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Energy Levels and Electromagnetic Transition of 90-94mo Nuclei Using Ibm-1

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***Abstract*- The energy states for the ϑ , β , ɤ bands and electromagnetic transitions B (E2) values for even – even molybdenum $^{90-94}$Mo nuclei are calculated in the present work of "the interacting boson model (IBM-1)" . The parameters of the equation of IBM-1 Hamiltonian are determined which yield the best excellent suit the experimental energy states . The positive parity of energy states are obtained by using IBS1. for program for even $^{90-94}$Mo isotopes with bosons number 5 , 4 and 5 respectively. The" reduced transition probability B(E2)" of these neuclei are calculated and compared with the experimental data . The ratio of the excitation energies of the 4_1^+ to 2_1^+ states (R4/2) are also calculated . The calculated and experimental (R4/2) values showed that the $^{90-94}$ Mo nuclei have the vibrational dynamical symmetry U(5). Good agreement was found from comparison between the calculated energy states and electric quadruple probabilities B(E2) transition of the $^{90-94}$Mo isotopes with the experimental data .**

***Keywords* - Energy states, Symmetry U(5), B(E2) value , molybdenum Mo isotopes.**

I. Introduction

The" interacting bosons model (IBM-1)" is describe the nuclear states of isotopes (arima and Iachello,1975; Hasan *et al.*, 2017 ;Islam and Hossain2016). This model used to descripe the nuclear excitation states and calculate it . The IBM-1 is successful in reproducing the energy states of isotopes (Iachello and Arima, 1987; Hummadi, 2017; kassim.*et al.*, 2016) .This model not distinguished between the proton and neutron bosons . The Hamiltonian of IBM-1 has three symmetries of U(5), O(6) and SU(3) (kassim *et al.*, 2016; Su Youn Lee *et al.*, 2017). The even – even $^{90-94}$ Mo isotopes are near to the magic number for neutrons at z =50, while the number of protons in the open shell less than the magic number 50. The $^{92}_{42}Mo_{\ 50}$ isotope has the magic number 50 that's make it more stable than $^{90}_{42}Mo_{\ 48}$ and $^{90}_{42}Mo_{\ 52}$.

II. The Interacting Boson Model

In term of s and d bosons operators the IBM-1 Hamiltonian can be written as (Arima and Iachello, Ann, 1978; Arima and Iachello, phys, 1978; Kassim *et al.*, 2018; Arima and Iachello, 1981; Iachello ,1981; Mohammed and Al-shimmery, 2011):

$$H = \varepsilon_s\ (s^+ .\ s^{\sim}) + \varepsilon_d\,(d^+ .\ d^{\sim}) + \sum_{L=0,2,4} 1/2\,(2L+1)^{1/2}$$
$$C_L \times [[\ d^+ \times d^+]^{(L)} \times [\tilde{d} \times \tilde{d}\,]^{(L)}\,]^{(0)} + \frac{1}{\sqrt{2}}\,\nu_2[\,[d^+ \times d^+\,]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)}$$
$$+ [d^+ \times s^+\,]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} + \frac{1}{2}\,\nu_0[\,[d^+ \times d^+\,]^{(0)} \times [\,\tilde{s} \times \tilde{s}\,]^{(0)}$$
$$+[s^+ \times s^+]^{(0)} \times [\,\tilde{d} \times \tilde{d}\,]^{(0)}\,]^{(0)} + \frac{1}{2}\,u_0\,[[s^+ \times s^+]^{(0)} \times [\,\tilde{s} \times \tilde{s}\,]^{(0)}]^{(0)}$$
$$+u_2\,[[d^+ \times s^+\,]^{(2)} \times [\,\tilde{d} \times \tilde{s}]^{(2)}\,]^{(0)} \quad \ldots\ldots (1)$$

III. Dynamical Symmetry U(5)

Hamiltonian operator for this dynamical symmetry according to the following equation (Casten and Warner, 1988; sharrad *et al.*, 2013)

$$\hat{H} = \varepsilon\hat{n}_d + a_1(\hat{L}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4\,.\hat{T}_4) \quad \ldots\ldots(2)$$

The eigen values for this symmetry can be given as (Bonatsos, 1988) :

$$U(5) : E(n_d\,, \nu\,, L\,) = \varepsilon\, n_d + k_1 n_d\,(n_d + 4) + k_4\,\nu\,(\nu + 3) + k_5\,L\,(L+1) \quad \ldots\ldots(3)$$

The vibrational dynamical symmetry represented by the sub – group U(5) and its quantum numbers that make it has diagonal attribute could be described as : (Shelley *et al.*, 2015).

$$U(6) \supset U(5) \supset O(5) \supset O(3) \supset O(2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$M_L \quad L$

$\nu, n\Delta \quad n_d \quad]\, N\, [$

Where [N] is number of boson (N= Nπ + Nν) .

$$n_d = N, N-1, \text{—}, 1,0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\nu = n_d, n_d - 2, \text{—}, 1 \text{ or } 0 \ (n_d \text{ odd or even}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

(ν) is called seniority and it represents the number of d – type bosons that are not coupled to zero angular momentum . (nΔ) is the number of triplet boson that are related to zero angular momentum this quantum number is added because there are many levels having the same angular momentum . So , it would be a transition from partial group O(5) to O(3) not fully decomposable .

$$I=\lambda,\lambda+1,\ldots 2\lambda-2,2\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

(λ) wave function symbol ; the term (2λ–1) is not shown because it doesn't full fill the symmetry attribute :

$$\lambda=\nu-3n\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

$$-I \le M_I \le I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

IV. Results and Discussion

1. Energy levels

The experimental values of "$R_{4/2}$= (E_4^+/E_2^+) of low lying energy levels" for ^{90}Mo , ^{92}Mo and ^{94}Mo isotopes are 2. 1 , 1.51 and 1.81, respectively. From this values we have identified U(5) symmetry in the ^{90}Mo , ^{92}Mo and ^{94}Mo neuclei . Table(1) shows theoretical and experimental values for R = ($E\,4_1^+$ / E 2_1^+) . The energy levels band (0^+ , 2^+ ,4^+ ,6^+ ,8^+) for $^{90-94}$Mo isotopes have been calculated by using the equation (2) .

The $^{90}_{42}Mo_{48}$ nucleus has 42 protons and 48 neutrons . thus both its proton bosons and neutron bosons correspond to pairs of holes , taking the values Nπ = (50 – 42) /2 = 4 and Nν = (50 – 48) /2 = 1

The $^{90}_{42}Mo_{50}$ nucleus has 42 protons and 50 neutrons (magic number) that's make it more stable than $^{90}_{42}Mo_{48}$ and $^{94}_{42}Mo_{52}$ nuclei . The $^{94}_{42}Mo_{52}$ nucleus has 42 proton and 52 neutrons, thus its proton bosons correspond to pairs of holes , while its neutron bosons correspond to pairs of particles , taking values Nπ = (50 – 42) /2 = 4 and Nν = (52 – 50) /2 = 1

Table 1 . Theoretical and experimental values of " R4/2 =($E\,4_1^+$ / E 2_1^+)" For $^{90-94}$Mo isotopes

Nucl	IBM–1			Exp		
	E (2_1^+)	E (4_1^+)	E (4_1^+)/ E (2_1^+)	E (2_1^+)	E (4_1^+)	E (4_1^+)/ E (2_1^+)
^{90}Mo	0.896	1.905	2.12	947.97	2002.06	2.1
^{92}Mo	1.509	1.824	1.21	1509.51	2282.61	1.51
^{94}Mo	0.873	1.846	2.1	871.1	1573.76	1.81

pairs of particles , taking values Nπ = (50 – 42) /2 = 4 and Nν = (52 – 50) /2 = 1 .

The parameters have been calculated to find the energy levels (2^+ , 4^+ , 6^+ , 8^+) and compard with the experimental values [NDS ENSDF for experimental data] . Table 2 shows the values of these parameters .

Table 2 .The values of the parameters that used in IBM- 1 calculations for the energy states of $^{90-94}$Mo isotopes .

Nucleus	EPS	P .P	I . I	Q . Q	T3.T3	T4. T4
^{90}Mo	0.7856	0.000	0.0172	0.000	– 0.1530	0.0172
^{92}Mo	0.7510	0.000	0.0325	0.000	– 0.1924	0.1811
^{94}Mo	0.7556	0.000	0.0172	0.000	– 0.1530	0.1270

The calculated states of g – band , β – band and γ – band and the experimental data of the states for $^{90-94}$Mo isotopes are shown in figures . 1 , 2 and 3 . There are good agreements for calculated states with the experimental data [NDS ENSDF for experimental data] . The" interacting boson model (IBM-1)" was successfuled in predicting the g –,β –and γ–bands for $^{90-94}$Mo isotopes as presented in Tables 3,4 and 5 .

Table 3 . The g– band for $^{90-94}$Mo nuclei (in MeV)

Jπ	$^{90}_{42}Mo_{48}$		$^{92}_{42}Mo_{50}$		$^{94}_{42}Mo_{52}$	
	IBM–1	Exp.	IBM–1	Exp.	IBM–1	Exp.
0^+	0 .0	0 .0	0 .0	0 .0	0 .0	0 .0
2^+	0.896	0.947	1.509	1.509	0.873	0.871
4^+	1.905	2.002	1.824	2.282	1.846	1.573
6^+	2.937	2.811	2.668	2.612	2.417	2.423
8^+	3.634	2.874	3.828	2.760	3.113	2.955

Table 4 . The β– band for $^{90-94}$Mo nuclei (in MeV)

Jπ	$^{90}_{42}Mo_{48}$		$^{92}_{42}Mo_{50}$		$^{94}_{42}Mo_{52}$	
	IBM–1	Exp.	IBM–1	Exp.	IBM–1	Exp.
0^+	2.461	2.450	2.796	2.519	1.962	1.741
2^+	2.702	2.613	2.807	3.091	2.131	1.864
4^+	2.959		3.044		2.416	2.294
6^+	3.474		3.899		2.925	2.872
8^+	4.237		5.059		3.675	

Table 5 . The γ– band for $^{90-94}$Mo nuclei (in MeV)

Jπ	$^{90}_{42}Mo_{48}$		$^{92}_{42}Mo_{50}$		$^{94}_{42}Mo_{52}$	
	IBM–1	Exp.	IBM–1	Exp.	IBM–1	Exp.
0^+	1.953	1.979	3.352	3.841	2.898	3.320
2^+	1.571	1.896	3.239	3.542	2.613	2.067
3^+	2.235	2.432	2.530	2.849	3.175	2.805
4^+	2.417		3.363	3.876	2.916	2.564
5^+	3.189	2.548	3.431	2.526	3.184	3.447
6^+	2.932		4.208		3.424	3.165
7^+	3.255	3.355	3.869	3.624	3.781	
8^+	4.159		5.367		4.156	

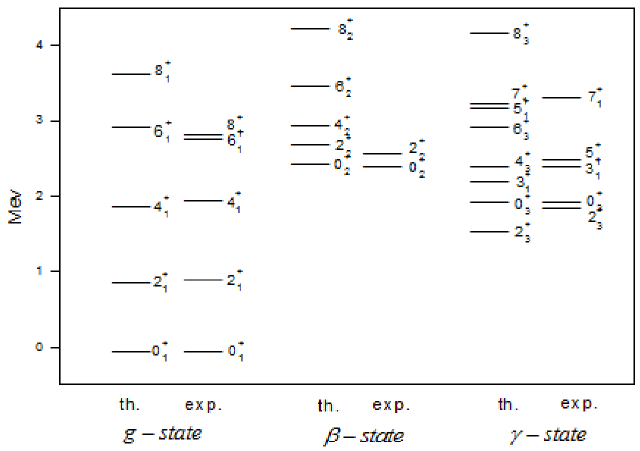

Fig 1. The calculated states and available experimental data [NDS ENSDF for experimental data] for $^{90}_{42}$Mo48 nucleus .

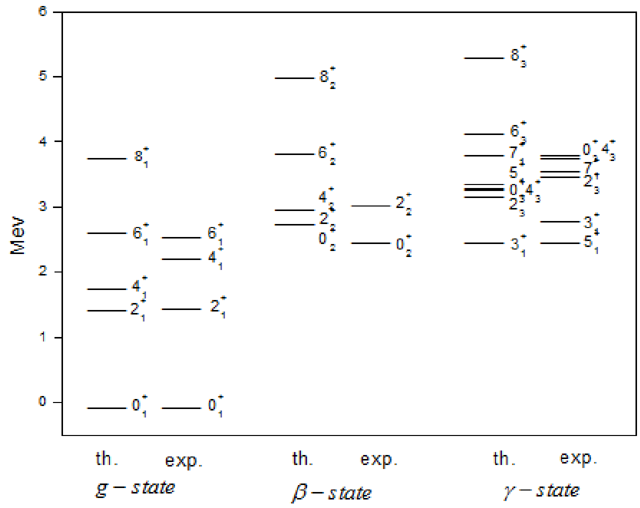

Fig 2 The calculated states and available experimental data [NDS ENSDF for experimental data] for $^{92}_{42}$Mo50 nucleus .

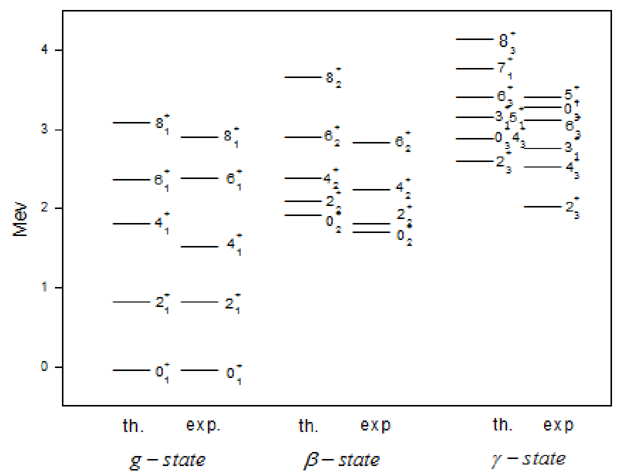

Fig 3. The calculated states and available experimental data [NDS ENSDF for experimental data] for $^{94}_{42}Mo_{52}$ nucleus .

2.The B(E2) transitions value in U(5) symmetry

The transition is taken between the states| [N] (n_d) $\nu n\Delta$ IM_I > of this line where the transition operator (T^{E2}) can be obtain form :

$$T^{E2} = \alpha 2\ [\ d^{*}\ s + s^{*}\ d\ ]^{(2)} + \beta 2\ [d^{*}\ d]^{(2)} \qquad \ldots\ldots\ldots(10)$$

Where α_2 and β_2 are parameters related to α_2. (Kassim and Sharrad, 2014; Hossain *et al.*, 2015) .

B (E2 ; n_d+1 , $\nu = n_d$+1 , $n\Delta = 0$; $I = 2n_d + 2 \rightarrow n_d$, $\nu = n_d$, $n\Delta$=0,I=2n_d)$\alpha_2^2\ \frac{I+2}{2} \times \frac{2N-1}{2}$……….................….(11)

Notice that only the first term of $\hat{T}^{E2}$ is contributes . This is due to the fact that the tates in this limiting symmetry are characterized by the fixed number of d – bosons .

B (E2; $2_1^+ \rightarrow o_1^+$)=$\alpha_2^2\ N$ ………..…..............….(12)

Table (6) . shows the calculated parameters ($\alpha_2$and β2) which were obtained in the present work . The calculated values of "the reduced probability B(E_2) transitions" and the experimental data(Bonatsos, 1988) are presented in Table (7) . for $^{90-94}$Mo .

Table 6. The parameters of T^{E2} (in e b)

Nucleus	α_2	β2
^{90}Mo	0 . 13	– 0 . 065
^{92}Mo	0 . 13	– 0 . 06
^{94}Mo	0 . 13	– 0 . 07

Table 7 . B(E2) values for $^{90-94}$Mo nuclei (in e^2 b^2) [NDS ENSDF for experimental data]

Ji – Jf	^{90} Mo		^{92}Mo		^{94}Mo	
	IBM-1	Exp.	IBM-1	Exp.	IBM-1	Exp.
$2_1^+ \rightarrow o_1^+$	1.450		1.484	1.493	1.41	1.40
$4_1^+ \rightarrow 2_1^+$	2.403		2.18	4.267	2.10	2.26
$2_2^+ \rightarrow 2_1^+$	0.356		0.343	0.01	0.366	
$2_2^+ \rightarrow o_1^+$	0.217		0.210		0.223	0.035
$2_3^+ \rightarrow 2_1^+$	0.0001		0.0	0.003	0.002	0.027
$2_3^+ \rightarrow o_1^+$	0.0		0.0		0.0	0.192

V. Conclusion

The energy states for positive parity for ^{90}MO , ^{92}MO and ^{94}MO neuclei have been calculated by "using interacting boson model-1"and then used the same model (IBM-1)to calculate "the reduced probability of B(E2)" transition for the isotopes above. The calculated energy levels and " B(E2)" of $^{90-94}$ Mo nuclei shows a good agreement with the experimental data . From the ratio $R_{4/2}$ the symmetry of $^{90–94}$Mo isotopes are the vibrational symmetry U(5). The calculations of the energy levels showed that the energy levels of isotope $^{92}Mo_{50}$ was highest than the energy levels of the isotopes ^{90} Mo_{48} and $^{94}Mo_{52}$ because the isotope $^{92}Mo_{50}$ have the magic number for the Neutrons and cause to high stable for this isotope .

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