BUCKLING OF SLENDER CRACKED FIXED-FREE ENDED COLUMNS EXPOSED TO TWO CRACKS UNDER CONCENTRIC VERTICAL LOADS +

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Abstract:

An equation is derived to represent the buckling load of a slender column exposed to two cracks under concentric vertical load. The case which was studied is for columns fixed at one end and free at the other. The equation includes a set of sine –cosine functions but the main variables are buckling load parameter, depth of crack and positions of cracks. The main source of derivation is the elastic buckling theory using a transfer matrix method by representing cracks by a massless rotational springs. The governed equation was solved by a visual basic program and trial and error method to reach the most accurate result. The equation is checked for perfect columns to see its validity and it gave hundred percent comparison. The results show that the buckling load is reduced generally by different ratios depending on the position of the cracks and their depths. The effect of the crack depth ratio is severely affecting the buckling load especially between (0.15-0.20) ratios. The effect of position of cracks is also studied and it is found that the maximum decrease in buckling load was at position ratios β1 and β2 equal to 0.9 and 1.0 respectively. .

Keywords: Buckling load, cracks, columns ,rotational spring.

الانبعاج في الأعمدة النحيفة المعرضة إلى شقين تحت تأثير أحمال عمودية مركزية

حاكم سعيد محمد القريشي

المستخلص:

تم في هذا البحث اشتقاق معادلة للأعمدة المثبتة من جهة وحرة من جهة أخرى واقعة تحت تأثير حمل مركزي عمودي. المعادلة التي تم اشتقاقها تتعلق بإيجاد حمل الانبعاج الحرج للأعمدة المتعرضة إلى وجود شقين. تتضمن هذه المعادلة دوال الجيب وجيب التمام وقد أخذ بنظر الاعتبار عمق الشق وموضعه أي بعده عن رأس العمود.تم اعتماد نظرية الانبعاج المرن واستخدام طريقة المصفوفة الناقلة وتمثيل الشقوق بنوابض دوران عديمة الكتلة. فحصت المعادلة من خلال مقارنتها مع معادلة أويلر(عند عدم وجود شقوق) وكانت مقارية ومطابقة الى درجة كبيرة من معادلة اويلر. بينت النتائج أن هنالك انخفاض في حمل الانبعاج بمعدلات مختلفة تعتمد بالدرجة الأساس على عمق الشقوق مقارنة مع العمود السليم بدون تشققات. كما تم ملاحظة أن أعلى نقصان يحدث في حمل الانبعاج عندما تكون نسبة عمق الشق الى عرض العمود بحدود(0.15−0.20).

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Introduction :

Load carrying capacity of columns and beam-columns are reduced when imperfections such as cracks and initial crookedness existed[1].Cracks also introduce a zone of discontinuity in the region of interest, which makes choice of solutions and analytical modeling of the problem difficult[2]. Buckling of circular rings and columns with cracks have been studied analytically by Dimarogonas[3] using perturbation method. Liebowitz et al [4] and Liebowitz and Claus [5]used sine and cosine functions to model buckling loads of pre-cracked columns. Their model was approximate because it disregarded the discontinuity of the sine and cosine functions in their study. Analytical studies on conservatively loaded beam columns have been reported by Jiki[6] who used Liapunov's second method of stability analysis in conjunction with eigenvalue in equalities to obtain reduced load carrying capacities of pre-cracked beam-columns and concludes that the presence of cracks accelerate the process of failure either by buckling or by fracture.

A numerical model for stability analysis of pre-cracked beam-column had been reported by Jiki P.N.& Karim U.[2] using newly developed java code to process the stability characteristics of the pre-cracked columns. They concluded that a stiffness reduction parameter k is proposed and used to calculate reduced buckling loads due to the presence of an edge crack in the pre-cracked beam column.

The buckling problem of cracked columns has been the subject of numerous investigations. Among these, Anifantis, Dimarogonas A.D.[7] investigated the stability of columns with cracks subjected to follower and vertical loads. A new method for buckling analysis of non-uniform cracked columns was presented by Ranjabran A. et al [8]. A crack is modeled by a massless rotational spring and included in the governing equations with the help of Heaviside step and Dirac delta functions. They also studied a uniform column with internal spring by the same method. Okamura H. ,Liu H.W. and Chu C.S. [9] studied a cracked column under compression based on the column theory together with the well-known relationship between the compliance and the stress intensity factor of a cracked column. They concluded that a crack reduces the flexural rigidity and the load carrying capacity of a column under an eccentric compression.

The buckling of slender prismatic columns with a single edge crack under concentric vertical loads had been studied by Curel M.A.&Kisa M.[10] to find out the buckling load. They replaced the cracked section by a massless rotational spring. They find out a set of equations for all the types of column supports or restraints from which buckling load can be obtained.

In this paper, a transfer matrix method is used to perform and obtain the buckling load of a fixed-free ended column exposed to two cracks. A formula has been constructed depending upon the position of the cracks, distance between cracks and depth of crack in the column.

Formulation of the problem :

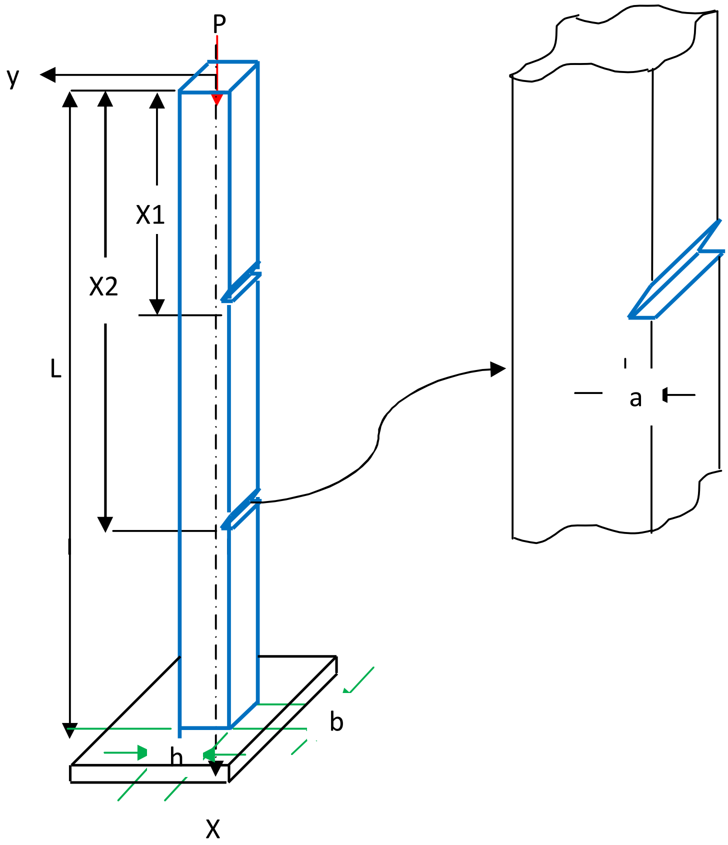
A prismatic column of length **L** and rectangular cross section with two cracks is shown in Figure 1(a).Figure 1(b) shows the mathematical model of this column where a massless two rotational springs are supplied after the local flexibility caused by these cracks were considered. The flexibility of these cracks are C_1 and C_2 respectively.The flexibility C can be written as(Shifrin and Ruotolo [11]).

h *is the height of the cross section* *f(ξ) is the local flexibility function and is given by Shifrin and Ruotolo(1999)*

$$C = 5.346\, hf\,(\xi) \text{......... ....}(1)$$

$\xi = a/h$, a is the depth of crack in direction of h.

$$f(\xi) = 1.862\,\xi^{2} - 3.95\,\xi^{3} + 16.375\,\xi^{4} - 37.226\,\xi^{5} + 76.81\,\xi^{6}$$
$$-126.9\,\xi^{7} + 172\,\xi^{8} - 143.97\,\xi^{9} + 66.56\,\xi^{10} \text{.......... ....}(2)$$



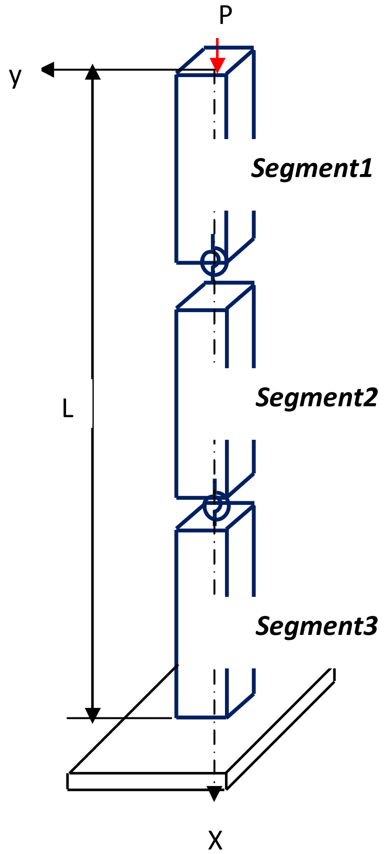


Figure 1(a) slender column with two cracks. **Fig. 1(b)Mathematical model of slender column with two cracks.**

The differential equation for buckling of segment 1 can be written as (Timoshenko and Gere 1961[12])

$$\frac{d^{4}y_{1}}{dx^{4}} + k^{2}\frac{d^{2}y_{1}}{dx^{2}} = 0 \text{.......... ....}(3)$$

where

$$k^{2} = \frac{P}{EI}$$

The relationship among displacement ,slope ,bending moment and shear force are as follows:

$$\theta_1(x) = \frac{dy_1}{dx}$$
$$M_1(x) = -EI\frac{d^2y}{dx^2} \quad \text{...(4)}$$
$$V_1(x) = \frac{dM_1}{dx} - P\frac{dy_1}{dx}$$

The general solution of Eq.(3) is given by:

$$y_1(x) = A_1 + A_2 x + A_3 \sin(kx) + A_4 \cos(kx) \quad \text{......... ....(5)}$$

Using Eqs.(4) and (5), the following relationship can be written:

$$\begin{Bmatrix} y_1(x) \\ \theta_1(x) \\ M_1(x) \\ V_1(x) \end{Bmatrix} = [B(x)] \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \quad \text{..............................(6)}$$

In which

$$[B(x)] = \begin{bmatrix} 1 & x & \sin(kx) & \cos(kx) \\ 0 & 1 & k\cos(kx) & -k\sin(kx) \\ 0 & 0 & P\sin(kx) & P\cos(kx) \\ 0 & -P & 0 & 0 \end{bmatrix} \quad \text{..............................(7)}$$

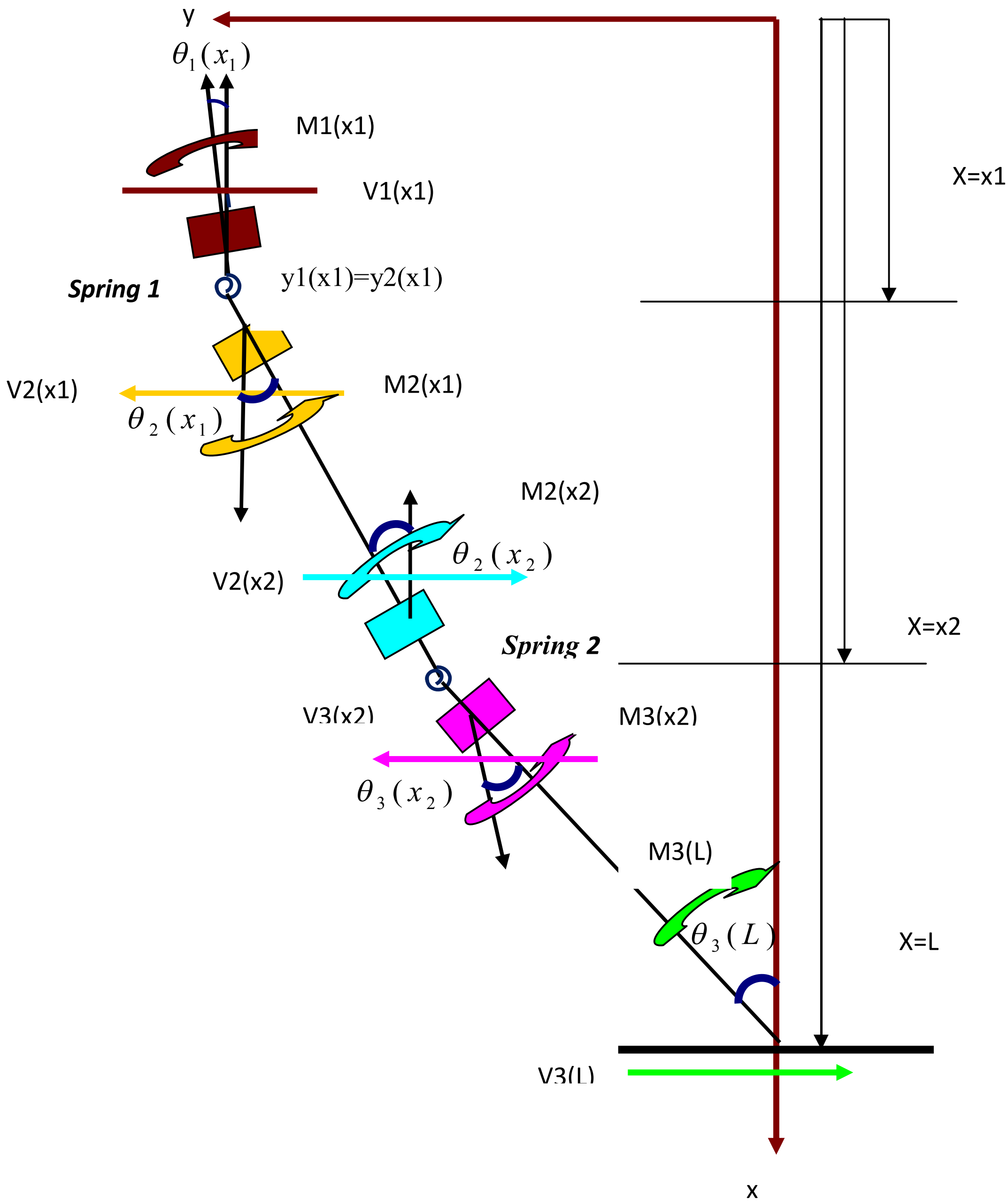
When these parameters are used at the two ends of segment 1 the following relationship will be governed,

$$\begin{Bmatrix} y_1(x_1) \\ \theta_1(x_1) \\ M_1(x_1) \\ V_1(x_1) \end{Bmatrix} = [T_1] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix} \quad \text{..(8)}$$

Here $[T_1] = [B(x_1)][B(0)]^{-1}$

$[T_1]$ Is called the transfer matrix because this matrix transfers the parameters at the upper end X=0 to those at the lower end X=Xc of segment 1.

There is continuity among the displacements, bending moments and shear forces at the boundary of segment 1 and 2,but there is discontinuity between slopes at this point, caused by the bending moment and rotation of the spring representing the cracked section(Figure2),



Figure(2):Continuity and discontinuity of displacements and forces at first and second cracks.

$$y_1(x_1) = y_2(x_1)$$

$$y_1''(x_1) = y_2''(x_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10a)$$

$$y_1'''(x_1) = y_2'''(x_1)$$

$$\theta_2(x_1) - \theta_1(x_1) = y_2'(x_1) - y_1'(x_1) = \Delta\theta(x_1) = C_1 y_1''(x_1) = -C_1 \frac{M_1(x_1)}{EI} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10b)$$

According to Eqs.(10a)&(10b),the following matrix form equation will be governed,

$$\begin{Bmatrix} y_2(x_1) \\ \theta_2(x_1) \\ M_2(x_1) \\ V_2(x_1) \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \dfrac{-C_1}{EI} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_1(x_1) \\ \theta_1(x_1) \\ M_1(x_1) \\ V_1(x_1) \end{Bmatrix} \text{.......... ....}(11)$$

Substitute Eq.(8) into(11),

$$\begin{Bmatrix} y_2(x_1) \\ \theta_2(x_1) \\ M_2(x_1) \\ V_2(x_1) \end{Bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \dfrac{-C_1}{EI} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [T_1]}_{T_{1C}} \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix} \text{.................................}(12)$$

$$[T_{1C}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \dfrac{-C_1}{EI} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [T_1] \text{...}(13)$$

The equation for segment 2 can be obtained by using Eqs.(12) and (8),

$$\begin{Bmatrix} y_2(x_2) \\ \theta_2(x_2) \\ M_2(x_2) \\ V_2(x_2) \end{Bmatrix} = [T_2] \begin{Bmatrix} y_2(x_1) \\ \theta_2(x_1) \\ M_2(x_1) \\ V_2(x_1) \end{Bmatrix} = [T_2][T_{1C}] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix} \text{.......... ....}(14)$$

Where $[T_2] = [B(x_2)][B(x_1)]^{-1}$ ....(15)

The equations of segment 3 can be obtained as follows:

$$\begin{Bmatrix} y_3(x_2) \\ \theta_3(x_2) \\ M_3(x_2) \\ V_3(x_2) \end{Bmatrix} = [T_{2C}] \begin{Bmatrix} y_2(x_2) \\ \theta_2(x_2) \\ M_2(x_2) \\ V_2(x_2) \end{Bmatrix} = [T_{2C}][T_2] \begin{Bmatrix} y_2(x_1) \\ \theta_1(x_1) \\ M_1(x_1) \\ V_1(x_1) \end{Bmatrix}$$

$$= [T_{2C}][T_{1C}] \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix} \text{.......... ....}(16)$$

$$[T_{2C}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \dfrac{-C_2}{EI} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [T_2] \text{.........} \quad \text{....(17)}$$

Where,

$$\begin{Bmatrix} y_3(L) \\ \theta_3(L) \\ M_3(L) \\ V_3(L) \end{Bmatrix} = [T_3] \begin{Bmatrix} y_3(x_2) \\ \theta_3(x_2) \\ M_3(x_2) \\ V_3(x_2) \end{Bmatrix} = \underbrace{[T_3][T_{2C}][T_{1C}]}_{[T]} \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ M_1(0) \\ V_1(0) \end{Bmatrix} \text{.......... ....(18)}$$

In which,

$$[T] = [T_3][T_{2C}][T_{1C}]$$

And $\mathbf{[T_3] = [B(L)][B(x_2)]^{-1}}$

To arrange the solution follow the following steps:

1.Find out $[T_1]$ 2.Find out $[T_{1C}]$ 3.Find out $[T_2]$

4.Find out $[T_{2C}]$ 5.Find out $[T_3]$ 6. Find out $[T]$

After so many calculations and simplifications, these T parameters will be as follows:

$$[T_1] = \begin{bmatrix} 1 & \dfrac{\sin(kx_1)}{k} & \dfrac{\cos(kx_1)-1}{P} & \dfrac{\sin(kx_1)-kx_1}{kP} \\ 0 & \cos(kx_1) & \dfrac{-k\sin(kx_1)}{P} & \dfrac{\cos(kx_1)-1}{P} \\ 0 & \dfrac{P\sin(kx_1)}{k} & \cos(kx_1) & \dfrac{\sin(kx_1)}{k} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{..................................(19)}$$

$$[T_{1C}] = \begin{bmatrix} 1 & \dfrac{\sin(kx_1)}{k} & \dfrac{\cos(kx_1)-1}{P} & \dfrac{\sin(kx_1)-kx_1}{kP} \\ 0 & \cos(kx_1) - C_1 k\sin(kx_1) & \dfrac{-k\sin(kx_1)}{P} - \dfrac{C_1\cos(kx_1)}{EI} & \dfrac{\cos(kx_1)-1}{P} - \dfrac{C_1\sin(kx_1)}{kEI} \\ 0 & \dfrac{P\sin(kx_1)}{k} & \cos(kx_1) & \dfrac{\sin(kx_1)}{k} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{..........(20)}$$

$$\left[T_2\right]=\begin{bmatrix} 1 & \frac{[\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)]}{k} & \frac{[\sin(kx_2)\sin(kx_1)+\cos(kx_2)\cos(kx_1)-1]}{P} & \frac{x_1-x_2}{P}+\frac{\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)}{kP} \\ 0 & \cos(kx_2)\cos(kx_1)+\sin(kx_2)\sin(kx_1) & \frac{k[\cos(kx_2)\sin(kx_1)-\sin(kx_2)\cos(kx_1)]}{P} & \frac{\cos(kx_2)\cos(kx_1)+\sin(kx_2)\sin(kx_1)-1}{P} \\ 0 & \frac{P[\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)]}{k} & \sin(kx_2)\sin(kx_1)+\cos(kx_2)\cos(kx_1) & \frac{\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)}{k} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{...........(21)}$$

$$\left[T_{2C}\right]=\begin{bmatrix} 1 & \frac{[\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)]}{k} & \frac{[\sin(kx_2)\sin(kx_1)+\cos(kx_2)\cos(kx_1)-1]}{P} & \frac{x_1-x_2}{P}+\frac{\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)}{kP} \\ 0 & \begin{matrix}[\cos(kx_2)\cos(kx_1)+\sin(kx_2)\sin(kx_1)] \\ -\frac{c_2 p}{EI}\left[\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1\right]\end{matrix} & \begin{matrix}\frac{k[\cos(kx_2)\sin(kx_1)-\sin(kx_2)\cos(kx_1)]}{P} \\ -\frac{C_2}{EI}\left[\sin(kx_2)\sin(kx_1)+\cos(kx_2)\cos(kx_1)\right]\end{matrix} & \begin{matrix}[\frac{\cos(kx_2)\cos(kx_1)+\sin(kx_2)\sin(kx_1)-1}{P}] \\ -\frac{C_2}{kEI}\left[\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)\right]\end{matrix} \\ 0 & \frac{P[\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)]}{k} & \sin(kx_2)\sin(kx_1)+\cos(kx_2)\cos(kx_1) & \frac{\sin(kx_2)\cos(kx_1)-\cos(kx_2)\sin(kx_1)}{k} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{...(22)}$$

$$\left[T_3\right]=\begin{bmatrix} 1 & \frac{[\sin(kL)\cos(kx_2)-\cos(kL)\sin(kx_2)]}{k} & \frac{[\sin(kL)\sin(kx_2)+\cos(kL)\cos(kx_2)-1]}{P} & \frac{x_2-L}{P}+\frac{\sin(kL)\cos(kx_2)-\cos(kL)\sin(kx_2)}{kP} \\ 0 & \cos(kL)\cos(kx_2)+\sin(kL)\sin(kx_2) & \frac{k[\cos(kL)\sin(kx_2)-\sin(kL)\cos(kx_2)]}{P} & \frac{\cos(kL)\cos(kx_2)+\sin(kL)\sin(kx_2)-1}{P} \\ 0 & \frac{P[\sin(kL)\cos(kx_2)-\cos(kL)\sin(kx_2)]}{k} & \sin(kL)\sin(kx_2)+\cos(kL)\cos(kx_2) & \frac{\sin(kL)\cos(kx_2)-\cos(kL)\sin(kx_2)}{k} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{....(23)}$$

From the previous relationships it was found that,

$$[T]=[T_3][T_{2C}][T_{1C}]$$

Now for simplification assume that $[T_3][T_{2C}]=[A]$ and $[T]=[A][B]$

I.e. $[T_{1C}]=[B]$

$$[A]=\begin{bmatrix} a11 & a12 & a13 & a14 \\ a21 & a22 & a23 & a24 \\ a31 & a32 & a33 & a34 \\ a41 & a42 & a43 & a44 \end{bmatrix} and [B]=\begin{bmatrix} b11 & b12 & b13 & b14 \\ b21 & b22 & b23 & b24 \\ b31 & b32 & b33 & b34 \\ b41 & b42 & b43 & b44 \end{bmatrix} then, [T]=\begin{bmatrix} T11 & T12 & T13 & T14 \\ T21 & T22 & T23 & T24 \\ T31 & T32 & T33 & T34 \\ T41 & T42 & T43 & T44 \end{bmatrix} \text{....................(24)}$$

Eigen value Equations and Eigen values :

The eigenvalue equations can be established by using Eq.(18) and the end conditions of a fixed-free ended column which will be studied alone in this study. Equation(18) becomes:

$$\begin{Bmatrix} 0 \\ 0 \\ M_3(L) \\ V_3(L) \end{Bmatrix}=\begin{bmatrix} T11 & T12 & T13 & T14 \\ T21 & T22 & T23 & T24 \\ T31 & T32 & T33 & T34 \\ T41 & T42 & T43 & T44 \end{bmatrix} . \begin{Bmatrix} y_1(0) \\ \theta_1(0) \\ 0 \\ 0 \end{Bmatrix} \text{..................(25)}$$

Equation(25) can be reduced to the following form:

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} T11 & T12 \\ T21 & T22 \end{bmatrix} \begin{bmatrix} \begin{Bmatrix} y_1(0) \\ \theta_1(0) \end{Bmatrix} \end{bmatrix} \text{.......... ....}(26)$$

For nontrivial solution, the determinant of the matrix in Eq.(26) must equal to zero. This will lead to:

$$T_{11}T_{22} - T_{12}T_{21} = 0\text{.......... ....}(27)$$

Substitute the values of the T,s :

$$[a11b11 + a12b21 + a13b31 + a14b41][a21b12 + a22b22 + a23b32 + a24b42] - [a11b12 + a12b22 + a13b32 + a14b42]$$
$$[a21b11 + a22b21 + a23b31 + a24b41] = 0$$

This Equation will be reduced to :

$$a22b22 + a23b32 = 0\text{...}(28)$$

After so many simplifications the equation of buckling of fixed-free ended column exposed to two cracks will be as follows:

$$(EI)^2 * k^2 \left[C_1C_2 \sin(X_1k)\sin(X_2 - X_1)kL * \cos(1-\beta_2)k\right] - EI\left[C_1 \sin(X_1k) * \cos(1-X_1)k + C_2 \sin(X_2k) * \cos(1-X_2)k\right] + \cos(kL) = 0\text{......}(29)$$

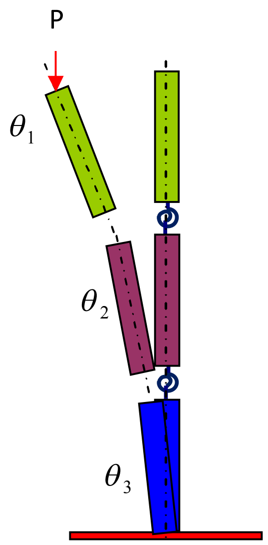
When there are no imperfections(for the current state no cracks) i.e. $C_1=C_2=X_1=X_2=0$,

Euler load is governed as [$P_E = \dfrac{\pi^2 EI}{4L^2}$] To simplify more, let $\dfrac{x_1}{L} = \beta_1$, and $\dfrac{x_2}{L} = \beta_2$

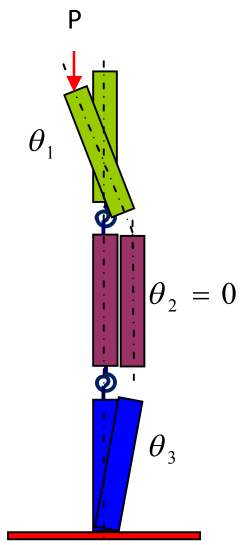
Now Eq.(29) will be as:

$$(EI)^2 * k^2 \left[C_1C_2 \sin(\beta_1 kL)\sin(\beta_2 - \beta_1)kL * \cos(1-\beta_2)kL\right] - EI\left[C_1 \sin(\beta_1 kL) * \cos(1-\beta_1)kL + C_2 \sin(\beta_2 kL) * \cos(1-\beta_2)kL\right] + \cos(kL) = 0\text{......}(30)$$

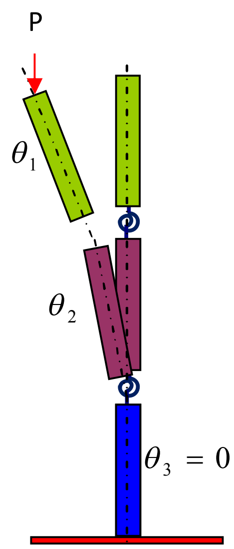
All of the expected mechanisms of buckling for fixed-free ended column can be shown in Figure (3).



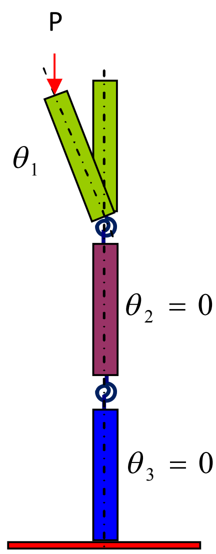
(a)First mechanism



(b) Second mechanism



(c)Third mechanism



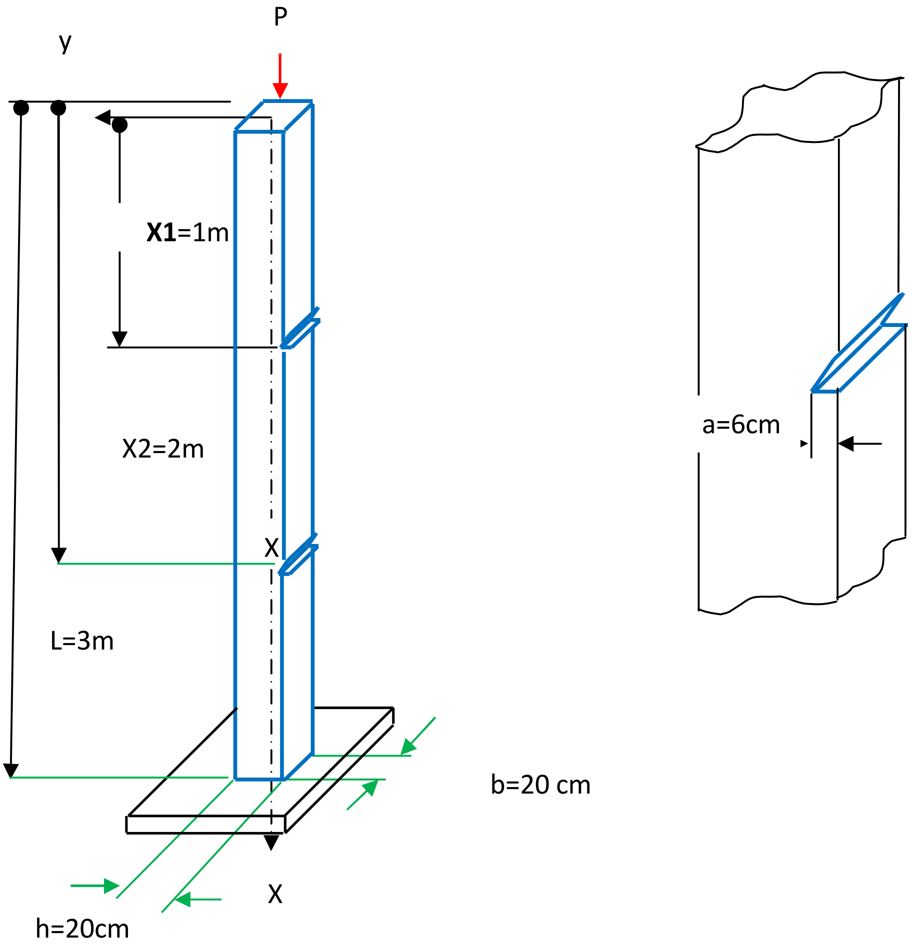
(d)Fourth mechanism

Figure(3):Expected buckling mechanisms of column exposed to two cracks.

Examples and discussion of the results:

To study the effects of existing two cracks in the columns on the buckling load of fixed-free ended column, a column its sketch is shown in Fig.(4) was considered .The following column data are given as in Ref(10):

h=b=20cm , L=3m , E=2x10^6 N/cm^2=2x10^7 KN/m^2, a=0.3h=6cm , X1=L/3=1m, X2=2L/3=2m



Figure(4): Fixed free ended column exposed to two cracks and external applied axial load.

Substituting these data in Eq.(30) and solving this eigenvalue equation. This equation was solved by the help of visual basic language program and using trial and error to reach the final result . The aim of this program is to find out(k) from which P_E governed. One of the runs of this program taking the values of the mentioned column parameters can be seen as mentioned below.

```
Private Sub Command1_Click()
Dim K As Single
Dim K As Single
Dim y As Single
Dim L As Single
Dim B1 As Single
Dim B2 As Single
Dim C1 As Single
Dim C2 As Single
Dim E As Single
Dim I As Single
K = Val(Text1.Text)
L = 3
```

B2 = 1
B1 = 0.9
C1 = 0.0171125
C2 = 0.0171125
E = 20000000
I = 0.0001333333333

$$y = \begin{array}{l} (EI)^2 * k^2\left[C_1 C_2 \sin(\beta_1 kL)\sin(\beta_2-\beta_1)kL * \cos(1-\beta_2)kL\right] - \\ EI\left[C_1 \sin(\beta_1 kL) * \cos(1-\beta_1)kL + C_2 \sin(\beta_2 kL) * \cos(1-\beta_2)kL\right] + \cos(kL) = 0 \end{array}$$

Text2.Text = y
End Sub

The following cases were governed taking into consideration in comparison, that for perfect column or Euler's column buckling load is P_E=0.274155EI.

(1) Effect of crack depth variation "a"(i.e. zeta ξ) on critical buckling load:

Firstly, it must be understood that the effect on buckling load will be represented by" K^2" so increasing in this parameter means increasing in buckling load and vice versa. It is clearly shown that from Fig.(5) that as the crack depth increases the buckling load decreases and when there are no cracks Euler's load is governed and that is the logic. Other index which supports this logic is the reduction in buckling load which is represented in Fig.(6) .It is seen that as crack depth increases reduction in buckling load increases. Another important note that can be seen from this figure is the jump in the reduction of buckling load between ξ=0 to ξ=0.1.

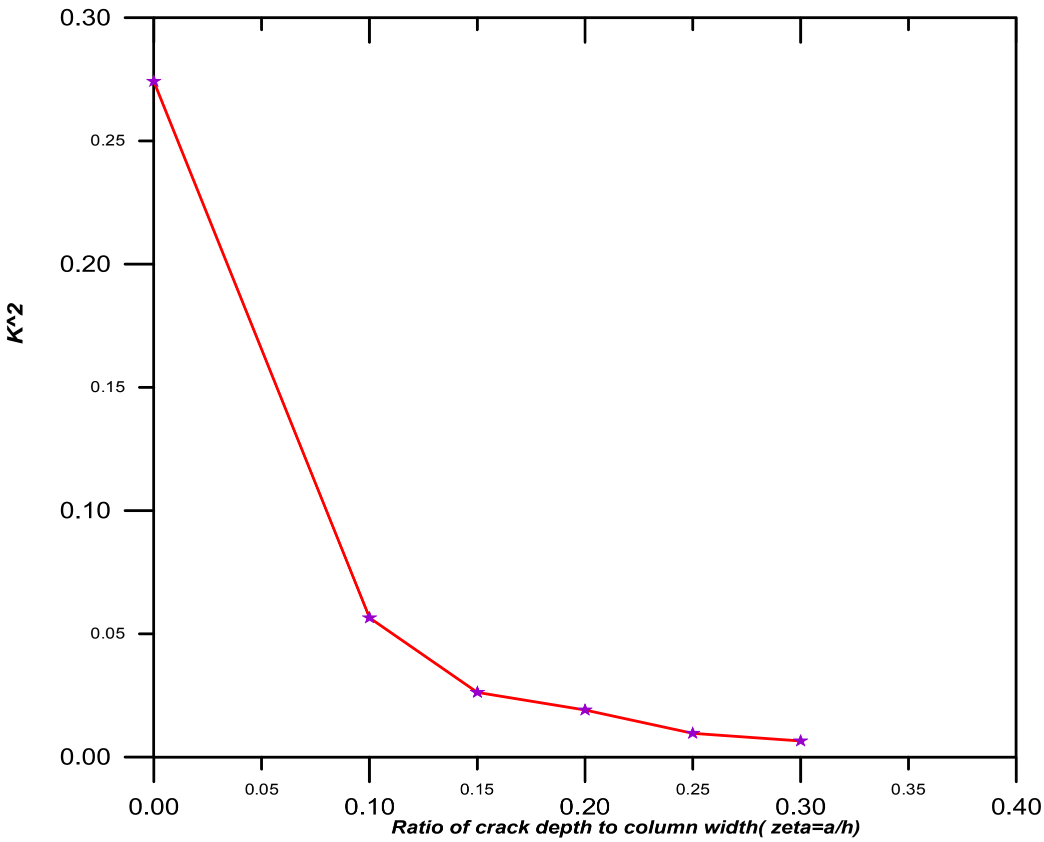


Fig.(5) Relation between K^2 and crack depth ratio (zeta)

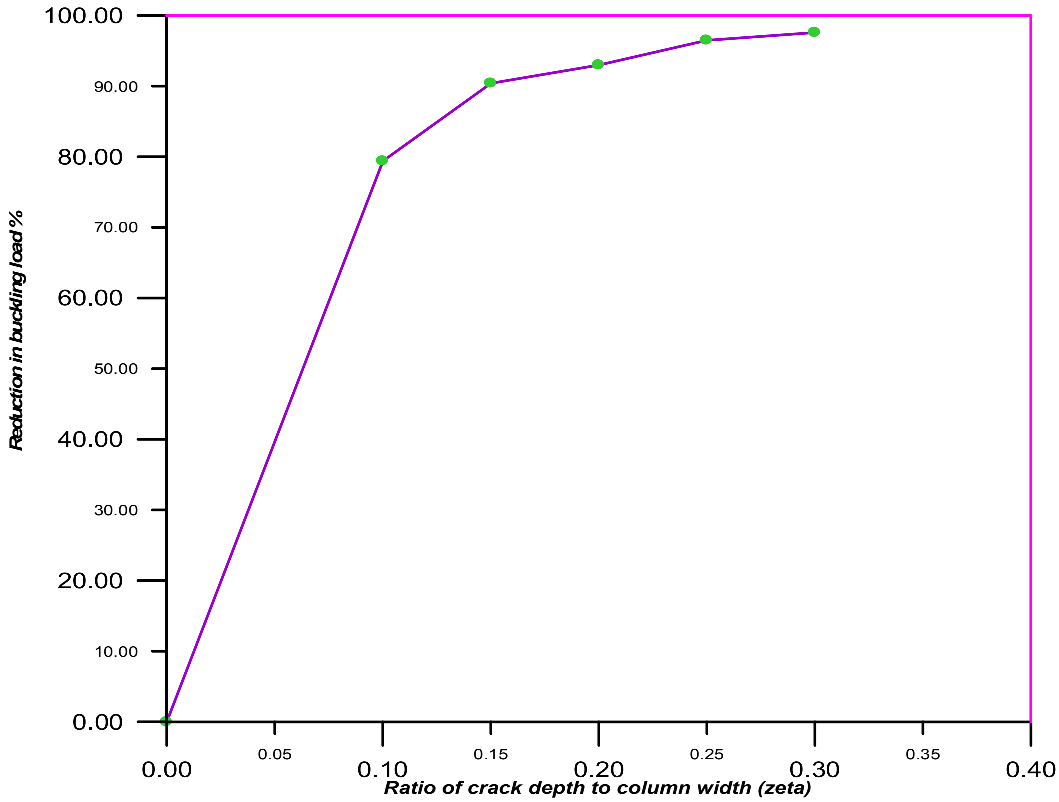


Fig.(6) Effect of crack depth ratio (zeta) on reduction in buckling load.

(2)Effect of variation of cracks positions "X1" and "X2" on critical buckling load:

The positions of cracks in the body of the column were represented by the ratio to the total height of the column therefore X1 will be X1/L and X2 will be X2/L and the two were assumed to be $\beta 1$ and $\beta 2$ respectively. Equation(30) is used to find out the values of the parameter K. The distance between the two cracks is taken to be 0.1L. Fig.(7) shows the relationship between K and $\beta 1$ where the values of $\beta 1$ are taken to be from 0.1 to 0.9 for different values of ξ (0.1,0.15,0.2 , 0.25 and 0.3). The values of K are decreased as $\beta 1$ increased. Same relation and variation can be shown in Fig.(8) with $\beta 2$ values starts from 0.2 to 1 for different values of ξ (0.1,0.15,0.2 , 0.25 and 0.3). The effect of the values of ξ hardly affecting the results of K because as the depth of crack increases stiffness of the column decrease as shown in figures (7 and 8).

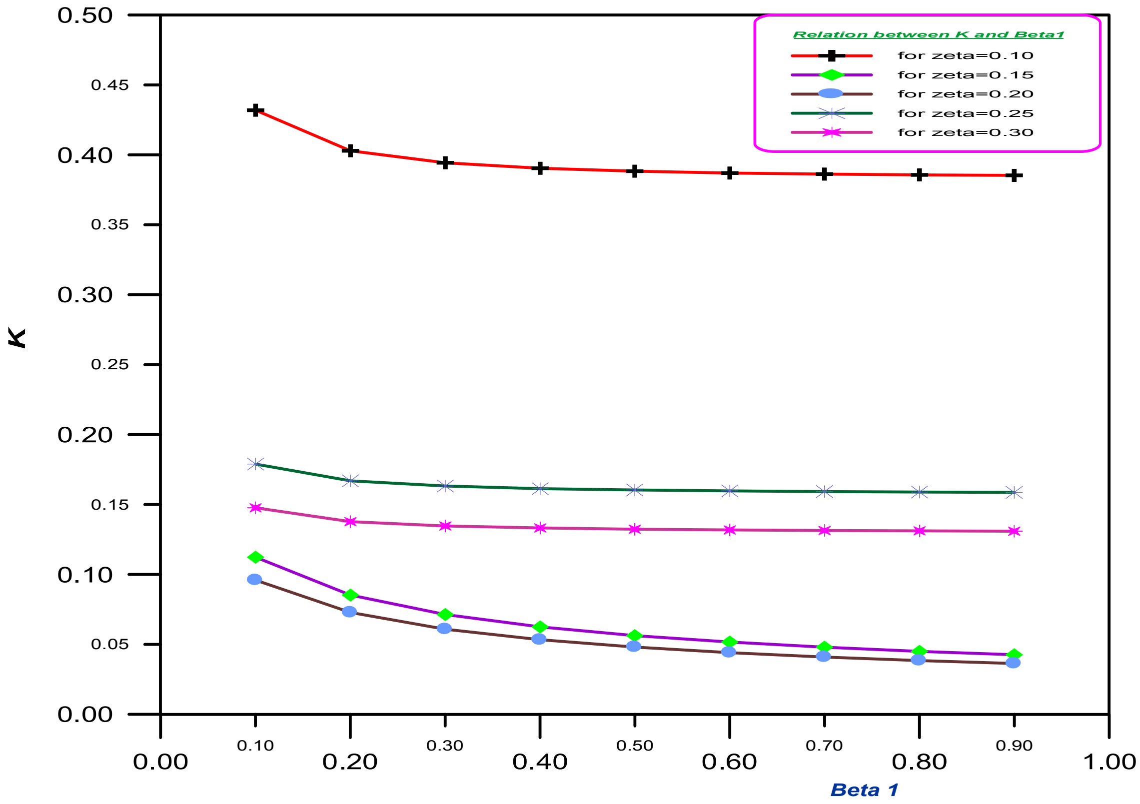


Fig.(7) :Relation between K and distance from the free end of the column (Beta1)

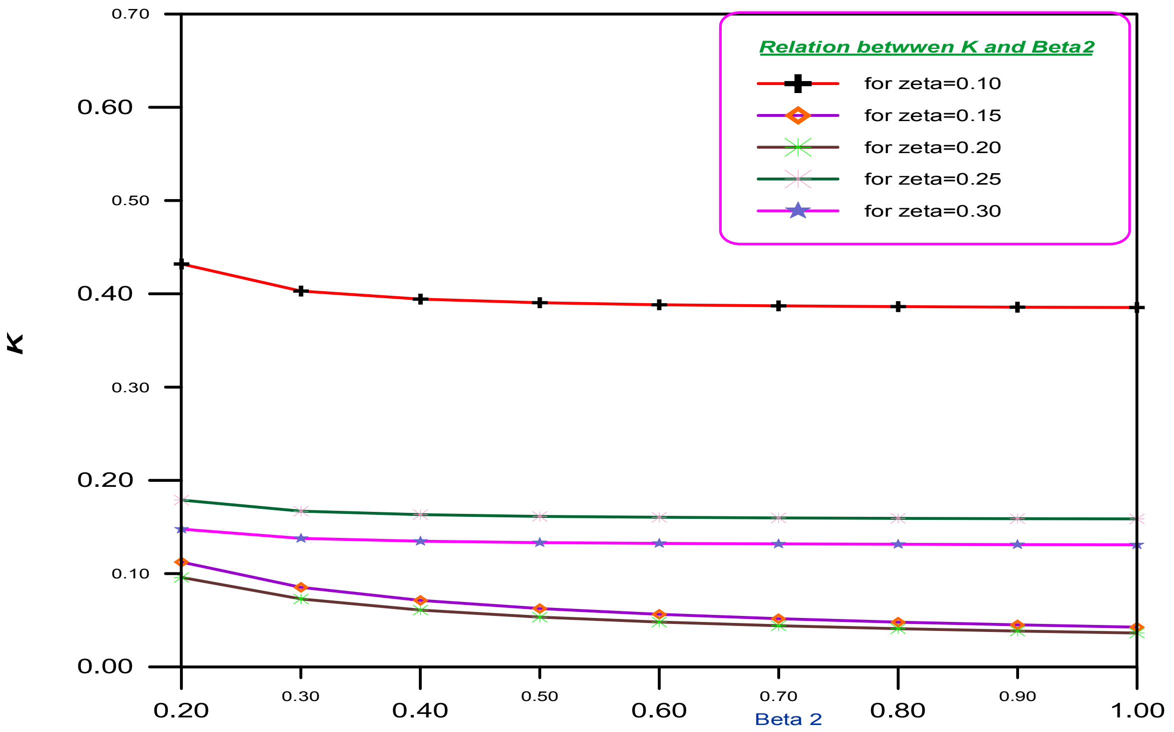


Fig.(8): Relation between K and disance X2 from the free end of the column (Beta2).

Conclusions and recommendations:

The following points can be concluded in studying the effect of existing two cracks in the buckling of fixed-free ended column:

(1)An equation was derived to find out the buckling load by easily specifying the depth of cracks and their positions.

(2) It is noticed that a reduction of more than 96% in the buckling load is registered compared to Euler's load when the ratio of crack depths exceeds 0.3 which informs us to a danger state .

(3)There is a sever inverse linear relation between the reduction in buckling load and depth of crack index(ξ) till 0.1 after which the effect will be slightly small.

(4) The effect of cracks positions show that a slightly decreasing in buckling load is shown as $\beta1$ and $\beta2$ are increased.

(5) A more study is needed to know the effect of variation of the flexural rigidity of the column material on the carrying capacity of the column.

(6) Study of applying the governed equation for other types of columns.

(7) The current study takes into account that the depths of the two cracks are same, so what will be the results when they are different . A statistical study must be done.

(8) The study takes a regular distance between cracks and regular distances from ends of column ,therefore a study must done if these distances are irregular.

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