



Nearly Quasi 2-Absorbing submodule

Haibt K. Mohammadali , Khalaf H Alhabeeb

Department of Mathematics , College of Computer Science and Mathematics , Tikrit University , Tikrit , Iraq

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Corresponding Author:

Name: Khalaf H Alhabeeb

E-mail:

khalif.alhabeeb@gmail.com

Tel:

1 – Introduction

Al-Mothafar and Abdul-Al kalik in 2016 introduce the concept of nearly quasi prime submodules as generalization of prime (quasi prime) submodules, where "a proper submodule E of an R -module X is called nearly quasi prime, if whenever $abx \in E$, with $a, b \in R, x \in X$, implies that, either $ax \in E + J(X)$, $bx \in E + J(X)$ [1]", and "a proper submodule E of an R -module X is called quasi prime if whenever $abx \in E$ with $a, b \in R, x \in X$, implies that, either $ax \in E$ or $bx \in E$ [2]", and a proper submodule E of an R -module X is called prime if whenever $ax \in E, a \in R, x \in X$, implies that $x \in E$ or $a \in [E : X]$ [3], where $[E : X] = \{ r \in R : rX \subseteq E \}$. Many basic properties of nearly quasi-2-absorbing submodules are given, also many characterizations of this concept are establish see proposition (2.6) and corollary (2.7). Also, we show that the intersection of two nearly quasi-2-absorbing submodules need not to be nearly quasi-2-absorbing, but under certain condition it satisfy see Remark (2.8), proposition (2.11).

2 - Nearly quasi-2-absorbing submodules

Recall that a proper submodule E of an R -module X is 2-absorbing if whenever $abx \in E$ with $a, b \in R, x \in X$, implies that either $ax \in E$ or $bx \in E$ or $ab \in [E : X]$ [4]. We introduce in this section the definition of nearly quasi 2-absorbing submodules as generalization of prime , quasi prime , nearly quasi prime submodules, and 2-absorbing submodules.

Definition (2 . 1)

Abstract

All rings in this note are commutative rings with identity, and all R -modules are left unitary. "A proper submodule E of an R -module X is called nearly quasi prime submodule, if whenever $abx \in E$, with $a, b \in R, x \in X$, implies that, either $ax \in E + J(X)$ or $bx \in E + J(X)$ ", where $J(X)$ is the Jacobson radical of X . this led us to introduce the concept a nearly quasi 2-absorbing submodule as a generalization of nearly quasi prime submodules and 2-absorbing submodules, where a proper submodule E of an R -module X is called nearly quasi 2-absorbing submodule of X , if whenever $abcx \in E$, where $a, b, c \in R, x \in X$ implies that either $abx \in E + J(X)$ or $acx \in E + J(X)$ or $bcx \in E + J(X)$. We study the basic properties of nearly quasi 2-absorbing. Moreover, the relations of nearly quasi 2-absorbing submodule with other classes of modules are established. Also, characterization, and examples are given.

A proper submodule E of an R -module X is said to be nearly quasi 2-absorbing , if whenever $rstx \in E, r, s, t \in R, x \in X$, implies that $rsx \in E + J(X)$ or $rtx \in E + J(X)$ or $stx \in E + J(X)$.

And a proper ideal I of a ring R is called nearly quasi 2-absorbing ideal if I is nearly quasi-2-absorbing submodule , of R -module R .

Remarks and Examples (2.2)

1– Every prime submodule of an R -module X is nearly quasi-2-absorbing, while the converse is not true in general.

Proof

Assume that E is a prime submodule of an R -module X , and $rstx \in E, r, s, t \in R, x \in X$, that is $rs(tx) \in E$, suppose that $tx \notin E$ and $stx \notin E + J(X)$, $rstx \notin E + J(X)$. since E is a prime and $tx \notin E$, it follows that $rs \in [E : X]$, that is $rsx \in E \subseteq E + J(X)$ for each $x \in X$, hence $rsx \in E + J(X)$.

For the converse consider the following example:- let $X = Z, R = Z$ and $E = 6Z$ is a submodule of X , $6Z$ is nearly quasi-2-absorbing submodule of X but $6Z$ is not prime submodule of X .

2– it is clear that every nearly quasi prime submodule of X is nearly quasi-2-absorbing , while the converse need not to be true.

For the converse consider the following example:-

Let $X = Z \oplus Z, R = Z, E = 10Z \oplus (0)$ it is clear that E is nearly quasi-2-absorbing submodule of X , but E is not nearly quasi prime submodule of X , since $2 \cdot 5$

$(1,0) \in E$, $2, 5 \in Z$, $(1, 0) \in X$, but $2 \cdot (1,0) \notin E + J(X)$ and $5 \cdot (1,0) \notin E + J(X)$.

3 – It is clear that every 2-absorbing submodule is nearly quasi-2-absorbing. For the converse consider the following example:-

Let $X = Z \oplus Z$, $R = Z$, $E = (0) \oplus 35Z$, E is nearly quasi-2-absorbing but not 2-absorbing, since $5 \cdot 7(0,1) \in E$, $5(0,1) \notin E$ and $7(0,1) \notin E$ and $5 \cdot 7 \notin [E : X] = (0)$.

4 – It is clear that every quasi prime submodule of X is nearly quasi-2-absorbing, but the converse is not true in general.

For the converse consider the following example:- let $X = Z_8 \oplus Z$ and $R = Z$, $E = \langle 4 \rangle \oplus Z$, it is clear that E is nearly quasi-2-absorbing, but not quasi prime, since $2 \cdot 2(1,1) \in E = \langle 4 \rangle \oplus Z$, but $2(1,1) \notin E$.

Proposition (2.3)

If E and K are two submodules of an R -module X with $E \not\subseteq K$, and E is nearly quasi-2-absorbing submodule of X such that $J(X) \subseteq J(K)$, then E is a nearly quasi-2-absorbing submodule of K .

Proof

Let $rstx \in E$, $r, s, t \in R$, $x \in K \subseteq X$, then either $rsx \in E + J(X)$ or $rtx \in E + J(X)$ or $stx \in E + J(X)$. But $J(X) \subseteq J(K)$, hence either $rsx \in E + J(K)$ or $rtx \in E + J(K)$ or $stx \in E + J(K)$.

Proposition (2.4)

If H is nearly quasi-2-absorbing submodule of X and $J(X) \subseteq H$, then H is a weakly quasi-2-absorbing in X .

Proof

It is clear

Proposition (2.5)

Let H be a proper submodule of an R -module X , if $[H+J(X):(x)]$ is quasi prime ideal of R for each $x \in X$, then H is nearly quasi-2-absorbing in X .

Proof

Let $rstx \in H$, $r, s, t \in R$, $x \in X$, and $rsx \notin H + J(X)$, implies that $rstx \in H + J(X)$, hence $rst \in [H+J(X):(x)]$, it follows that either $rt \in [H+J(X):(x)]$ or $st \in [H+J(X):(x)]$, that is either $rtx \in H + J(X)$ or $stx \in H + J(X)$.

The following proposition are characterization of nearly quasi 2-absorbing submodules.

Proposition (2.6)

A submodule H of an R -module X is nearly quasi 2-absorbing in X if and only if for every ideals I_1, I_2, I_3 of R , and submodules F of X , with $I_1 I_2 I_3 F \subseteq H$, implies that either $I_1 I_2 F \subseteq H + J(X)$ or $I_1 I_3 F \subseteq H + J(X)$ or $I_2 I_3 F \subseteq H + J(X)$.

Proof

$\Rightarrow$

Assume that $I_1 I_2 I_3 F \subseteq H$, where I_1, I_2, I_3 are ideals and F is a submodule of X , and suppose that $I_1 I_2 F \not\subseteq H + J(X)$ or $I_1 I_3 F \not\subseteq H + J(X)$ or $I_2 I_3 F \not\subseteq H + J(X)$. So there exists $x_1, x_2, x_3 \in F$ and $a_1 \in I_1$, $a_2 \in I_2$, $a_3 \in I_3$ such that $a_1 a_2 x_1 \notin H + J(X)$ and $a_1 a_3 x_2 \notin H + J(X)$ and $a_2 a_3 x_3 \notin H + J(X)$. Since H is a nearly quasi 2-absorbing submodule of X and $a_1 a_2 a_3 x_1 \in H$ and $a_1 a_2 x_1 \notin H + J(X)$, then we have $a_1 a_3 x_1 \in H + J(X)$ or $a_2 a_3 x_1 \in H + J(X)$. Also $a_1 a_2 a_3 x_2 \in H$

and $a_1 a_3 x_2 \notin H + J(X)$, implies that either $a_1 a_2 x_2 \in H + J(X)$ or $a_2 a_3 x_2 \in H + J(X)$. Also $a_1 a_2 a_3 x_3 \in H$, and $a_2 a_3 x_3 \notin H + J(X)$, implies that either $a_1 a_2 x_3 \in H + J(X)$ or $a_1 a_3 x_3 \in H + J(X)$. Thus either $I_1 I_2 F \subseteq H + J(X)$ or $I_1 I_3 F \subseteq H + J(X)$ or $I_2 I_3 F \subseteq H + J(X)$.

$\Leftarrow$

Assume that $a_1 a_2 a_3 x \in H$, $a_1, a_2, a_3 \in R$, $x \in X$, then $(a_1)(a_2)(a_3)(x) \subseteq H$, so either $(a_1)(a_2)(x) \subseteq H + J(X)$ or $(a_1)(a_3)(x) \subseteq H + J(X)$ or $(a_2)(a_3)(x) \subseteq H + J(X)$, hence either $a_1 a_2 x \in H + J(X)$ or $a_1 a_3 x \in H + J(X)$ or $a_2 a_3 x \in H + J(X)$.

Proposition (2.7)

A submodule H of an R -module X is nearly quasi 2-absorbing in X if and only if for each submodule F of X and for each $a_1, a_2, a_3 \in R$, such that $a_1 a_2 a_3 F \subseteq H$, implies that either $a_1 a_2 F \subseteq H + J(X)$ or $a_1 a_3 F \subseteq H + J(X)$ or $a_2 a_3 F \subseteq H + J(X)$.

Proof

Direct

Remark (2.8)

The intersection of two nearly quasi 2-absorbing submodules of an R -module X need not to be nearly quasi 2-absorbing submodule of X , as the following example shows that:-

Let $X = Z$ and $R = Z$, $H = 9Z$, $K = 2Z$, H and K are nearly quasi 2-absorbing submodules of X , but $H \cap K = 18Z$ is not nearly quasi 2-absorbing in X , since $2 \cdot 3 \cdot 3 \cdot 1 \in 18Z$, but $2 \cdot 3 \cdot 1 = 6 \notin 18Z + J(X)$ and $3 \cdot 3 \cdot 1 = 9 \notin 18Z + J(X)$. Hence $18Z$ is not nearly quasi 2-absorbing submodule of X .

"Recall that a ring R is a good ring if $J(R)X = X$, where X is an R -module [5]".

Lemma (2.9) [5]

If R is a good ring and N is a submodule of an R -module X , then $J(X) \cap N = (N)$.

Lemma (2.10) [5, lemma 2.3.15]

Let X be an R -module, and A, B and C are submodules of X with $B \not\subseteq C$. Then $(A+B) \cap C = (A \cap C) + B = (A \cap C) + (B \cap C)$.

Proposition (2.11)

let X be an R -module over a good ring R , and H is nearly quasi 2-absorbing submodule of X and F is a submodule of X with $J(X) \subseteq F$ and $F \not\subseteq H$, then $F \cap H$ is nearly quasi 2-absorbing submodule of F .

proof

since $F \not\subseteq H$, then $F \cap H$ is a proper submodule of F , Now let $rstx \in F \cap H$, where $r, s, t \in R$, $x \in F$, implies that $rstx \in F$ and $rstx \in H$, but H is a nearly quasi 2-absorbing in X , then either $rsx \in H + J(X)$ or $rtx \in H + J(X)$ or $stx \in H + J(X)$, since $x \in F$, implies that $rsx \in (H + J(X)) \cap F$ or $rtx \in (H + J(X)) \cap F$ or $stx \in (H + J(X)) \cap F$. But $J(X) \subseteq F$, so by lemma (2.10), we have either $rsx \in (H \cap F) + (J(X) \cap F)$ or $rtx \in (H \cap F) + (J(X) \cap F)$ or $stx \in (H \cap F) + (J(X) \cap F)$. But R is a good ring then by lemma (2.9), we have $J(X) \cap F = J(F)$. Thus either $rsx \in (H \cap F) + J(F)$ or $rtx \in (H \cap F) + J(F)$ or $stx \in (H \cap F) + J(F)$.

Corollary (2.12)

let X be an R -module over a good ring R , and H, F are submodules of X with $F \not\subseteq H$ if H is nearly quasi 2-absorbing in X , and F is a maximal submodule of X , then $H \cap F$ is nearly quasi 2-absorbing submodule of F .

Proof

Since F is maximal submodule of X , then $J(X) \subseteq F$. Hence the proof follows by proposition (2.11)

The following proposition shows that the intersection of two nearly quasi 2-absorbing submodules is nearly quasi 2-absorbing under certain conditions.

Proposition (2.13)

If H and F are two nearly quasi-2-absorbing submodules of an R -module X , with $H \subseteq J(X)$ or $F \subseteq J(X)$, then $H \cap F$ is nearly quasi-2-absorbing submodule of X .

Proof

Assume that $rstx \in H \cap F$, $r, s, t \in R$ and $x \in X$ it follows that $rstx \in H$ and $rstx \in F$, but both H and F are nearly quasi 2-absorbing submodules of X . So, either $rsx \in H + J(X) = J(X)$ or $rtx \in H + J(X) = J(X)$ or $stx \in H + J(X) = J(X)$ and $rsx \in F + J(X) = J(X)$ or $rtx \in F + J(X) = J(X)$ or $stx \in F + J(X) = J(X)$. Thus either $rsx \in H \cap F + J(X)$ or $rtx \in H \cap F + J(X)$ or $stx \in H \cap F + J(X)$ (because $H \cap F \subseteq J(X)$). Thus $H \cap F$ is nearly quasi 2-absorbing of X .

Proposition (2.14)

Let H and F be two submodules of an R -module X , with $F \not\subseteq H$ and $J(X) = J(F)$. If H is nearly quasi 2-absorbing submodule of X , then $H \cap F$ is nearly quasi 2-absorbing of F .

Proof

It is clear that $F \cap H$ is a proper submodule of F . Let $rstx \in F \cap H$, $r, s, t \in R$, $x \in F$, so $rstx \in F$ and $rstx \in H$. But H is a nearly quasi 2-absorbing in X , and $x \in F \subseteq X$, then either $rsx \in H + J(X)$ or $rtx \in H + J(X)$ or $stx \in H + J(X)$. Since $x \in F$, implies that either $rsx \in (H + J(X)) \cap F$ or $rtx \in (H + J(X)) \cap F$ or $stx \in (H + J(X)) \cap F$. Thus since $J(X) = J(F)$, we get either $rsx \in (H + J(F)) \cap F$ or $rtx \in (H + J(F)) \cap F$ or $stx \in (H + J(F)) \cap F$. Hence by lemma (2.10), we have either $rsx \in (H \cap F) + J(F)$ or $rtx \in (H \cap F) + J(F)$ or $stx \in (H \cap F) + J(F)$. Thus $H \cap F$ is nearly quasi 2-absorbing in F .

Proposition (2.15)

Let H be a submodule of an R -module X , and $H + J(X)$ is nearly quasi-2-absorbing submodule of X , then H is nearly quasi-2-absorbing submodule in X .

Proof

To prove that H is nearly quasi 2-absorbing submodule let $rstx \in H$, $r, s, t \in R$, $x \in X$, since $H \subseteq H + J(X)$, then $rstx \in H + J(X)$. But $H + J(X)$ is nearly quasi-2-absorbing in X , so either $rsx \in H + J(X) + J(X) = H + J(X)$ or $rtx \in H + J(X) + J(X) = H + J(X)$ or $stx \in H + J(X) + J(X) = H + J(X)$. Thus H is nearly quasi-2-absorbing in X .

Proposition (2.16)

Let H be proper submodule of an R -module X such that H is nearly quasi 2-absorbing submodule of X ,

then $S^{-1}H$ is nearly quasi 2-absorbing submodule of $S^{-1}R$ -module $S^{-1}X$.

Proof

Assume that $\frac{a}{s_1} \frac{b}{s_2} \frac{c}{s_3} \frac{x}{s_4} \in S^{-1}H$, where $\frac{a}{s_1}, \frac{b}{s_2}, \frac{c}{s_3} \in S^{-1}R$ and $\frac{x}{s_4} \in S^{-1}X$, $a, b, c \in R$, $x \in X$, $s_1, s_2, s_3, s_4 \in S$, it follows that $\frac{abcx}{t} \in S^{-1}H$ where $s_1 s_2 s_3 s_4 = t \in S$. then there exists $t_1 \in S$ such that $t_1 abcx \in H$, since H is nearly quasi 2-absorbing in X then either $abxt_1 \in H + J(X)$ or $acxt_1 \in H + J(X)$ or $bcxt_1 \in H + J(X)$, which implies that either $\frac{a}{s_1} \frac{b}{s_2} \frac{x}{s_4} \frac{t_1}{t} \in S^{-1}[H + J(X)] = S^{-1}H + J(S^{-1}X)$ or $\frac{a}{s_1} \frac{c}{s_3} \frac{x}{s_4} \frac{t_1}{t} \in S^{-1}[H + J(X)] = S^{-1}H + J(S^{-1}X)$ or $\frac{b}{s_2} \frac{c}{s_3} \frac{x}{s_4} \frac{t_1}{t} \in S^{-1}[H + J(X)] = S^{-1}H + J(S^{-1}X)$. Thus $S^{-1}H$ is nearly quasi 2-absorbing submodule of $S^{-1}X$.

"Recall that a submodule E of an R -module X is small in X , if for any submodule F of X such that $X = E + F$ then $F = X$ [5]."

Also, "recall that an R -epimorphism $\varphi: X \rightarrow X'$ is called small epimorphism if $\text{Ker } \varphi$ small submodule of X [5]."

Lemma (2.17) [5, corollary 9.1.5]

If an R -epimorphism $\varphi: X \rightarrow X'$ is small, then $\varphi(J(X)) = J(X')$ and $\varphi^{-1}(J(X')) = J(X)$.

Proposition (2.18)

Let $\varphi: X \rightarrow X'$ be a small R -epimorphism, and E is a nearly quasi-2-absorbing submodule of X' , then $\varphi^{-1}(E)$ is a nearly quasi-2-absorbing of X .

Proof

Assume that $rstx \in \varphi^{-1}(E)$, $r, s, t \in R$, $x \in X$, so $rst\varphi(x) \in E$. But E is a nearly quasi 2-absorbing submodule of X' , then either $rs\varphi(x) \in E + J(X')$ or $rt\varphi(x) \in E + J(X')$ or $st\varphi(x) \in E + J(X')$. Thus either $rsx \in \varphi^{-1}(E) + \varphi^{-1}(J(X'))$ or $rtx \in \varphi^{-1}(E) + \varphi^{-1}(J(X'))$ or $stx \in \varphi^{-1}(E) + \varphi^{-1}(J(X'))$. But φ is small epimorphism, implies that $\varphi^{-1}(J(X')) = J(X)$ by lemma (2.17). Hence we have either $rsx \in \varphi^{-1}(E) + J(X)$ or $rtx \in \varphi^{-1}(E) + J(X)$ or $stx \in \varphi^{-1}(E) + J(X)$.

Proposition (2.19)

Let $\varphi: X \rightarrow X'$ be a small R -epimorphism, and E be a proper nearly quasi-2-absorbing submodule of X such that $\text{Ker } \varphi \subseteq E$, then $\varphi(E)$ is a nearly quasi-2-absorbing submodule of X' .

Proof

It is clear that $\varphi(E)$ is proper submodule of X' , if not suppose that $\varphi(E) = X'$, let $x \in X$ such that $\varphi(x) \in X' = \varphi(E)$, implies that there exist $e \in E$ such that $\varphi(e) = \varphi(x)$. implies that $\varphi(e - x) = 0$, it follows that $e - x \in \text{Ker } \varphi \subseteq E$, implies that $x \in E$, thus $E = X$ contradiction. Now assume that $rstx \in \varphi(E)$, $r, s, t \in R$, $x \in X$, since φ is an epimorphism, then there exist $x' \in X$ such that $\varphi(x) = x'$. Hence $rst\varphi(x) \in \varphi(E)$, implies that there exist $e \in E$ such that $rst\varphi(x) = \varphi(e)$, hence $\varphi(rstx - e) = 0$, so $rstx - e \in \text{Ker } \varphi \subseteq E$, implies that $rstx \in E$. But E is a nearly quasi 2-

absorbing in X , so either $rsx \in E + J(X)$ or $rtx \in E + J(X)$ or $stx \in E + J(X)$, it follows that either $rsx \in \phi(E) + \phi(J(X))$ or $rtx \in \phi(E) + \phi(J(X))$ or $stx \in \phi(E) + \phi(J(X))$. Thus by lemma (2.17), we have either $rsx \in \phi(E) + J(X')$ or $rtx \in \phi(E) + J(X')$ or $stx \in \phi(E) + J(X')$.

Corollary (2.20)

Let E be a submodule of an R -module X , and F be a small submodule of X contained in E . Then $\frac{E}{F}$ is a nearly quasi 2-absorbing submodule of $\frac{X}{F}$ if and only if E is a nearly quasi 2-absorbing in X .

Proof

Let $\pi: X \rightarrow \frac{X}{F}$ be a natural R -epimorphism, then the result follow by proposition (2.18) and proposition (2.19).

Proposition (2.21)

Let X' and X'' be R -modules and $X = X' \oplus X''$ if $E = E' \oplus E''$ is a nearly quasi 2-absorbing

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submodule in X , with $E \subseteq J(X)$, where E', E'' are submodules of X', X'' respectively. Then E' and E'' are nearly quasi 2-absorbing submodules in X' and X'' respectively.

Proof

Assume that $rstx' \in E', r, s, t \in R, x' \in X'$. Then $rst(x', 0) \in E = E' \oplus E''$. But E is a nearly quasi 2-absorbing in X , then either $rs(x', 0) \in E + J(X) = J(X)$ or $rt(x', 0) \in E + J(X) = J(X)$ or $st(x', 0) \in E + J(X) = J(X)$. But $J(X) = (X') \oplus (X'')$. It follows that either $rs(x', 0) \in J(X') \oplus J(X'')$ or $rt(x', 0) \in J(X') \oplus J(X'')$ or $st(x', 0) \in J(X') \oplus J(X'')$. Thus either $rsx' \in J(X') \subseteq E' + J(X')$ or $rtx' \in J(X') \subseteq E' + J(X')$ or $stx' \in J(X') \subseteq E' + J(X')$. Hence E' is a nearly quasi 2-absorbing submodule in X' .

By similar way E'' is a nearly quasi 2-absorbing submodule of X'' .

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المقاسات الجزئية المستحوذة من النمط - 2 الظاهرية القريبة

هيبية كريم محمد علي ، خلف حسن الحبيب

قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

كل الحلقات المستخدمة في هذا البحث هي حلقات ابدالية بمحايد و كل المقاسات المعرفة عليها هي مقاسات يسارية أحادية. المقاس الجزئي الفعلي E من المقاس X يدعى مقاس جزئي اولي ظاهري قريب اذا كان $abx \in E$ حيث $a, b \in R, x \in X$ اما $ax \in E + J(X)$ او $bx \in E + J(X)$ حيث ان $J(X)$ هو جذر جاكوبسن، هذا التعريف قادنا لكي نقدم مفهوم المقاس الجزئي المستحوذ من النمط - 2 الظاهري كأعمام للمقاس الجزئي الاولي الظاهري القريب و المقاس الجزئي المستحوذ من النمط - 2، حيث انه يدعى المقاس الجزئي الفعلي E بأنه مقاسا جزئيا مستحوذا من النمط - 2 ظاهريا قريبا اذا كان $abcm \in E$ حيث $a, b, c \in R, m \in X$ اما $abm \in E + J(X)$ او $acm \in E + J(X)$ او $bcm \in E + J(X)$. درسنا معظم المفاهيم الأساسية لهذا المفهوم ، بالإضافة لذلك درسنا علاقة هذا المفهوم مع أصناف أخرى من المقاسات. كذلك اعطينا مكافئات و امثلة عليه.