ELEMENTARY DESIGN CURVES FOR CYLINDRICAL TANKS OF VARIABLE THICKNESS +

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Abstract :

A meaningful analysis of Cylindrical tanks of variable thickness requires an analytical model and dealing with advanced mathematical procedures which may constitute a barrier or dis advantage to the design engineer in dealing with such problems.
This paper is aimed and to present a parametric study of open cylindrical tanks of linearly changing wall thickness with geometrical variables, in the form of wall slope (α), wall height (H) and radians to wall height (a/H).
On using Bending Theory of Cylindrical Shells the G.D.E is solved to obtain the maximum values of internal forces leading to bending moment (Mx), the hoop (Nθ) and the radial deflection (Wx). The results are summarized in very simple curves, and are tested for solving practical problems. The curves are seemed to be very handy at the least in the preliminary design of Cylindrical reinforced concrete tank.

التصميم الأولي لخزانات اسطوانية ذات سمك جدار متغير

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المستخلص :

يتطلب التحليل الكفوء للخزانات الأسطوانية ذات سمك جدار متغير افتراض وتمثيل تحليلي والتعامل مع معادلات رياضية معقدة ومتقدمة قد تشكل صعوبة أو مساوئ لمهندس التصميم لدى التعامل مع هذه المسائل.
يهدف البحث إلى تمثيل وتقديم ومعاملات لخزانات اسطوانية مفتوحه ذات سمك جدار متغير خطياً مع متغيرات هندسية تتضمن ميل الجدار (α)، ارتفاع الجدار (H) ونسبة نصف القطر إلى الارتفاع (a/H). باستخدام نظرية الانحناء للصفائح القشرية الاسطوانية تم حل المعادلات التفاضلية العامة لإيجاد أقصى القيم للقوى الداخلية المؤثرة على الجدار وبالتحديد عزم الانحناء المحوري (Mx) والقوى المحيطية (Nθ) والحيود القطري الشعاعي(Wx)، وتنظيم النتائج على شكل منحنيات مبسطة جدا.
أظهرت المنحنيات من خلال عرض النتائج لمسائل تتضمن خزانات اسطوانية ذات أبعاد محددة كفاءة الطريقة وفائدتها على الأقل في التصميم الأولي للخزان الكونكريتي الأسطواني.

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Introduction :

Liquid storage tanks are used extensively in different applications for the purpose of large volume storage and pressure maintenance in pipe networks.
Tanks are usually classified in accordance to shape, volume, stored material and material of construction. Tanks are usually made of steel or Reinforced concrete and constructed in circular or rectangular forms, where they may be constructed underground, over ground or elevated.
Circular cylindrical tanks are used very frequently and utilized in different industries for the storage of liquid, gas and solid materials. Open circular cylindrical tanks are commonly used for water storage, which they come in different sizes and capacities, where for medium and large size capacities it may be advantageous to construct these walls in variable thickness form.
Analysis of circular cylindrical tanks of variable thickness require dealing with advanced mathematical procedures which may constitute a barrier or a disadvantage to the design engineer in dealing with such problems, where few attempts were made to produce the required bending moments and hoop forces in the form of tables and curves [1,2].
Analysis of such shells using mixed finite element method proved to produce very accurate results [3].
This study presents a treatment of over ground open circular cylindrical tanks made of monolithic concrete which are used very extensively for portable water or in industrial and agricultural purposes. Using the linear bending theory of shells, the research presents a parametric study of open circular cylindrical tanks of linearly changing wall thickness with geometrical variables in the form of starting wall thickness at the free end (h_0), wall slope (α), wall height (H) and tank radius to wall height ratio (a/H).
The obtained results are summarized in very simple curves coupled with simple equations for obtaining the maximum values of the internal forces and deformation in the shell, namely omprising l bending moment, hoop force, and radial deflection.
The obtained curves are tested for solving few practical problems and comparing the results obtained from curves with those obtained using shell theory and mixed finite element method, where very good agreement were obtained for such results.

Bending Theory of Cylindrical Shells :

It is well understood that the linear bending theory of thin shells is based on certain basic assumptions and approximations, which are mainly stated as:

1. Material of the shell is homogeneous.
2. Ratio of the thickness to the smallest radius of curvature is very small.
3. Normal to the middle surface before deformation remains normal after deformation.
4. Stress components normal to the middle surface are very small compared to the inplane components and may be neglected.
5. Components of displacements and strains are small.

Based on the above assumptions and utilizing the equilibrium, strain-displacement relations and stress-strain relationships the general governing equation for thin shells may be formulated which will be in the form of a high order partial differential equation. (8^{th} order).

It was found that it is much easier to produce the governing differential equation for particular geometric as in the case with shells of revolutions and more precisely cylindrical shells fig. (1), assuming the condition of axisymmetric behavior which is the outcome of internal liquid pressure. Using the required equilibrium conditions and elasticity relationship the General differential equations for the axisymmetric bending of cylindrical shells is obtained in the following form: (3)

$$\frac{d^2}{dx^2}\left[D\left(\frac{d^2 w}{dx^2}\right)\right]+\left(\frac{Ehw}{a^2}\right)=q$$

Where:

h	is the thickness of the cylinder.
w	is the radial deflection.
Q	is the applied axisymmetric radial load.
a	is the shell radius.
D	is the flexural rigidity $=\dfrac{Eh^3}{12(1-v^2)}$
E , v	are the elasticity constants

Solving the G.D.E with the use of the proper physical boundary conditions of the problem leads to obtaining expressions for the shell radial deflection (Wx), the bending moment (Mx), the hoop force (Nθ) and the transverse shear force (Qx).

Circular Cylindrical Tanks of Constant Thickness :

For tanks of constant thickness as shown in fig. (2) the flexural rigidity of the shell (D) becomes constant, and the fourth order ordinary differential equation of the shell becomes easy to solve.

$$\frac{d^4 w}{dx^4}+4\beta^4 w=-\frac{\gamma(H-x)}{D}$$

Where:

$$\beta^4=\frac{Eh}{4a^2 D}$$

$$q=-\gamma(H-x)$$

The general solution of this problem as well as obtaining expressions for the corresponding internal forces (Mx, Nθ, Qx) is a straight forward procedure and available in many reference books [4].

Circular Cylindrical Tanks of Variable Thickness :

In the case of tanks where they have a relatively large value of the ratio of radius to height, it becomes economical to consider the linear variations of thickness. In such cases the following variables are considered.

$$h = \alpha \ x$$

$$D = \frac{E \alpha^3 x^3}{12(1 - v^2)}$$

Where (α) representing the slope of the wall as shown in fig. (3). The G.D.E becomes more complex in the form and nature.

$$\left(\frac{d^2}{dx^2}\right)\left(\frac{x^2 d^2 w}{dx^2}\right) + \frac{12(1 - v^2)}{\alpha^2 a^2 xW} = \left[\frac{12(1 - v^2)}{E \alpha^3}\right] \gamma (x - x_0)$$

The solution of such a equation requires the generation of Bessel function or comparable functions. Timoshenko & Win[4] treats this problem in his well known procedures of obtaining the required parameters in the form of tables, where in such cases it my be difficult to follow and use. The aim of this paper is to consider this formulation in depth and carry out a parametric study producing simple equations and curves for the most important internal forces which are used as design parameters of such tanks.

Parametric Study :

A computer program has been developed to compute the radial deflection (Wx) and the internal forces (Mx, NQ, Qx) for both constant and linearly varying wall thickness. The parametric study is based on computing the maximum values of the deflection and internal forces, which are the critical values in design procedures. The main geometrical parameters, which are varied in this particular study are the wall height (H), the tank radius (a), the wall slope (α) and the tank free end thickness (h_0), where the height varid from 3 to 8 meters in steps of one meter, the wall slope varies from 0.02 to 0.035 in steps of 0.005, and the ratio (R = a/H) varies from 0.5 to 3 in steps of 0.5.

It is clear that range of geometrical data covers the scope of most reasonably size of reinforced concrete tanks, where all those geometries are solved for the internal forces using a minimum thickness at the top free end of 10, 12, 14 cm which are considered to be quite acceptable for reinforced concrete tanks.

The obtained results of about six hundred tank problems are tabulated for different internal forces and for every height a table is constructed showing the maximum values against the given range of R_1 where table (1) is shown as a representative sample of the obtained information omprising 54 table.

To avoid such a large number of tables and to make it easier for the design engineer to utilize such important information of the maximum values of the internal forces in the tank, and possibly change the geometrical parameters which may coincide with acceptable internal forces, a very detailed analysis of the obtained results are carried out.

A systematic checking of the variation of every internal force with the geometric variables led to producing a single curve for the maximum values of every internal force, coupled with a simple equation and a small table as shown in the curves given in Figs. (4-6), where they are constructed to appear in non-dimensional form with unity values for the specific gravity of the water and the modulus of elasticity of the tank material.

Fig. (4) shows the curve representing the variation of the maximum value of bending moment at the base of the tank as a function of (ξ) and the bending moment of 3m. height and ($\alpha = 0$) tanks, given as Mx_0, in relation to the variation of ($R = a/H$) ratio, where the bending moments for any given ratio of (a/H) and (h_0) is given as

$$Mx = \xi(1 + 9\alpha H)Mx_0 \quad , \quad \alpha = 0.02 - 0.035$$

Fig. (5) shows the curve representing the variation of the maximum value of the hoop force as a function of (ξ) and the maximum hoop force for 3m height and ($\alpha = 0$) tank being given as ($N\theta$) in relation to the variation of (a/H), where the maximum hoop force for any given ratio of (a/H) and (h_0) is given as:

$$N\theta = \xi(1 - 4\alpha)N\theta o \quad , \quad \alpha = 0.02 - 0.035$$

Bearing in mind that the maximum hoop force for different geometric parameters is not at a fixed point but varies within a certain range from 0.20H to 0.47H from tank bottom.

Fig. (6) shows the curve representing the maximum deflection obtained for 3m height and ($\alpha = 0$) tank, given as (Wx_0), in relation to the variation of (a/H) ratio, where the maximum deflection for any given geometry is represented by the following equation:

$$Wx = \xi\left(1 - 4\alpha H + \frac{\alpha H^2}{6}\right)Wxo \quad , \quad \alpha = 0.02 - 0.035$$

It can also be noted that, the position of maximum deflection is not static but varies with the geometric parameters and lies within the range of 0.2H to 0.5H from tank bottom.

As was mentioned, using the curves represent a simple process which is reflected in the following example:

The parameters are given as:

$$H = 4m \quad , \quad R = \frac{a}{H} = 1.5 \quad , \quad \alpha = 0.027 \quad , \quad h_o = 0.1m$$

$$Mx = \xi(1 + 9\alpha H)Mx_0$$
$$= 2.75(1 + 9 * 0.027 * 4) * 215 = 1166$$
$$N\theta = \xi(1 - 4\alpha)N\theta o$$
$$= 2.2(1 - 4 * 0.027) * 64.4 = 126.4$$

$$Wx = \xi\left(1 - 4\alpha H + \frac{\alpha H^2}{6}\right)Wxo$$

$$= 6\left(1 - 4 * 0.027 * 4 + \frac{0.027 * 4 * 4}{6}\right) * (-1288) = -4946$$

To make sure that the obtained curves satisfy different geometrical parameters, they are tested for many cases, where table (2) gives a representative number of problems being solved analytically and the obtained results are compared with those obtained using the curves.

Noting that the deflection curves (Wx) give higher errors than the internal forces (Mx, Nθ), but such errors are not critical compared to those of internal forces which are considered as controlling factors in the design process.

Through the mentioned three curves, it is thought that a design engineer, in a simple and efficient process can decide about the dimensions of the tank related to the capacity required, as We ll as stresses and deflection limitation of the code at hand.

These curves may can be utilized with a good reliability for preliminary design of open circular tanks, with an average range of error not exceeding 4%.

Conclusions , and Recommendations :

Circular cylindrical tanks of variable thickness are well known for their wide application and particularly those made of concrete of a relatively large height. The paper presents a parametric study taking care of the changes in the geometric variables, namely height, diameter and wall slope, from which it is possible to draw the following conclusions:

1. The study covered a good and wide range of geometric variables.
2. The presented curves for the maximum values of internal forces and deflection are very simple to use with good reliability.
3. Limiting the wall thickness of the top free and to three values may constitute a draw back in certain circumstances and its recommended that wide range of thicknesses should be studied and may be presented as consistent curves with the available curves.
4. The minimum height of three meters used in this study may also be reduced to two meters to wider the range of applications.
5. The study should be extended to step type of thickness variation (discontinues wall thickness) to be applied to steel tanks.

Reference :

1. EL-Bhairy, S. “Reinforced Concrete Design Handbook”, Cairo, 1974.
2. Hilal, M. “Theory and design of reinforced concrete tanks”, Cairo, 1984.
3. Barony, S.Y. and Tottenham, H. “The analysis of rotational shells using a curved ring. Element and mixed Finite Element formulation”. Int. J. Num. methods in Eng., Vol. 10, 861-872,1976.
4. Timoshenko, S. P. and Win. Theory of plates and shells, McGraw Hill., 2nd ed. 1969.

Table (1): Maximum Values of (Mx) for ho = 0.1m, H = 3m and the Shown Variation of R and α

α \ R	0.5	1.0	1.5	2.0	2.5	3.0
0	119	228	329	425	516	602
0.02	188	353	503	642	772	809
0.025	203	382	543	691	828	954
0.030	220	411	583	740	883	1015
0.035	236	439	621	786	936	1073

Table (2): Comparison Between Analytical and curve results for different geometrical parameters

	Mx		Nθ		Wx	
	Analy.	Curve	Analy.	Curve	Analy.	Curve
Tank parameters	H = 4.0 , h_o = 0.1m					
R = 1 α = 0.032	887.1 £	879.1 = 1.9	97.7 £	98.3 = 1.75	-2198 £	-2215.4 = 3
R= 1.5 α = 0.027	1167 £	1166 = 2.75	133.1 £	126.4 = 2.2	-4956 £	-4946 = 6
Tank parameters	H = 5.0 , h_0 = 0.12m					
R = 1 α = 0.027	1549 £	1613 = 2.1	156.3 £	161.6 = 1.75	-3776 £	-3963 =2.75
R= 1.5 α = 0.023	2089 £	2041 =2.95	214.3 £	217.2 = 2.3	-8604 £	-9054 = 5.5
Tank parameters	H = 6.0 , h_0 = 0.14m					
R= 1 α = 0.02	2342 £	2358.5 = 2.8	234.2 £	244.7 = 1.75	-6468 £	-6394 = 2.0
R = 2.5 α = 0.025	5693 £	5908.1 = 5.1	425.3 £	410.4 = 3	-29540 £	-28886 = 11.5

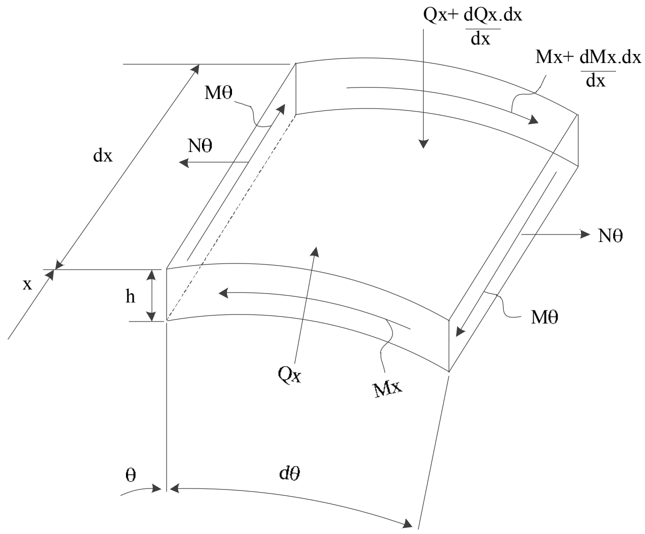

Figure (1): Cylindrical Shell element

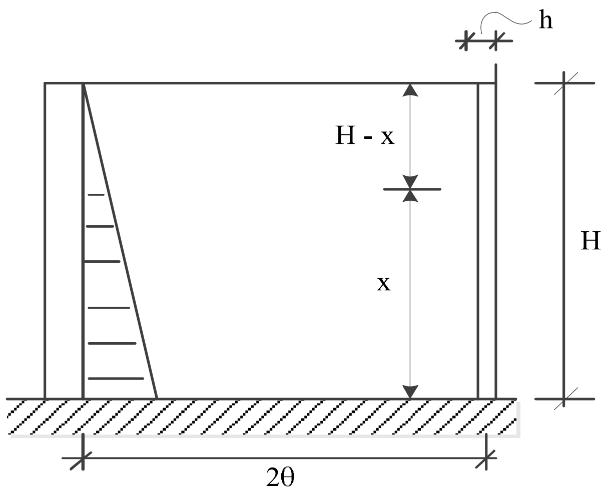

Figure (2): Cylindrical tank of constant thickness

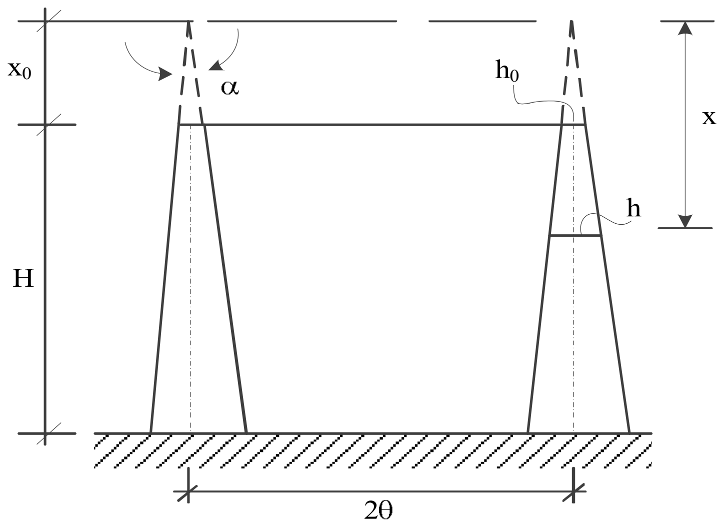

Figure (3): Cylindrical tank of variable thickness

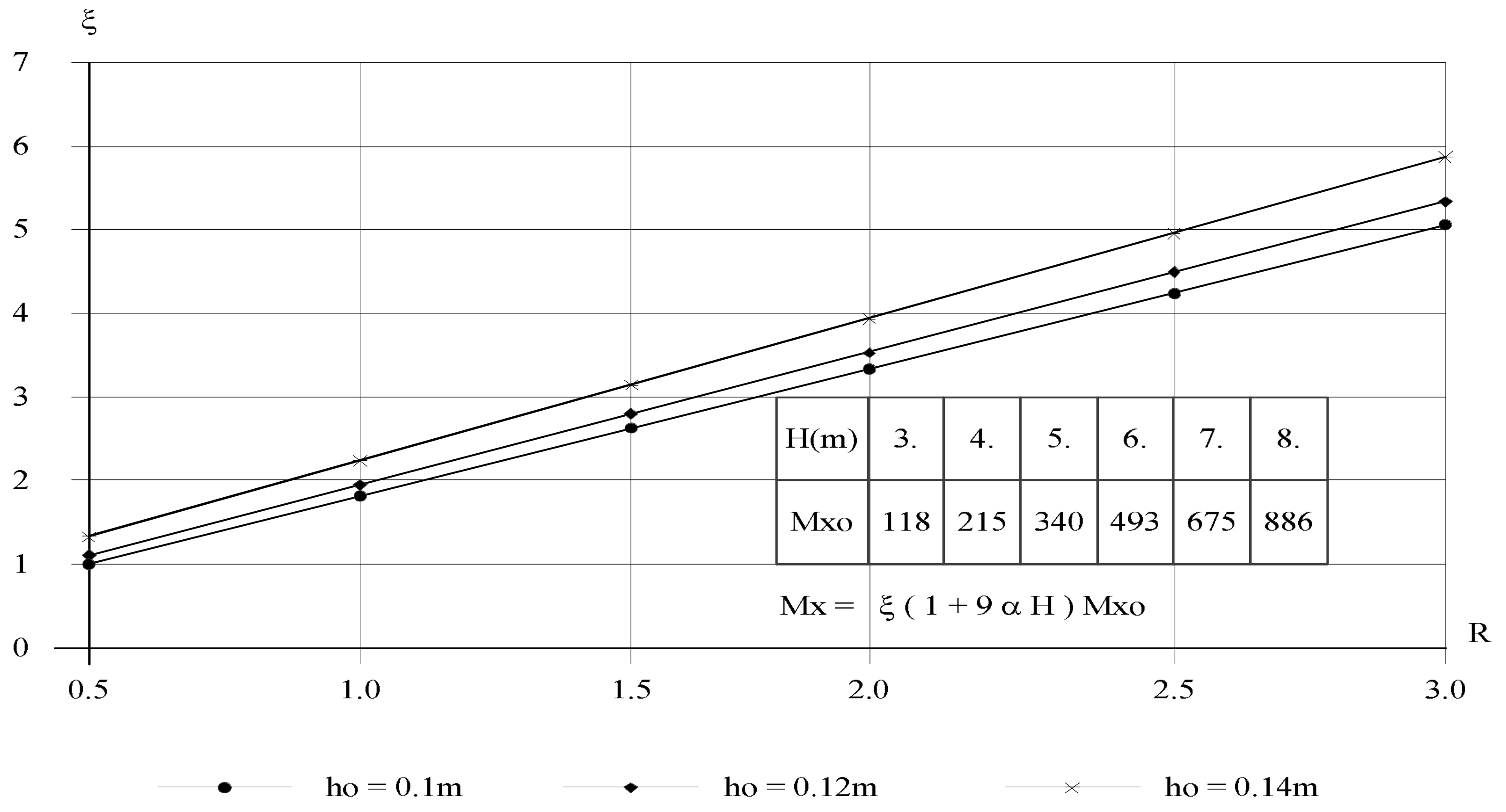

Figure (4): Curves for mertidional Bending moments in the tank

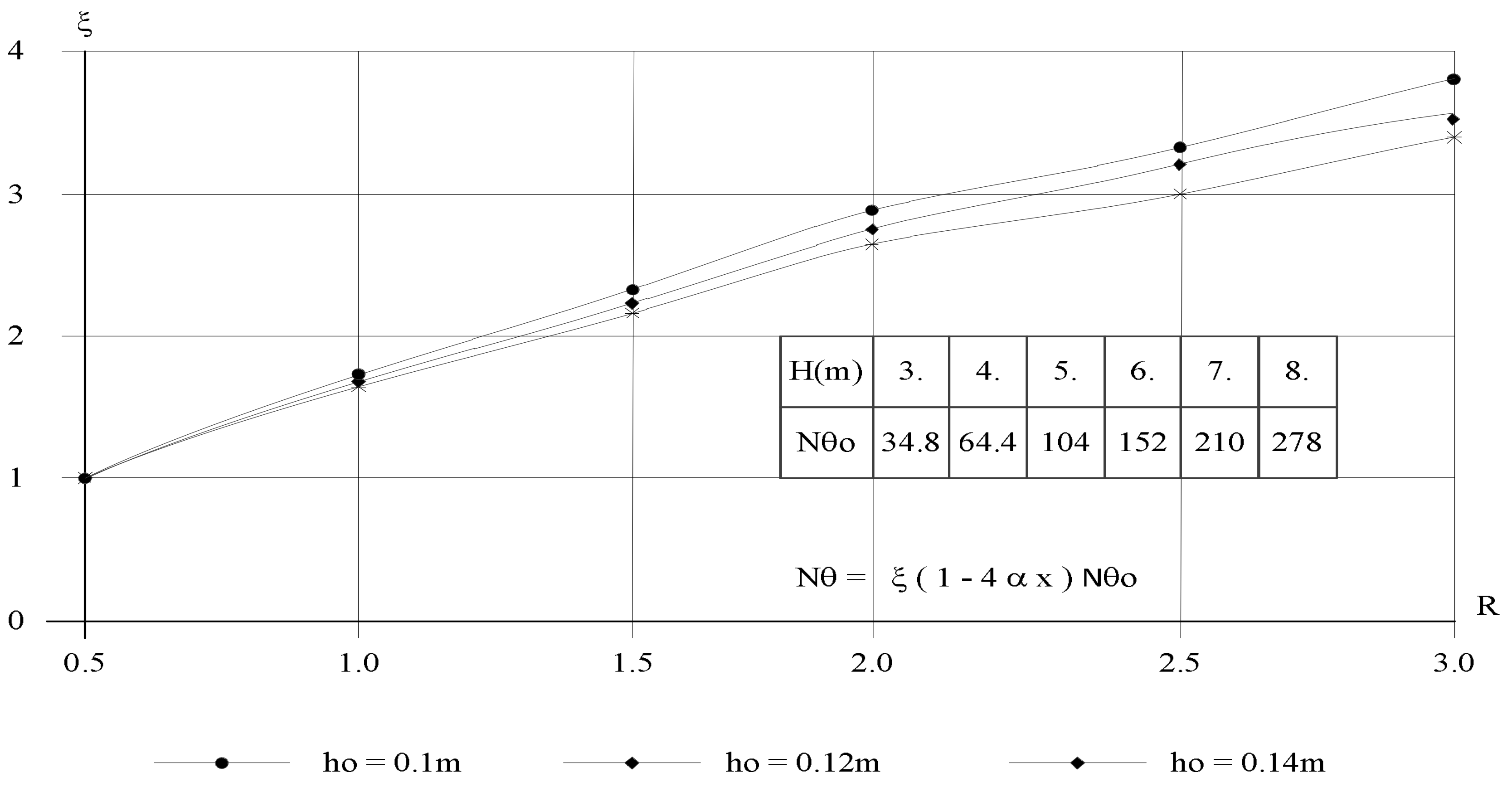

Figure (5): Curves for the hoop in the tank

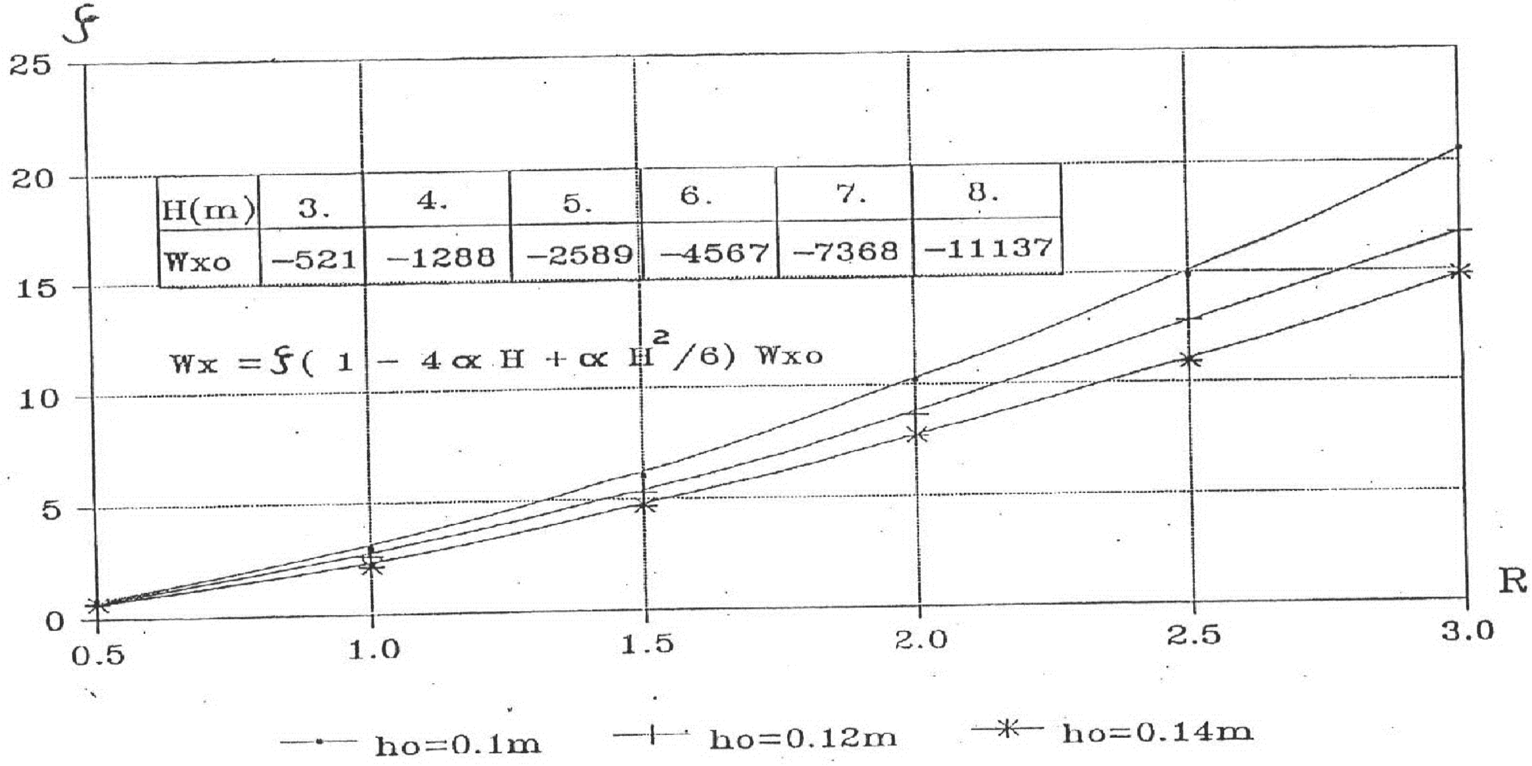

Figure (6): Curves for the radial deflection in the tank