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On Generalize Some Weak Forms of Supra Mappings in Intuitionistic Topological Spaces

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Abstract

The aim of this paper is to introduce a new classes of supra mapping**s** called Intuitionistic Generalized Pre supra mapping, Intuitionistic Generalized Semi supra mapping, Intuitionistic Generalized **α**- supra mapping and Intuitionistic Generalized **β**-supra mapping. At last we studied some of their properties and investigate relationships among this concepts.

1. Introduction

Zadeh [1] was introducing **the** concept of "fuzzy set" in 1965**.** After that in 1968 Chang [2] introduced the concept of "fuzzy topology" .Also in 1983, Atanassov introduced the concept of " Intuitionistic fuzzy set " [3,4] . Finally Coker [5], introduced concept of " intuitionistic sets" and using it to introduce the concept of " intuitionistic topological spaces" [6] .

In this paper we generalize some weak forms of supra mappings in Intuitionistic topological spaces and studied some of their properties and relationships among them.

2. Preliminaries

Definition 2.1 [5] Let $M \subseteq X \neq \emptyset$ and. The Intuitionistic set $\tilde{M}$ (IS, for short) is the form $\tilde{M}=\langle x, M_1, M_2 \rangle$ and M_1 , $M_2 \subseteq X$ with condition $M_1 \cap M_2=\emptyset$ **.** The set M_1 is called "the set of members" of $\tilde{M}$ and M_2 is called " the set of non-members**"** of $\tilde{M}$.

Definition 2. 2 [5] Let $X \neq \emptyset$, and let $\tilde{M} = \langle x, M_1, M_2 \rangle$, $\tilde{N} =\langle x, N_1, N_2\rangle$ are two Intuitionistic sets respectively. Also, let$\{\tilde{M}_s; s \in S\}$ be a collection of "Intuitionistic sets in X" , and $\tilde{M}_i =\langle x, M_s^{(1)}, M_s^{(2)}\rangle$, the following is valid .

1) $\tilde{M}\ \tilde{M} \subseteq \tilde{N}$ iff $M_1 \subseteq N_1$ and $N_2 \subseteq M_2$,

2) $\tilde{M} = \tilde{N}$ iff $\tilde{M} \subseteq \tilde{N}$ and $\tilde{N} \subseteq \tilde{M}$,

3) The complement of $\tilde{M}$ is denoted by $\bar{\tilde{M}}$ and defined by $\bar{\tilde{M}} =\langle x, M_2, M_1\rangle$,

4) $\cup \tilde{M}_i =\langle x, \cup M_s^{(1)}, \cap M_s^{(2)} \rangle$, $\cap \tilde{M}_i=\langle x, \cap M_s^{(1)}, \cup M_s^{(2)} \rangle$,

5) $\dot{\emptyset} =\langle x, \emptyset, X\rangle$, $\dot{X} =\langle x, X, \emptyset\rangle$**.**

Definition 2.3 [6] Let $X \neq \emptyset$, $w \in X$ and let $\tilde{M} = \langle x, M_1, M_2\rangle$ be an Intuitionistic set the Intuitionistic point (IP f, for briefly) "Is $\dot{w}$ " defined by $\dot{w}= \langle x, \{w\}, \{w\}^c\rangle$ in X**.** Also a Vanishing Intuitionistic point defined by Is $\ddot{w} =\langle x, \emptyset, \{w\}^c\rangle$ in X **.** The Is $\dot{w}$ is said belong in $\tilde{M}$ ($\dot{w} \in M$, for brief) iff $w \in M_1$, also Is $\ddot{p}$ contained in $\tilde{M}$

($\ddot{w} \in \widetilde{M}$, for short) iff $w \notin M_2$.

Definition 2.4 [5] Let $X, Y \neq \emptyset$, and $r : (X, \mu) \longrightarrow (Y, \gamma)$ be a mapping.

a) If $\widetilde{N} = \langle y, N_1, N_2 \rangle$ is an Is in Y, then the inverse image of $\widetilde{N}$ under r defined by

$r^{-1}(\widetilde{N}) = \langle x, r^{-1}(N_1), r^{-1}(N_2) \rangle$.

b) If $\widetilde{M} = \langle x, M_1, M_2 \rangle$ is an Is in X, then $r(\widetilde{V}) = \langle y, r(M_1), \vec{r}(M_2) \rangle$ is an Is in Y

where $\vec{r}(\widetilde{M}) = \left(\overline{r(\overline{M_2})}\right)$.

Definition 2.5 [7] Let $X \neq \emptyset$. An Intuitionistic topology (ITS, for short) on X is a collection μ of an "Intuitionistic sets" in X satisfying:

(1) $\dot{\emptyset}, \dot{X} \in \mu$.

(2) μ is closed under finite intersections.

(3) μ is closed under arbitrary unions.

Each element in μ is called "Intuitionistic open set" and denoted by "IOS"

The complement of an " Intuitionistic open set" is called "Intuitionistic closed set" denoted by "ICS".

Definition 2.6 [7] Let (X, μ) be an ITS and let $\widetilde{M} = \langle x, M_1, M_2 \rangle \subseteq X$. The "interior" (namely, $int(\widetilde{M})$) and the "closure"(namely, $cl(\widetilde{M})$) are defined:

$int(\widetilde{M}) = \cup\{ \widetilde{V} : \widetilde{V} \subseteq \widetilde{M}, V \in \mu \}$,

$cl(\widetilde{M}) = \cap\{ \widetilde{J} : \widetilde{M} \subseteq \widetilde{J}, \bar{\widetilde{J}} \in \mu \}$. Also

1- $sint(\widetilde{M}) = \cup\{ \widetilde{V} : \widetilde{V} \subseteq \widetilde{M}, V \in ISOX \}$,

$scl(\widetilde{M}) = \cap\{ \widetilde{J} : \widetilde{M} \subseteq \widetilde{J}, \bar{\widetilde{J}} \in ISCS \}$.

2- $pint(\widetilde{M}) = \cup\{ \widetilde{V} : \widetilde{V} \subseteq \widetilde{M}, V \in IPOX \}$,

$pcl(\widetilde{M}) = \cap\{ \widetilde{J} : \widetilde{M} \subseteq \widetilde{J}, \bar{\widetilde{J}} \in IPCS \}$.

3- $\alpha int(\widetilde{M}) = \cup\{ \widetilde{V} : \widetilde{V} \subseteq \widetilde{M}, V \in I\alpha OX \}$,

$\alpha cl(\widetilde{M}) = \cap\{ \widetilde{J} : \widetilde{M} \subseteq \widetilde{J}, \bar{\widetilde{J}} \in I\alpha CS \}$.

4- $\beta int(\widetilde{M}) = \cup\{ \widetilde{V} : \widetilde{V} \subseteq \widetilde{M}, V \in I\beta OX \}$,

$\beta cl(\widetilde{M}) = \cap\{ \widetilde{J} : \widetilde{M} \subseteq \widetilde{J}, \bar{\widetilde{J}} \in I\beta CS \}$.

Remark 2-7.[7] This implications are valid:

$sint(\widetilde{M}) \subseteq \widetilde{M}$, $scl(\widetilde{M}) = \widetilde{M}$, $pint(\widetilde{M}) \subseteq \widetilde{M}$, $pcl(\widetilde{M}) = \widetilde{M}$, $\alpha int(\widetilde{M}) \subseteq \widetilde{M}$, $\alpha cl(\widetilde{M}) = \widetilde{M}$, $\beta int(\widetilde{M}) \subseteq \widetilde{M}$ and $\beta cl(\widetilde{M}) = \widetilde{M}$.

Definition 2.8. [8]

Let (X,μ) be an ITS. IS $\widetilde{M}$ of X is said to be

1. ISOS if $\widetilde{M} \subseteq Icl(Iint(\widetilde{M}))$,

2. IPOS if $\widetilde{M} \subseteq Iint\left(Icl(\widetilde{M})\right)$,

3. IαOS if $\widetilde{M} \subseteq$ □$Iint(Icl(Iint(\widetilde{M})))$,

4. IβOS if $\widetilde{M} \subseteq Icl(Iint(Icl(\widetilde{M})))$.

The family of all intuitionistic semi-open, pre-open, α-open and β-open sets of (X, μ) are denoted by "ISOS(X)","IPOS(X)","IαOS(X)" and "IβOS(X)" respectively. Also the complement of all intuitionistic semi-open, pre-open , , α-open and β-open sets of (X, μ) are denoted by "ISCS(X)","IPCS(X)","IαCS(X)" and "IβCS(X)" respectively.

Definition 2.9. [8]

Let (X, μ) be an ITS. An intuitionistic set $\widetilde{M}$ of X is said to be:

1) Intuitionistic generalizes-open set(Igos , for short) if $\forall$ U is ICS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$.

2) Intuitionistic generalizes semi-open set(Igsos , for short) if $\forall$ U is ISCS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$.

3) Intuitionistic generalizes pre-open set (IgPos , for short) if $\forall$ U is IPCS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$.

4) Intuitionistic generalizes α-open set (Igαos , for short) if $\forall$ U is IαCS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$.

5) Intuitionistic generalizes β-open set (Igβos , for short) if $\forall$ U is IβCS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$.

Definition 2.10. [8]

A map $r : (X,\mu) \rightarrow (Y ,\delta)$ is said to be:

1- intuitionistic continuous if the pre-image $f^{-1}(\widetilde{M})$ is IOS in X for every IOS $\widetilde{M}$ in Y.

2. Intuitionistic pre continuous if the pre image $f^{-1}(\widetilde{M})$ is IPOS in X for every IOS $\widetilde{M}$in Y.

3. Intuitionistic semi continuous if the pre image $f^{-1}(\widetilde{M})$ is ISOS in X for every IOS $\widetilde{M}$ in Y.

4- Intuitionistic α-continuous if the pre image $f^{-1}(\widetilde{M})$ is IαOS in X for every IOS $\widetilde{M}$ in Y .

5- Intuitionistic β-continuous if the pre image $f^{-1}(\widetilde{M})$ is IβOS in X for every IOS $\widetilde{M}$ in Y .

Now , we give this definition .

Definition 2.11.

A map $r : (X,\mu) \rightarrow (Y ,\delta)$ is said to be:

1. IgP continuous mapping if the pre image $f^{-1}(\widetilde{M})$ is IPCS in X for every IOS $\widetilde{M}$in Y.

2. IgS continuous mapping if the pre image $f^{-1}(\widetilde{M})$ is ISCS in X for every IOS $\widetilde{M}$ in Y.

3- Igα-continuous mapping if the pre image $f^{-1}(\widetilde{M})$ is IαCS in X for every IOS $\widetilde{M}$ in Y .

4- Igβ-continuous mapping if the pre image $f^{-1}(\widetilde{M})$ is IβCS in X for every IOS $\widetilde{M}$ in Y .

Section 2 INTUITIONISTIC GENERALIZED PRE, SEMI, β & α- of SUPRA MAPPINGS

In this section we have introduced intuitionistic this concepts: generalized pre supra mapping, Intuitionistic generalized semi supra mapping, Intuitionistic generalized β supra mappings, Intuitionistic generalized α- supra mapping and studied some from its properties.

Definition 2.1: A mapping $r: (X, \mu) \rightarrow (Y, \gamma)$ be an "Intuitionistic generalized pre supra mapping " ("IgPsm", for short) (resp., 'Intuitionistic generalized semi supra mapping "("IgSsm", for short), "Intuitionistic generalized α supra mapping " (Igαsm, for short), "Intuitionistic generalized β supra mapping" ('Igβsm", for short) if $r^{-1}(\widetilde{M})$ is an "IgPOS" (resp., is an "IgSOS", "IgαOS", "IgβOS") in (X, μ) for every "IgPOS" $\tilde{M}$ of (Y, γ) ((resp., for every "IgSOS" $\tilde{M}$, "IgαOS" $\tilde{M}$ "IgβOS" $\tilde{M}$ of (Y, γ)).

Proposition 2.2: Let $r: (E, \mu) \to (D, \gamma)$ and $p: (D, \gamma) \to (J, \delta)$ be IgPsm. Then $p \circ r: (E, \mu) \to (J, \delta)$ is IgPsm.

Proof: Let $\tilde{M}$ be IgPOS in J. Then $p^{-1}(\tilde{M})$ is IgPOS in D, since r is IgPsm, so $r^{-1}(p^{-1}(\tilde{M}))$ is IgPOS in E. Therefore $p \circ r$ is an IgPsm.

Proposition 2.3: Let $r: (E, \mu) \to (D, \gamma)$ and $p: (D, \gamma) \to (J, \delta)$ be Igαsm. Then $p \circ r: (E, \mu) \to (J, \delta)$ is IgSsm.

Proof: Let $\tilde{M}$ be IgαOS in J. So that $p^{-1}(\tilde{M})$ is IgαOS in D, since r is Igαsm and every IgαOS is IgSOS. Thus $r^{-1}(p^{-1}(\tilde{M}))$ is IgSOS in E. Therefore $p \circ r$ is IgSsm.

Proposition 2.4: : Let $r: (E, \mu) \to (D, \gamma)$ be an IgPsm and $p: (D, \gamma) \to (J, \delta)$ be IgP continuous supra mapping, then $p \circ r: (E, \mu) \to (J, \delta)$ is IgP continuous supra mapping.

Proof: Let $\tilde{M}$ be IOS in J. Then $p^{-1}(\tilde{M})$ is IgPOS in Y. Since r is IgPsm, then $r^{-1}(p^{-1}(\tilde{M}))$ is IgPOS in X. Therefore $p \circ r$ is IgP continuous supra mapping.

Proposition 2.5: : Let $r: (E, \mu) \to (D, \gamma)$ be an Igsm and $p: (D, \gamma) \to (J, \delta)$ be IgP continuous supra mapping, then $p \circ r: (E, \mu) \to (J, \delta)$ is IgP continuous supra mapping.

Proof : it's obvious.

Proposition 2.6: Let $r: (E, \mu) \to (D, \gamma)$ be an Igαsm and $p: (D, \gamma) \to (J, \delta)$ be Igα continuous mapping, then $p \circ r: (E, \mu) \to (J, \delta)$ is an IgS continuous mapping.

Proof: Let $\tilde{M}$ be IOS in J. Then $p^{-1}(\tilde{M})$ is an IgαOS in D. Since r is Igαsm, then $r^{-1}(p^{-1}(\tilde{M}))$ is IgSOS in E and every IgαOS is IgSOS. Hence $p \circ r$ is IgS continuous mapping.

Proposition 2.7: Let $r: (E, \mu) \to (D, \gamma)$ be IgPsm and $p: (D, \gamma) \to (J, \delta)$ be IgP continuous mapping, then $p \circ r: (E, \mu) \to (J, \delta)$ is Igβ continuous mapping.

Proof : it's obvious.

Proposition 2.8: Let $r: (E, \mu) \to (D, \delta)$ be Igαsm. Then this implications are equivalent:

(i) $r^{-1}(\tilde{M})$ is IgSOS in E for each IgSOS $\tilde{M}$ in D, (ii) $r^{-1}\,\alpha\mathrm{int}(\tilde{M}) \subseteq \mathrm{sint}\, r^{-1}(\tilde{M})$ for every "Is $\tilde{M}$" of D, (iii) $\alpha\mathrm{cl}\, r^{-1}(\tilde{M}) \subseteq r^{-1}\,\mathrm{scl}(\tilde{M})$ $\forall$ " Is $\tilde{M}$" of D.

Proof: (i) $\Rightarrow$ (ii) Let $\tilde{M}$ be ISOS in D and $\alpha\mathrm{int}(\tilde{M}) \subseteq \tilde{M}$, so $r^{-1}\alpha\mathrm{int}(\tilde{M}) \subseteq r^{-1}(\tilde{M})$. Since $\alpha\mathrm{int}(\tilde{M})$ is IαOS in D, and every IαOS is ISOS. So IgSOS in D. Therefore $r^{-1}\,\alpha\mathrm{int}(\tilde{M})$ is IgSOS in E, and $r^{-1}\,\alpha\mathrm{int}(\tilde{M})$ is ISOS in E., since $r^{-1}\alpha\mathrm{int}(\tilde{M}) \subseteq r^{-1}\mathrm{sint}(\tilde{M})$. Thus $r^{-1}\alpha\mathrm{int}(\tilde{M}) = \alpha\mathrm{int}\, r^{-1}\,\alpha\mathrm{int}(\tilde{M}) \subseteq \alpha\mathrm{int}\, r^{-1}(\tilde{M}) \subseteq \mathrm{sint}\, r^{-1}\,\mathrm{sint}(\tilde{M}) \subseteq \mathrm{sint}\, r^{-1}(\tilde{M})$.

(ii) $\Rightarrow$ (iii) by taking complement of (ii) we get the result of (iii).

(iii) $\Rightarrow$ (i) Let $\tilde{M}$ be IgSCS in D. Since $\tilde{M}$ is ISCS in D and $\mathrm{scl}(\tilde{M}) = \tilde{M}$. So $r^{-1}(\tilde{M}) = \alpha\mathrm{cl}\, r^{-1}(\tilde{M}) \subseteq r^{-1}\,\mathrm{scl}(\tilde{M})$. Therefore $r^{-1}(\tilde{M})$ is IgSCS in E.

Proposition 2.9: Let $r: (E, \mu) \to (D, \delta)$ be Igαsm. Then this implications are equivalent:

(i) $r^{-1}(\tilde{M})$ is IgαOS in E $\forall$ IgαOS $\tilde{M}$ in D, (ii) $r^{-1}\,\mathrm{pint}(\tilde{M}) \subseteq \beta\mathrm{int}\, r^{-1}(\tilde{M})$ for every Is $\tilde{M}$ of D, (iii) $\mathrm{pcl}\, r^{-1}(\tilde{M}) \subseteq r^{-1}\,\beta\mathrm{cl}(\tilde{M})$ $\forall$ IS $\tilde{M}$ of D.

Proof: (i) $\Rightarrow$ (ii) Let $\tilde{M}$ be IαOS in D and $\alpha\mathrm{int}(\tilde{M}) \subseteq \tilde{M}$, and $r^{-1}\alpha\mathrm{int}(\tilde{M}) \subseteq r^{-1}(\tilde{M})$. Since $\alpha\mathrm{int}(\tilde{M})$ is IβOS in D, and every IαOS is IβOS. So IgβOS in D. Therefore $r^{-1}\,\alpha\mathrm{int}(\tilde{M})$ is IgβOS in E, and $r^{-1}\,\alpha\mathrm{int}(\tilde{M})$ is IβOS in E ,since $r^{-1}\alpha\mathrm{int}(\tilde{M}) \subseteq r^{-1}\mathrm{pint}(\tilde{M}) \subseteq r^{-1}\beta\mathrm{int}(\tilde{M})$. Thus $r^{-1}\mathrm{pint}(\tilde{M}) = \mathrm{pint}\, r^{-1}\mathrm{pint}(\tilde{M}) \subseteq \mathrm{pint}\, r^{-1}(\tilde{M}) \subseteq \beta\mathrm{int}\, r^{-1}\,\beta\mathrm{int}(\tilde{M}) \subseteq \beta\mathrm{int}\, r^{-1}(\tilde{M})$.

(ii) $\Rightarrow$ (iii) by taking complement of (ii) we get the result of (iii).

(iii) $\Rightarrow$ (i) Let $\tilde{M}$ be IgαOS in D, since $\tilde{M}$ is IαOS in D and $\alpha\mathrm{cl}(\tilde{M}) = \tilde{M}$. Thus $r^{-1}(\tilde{M}) = \alpha\mathrm{cl}\, r^{-1}(\tilde{M}) \subseteq r^{-1}\alpha\mathrm{cl}(\tilde{M})$. Therefore $r^{-1}(\tilde{M})$ is IgαOS in E.

Proposition 2.10: Let $r: (E, \mu) \to (D, \delta)$ be IgPsm. Then this implications are equivalent:

(i) $r^{-1}(\tilde{M})$ is IgPOS in E $\forall$ IgPOS $\tilde{M}$ in D, (ii) $r^{-1}\,\mathrm{pint}(\tilde{M}) \subseteq \beta\mathrm{int}\, r^{-1}(\tilde{M})$ for every Is $\tilde{M}$ of D, (iii) $\mathrm{pcl}\, r^{-1}(\tilde{M}) \subseteq r^{-1}\,\beta\mathrm{cl}(\tilde{M})$ $\forall$ IS $\tilde{M}$ of D.

Proof: it's obvious.

Proposition 2.11: Let $r: (E, \mu) \to (D, \gamma)$ and $p: (D, \gamma) \to (J, \delta)$ are two Igαsm. Then $p \circ r: (E, \mu) \to (J, \delta)$ is IgPsm.

Proof: Let $\tilde{M}$ be IgαOS in J. Thus $p^{-1}(\tilde{M})$ is IgαOS in D, since r is Igαsm ,then $r^{-1}(p^{-1}(\tilde{M}))$ is IgαOS in E. Since every IgαOS is IgPOS. Thus $r^{-1}(p^{-1}(\tilde{M}))$is IgPOS in E. Therefore $p \circ r$ is IgPsm.

Proposition 2.12: Let $r: (E, \mu) \to (D, \gamma)$ be IgPsm and $p: (D, \gamma) \to (J, \delta)$ be Igα continuous mapping, then $p \circ r: (E, \mu) \to (J, \gamma)$ is Igβ continuous mapping.

Proof: Let $\tilde{M}$ be IgαOS in J. So that $p^{-1}(\tilde{M})$ is IgαOS in D, since every IgαOS is IgPOS and r is IgPsm, then $r^{-1}(p^{-1}(\tilde{M}))$ is IgPOS in E. Since every IgPOS is IgβOS. Thus $r^{-1}(p^{-1}(\tilde{M}))$is Ig$\beta$OS in E. Thus $p \circ r$ is Igβ continuous mapping.

Proposition 2.13: Let $r: (E, \mu) \to (D, \gamma)$ be Igαsm and $p: (D, \gamma) \to (J, \delta)$ be Igα continuous mapping, then $p \circ r: (E, \mu) \to (J, \gamma)$ is Igβ continuous mapping.

Proof: Let $\tilde{M}$ be IαOS in J. So that $p^{-1}(\tilde{M})$ is IgαOS in D, since r is Igαsm, $r^{-1}(p^{-1}(\tilde{M}))$ is IgαOS in E. Since every IgαOS is IgβOS. Thus $r^{-1}(p^{-1}(\tilde{M}))$ is IgβOS in E. Therefore $p \circ r$ is Igβ continuous mapping.

Proposition 2.14: Let $r : (E, \mu) \to (D, \gamma)$ be IgSsm and $p: (D, \gamma) \to (J, \delta)$ be Ig continuous mapping , then $p \circ r : (E, \mu) \to (J, \gamma)$ is IgS continuous mapping .

Proof: Let $\tilde{M}$ be IOS in J . So that $p^{-1}(\tilde{M})$ is IgSOS in D, since r is IgSsm, then $r^{-1}(p^{-1}(\tilde{M}))$ is IgOS in E . Since every IgOS is IgSOS . Thus $r^{-1}(p^{-1}(\tilde{M}))$ is IgSOS in E . Therefore $p \circ r$ is IgS continuous mapping.

Proposition 2.15: Let $r : (E, \mu) \to (D, \gamma)$ be IgPsm and $p: (D, \gamma) \to (J, \delta)$ be Igp continuous mapping , then $p \circ r: (E, \mu) \to (J, \gamma)$ is Igβ continuous mapping .

Proof: Let $\tilde{M}$ be IPOS in J.Thus $p^{-1}(\tilde{M})$ is IgPOS in D, since r is IgPsm, then $r^{-1}(p^{-1}(\tilde{M}))$ is IgPOS in E . Since every IgPOS is IgβOS . Hence $r^{-1}(p^{-1}(\tilde{M}))$is IgβOS in E . Therefore $p \circ r$ is Igβ continuous mapping.

Proposition 2.16: Let $r : (E, \mu) \to (D, \gamma)$ be Igαsm and $p: (D, \gamma) \to (J, \delta)$ be Igs continuous mapping , then $p \circ r: (E, \mu) \to (J, \gamma)$ is Igβ continuous mapping .

Proof: it obvious .

Proposition 2.17: Let $r : (E, \mu) \to (D, \gamma)$ be Igβsm and $p: (D, \gamma) \to (J, \delta)$ be Igα continuous mapping , then $p \circ r: (E, \mu) \to (J, \gamma)$ is Igβ continuous mapping .

Proof: Let $\tilde{M}$ be IαOS in J.Thus $p^{-1}(\tilde{M})$ is IgβOS in D, since r is Igβsm, then $r^{-1}(p^{-1}(\tilde{M}))$ is IgβOS in E . Since every IgαOS is IgβOS . Hence $r^{-1}(p^{-1}(\tilde{M}))$is IgβOS in E . Therefore $p \circ r$ is Igβ continuous mapping.

Proposition 2.18: Let $r : (E, \mu) \to (D, \gamma)$ be Igαsm and $p: (D, \gamma) \to (J, \delta)$ be Ig continuous mapping , then $p \circ r: (E, \mu) \to (J, \gamma)$ is IgP continuous mapping .

Proof: it obvious .

Proposition 2.19: Let $r : (E, \mu) \to (D, \gamma)$ be Igαsm and $p: (D, \gamma) \to (J, \delta)$ be Ig continuous mapping , then $p \circ r: (E, \mu) \to (J, \gamma)$ is IgS continuous mapping .

Proof: it obvious .

Section 3 The RELATIONS AMONG INTUITIONISTIC GENERALIZED PRE SUPRA MAPPING, INTUITIONISTIC GENERALIZED β −SUPRA MAPPING, INTUITIONISTIC GENERALIZED SEMI SUPRA MAPPING AND INTUITIONISTIC GENERALIZED α − SUPRA MAPPING .

Now, We give this important theorem .

Theorem 3.1. The implication among some types of mappings are given by the following diagram.

$$\begin{tikzcd}
\mathbf{IgPsm} \arrow[rr] & & \mathbf{Ig\beta sm} & \\
\mathbf{Igsm} \arrow[r] & \mathbf{Ig\alpha sm} \arrow[ul] \arrow[rr] & & \mathbf{IgSsm} \arrow[ul]
\end{tikzcd}$$

Proof : IgPsm $\longrightarrow$ **Igβsm**

Let $r : (E, \mu) \to (D, \delta)$ be a mapping and $\tilde{M}$ be IPOS in D, since r is IgPsm , then $r^{-1}(\tilde{M})$ is IgPOS in E . Since each IPO(Y) is IβO(Y) .Hence $r^{-1}(\tilde{M})$ is IgβOS in E for each $\tilde{M}$ be IβOS in D. Therefore r is Igβsm .

Igsm $\longrightarrow$ **Igαsm**

Let $r : (E, \mu) \to (D, \delta)$be a mapping and $\tilde{M}$ be IOS in D. Since r is Igsm , then $r^{-1}(\tilde{M})$ is IgOS in E . Since each IO(Y) is IαO(Y) .So that $r^{-1}(G)$ is IgαOS in E $\forall$ $\tilde{M}$ be IαOS in Y. Therefore r is Igαsm .

Igαsm $\longrightarrow$ **IgSsm**

Let $r : (E, \mu) \to (D, \delta)$be a mapping and let $\tilde{M}$ be IαOS in D. Since r is Igαsm , then $r^{-1}(\tilde{M})$ is IgαOS in E . Since each IαO(Y) is ISO(Y) .Thus $r^{-1}(G)$ is IgSOS in E $\forall$ $\tilde{M}$ be ISOS in Y. Therefore r is Igαsm .

IgSsm $\longrightarrow$ **Igβsm**

Let $r : (E, \mu) \to (D, \delta)$be a mapping and let $\tilde{M}$ be ISOS in D, since r is IgSsm , then $r^{-1}(\tilde{M})$ is IgSOS in E . Since each ISO(Y) is IβO(Y) .Hence $r^{-1}(G)$ is IgβOS in E $\forall$ $\tilde{M}$ be an IβOS in D. Therefore r is Igβsm .

Igαsm $\longrightarrow$ **IgPsm**

Let $r : (E, \mu) \to (D, \delta)$be a mapping and let $\tilde{M}$ be ISOS in D, since r is IgSsm , then $r^{-1}(\tilde{M})$ is IgSOS in E . Since each ISO(Y) is IβO(Y) .Hence $r^{-1}(G)$ is IgβOS in E $\forall$ $\tilde{M}$ be an IβOS in D. Therefore r is Igβsm .

Remark 3.2 By transitivity we get this result:

1- **Igαsm** $\longrightarrow$ **Igβsm**

2- **Igsm** $\longrightarrow$ **IgPsm**

3- **Igsm** $\longrightarrow$ **IgSsm**

The converse of the Theorem 3.1. is not true, the following examples are shown the cases.

Example 3.3. Let $E = \{a, m, n\}$ with topology $\mu = \{\dot{E}, \dot{\emptyset}, \tilde{S}, \tilde{R}, \tilde{K}, \tilde{U}\}$, where $\tilde{S} = \langle e, \{w\}, \{t, z\}\rangle$, $\tilde{R} = \langle e, \{w, z\}, \emptyset\rangle$, $\tilde{K} = \langle e, \{w\}, \emptyset\rangle$, $\tilde{U} = \langle e, \{w\}, \{z\}\rangle$ & $D = \{5,6,7\}$ with topology $\delta = \{\dot{Y}, \dot{\emptyset}, \tilde{W}, \tilde{Q}\}$, where $\tilde{W} = \langle d, \{5\}, \{6,7\}\rangle$, $\tilde{Q} = \langle d, \{5\}, \emptyset\rangle$. Let a mapping $r : (E, \mu) \to (D, \delta)$ defined by $r(\{a\}) = \{5\}$, $r(\{m\}) = \{7\}, r(\{n\}) = \{6\}$. Then

1- r is Igβsm , because $\forall$ $\tilde{M}$ be IgβOS in D, $r^{-1}(\tilde{M})$ is IgβOS in E. But r is not IgPsm, because $r^{-1}(\{5,7\}) = \{a, m\}$ is not IgPOS in E .

2- Also r is Igβsm , because $\forall$ $\tilde{M}$ be IgβOS in D, $r^{-1}(\tilde{M})$ is IgβOS in E .But r is not IgSsm, because $r^{-1}(\{6,7\}) = \{m, n\}$ is not IgSOS in E .

Example 3.4. Let $E = \{c, d\ e\}$ with topology $\mu = \{\dot{E}, \dot{\emptyset}, \tilde{M}, \tilde{N}\}$, where $\tilde{M} = \langle e, \{c\}, \{d, e\}\rangle$, $\tilde{N} = \langle e, \{c\}, \emptyset\rangle$ & $D = \{1,2,3\}$ with topology $\delta = \{\dot{D}, \dot{\emptyset}, \tilde{O}, \tilde{F}\}$, where $\tilde{O} = \langle d, \{1\}, \{3\}\rangle$, $\tilde{F} = \langle d, \{1\}, \emptyset\rangle$. Let a mapping $r : (E, \mu) \to (D, \gamma)$ defined by $r(\{c\}) = \{2\}$, $r(\{d\}) = \{3\}$, $r(\{e\}) = \{1\}$. Thus r is IgPsm, because $\forall \tilde{M}$ be IgPOS in D, $r^{-1}(\tilde{M})$ is IgPOS in E. But r is not Igsm, because $r^{-1}(\{1,3\}) = \{e, d\}$ is not IgOS in E.

Example 3.5. Let $E = \{d, v, p, t\}$ with topology $\mu = \{\dot{E}, \dot{\emptyset}, \tilde{B}, \tilde{J}, \tilde{X}, \tilde{N}\}$, where $\tilde{B} = \langle e, \{d\}, \{v, p, t\}\rangle$, $\tilde{J} = \langle e, \emptyset, \emptyset\rangle$, $\tilde{X} = \langle e, \emptyset, \{v, p, t\}\rangle$, $\tilde{N} = \langle e, \{d\}, \emptyset\rangle$, $Y = \{1,3,5\}$ with topology $\gamma = \{\dot{D}, \dot{\emptyset}, \tilde{L}, \tilde{P}\}$, where $\tilde{L} = \langle d, \{1\}, \{3,5\}\rangle$, $P = \langle d, \{1\}, \emptyset\rangle$. Let mapping $r : (E, \mu) \to (D, \gamma)$ defined by $r(\{d\}) = \{1\}$, $r(\{v\}) = r(\{p\}) = \{3\}$, $r(\{t\}) = \{5\}$.
Therefore r is IgSsm, because $\forall \tilde{M}$ be IgSOS in D, $r^{-1}(\tilde{M})$ is IgSOS in E.
But r is not Igαsm, because $r^{-1}(\{3,5\}) = \{p, v, t\}$ is not IgαOS in E.

Example 3.6. Let $E = \{i, j\}$ with topology $\mu = \{\dot{X}, \dot{\emptyset}, \tilde{M}, \tilde{N}\}$, where $\tilde{M} = \langle e, \{i\}, \{j\}\rangle$, $\tilde{N} = \langle e, \{i\}, \emptyset\rangle$ and $D = \{3,4\}$ with topology $\gamma = \{\dot{D}, \dot{\emptyset}, \tilde{M}, \tilde{W}\}$, where $\tilde{M} = \langle x, \emptyset, \{3\}\rangle$, $\tilde{W} = \langle x, \emptyset, \emptyset\rangle$. Let a mapping $r : (E, \mu) \to (D, \gamma)$ defined by as $r(\{i\}) = \{3\}$, $r(\{j\}) = \{4\}$. So that r is Igαsm, because $\forall \tilde{M}$ be IgαOS in D, $r^{-1}(\tilde{M})$ is IgαOS in E. But r is not Igsm, because $r^{-1}(\{4\}) = \{j\}$ is not IgOS in E.

Remark 3-7. IgPsm and IgSsm is independent notions .The following two examples shows this two cases .

Example 3.8. Let $E = \{w, r, i\}$ with topology $\mu = \{\dot{E}, \dot{\emptyset}, \tilde{K}, \tilde{S}, \tilde{G}, \tilde{Z}, \tilde{I}\}$, where $\tilde{K} = \langle e, \{w\}, \{r, i\}\rangle$, $\tilde{S} = \langle e, \{w\}, \emptyset\rangle$, $\tilde{G} = \langle e, \{w, r\}, \{i\}\rangle$, $\tilde{Z} = \langle e, \{w\}, \{r\}\rangle$, $\tilde{I} = \langle e, \{w, r\}, \emptyset\rangle$ and $D = \{2,4,6\}$ with topology $\gamma = \{\dot{D}, \dot{\emptyset}, \tilde{T}, \tilde{H}, \tilde{C}, \tilde{F}, \tilde{R}\}$, where $\tilde{T} = \langle d, \{2\}, \{4,6\}\rangle$, $\tilde{H} = \langle d, \{2\}, \emptyset\rangle$, $\tilde{C} = \langle d, \{2,4\}, \{6\}\rangle$, $\tilde{F} = \langle d, \{2\}, \{6\}\rangle$, $\tilde{R} = \langle d, \{2,4\}, \emptyset\rangle$. Let a mapping $r : (E, \mu) \to (D, \gamma)$ defined by $r(\{w\}) = \{2\}$, $r(\{w\}) = \{4\}$, $r(\{i\}) = \{6\}$. So r is IgPsm, because $\forall \tilde{M}$ be an IgPOS in D, $r^{-1}(\tilde{M})$ is IgPOS in E. But r is not IgSsm, because $r^{-1}(\{4,6\}) = \{w, i\}$ is not IgSOS in E.

Example 3.9. Let $E = \{o, p, u\}$ with topology $\mu = \{\dot{E}, \dot{\emptyset}, \tilde{W}, \tilde{R}, \tilde{K}, \tilde{P}, \tilde{T}, \tilde{H}, \tilde{Z}\}$, where $\tilde{W} = \langle e, \{o\}, \{p, u\}\rangle$, $\tilde{R} = \langle e, \{o\}, \{p\}\rangle$, $\tilde{K} = \langle e, \{o\}, \emptyset\rangle$, $\tilde{P} = \langle e, \{o, p\}, \emptyset\rangle$, $\tilde{T} = \langle e, \emptyset, \emptyset\rangle$, $\tilde{H} = \langle e, \emptyset, \{p, u\}\rangle$, $\tilde{Z} = \langle e, \emptyset, \{p\}\rangle$ and $D = \{1,4,7\}$ with topology $\gamma = \{\dot{D}, \dot{\emptyset}, \tilde{V}, \tilde{G}, \tilde{Q}\}$, where $\tilde{V} = \langle d, \{1\}, \{4,7\}\rangle$, $\tilde{G} = \langle d, \{1,4\}, \emptyset\rangle$, $\tilde{Q} = \langle d, \{1\}, \emptyset\rangle$. Let a mapping $r : (E, \mu) \to (D, \gamma)$ defined by $r(\{o\}) = \{1\}$, $r(\{p\}) = \{4\}$, $r(\{u\}) = \{7\}$. So r is IgSsm, because for each $\tilde{M}$ is IgSOS in D, $r^{-1}(\tilde{M})$ is IgSOS in E. But r is not IgPsm, because $r^{-1}(\{4,7\}) = \{p, u\}$ is not IgPOS in E.

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حول تعميم بعض الاشكال الضعيفة للتطبيقات الفوقية في الفضاءات التبولوجية الحدسية

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الملخص

في هذا البحث قدمنا صفوف جديدة من التطبيقات اسميناها : intuitionistic generalized Pre supra mapping, intuitionistic generalized Semi supra mapping, intuitionistic generalized α-supra mapping , intuitionistic generalized β-supra mapping ودرسنا بعض خواصها.

واخيرا درسنا واستقصينا العلاقات بين هذه المفاهيم.