Tikrit Journal of Pure Science 23 (7) 2018 *ISSN: 1813 – 1662 (Print)*
E-ISSN: 2415 – 1726 (On Line)

TJPS

TIKRIT JOURNAL OF PURE SCIENCE

Journal Homepage: http://main.tu-jo.com/ojs/index.php/TJPS/index

Weakly Quasi 2-Absorbing submodule

Haibt K . Mohammadali , Khalaf H Alhabeeb

Department of mathematics , College of computer science and mathematics , Tikrit University , Tikrit , Iraq

DOI: http://dx.doi.org/10.25130/tjps.23.2018.118

ARTICLE INFO.

Article history:

-Received: 14 / 3 / 2018

-Accepted: 10 / 4 / 2018

-Available online: / / 2018

Keywords: weakly quasi-prime submodule, 2-absorbing submodule, weakly quasi 2-absorbing submodule .

Corresponding Author:

Name: Khalaf H Alhabeeb

E-mail:

khalif.alhabeeb@gmail.com

Tel:

Abstract

"Let R be a commutative ring with identity , and M is a unitary left R-module", "A proper submodule E of an R-module M is called a weakly quasi-prime if whenever r, s $\in$ R, m $\in$ M, with $0 \neq rsm \in E$, implies that rm $\in$ E or sm $\in$ E". "We introduce the concept of a weakly quasi 2-absorbing submodule as a generalization of weakly quasi-prime submodule", where a proper submodule E of M is called a weakly quasi 2-absorbing submodule if whenever r,s,t $\in$ R, m $\in$M with $0\neq rstm \in E$, implies that rsm $\in$ E or rtm $\in$ E or stm $\in$ E. we study the basic properties of weakly quasi 2-absorbing. Furthermore, the relationships of weakly quasi 2-absorbing submodule with other classes of module are elistraited.

1- Introduction

The concept of weakly quasi-"prime submodule which is a generalization of" a weakly" prime submodule was introduce by in [1], " where a proper submodule E of an R-module M is called weakly quasi-prime submodule if whenever r, s $\in$ R, m $\in$ M , with $0 \neq rsm \in E$, implies that either rm $\in$ E or sm $\in$ ". And " a proper submodule E of an R-module M is called weakly prime submodule if whenever r $\in$ R, m $\in$ M, with rm $\in$ E then m $\in$ E or r $\in$ [E : M] [2]", where "[E : M] = { r $\in$ R ; r M $\subseteq$ E }"." A proper submodule E of an R-module M is called 2-absorbing (weakly 2-Absorbing) if whenever r , s $\in$ R, m $\in$ M with rsm $\in$ E ($0\neq rsm \in E$) , then rm $\in$ E or sm $\in$ E or rs$\in$[E:M] [3] " .

In this work we generalized the concept of weakly quasi-prime submodule to a weakly quasi 2-absorbing submodule , a proper submodule E of an R-module M is called weakly quasi 2-absorbing if whenever r , s , t $\in$ R, m $\in$ M , with $0 \neq rstm \in E$ imples that either rsm $\in$ E or stm $\in$ E or stm $\in$ E . Furthermore, we prove that every "2-absorbing (weakly 2-absorbing) submodule is weakly" quasi 2-absorbung submodule .

2 – Weakly quasi 2-absorbing submodules

In this section we introduce, the definition of weakly quasi 2-absorbing submodule , as a generalization of weakly quasi prime submodule.

Definition 2.1

A Proper submodule E of an R- module M is called a weakly quasi 2-absorbing, if whenever r , s , t $\in$ R , m $\in$ M , with $0 \neq rstm \in E$, then rsm $\in$ E or rtm $\in$ E or stm $\in$ E

And an ideal of a ring R is weakly quasi 2-absorbing if if it is a weakly quasi 2-absorbing submodule of an R-module R .

Remark 2.2

Every weakly quasi-prime "submodule of an R-module M is a weakly" quasi 2-"absorbing submodule of M" but the converse need not to be true.

Proof

Assume E is a weakly quasi-prime submodule of an R-module M , and let $0 \neq rstm \in E$, where r,s,t $\in$ R , m $\in$ M , and suppose that rsm $\notin$ E. Then $0 \neq rs(tm) \in E$. By hypothesis we have r(tm) $\in$ E or s (tm) $\in$ E . That is rtm $\in$ E or stm $\in$ E . Hence E is weakly quasi 2-absorbing submodule in M .

For the converse consider the following example :- let M = Z , R = Z and E =4Z. E is a weakly quasi 2-absorbing submodule , but not weakly quasi-prime submodule of Z since 2 . 2 . 3 $\in$ 4Z where 2 , 3 $\in$ Z , but 2 .3 = 6 $\notin$ 4Z , and 4Z is weakly quasi 2-absorbing since $0 \neq 1 . 2 . 2 . 1 \in 4Z$, we have 2 .2 .1 $\in$ 4Z .

PROPOSITION 2.3

"Let E and D be a submodules of an R-module M with E$\subseteq$D" . If E is a weakly quasi 2-absorbing submodule in M . then E is a weakly quasi 2-absorbing submodule in D .

Proof

Assume that 0 ≠ rstm ∈ E with r , s , t ∈ R , m∈ D ⊆ M , hence m ∈M . Since E is weakly quasi 2-absorbing submdule in M , then rsm ∈ E or rtm ∈ E or stm ∈ E . Hence E is weakly quasi 2-abosrbing in D .

REMARK 2.4

A submodule of a weakly quasi 2-absorbing submodule need not to be a weakly quasi 2-absorbing . The following example explain that .

Let M = Z , R = Z , and E = 4Z , D = 36Z . 4Z is a weakly quasi 2-absorbing in Z , and we have 36Z ⊆ 4Z , 36Z is not weakly quasi 2-absorbing in Z , since 0 ≠ 2 .2 .3 .3 ∈ 36Z , but 2 .2 .3=12 ∉ 36Z ,

PROPOSITION 2.5

Let E and D are a submodules of a module M with D⊆E . Then E is a weakly quasi 2-absorbing in M if and only if $\frac{E}{D}$ is weakly quasi 2-absorbing in $\frac{M}{D}$.

Proof

Let E weakly quasi 2-absorbing in M , and let 0 ≠ rst(m+D) ∈ $\frac{E}{D}$, where r , s , t ∈ R , m+D ∈ $\frac{M}{D}$, m ∈ M . It follows that 0 ≠ rstm ∈ E . Since E is a weakly quasi 2-absorbing submodule in M , then either rsm ∈ E or rtm ∈ E or stm ∈ E . It follows that either rsm+D ∈ $\frac{E}{D}$ or rtm+D ∈ $\frac{E}{D}$ or stm+D ∈ $\frac{E}{D}$. That is either rs(m+D) ∈ $\frac{E}{D}$ or rt(m+D) ∈ $\frac{E}{D}$ or st(m+D) ∈ $\frac{E}{D}$. Thus $\frac{E}{D}$ is weakly quasi 2-absorbing submodule in $\frac{M}{D}$.

Conversly: suppose that $\frac{E}{D}$ is weakly quasi "2-absorbing submodule" in $\frac{M}{D}$, with let 0 ≠ rstm ∈ E , where r , s , t ∈ R , m ∈ M . Hence 0 ≠ rstm+D ∈ $\frac{E}{D}$. That is 0≠ rst(m+D) ∈ $\frac{E}{D}$. Since $\frac{E}{D}$ is a weakly quasi 2-absorbing in $\frac{M}{D}$, then either rs(m+D) ∈ $\frac{E}{D}$ or rt(m+D) ∈ $\frac{E}{D}$ or st(m+D) ∈ $\frac{E}{D}$. It follows that either rsm+D ∈ $\frac{E}{D}$ or rtm+D ∈ $\frac{E}{D}$ or stm+D ∈ $\frac{E}{D}$. Thus either rsm ∈ E or rtm ∈ E or stm ∈ E . Hence E is a weakly quasi "2-absorbing submodule".

REMARK 2.6

The intersection of two weakly quasi 2-absorbing "submodules of an R-module M" need not to be weakly quasi 2-absorbing submodule in M as the following example explain that:

Let M = Z , R = Z , E = 4Z , D = 9Z , are weakly quasi 2-absorbing submodules in Z . But E ∩ D = 36Z is not weakly quasi 2-absorbing submodule in Z.

PROPOSITION 2.7

The intersection of two quasi-prime submodules "of an R-module M" is weakly quasi 2-absorbing submodule"

proof

Let E, D be two quasi-prime " submodules of M" , with 0≠rstm∈E∩D , where r , s, t ∈ R , m∈ M , since E is a quasi-prime submodule in M we assume that rm ∈ E , also since D is quasi-prime in M , we assume that sm ∈ D , it follows that rsm ∈ E∩D . Hence E∩D is weakly quasi 2-absorbing submodule in M .

"Since every" weakly "prime submodule is a weakly quasi-prime submodule[1"]. Hence we get the following result.

COROLLARY 2.8

The intersection of two weakly prime "submodule of an R-module M" is weakly quasi "2-absorbing".

PROPOSITION 2.9

The inverse image of weakly quasi 2-absorbing submodule is weakly quasi 2-absorbing submodule.

Proof

We assume that f is an R-epimorphism from M to $M^`$ and E is weakly quasi 2-absorbing of $M^`$. Let 0 ≠ rstm∈ f^{-1}(E) , where r , s , t ∈R , m ∈ M . It follows that 0 ≠ rstf(m) ∈ E , but E is weakly quasi 2-absorbing in $M^`$, then either rsf(m)∈E or rtf(m)∈E or stf(m) ∈ E , then it follows that rsm∈f^{-1}(E) or rtm∈f^{-1}(E) or stm ∈ f^{-1}(E). Thus f^{-1}(E) is a weakly quasi 2-absorbing in M .

PROPOSITION 2.10

"Let f : M → $M^`$ be an R-epimorphism, and E be a proper submodule of M with Ker(f) ⊆ E". Then E is a weakly quasi 2-absorbing submodule in M iff f( E ) is a weakly quasi 2-absorbing submodule in $M^`$.

Proof (→)

Assume that 0≠rst$m^`$∈ f(E) , where r , s , t ∈R , $m^`$ ∈$M^`$ , since f is onto , then $m^`$ =f(m) for some m∈M . Hence 0 ≠ rstf(m)∈f(E) , implies that 0≠f(rstm) = f(e) for some nonzero e ∈ E . Hence f(rstm - e) = 0 , implies that 0≠rstm ∈ Kerf ⊆ E . That is 0 ≠ rstm ∈ E , since E is a weakly quasi 2-absorbing in M , then either rsm∈E or rtm∈ E or stm∈E . It follows that either rsf(m)∈ f(E) or rtf(m) ∈ f(E) or stf(m)∈f(E) . That is either rs$m^`$∈ f(E) or rt$m^`$∈ f(E) st$m^`$∈ f(E) . Hence f(E) is a weakly quasi 2-absorbing.

← Let 0 ≠ rstm∈E, where r , s , t ∈ R , m ∈ M , then 0≠ f(rstm) ∈ f(E) , it follows that 0 ≠ rstf(m) ∈ f(E) . But f(E) is a weakly quasi 2-absorbing in $M^`$, then either rsf(m) ∈ f(E) or stf(m)∈f(E) or rtf(m)∈f(E). If rsf(m)∈f(E) , implies that f(rsm) = f(e_1) for some nonzero e_1∈ E , hence f(rsm - e_1) = 0 , implies that rsm - e_1 ∈ Kerf ⊆ E , it follows that rsm ∈ E . Similarly we get stm ∈ E or rtm ∈ E , Hence E is a weakly quasi 2-absorbing in M .

REMARK 2.11

If K, L are submodules M with K isomorphic to L and K is a weakly quasi 2-absorbing submodule, then L is not weakly quasi 2-absorbing submodule. The following example shows that;

Let M = Z , R = Z , K = 2Z , L = 8Z , are submodule of M , 2Z≅8Z , we have 2Z weakly quasi 2-absorbing submodule, but 8Z is not weakly quasi 2-absorbing in Z , since if 0≠ 2 . 2 . 2 . 1 ∈ 8Z , 2 . 2. 1 ∉ 8Z .

PROPSITION2.12

Every weakly 2-absorbing submodules of M is weakly quasi 2-absorbing submodule .

Proof

Assume that E is a weakly 2-absorbing in M , and let $0 \neq$ rstm∈ E with r , s , t ∈ R , m ∈ M . Therefore either r(tm)∈ E or s(tm) ∈ E or rs ∈ [E:M]. The first two cases lead us to that E is a weakly quasi 2-absorbing in M.

PROPOSITION 2.13

Assume that M is cyclic module, and E be a proper submodule of M . Then E is a weakly 2-absorbing in M iff E is a weakly quasi 2-absorbing in M .

Proof

The first part follows by proposition (2.12)

The second part: suppose that M is a weakly quasi 2-absorbing in M , it is given that M is cyclic , mean that M=Rx for some x∈ M . Let $0 \neq$ rsm ∈ E , with r , s in R , m in M ,m = tx , where t ∈ R . Thus $0 \neq$ rstx ∈ E , since E is a weakly quasi 2-absorbing in M , then either rsx∈ E or rtx ∈ E or stx ∈ E and hence either rs ∈ [E : x] = [E : M] or rm ∈ E or sm ∈ E .

Since "every 2-absorbing submodule is weakly 2-absorbing [5]", we get the following corollary.

Corollary 2.14

Let E be 2-absorbing "submodule of an R-module M". Then E is a weakly quasi 2-absorbing.

PROPOSITION 2.15

"Let M be an R-module, and E be a proper submodule of M" . Then E is a weakly quasi 2-absorbing in M iff [E: m] is a weakly quasi 2-absorbing ideal of R for every m ∉ E .

Proof (→)

We have [E : m] proper ideal in R , since m ∈ M , m ∉ E . Let 0≠rst ∈ [E: m] , where r , s , t ∈ R then $0 \neq$ rstm∈E , but E is a weakly quasi 2-absorbing submodule in M , then either rsm ∈ E or rtm ∈ E or stm ∈ E hence rs ∈ [E : M] or rt ∈ [E :M] or st ∈ [E:M] , hence [E: M] is a weakly quasi 2-absorbing.

(←) $0 \neq$ rstm ∈ E, where r , s , t in R , m in M, with m dose not belong E, then $0 \neq$ rst ∈ [E : m] . But [E: m] is a weakly quasi 2- absorbing ideal in R, then rs ∈ [E : m] or rt ∈ [E : m] or st ∈ [E : m] , and hence rsm ∈ E or rtm∈ E or stm ∈ E .

PROPOSITION 2.16

Let E be proper weakly quasi 2-absorbind submodule of an R-module "M then S^{-1}E " weakly quasi 2-absorbing submodule of S^{-1}M as S^{-1}R – module.

Proof

Let $0 \neq \bar{a}\bar{b}\bar{c}\,\bar{m} \in S^{-1}E$, where $\bar{a} = \frac{a_1}{s_1}, \bar{b} = \frac{b_1}{s_2}, \bar{c} = \frac{c_1}{s_3}$ *are elements in* $S^{-1}E$ *, where* $a_1, b_1, c_1 \in R$ *, and* $\bar{m} = \frac{m_1}{s_4} \in S^{-1}M$ *, where* $m_1 \in M$ *,* $s_1, s_2, s_3, s_4 \in S$ *. Hence* $0 \neq \frac{a_1}{s_1}.\frac{b_1}{s_2}.\frac{c_1}{s_3}.\frac{m_1}{s_4} \in S^{-1}E$ *, that is* $0 \neq \frac{a_1\ b_1\ c_1\ m_1}{s_1\ s_2\ s_3\ s_4} \in S^{-1}E$, it follows that $0 \neq \frac{a_1\ b_1\ c_1\ m_1}{t} \in S^{-1}E$, where t = $s_1\ s_2\ s_3\ s_4$ ∈ S , then there exist t_1 ∈ S such that 0≠$t_1\, a_1\, b_1\, c_1\, m_1$ ∈ E , but E is a weakly quasi 2-absorbing in M , then either $t_1\, a_1\, b_1\, m_1$∈ E or $t_1\, a_1\, c_1\, m_1$ ∈ E or $t_1\, b_1\, c_1\, m_1$ ∈ E , then

If $t_1\, a_1\, b_1\, m_1$∈ E , then $\frac{t_1\, a_1\, b_1\, m_1}{t_1\, s_1\, s_2\, s_4} \in S^{-1}E$, so , $\frac{a_1}{s_1}\frac{b_1}{s_2}\frac{m_1}{s_4} \in S^{-1}E$

If $t_1\, a_1\, c_1\, m_1 \in E$*, then* $\frac{t_1\, b_1\, c_1\, m_1}{t_1\, s_2\, s_3\, s_4} \in S^{-1}E$ *, so ,* $\frac{b_1}{s_2}\frac{c_1}{s_3}\frac{m_1}{s_4} \in S^{-1}E$

If $t_1\, b_1\, c_1\, m_1$ ∈ E , then $\frac{t_1\, a_1\, c_1\, m_1}{t_1\, s_1\, s_3\, s_4} \in S^{-1}E$, so , $\frac{a_1}{s_1}\frac{c_1}{s_3}\frac{m_1}{s_4} \in S^{-1}E$

Thus either $\bar{a}\,\bar{b}\,\bar{m} \in S^{-1}E$ or $\bar{a}\,\bar{c}\,\bar{m} \in S^{-1}E$ or $\bar{b}\,\bar{c}\,\bar{m} \in S^{-1}E$.

Hence $S^{-1}E$ is a weakly quasi 2-absorbing in $S^{-1}M$.

PROPOSITION 2.17

Let E "be a proper submodule of an R-module M_1" Then E is a weakly quasi 2-absorbing submodule in M_1 iff E ⊕ M_2 Iis a weakly quasi 2-absorbing submodule of an R-module $M_1 \oplus M_2$, where M_2 is an R-module .

Proof

Let (0, 0) ≠ rst(m_1, m_2) ∈ E ⊕ M_2, where r , s , t ∈ R , $(m_1, m_2) \in M_1 \oplus M_2$ with m_1 is a nonzero elemest in M_1 and , m_2 is a nonzero element in M_2 , it follows that $0 \neq$ rstm_1 ∈ E or $0 \neq$ rstm_2 ∈ M_2 . But E is a weakly quasi 2-absorbing in M_1 , then either rsm_1 ∈ E or rtm_1 ∈ E or stm_1 ∈ E , it follows that either rs(m_1, m_2) ∈ E ⊕ M_2 or rt(m_1, m_2) ∈ E ⊕ M_2 or st(m_1, m_2) ∈ E ⊕ M_2 . Hence E ⊕ M_2 Is a weakly quasi 2-absorbung in $M_1 \oplus M_2$.

Conversely: suppose that E ⊕ M_2 is a weakly quasi 2-absorbing in $M_1 \oplus M_2$, and let $0 \neq$ rstm_1∈ E , where r , s , t ∈ R , m_1 is a nonzero element in M_1.

Then for each $m_2 \in M_2$, we have $0 \neq$ rst(m_1, m_2) ∈ E ⊕ M_2 , since E ⊕ M_2 is a weakly quasi 2-absorbing in $M_1 \oplus M_2$, then either rs(m_1, m_2) ∈ E ⊕ M_2 or st(m_1, m_2) ∈ E ⊕ M_2 or rt(m_1, m_2) ∈ E ⊕ M_2 . It follows that either rsm_1 ∈ E or rtm_1 ∈ E or stm_1 ∈ E , Hence E is a weakly quasi 2-absorbing submodule in M_1.

PROPOSITION2.18

Let E "be a proper submodule of an R-module" M_2 , then E is a weakly quasi 2-absorbing in M_2 if and only if in M_1 ⊕ E is a weakly quasi 2-absorbing in $M_1 \oplus M_2$.

Proof

Similarly as in proposition (2.17)

"PROPOSITION 2 .19

Let M be an R-module, and E be a proper" submodule of M. Then the statements are equivalents.

1 – E is a weakly quasi 2-absorbing submodule in M

2 – For each r , s in R , m in M if $0 \neq$ rsm ∉ E , then [E : rsm] = [E : rm] ∪ [E : sm]

3 – For each r , s in R , m in M if $0 \neq$ rsm ∉ E , then [E : rsm] = [E : rm] or [E : rsm] = [E:sm] .

Proof

1 ⇒ 2

Let t ∈ [E : rsm] , then 0 ≠rstm ∈ E . Since E is a weakly quasi 2-absorbing in M and 0≠rsm ∉ E , then either stm ∈ E or rtm ∈ E , then t ∈ [E:s m] or t ∈

[E :r m] , hence [E : rsm] ⊆ [E : sm] ∪ [E : rm] . Now , let t ∈ [E : rm] ∪ [E : sm] , then rtm ∈ E or stm ∈ E , implies that rstm ∈ sE⊆ E or rstm ∈ rE⊆ E , then we get rstm ∈ E.
Thus t ∈ [E : rsm] , hence [E : rm] ∪ [E : sm] ⊆ [E : rsm }.
Thus [E: rsm] = [E : rm] ∪ [E : sm] .
(2) ⇒ (3) : suppose that [E : rsm] = [E : rm] ∪ [[E : sm] and [E : rsm] is an ideal of R , then either [E : sm] ⊆ [E : rm] or [E : rm] ⊆ [E : sm] . It follows that [E : rsm] = [E: rm] or [E : rsm] = [E : sm] .
(3) ⇒ (1) : suppose that [E : rsm] = [E : rm] or [E : rsm] = [E : sm] , where r , s ∈ R , m ∈ M , and let 0 ≠ rstm ∈ E ,then we get t ∈ [E : rsm] . If [E:rsm] = [E : rm] , then t ∈ [E : rm] , implies that rtm ∈ E . If [E: rsm] = [E : sm] , then t ∈ [E : rsm] , implies that t ∈ [E : sm] , it follows that stm ∈ E . Hence E is a weakly quasi 2-absorbing in M .

" Recall that a proper submodule E of an R- module is called quasi prime if rsm ∈ E , where r , s ∈ R , m ∈ M implies that either rm ∈ E or sm ∈ E " [4] .
It is well known that every quasi prime submodule is a weakly quasi –prime [1] , we get the following result .

PROPOSITION 2.20

quasi prime submodule is weakly quasi 2-absorbing.

Proof

Follows by Remark (2 . 2)

“ Recall that a proper submodule N of an R-module M is a prime if rm ∈ N , with r ∈ R , m ∈ M , implies that either m ∈ N or r ∈ [N : M] [6]”.
“It is well known prime submodule is quasi-prime [4]” , we have the following corollary .

COROLLARY 2 .21

Every prime submodule is weakly quasi 2-absorbing.
Proof: Follows by proposition (2 . 20) .

References

[1] Waad K. H "Weakly Quasi-Prime Modules and Weakly Quasi-Prime Submodules", M. Sc. Thesis Tikrit University 2003.
[2] Ebrahimi S. Farzalpour F. "On Weakly Prime submodules", Tamkang Journal of Mathematics 38 (3) (2007), 247 – 252.
[3] Darani A., Soheilnia F. "2-Absorbing and Weakly 2-Absorbing submodules". Tahi Journal of Mathematic , 9 , (2011) , 577 – 584 .
[4] Muntaha A. R. " Quasi-Prime Modules and Quasi-Prime submodules", M.Sc. Thesis, Baghdad university 1999 .
[5] Moradi S., Azizi A. "Weakly 2-Absorbing Submoduie of Modules". Turkish Journal of Mathematics , 40 , (2016) 350 – 364 .
[6] Anderson D., Bataineh M. " Generalization of Prime Ideals " comm . Algebra , 39 (2008) 686 – 696 .

المقاساات الجزئية المستحوذة من النمط – 2 الظاهرية الضعيفة

هيبة كريم محمد علي ، خلف حسن الحبيب
قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

لتكن R حلقة ابدالية بمحايد و ليكن M مقاسا ايسر احادي على R . يقال للمقاس الجزئي الفصلي من المقاس Mمقاس ظاهري اولي ضعيف اذا كان r, s ∈ R و m ∈ M حيث ان 0≠ rsm ∈ E يؤدي الى ان r m ∈ E او sm∈ E، في هذا البحث قدمنا مفهوم المقاس الظاهري الاولي الضعيف ، حيث انه يدعى المقاس الجزئي الفصلي E مقاسا جزيئا مستحوذا من النمط – 2 ظاهريا ضعيفا اذا كان r , s , t ∈ R و m ∈M حيث ان 0≠ rstm ∈ E يؤدي الى rsm ∈ E او rtm ∈ E او stm ∈ E . درسنا الصفات الأساسية لهذا النوع من المقاسات الجزئية بالإضافة الى ذلك علاقة هذا المقاس الجزئي مع المقاسات الأخرى وضحت.