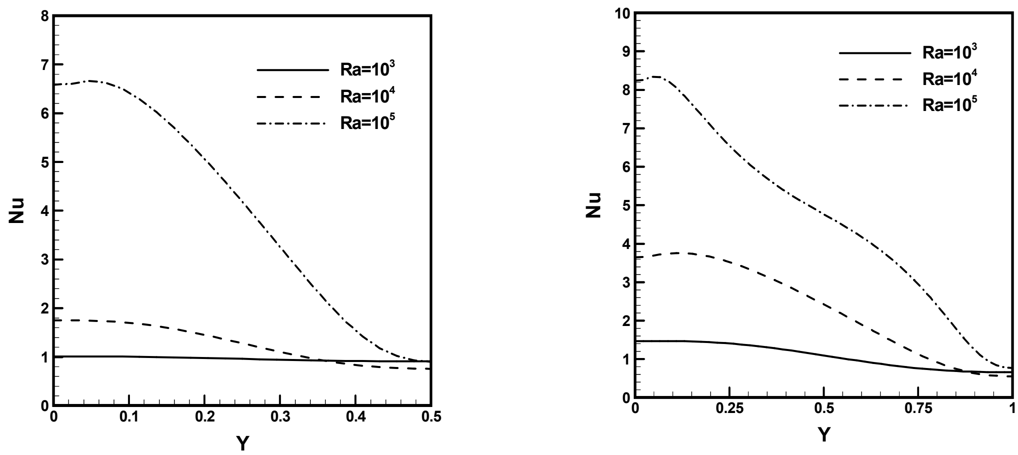


Fig (7): illustrated local Nusselt number for 0.5≤AR≤2 at different Rayleigh number.

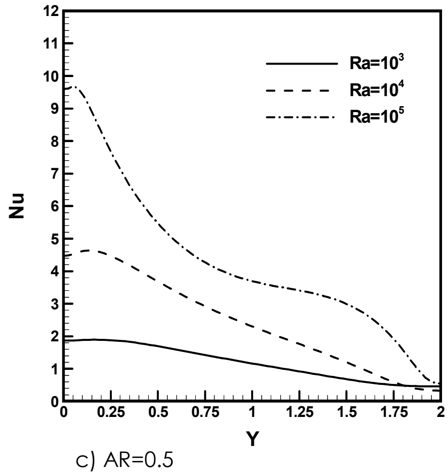


Fig (7): continued.

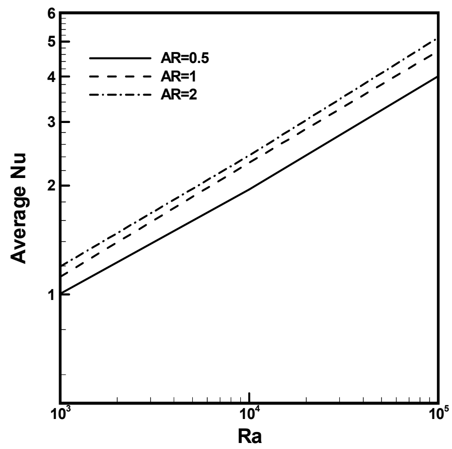


Fig (8): illustrated the relation between average Nusselt no. and Ra no.

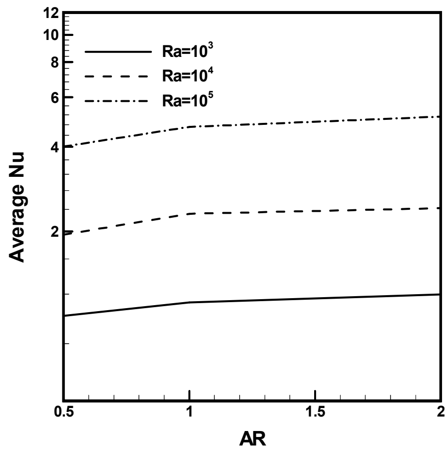


Fig (9): illustrated the relation between average Nusselt no. and AR for different Ra.

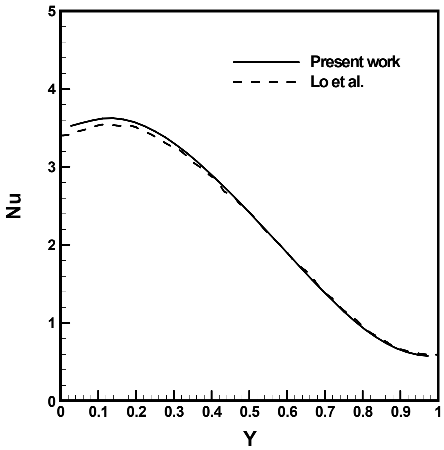


Fig (10): show comparison between present work and Lo et al. for variation Nu at Ra=10^4.

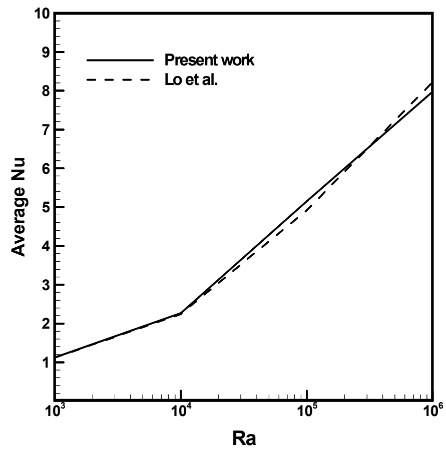


Fig (11): show comparison between present work and Lo et al. for relation between $\overline{Nu}$ and Ra.

Conclusions :

The present study examines the natural convection heat transfer in liquid (water, Pr=6.2) filled rectangular enclosure, the enclosure consist of top and bottom adiabatic surfaces, two isotherm vertical walls (left is hot and right is cold). Some important points can be drawn from the obtained results such as

- For ($Ra=10^3$) the heat transfer operation by conduction, and convection for ($Ra>10^3$). The streamlines shows a single cell form except at ($Ra=10^5$) where it shows a bicellular form.
- For $Ra=10^5$ and AR=1, show a bicellular flow in stream lines contour because the velocity nearby cold wall is larger than those which are faraway from cold wall. In isotherm lines contour shows plume region will be clearer.
- Average Nusselt number is increased with aspect ratio increasing for Ra ranging with (10^3-10^5) because increasing of heat transfer area.
- The solution scheme is validated by comparison with last research, where the agreement was excellent.

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