LIMITATION OF STOKES SECOND PROBLEM [+]

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Abstract:

A limit to the well-known Stokes second problem is tried in the present work. The limitation is presented mathematically and with the rough observation of the experimental procedure. The results show that the range of the Stokes second problem is related to the time required for the stroke motion of the plate as well as the stroke length itself in addition to the value of the dynamic viscosity of the liquid resting over the plate.

محدودية مشكلة ستوكس الثانية

ماجد حميد مجيد

المستخلص:

في البحث التالي تمت محاولة ايجاد حدود لمسألة ستوكس الثانية. و قد تم ايجـــاد هـــذه الحـــدود رياضـــيا بالاضافة الى الملاحظات النظرية للتجربة التي تم اجراءها لهذا الغرض. لقد اوجدت النتائج ان حدود مسألة ستوكس الثانية تتعلّق بالزمن اللازم لإتمام شوط الحركة لطح المعدن الذي يستقر عليه السائل بالاضافة الى طول ذلك الـــشوط مع مقدار اللزوجة الديناميكية لذلك السائل.

Key worlds: stokes equation, oscillation,

Nomenclatures

Symbol	Description	Units
a	Amplitude	M
B	Constant defined in equation (6)	
t	Time	sec.
u	Axial velocity	m/sec
U	Uniform Velocity	m/sec
x	Horizontal coordinate	M
y	Vertical coordinate	M

Greek symbols

Symbol	Description	Units
δ	Film thickness	M
μ	Viscosity	$N.s/m^2$
ν	Dynamic viscosity	
ρ	Density	Kg/m^3
ω	Wave frequency	rad/sec.

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Introduction:

Stokes second problem is the problem of the flow about an infinite wall which execute linear harmonic oscillation parallel to its self, and which was first treated by G. Stokes, and later by Lord Rayleigh [1]. This problem is widely treated both in Cartesian and polar coordinates, and many results are obtained to investigate the stability and other related phenomena. Figure (1) describes the problem.

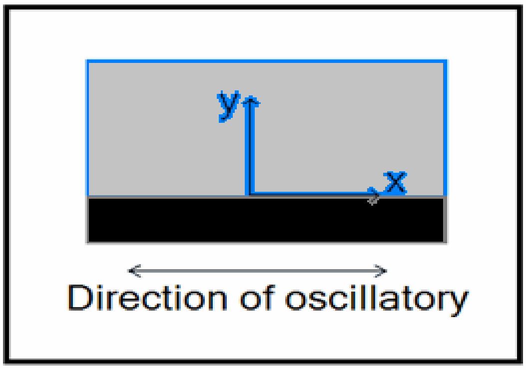

Figure (1): Representation of Stokes second problem.

All the treatments done before were dealt with the problem as the vertical coordinate is extending to infinity despite of the values of the wall oscillatory motion characteristics (stroke length, and time of oscillation). In the present work a trial is done to find a limit relating the oscillatory motion characteristics with the vertical coordinate in order to find where or where not such problems lying in that region.

Mathematical model:

In order to represent our problem mathematically consider the case of film with known thickness (δ) is resting over a horizontal wall has an oscillatory motion parallel to its direction. Now by dropping the gravity effect, the viscous forces will be balanced by transient acceleration term, or:

$$\mu \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial u}{\partial t} \tag{1}$$

With the boundary conditions:

$$\left.\begin{array}{l} u(0,t) = a\omega \\ \left.\frac{\partial u}{\partial y}\right|_{y=\delta} = 0 \end{array}\right\} \tag{2}$$

The solution of the above differential equation will be:

$$u = C_1(t) e^{y\sqrt{\frac{B}{2}}} \cos(\omega t + y\sqrt{\frac{B}{2}}) + C_2(t) e^{-y\sqrt{\frac{B}{2}}} \cos(\omega t - y\sqrt{\frac{B}{2}}) \tag{3}$$

Where

$$C_1(t) = \frac{1}{e^{\sqrt{2B}} \left[\dfrac{\cos(\omega t + \delta\sqrt{\frac{B}{2}}) - \sin(\omega t + \delta\sqrt{\frac{B}{2}})}{\cos(\omega t - \delta\sqrt{\frac{B}{2}}) - \sin(\omega t - \delta\sqrt{\frac{B}{2}})} \right] - 1}$$

(4)

$$C_2(t) = \frac{e^{\sqrt{2B}} \left[\dfrac{\cos(\omega t + \delta\sqrt{\frac{B}{2}}) - \sin(\omega t + \delta\sqrt{\frac{B}{2}})}{\cos(\omega t - \delta\sqrt{\frac{B}{2}}) - \sin(\omega t - \delta\sqrt{\frac{B}{2}})} \right] - 2}{e^{\sqrt{2B}} \left[\dfrac{\cos(\omega t + \delta\sqrt{\frac{B}{2}}) - \sin(\omega t + \delta\sqrt{\frac{B}{2}})}{\cos(\omega t - \delta\sqrt{\frac{B}{2}}) - \sin(\omega t - \delta\sqrt{\frac{B}{2}})} \right] - 1}$$

(5)

$$B = \frac{\omega \rho}{\mu}$$

(6)

The values of equation (3) is drawn in figure (2).

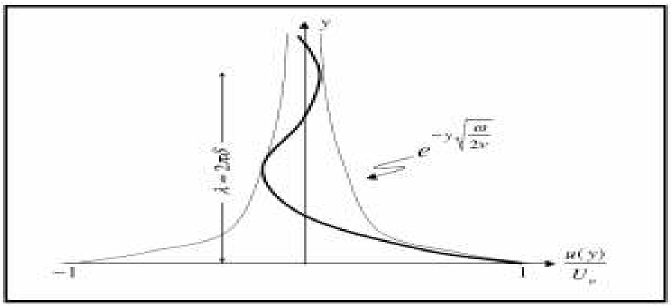

Figure (2): Values of equation (3).

These values are similar to that of the Stokes second problem which may give the impression of the similarity of the present problem with it, but the truth is that they are different, how? To answer that question, the problem will be divided by taking one stroke of the oscillatory motion. By taking the differential equation (1) with the same boundary conditions except for the first one will be replaced by:

$$u(0,t) = U_o \tag{7}$$

And by using Laplace transformation for the time independent variable, and by using the Laplace inversion charts [2], the following result obtained:

$$\frac{u}{U_o} = 1 - \mathrm{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \tag{8}$$

Now after what period of time the velocity distribution will vanish? Or in other words, after how long the whole boundary layer of the fluid will has the same velocity of the plate and looks like it stops. The following equation shows this phenomena :

$$\operatorname{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) = 1 \implies \frac{y}{2\sqrt{\nu t}} = \infty \tag{9}$$

From tables of error function [2]:

$$\operatorname{erf}(4) = 1 \implies \frac{y}{2\sqrt{\nu t}} = 4 \tag{10}$$

Which is the least time required for the boundary layer to stop. From (10) the following conclusions can be made:

1- In order to deal with problem lies in the field of Stokes second problem, the time of the stroke period should be:

$$t < \frac{\delta^2}{64\nu} \tag{11}$$

2- If the above condition did not be satisfied, or:

$$t > \frac{\delta^2}{64\nu} \tag{12}$$

Then it can be said that the problem is not Stokes second problem, but it lies in a new field.

After applying the above condition on moderate values of condensate film of water, with ω equal to 50 Hz, the time required for the film to fully develop is 2.1 $*10^{-4}$ sec, which is a very small value, and it can be said that the condition is about a surface carrying a static fluid on it.

To check the validity of the above conditions, one can back to the experimental work of Bull, J. L., and Grotberg, J. B [3], where a close boundary conditions are applied on a thin viscous film resting on a moving wall with V_{wall} , and the thickness of the film was 2 mm, where the final velocity will be:

$$u(x, y, t) = Bo\left[\frac{1}{2}\frac{\partial\delta}{\partial x}y^2 - \delta\frac{\partial\delta}{\partial x}y\right] + \frac{\partial\sigma}{\partial x}y + V_{wall} \tag{13}$$

Where the term between the brackets refers to the development of the velocity due to the wave created, and the middle term refers to the flow due to surfactant spreading, and the important is the last term, where the wall velocity added to the flow linearly and independently of any spatial or temporal independent variables.

Experimental checking:

In order to investigate and check the validity of the previous conclusions an experimental procedure with rough observation is done to check the behavior of a puddle resting over horizontal oscillating plate as shown in figure (3).

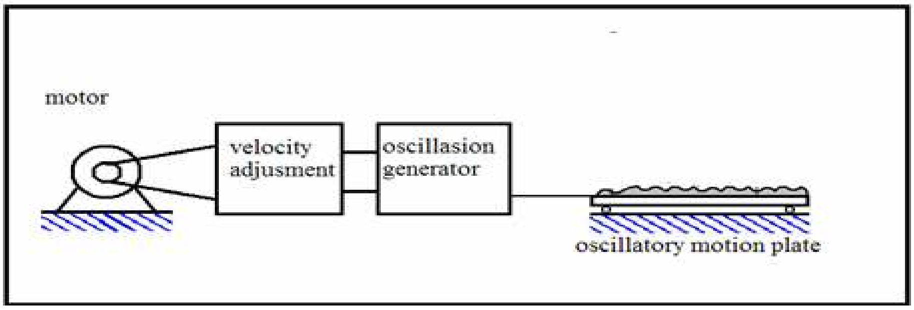

Figure (3): Experimental device.

According to Stokes second problem results a film of 1.5 mm thick and 16 rad/sec plate oscillation should appears constant for an observer over the plate, but the truth is that this film will appears as a moving wave film following the oscillatory plate with a lag time between them. This stunning result gave an indication about the discrepancy of the present work result from that of Stokes second problem.

Conclusion:

The previous sections discussed mathematically and experimentally an alternative description of the problem of a liquid film resting over an oscillating wall and with its limitation for the Stokes second problem in order to recognize the validity of the problem. The results shoed that the period length of the plat oscillation with a thickness of the film, which are given in equations (11) and (12) and can be shown in figure (4).

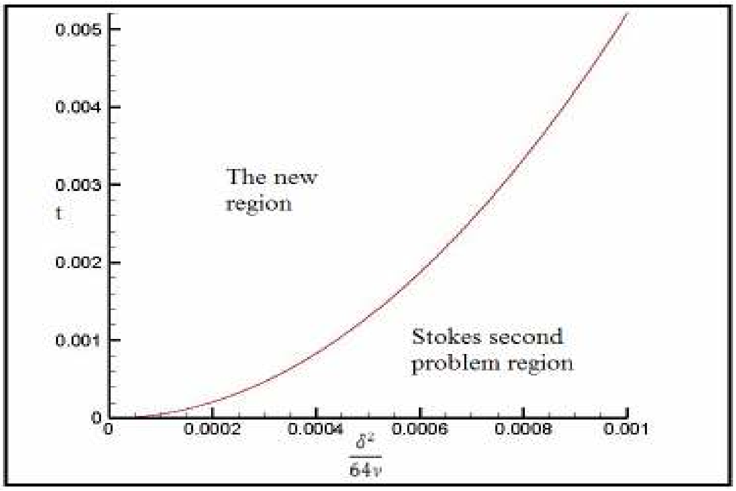

Figure (4): The limitation of Stokes second problem.

References:

1-Schlichting, , "*Boundary-Layer Theory*", 7th edition, McGraw-Hill book company, 1979.
2-Speigel, M., "*Handbook of Mathematica Formulas*", Schaum`s Outline Series, McGraw-Hill, 1968.
3-Bull, J. L., and Grotberg, J. B., "Surfactant Spreading on Thin Viscous Films: Film Thickness Evolution and Periodic Wall Stretch", *Experiments in Fluids*, vol. 34, pp. 1-15, 2003.