



w-extending and pw-extending Modules

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Abstract

In this work, we introduce and study two concepts of modules. The first one is a generalization of extending modules which is called a w-extending module. And the second concept is called pw-extending module which is a stronger property than w-extending. We give basic properties, example and characterization of these concepts. Moreover relationship of w-extending module and pw-extending modules with other types of modules are studied.

1.Introduction

Throughout this paper all rings are commutative with identity and all modules are left unital module. A module X is called extending, if every submodule of X is essential in a direct summand of X . Equivalently that every closed submodule of X is a direct summand of X [1], where a non zero submodule K of a module X is called essential if $K \cap L \neq (0)$ for every non zero submodule L of X [2]. And a submodule K of a module X is called closed if K has no proper essential extension in X [2]. A submodule C of a module X is called weak essential if $C \cap S \neq (0)$ for each non zero semi-prime submodule S of X [3]. A submodule K of a module X is called w-closed, if K has no proper weak essential extension in X [4]. A submodule L of a module X is called pw-closed if for each $m \in M, m \notin L$, there exists a w-closed submodule F of X with $L \leq F$ and $m \notin F$ [5]. Every w-closed submodule is a pw-closed submodule, while the converse is not true in general [5], an R -module M is called multiplication if every submodule N of M is of the form $N = IM$ for some ideal I of R [6]. An R -module Y is called fully semi-prime if every proper submodule of Y is a semi-prime [7]. An R -module M is called a completely essential if every non zero weak essential submodule of M is essential [7]. Let Y be an R -module $\text{ann}(y) = \{r \in R : ry = 0\}$ [2], $Z(y) = \{y \in Y : \text{ann}(y) \text{ essential in } R\}$ is called singular submodule of Y if $Z(y) = Y$, then Y is

called singular module, if $Z(y) = (0)$, then Y is called nonsingular module [2].

2. W- extending Modules

In this section, we introduce a new generalization of extending module called a w-extending module.

Definition (2.1)

An R -module Y is called w- extending, if every submodule of Y is a weakly essential in a direct summand of Y . A ring R is a w-extending, if R is a w-extending R -module.

The following is a characterization of w-extending module.

Proposition (2.2)

A module Y is a w- extending if and only if every w-closed submodule of Y is a direct summand of Y .

Proof

($\Rightarrow$) Let E be a w-closed submodule of Y , then E is a weak essential in a direct summand of Y say L . But E is a w-closed in Y , then by [4, Prop.2.4] $E=L$. Hence E is a direct summand of Y .

($\Leftarrow$) Let E be a submodule of Y If $E = (0)$, then clearly E is a weak essential in a direct summand of Y . If $E \neq (0)$, then by [4, Prop.2.6], there exists a w-closed submodule L in Y such that E is a weak essential in L . Thus, by hypothesis L is a direct summand of Y , so E is a weak essential in a direct summand of Y . Hence Y is a w- extending module.

Proposition (2.3)

A module Y is a w -extending if and only if every pw -closed submodule of Y is a weak essential in a direct summand of Y .

Proof

($\Rightarrow$) Let E be a pw -closed submodule of Y , then E is a weak essential in a direct summand of Y .

($\Leftarrow$) Direct.

Remarks and Examples (2.4)

1. Every extending module is a w -extending, however the converse is not true in general. As the following example shows: let $Y = Z_8 \oplus Z_2$ be a "Z-module", Y "is not extending since the submodule" $K = \langle \bar{2}, \bar{1} \rangle$ is closed but not a direct summand of Y . On the other hand Y is a w -extending since all w -closed submodules of Y are $\langle \bar{0}, \bar{1} \rangle$, $\langle \bar{4}, \bar{1} \rangle$ and Y itself and all of them are direct summand of Y . That is $Y = \langle \bar{1}, \bar{0} \rangle \oplus \langle \bar{4}, \bar{1} \rangle$ and $Y = \langle \bar{1}, \bar{0} \rangle \oplus \langle \bar{0}, \bar{1} \rangle$.

2. Z_n as a Z -module is a w -extending for each $n \geq 2$.

Proposition (2.5)

Let Y be a w -extending module and E, F are submodules of Y with $E \cap F$ is a w -closed submodule of Y . Then $E \cap F$ is a direct summand of E and F .

Proof

It is easy so is omitted. $E(Y)$

Proposition (2.6)

Let Y be an R -module, with $T \cap Y$ is a w -closed submodule of Y for all direct summand T of $E(Y)$. Then Y is a w -extending if and only if $T \cap Y$ is a direct summand of Y .

Proof

($\Rightarrow$) Is direct.

($\Leftarrow$) Let K be a submodule of Y , and B a relative complement of K in Y , then by [2, Prop.1.3] $K \oplus B$ is an essential submodule of Y , but Y is an essential in $E(Y)$, it follows that by [2, Prop.1.1] $K \oplus B$ is an essential in $E(Y)$, hence $E(Y) = E(K \oplus B) = E(K) \oplus E(B)$. That is $E(K)$ is a direct summand of $E(Y)$. Thus by assumption we have $E(K) \cap Y$ is a direct summand of Y . Now, we have K is essential in $E(K)$ and Y is essential in $E(Y)$, then by [2, Prop.1.1(2)] $K = K \cap Y$ is essential in $E(K) \cap Y$, thus by [3] K is a weak essential in $E(K) \cap Y$ which is a direct summand of Y .

From Proposition (2.2) and Proposition (2.6) we get the following.

Proposition (2.7)

Let Y be an R -module, with $T \cap Y$ is a w -closed submodule of Y for all direct summand T of $E(Y)$.

Then the following statements are equivalent:

1. Y is a w -extending module.
2. Every w -closed submodule of Y is a direct summand of Y .
3. $T \cap Y$ is a direct summand of Y .

Proof

(1) $\Leftrightarrow$ (2) Follows by Proposition (2.2).

(2) $\Rightarrow$ (3) Let T be a direct summand of $E(Y)$, then by hypothesis $T \cap Y$ is a w -closed submodule of Y . Thus by (2) $T \cap Y$ is a direct summand of Y .

(3) $\Leftrightarrow$ (1) Follows by Proposition (2.6).

Proposition (2.8)

Let Y be a non zero w -extending module, and E, F be a submodule of M such that $M = E \oplus F$ with $\text{ann}(E) + \text{ann}(F) = R$, and every weak essential extension of $K \oplus F$ (or $E \oplus K$) are completely essential modules, where K is any w -closed submodule in E (or F). Then E (or F) is a w -extending modules.

Proof

To prove first E is a w -extending. Let K be a w -closed submodule in E . Since F is a w -closed submodule in F , then by [4, Prop.2.26] we have $K \oplus F$ is a w -closed submodule in Y . But Y is a w -extending module, then $K \oplus F$ is a direct summand of Y . Thus there exists a submodule L of Y such that $Y = (K \oplus F) \oplus L = K \oplus (F \oplus L)$. That is K is a direct summand of Y . Now, K is a submodule of E , then by [2, Prop.1.10] K is a direct summand of E , hence E is a w -extending.

In similar way we can prove that F is a w -extending.

As a generalization of Proposition (2.8) we get the following.

Corollary (2.9)

Let $Y = \bigoplus_{i=1}^n E_i$ be a non zero w -extending module, where E_i is a submodule of Y for each $i = 1, 2, \dots, n$, and every weak essential extensions of $K \oplus E_1 \oplus E_2 \oplus \dots \oplus E_{j-1} \oplus E_{j+1} \oplus \dots \oplus E_n$ are completely essential modules, where K is a w -closed submodule of E_j and $\text{ann}(E_1) \oplus \text{ann}(E_2) \oplus \dots \oplus \text{ann}(E_n) = R$. Then E_j is a w -extending module.

Proposition (2.10)

Let Y be a finitely generated faithful, and multiplication R -module over a w -extending ring R . Then Y is a w -Extending R -module

Proof

Let F be a w -closed submodule of Y , then since Y is a multiplication, then $F = [F:Y]Y$. Thus by [4, Prop.3.7] we have $[F:Y]$ is a w -closed ideal in R . But R is a w -extending ring, then $[F:Y]$ is a direct summand of R . Thus $R = [F:Y] \oplus L$, where L is an ideal of R , hence $Y = RY = ([F:Y] \oplus L)Y = [F:Y]Y \oplus LY$. But Y is faithful and multiplication module so by [6, Theo.(1.6)], we have $[F:Y]Y \cap LY = ([F:Y] \cap L)Y$ and by definition of the direct sum, we have $[F:Y] \cap L = (0)$, thus $[F:Y]Y \cap LY = (0)M = (0)$, therefore $Y = [F:Y]Y \oplus LY$. That is $[F:Y]Y = F$ is a direct of Y .

The following proposition gives the converse of Proposition (2.10).

Recall that for any R -module M and any ideals I and J of R , if I is a semi-prime ideal of J , then IM is a semi-prime submodule of JM . This is called condition (*) [7].

Proposition (2.11)

Let Y be a finitely generated faithful and multiplication R -module such that Y satisfies the condition (*). Then Y is a w -extending module if and only if R is a w -extending ring.

Proof

($\Rightarrow$) Let L be a w -closed ideal in R . Since Y is a faithful multiplication and satisfies the condition (*), then by [4, Coro.3.8] LY is a w -closed submodule in Y . But Y is a w -extending module, then LY is a direct summand of Y . Thus $Y = LY \oplus N$ for some submodule N of Y . Since Y is a multiplication, then $N = [N:Y]Y$, so $Y = LY \cap [N:Y]Y = (L + [N:Y])Y$. Then by [6, Theo.1.6] $Y = LY \cap [N:Y]Y = (0) = (L \cap [N:Y])Y$, it follows that $L \cap [N:Y] \leq \text{ann } Y$. Since Y is a faithful and finitely generated multiplication modules, then by [6, Theo.3.1] we have $R = L \oplus [N:Y]$. Therefore L is a direct summand of R .

The following propositions show that under certain condition w -extending modules implies extending modules.

Proposition (2.12)

Let Y be a w -extending R -module provided that any non zero weak essential extensions of any submodule of Y is a completely essential. Then Y is an extending module.

Proof

Let E be a non zero submodule of Y , then E is a weak essential in a direct summand of Y , say K . That is K is a non zero weak essential extension of E , then K is a completely essential, then E is essential in K , hence Y is extending.

Proposition (2.13)

Let Y be a w -extending and a fully semi-prime module. Then Y is extending module

Proof

Let E be a non zero submodule of Y . Since Y is a w -extending, then E is a weak essential in a direct summand of Y say H . But Y is a fully semi-prime module then, by [7, Prop.2.4] E is essential submodule of H . That is Y is an extending.

As a direct application of Proposition (2.12) and Proposition (2.13) we get the following corollaries.

Corollary (2.14)

Let Y be a w -extending R -module, with every non zero direct summand of Y is completely essential module. Then Y is extending.

Corollary (2.15)

Let Y be a w -Extending module, with every nonzero direct summand of Y is a fully semi-prime module. Then Y is extending.

Proposition (2.16)

Let Y be a w -extending module such that for every submodule L of Y there exists a w -closed submodule K of Y with L is essential submodule of K . Then Y is extending.

Proof

Let L be a submodule of Y , then by assumption, there exists a w -closed submodule K of Y such that L is essential in K . But Y is a w -extending, then K is a direct summand of Y , hence Y is extending.

Proposition (2.17)

Let Y be a fully semi-prime and w -extending module. Then every direct summand of Y is a w -extending module.

Proof

Let N be a direct summand of Y , and let L be a w -closed submodule of N . Since N is a direct summand of Y , then N is a closed submodule of Y . Thus by [4, Prop.2.4] N is a w -closed submodule of Y . Then by [4, Prop.2.17] we have L is a w -closed of Y , but Y is a w -extending, then L is a direct summand of Y . That is $Y = L \oplus E$ for some submodule E of Y . $Y \cap N = N = (L \oplus E) \cap N = L \oplus (E \cap N)$, it follows that L is a direct summand of N . That is N is a w -extending.

3. pw-extending modules

In this part of the paper we introduced the definition of pw-extending module, which is a stronger form of w -extending module and give some of its basic properties.

Definition (3.1)

A module Y is called pseudo w -extending (for short pw-extending), if every pw-closed submodule of Y is a direct summand of Y . A ring R is called a pw-extending, if R is a pw-extending R -module.

Remarks and Examples (3.2)

1. It is clear that every pw-extending module is a w -extending, but the converse is not true in general. For example consider $Y = Z_{16}$ as a Z -module is a w -extending by Remarks and Examples (2.2)(2), while Z_{16} is not pw-extending since the submodule $\langle \bar{2} \rangle$ is a pw-closed submodule of Z_{16} , but not direct summand.

2. Every semi-simple R -module Y is a pw-extending but the converse not true.

Proof

Suppose that Y is semi-simple module, and let K be a pw-closed submodule of Y then K is a direct summand of M , hence Y is a pw-extending. For the converse consider the following example: let $Y = Z_6 \oplus Z_2$ as a z -module Y is a pw-extending because the only pw-closed submodule of M are $\langle \bar{0}, \bar{1} \rangle$, $\langle \bar{3}, \bar{1} \rangle$ and Y itself and are direct summand of Y . But Y is not semi-simple because not all submodules of Y is a direct summand of Y .

3. Z_{10} as a Z -module is a pw-extending Z -module.

4. Pw-extending module and extending module are in dependent concepts, as the following examples show: The Z -module Z_{16} is not pw-extending but it is extending since every submodule of Z_{16} is essential in a direct summand of Z_{16} , and Z_{16} is indecomposable, that is the only direct summand of Z_{16} is Z_{16} itself and (0) . Also, we notes in (2) the Z -module $Z_6 \oplus Z_2$ is a pw-extending, but not extending module since the submodule $\langle \bar{2}, \bar{1} \rangle$ is a closed submodule of $Z_6 \oplus Z_2$, but not direct summand. Hence $Z_6 \oplus Z_2$ is not extending.

Proposition (3.3)

A direct summand of a pw-extending module Y is a pw-extending, provided that Y is a fully semi-prime.

Proof

Let E be a direct summand of Y , then there exists a submodule L of Y such that $Y = E \oplus L$. Let K be a

pw-closed submodule of E . Since E is a direct summand of Y , then E is a closed submodule of Y . Then by [4, Prop.2.14] we get E is a w-closed submodule, thus by [5, Prop.2.7] K is a pw-closed submodule of Y . But Y is a pw-extending, then K is a direct summand of Y . Thus $Y = K \oplus T$ for some submodule T of Y . Now, $E = Y \cap E = (K \oplus T) \cap E = K \oplus (T \cap E)$. Thus K is a direct summand of E , hence E is a pw-extending.

Proposition (3.4)

Let Y be a module such that every weak essential extensions of any submodule of Y is a completely essential, and Y is a pw-extending. Then Y is extending module.

Proof

Let E be a nonzero closed submodule of Y . so by [5, Prop.2.6] E is a pw-closed submodule of Y , but Y is a pw-extending, then E is a direct summand of Y . Hence Y is extending.

Proposition (3.5)

Let Y be a fully semi-prime module, and pw-extending. Then Y is extending.

Proof

Follows by [5, Prop.2.7].

Proposition (3.6)

Let Y be a module, with every submodule E of Y there exists a pw-closed submodule L of Y with E essential submodule of L . If Y is a pw-extending then Y is extending.

Proof

It is easy, so we omitted.

4.Relationships of w-extending modules and pw-extending modules with some types of modules.

The main goal of this section is study the relation between w-extending and pw-extending modules with some types of modules.

“Recall that a module M is called CLS-module, if every y-closed submodule of M is a direct summand [8]”. Where a submodule A of an R -module M is called y-closed submodule of M , if $\frac{M}{A}$ is non-singular module [2].

Proposition (4.1)

Let Y be a fully semi-prime module. If Y is a w-extending module, then Y is a CLS-module.

Proof

Assume that Y is a w-extending, and E be a non zero y-closed submodule of Y , then by [4, Prop.2.33] E is a w-closed submodule of Y . But Y is a w-extending, then E is a direct summand of Y , hence Y is a CLS-module.

Proposition (4.2)

Let Y be a non-singular R -module. If Y is a CLS-module, then Y is a w-extending module.

Proof

Let E be a w-closed submodule of Y . Since Y is non-singular, then by [4, Prop.2.34] E is a y-closed submodule of Y . But Y is CLS-module, then E is a direct summand of Y , hence Y is a w-extending module.

Proposition (4.3)

Let Y be a fully semi-prime and non-singular R -module. Then the following statements are equivalent:

1. Y is CLS-module.
2. Y is Extending module.
3. Y is a w-Extending module.

Proof

(1) $\Leftrightarrow$ (2)

Let K be a closed submodule of Y , then by [9, Prop.2.1.2] K is a y-closed, hence K is a direct summand of Y . Thus Y is extending module.

(2) $\Rightarrow$ (3)

Let E be a w-closed submodule of Y , then by [4, Prop.2.13] E is a closed submodule of Y . Hence E is a direct summand of Y . Thus Y is a w-extending.

(3) $\Rightarrow$ (1)

Let L be a y-closed submodule of Y . Then by [4, Prop.2.33] L is w-closed. Hence L is a direct summand of Y . Thus Y is a CLS-module.

“Recall that a module M is called duo if every submodule K of M is a fully invariant. That is $f(N) \leq N$ for each $f \in \text{End}(M)$ [8]”.

“Recall that an R -module M is called FI-extending if every fully invariant submodule of M is essential in a direct summand of M [10]”.

Proposition (4.4)

Let Y be a duo R -module. If Y is FI-extending module, then Y is a w-Extending module.

Proof

Let E be a submodule of Y . Since Y is a duo, then E is fully invariant in Y . But Y is FI-extending, then E is an essential in a direct summand of Y . Hence E is a weak essential in a direct summand of Y . Thus Y is a w-extending module.

Proposition (4.5)

Let Y be a duo R -module, with every nonzero direct summand of Y is a completely essential. Then Y is a w-extending if and only if Y is FI-extending module.

Proof

($\Rightarrow$) Since Y is a w-extending, then by corollary (2.14) Y is extending, hence by [10] Y is FI-extending.

($\Leftarrow$) Follows by Proposition (4.4).

Theorem (4.6)

Let Y be a duo R -module, with every nonzero direct summand of Y is a completely essential. Then the following statements are equivalent :

1. Y is extending.
2. Y is FI-extending.
3. Y is a w-extending.

Proof

(1) $\Leftrightarrow$ (2) direct.

(2) $\Leftrightarrow$ (3) Follows by Proposition (4.4).

(3) $\Leftrightarrow$ (1) Follows Corollary (2.14).

“Recall that an R -module M is prime-extending if every non zero submodule of M is essential in a prim direct summand [11]”.

Proposition (4.7)

Every prime-extending module is a w-extending.

proof

Follows by [11, Remark 1.11]

Theorem (4.8)

Let Y be a fully semi-prime R -module, with every direct summand of Y is a prime. Then the following statements are equivalent :

1. Y is a prime extending.
2. Y is a extending.
3. Y is a w-extending.

Proof

(1) $\Leftrightarrow$ (2) by [11, Remark.1.11].

(2) $\Leftrightarrow$ (3) direct.

(3) $\Leftrightarrow$ (1) :Let K be a submodule of Y . Since Y is a w-extending, then there exists a direct summand say H of Y such that K is a weak essential in H . But M is a fully semi-prime, then by [7, Lemma 2.3] K is an essential in H . By hypothesis H is a prime, thus Y is a prime-extending.

"Recall that a module M is called strongly extending if every submodule N of M is essential stable direct summand of M ". "[12]. Where a submodule N is stable if $f(N) \leq N$ For each R -homomorphism $f: N \rightarrow M$ ".

Proposition (4.9)

Every strongly extending module is a w-extending.

Proof

Let Y be strongly-extending module then by [12, Rem 1.3]. Y is extending hence Y is a w-extending.

Theorem (4.10)

Let Y be a fully semi-prime module, with every direct summand of Y is stable. Then the following statements are equivalent:-

1. Y is strongly extending.
2. Y is extending.
3. Y is w-extending.

Proof

(1) $\Leftrightarrow$ (2) by Proposition (4.9).

(2) $\Leftrightarrow$ (3) direct.

(3) $\Leftrightarrow$ (1) suppose that Y is a w-extending and let K be a submodule of Y , then K is a weak essential in a direct summand of Y say H . But Y is a fully semi-prime, then K is essential in H . Thus by hypothesis H is stable, Hence Y is a strongly extending.

It is well-known sully invariant direct summand is stable [12, Lemma 2.1.6], we get the following result.

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Corollary (4.11)

Let Y be a fully semi-prime R -module, with every direct summand of Y is a fully invariant. Then the following statements are equivalents:

1. Y is a strongly extending.
2. Y is an extending.
3. Y is w-extending.

Recall that an R -module M is S-extending if every stable submodule of M is essential in a direct summand of M [2].

Proposition (4.12)

Let Y be a module, with every weak essential extending of any submodule of Y is a completely essential. Then the following statements hold:-

1. If Y is a pw-extending, then Y is FI-extending.
2. If Y is a pw-extending, then M is S-extending.

Proof

(1) By Proposition (3.4) Y is extending, hence Y is FI-extending.

(2) Again by Proposition (3.4) Y is extending hence by [12] Y is S-extending.

Recall that an R -module M is a purely-extending if every submodule of M is essential in a pure submodule [13].

Theorem (4.13)

Let X be a fully semi-prime R -module. Then the following statements hold:

1. If X is a pw-extending, X is FI-extending.
2. If X is a pw-extending, then X is S-extending.
3. If X is a pw-extending, X is CLS-extending.
4. If X is a pw-extending, then X is purely-extending.

Proof

(1) By Proposition (3.5) X is extending, hence by [10] X is IF-extending.

(2) Again by Proposition (3.5) X is extending, then by [12] X is S-extending.

(3) Let E be a non zero y -closed submodule of X . Since X is a fully semi-prime submodule, then by [5, Prop.2.14] E is a pw-closed submodule of X . But X is a pw-extending, then E is a direct summand of X . Hence X is CLS-module.

(4) By Proposition (3.5) X is extending, hence by [13] X is a pure extending.

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المقاسات الموسعة من النمط w والمقاسات الموسعة من النمط pw

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الملخص

في هذا العمل قدمنا ودرسنا مفهومين من المقاسات. الاول هو إعمام المقاسات الموسعة و الذي يدعى المقاس الموسع من النمط w - والثاني والذي يدعى المقاس الموسع من النمط pw - والذي هو أقوى حالة من المقاس الموسع من النمط w . أعطينا الصفات الاساسية، الامثلة و المكافئات لهذين المفهومين. بالإضافة لذلك درسنا علاقة المقاسات الموسعة من النمط w - والمقاسات الموسعة من النمط pw - مع بعض انواع المقاسات الاخرى.